Review of previous lecture (June 09)

Ginzburg - Landau - Wilson functional (from coarse graining)

$$Z = \int \mathcal{D}\phi_\Lambda \, e^{-H_0 - H_\Lambda} \approx \int \mathcal{D}\phi_\Lambda \, e^{-H_0} \left[1 - H_\Lambda + \frac{1}{2} H_\Lambda^2 - \dots\right]$$

perturbation theory
(need H_Λ to be small)

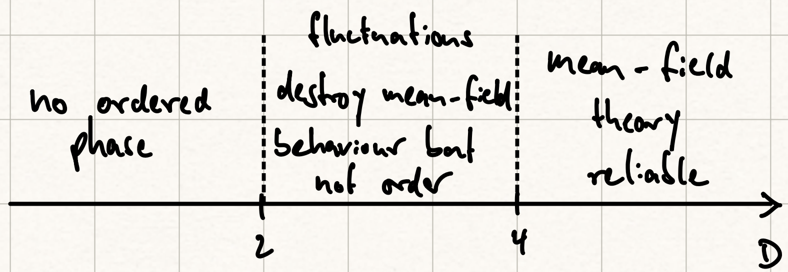
with $$H_0 = \int d^D\vec{x} \left(\frac{\phi_\Lambda(\vec{x})}{2} + \frac{(\nabla \phi_\Lambda(\vec{x}))^2}{2} \right)$$

$$H_\Lambda = \int d^D\vec{x} \; \frac{1}{4} \bar{u}_0 \, \phi_\Lambda^4(\vec{x}) \quad , \quad \bar{u}_0 = u_0 r_0^{\frac{D-4}{2}} \sim t^{\frac{D-4}{2}} \quad (*)$$

the case $H_\Lambda = 0$ is called Gaussian approximation, can be solved exactly $\Rightarrow$ same critical exponents like mean field theory

validity of perturbation theory and mean field theory depends on number of dimensions D (see $*$):

$D^* = 4$ upper critical dimension



anomalous dimension

study implications of the value of critical exponent ν :

$$\xi \sim |T-T_c|^{-\nu} \qquad \left(\text{mean-field result: } \nu = \tfrac{1}{2}\right)$$

define anomalous dimension θ: $\nu = \frac{1}{2} - \theta$

we know that different systems in a universality class behave the same around the critical point

$\Rightarrow$ natural conclusion, only macroscopic length scales matter $\rightarrow$ common scale : ξ, with $\xi \to \infty$ for $t \to 0$

use dimensional analysis : $[\xi] = L$ and $[r_0] = L^{-2}$

$r_0 \sim t$ (see above)

$$\Rightarrow \xi \sim r_0^{-\frac{1}{2}} \sim t^{-\frac{1}{2}} \quad \rightarrow \nu = \frac{1}{2}, \ \theta = 0$$

that means based on these general arguments ν should <u>always</u> take the mean-field value $\nu = \frac{1}{2}$, $\theta = 0$

however, based on table above, that is not the case

How is this possible?

$\rightarrow$ violation of naive dimensional analysis

$\rightarrow$ there must be <u>another length scale</u> in the system!

despite the fact that long wavelength phenomena dominate physics close to the critical point, also microscopic length scales show up in critical exponents!

consider as an example the lattice spacing as an additional scale:

$$[\xi] = L \,, \quad [a] = L \,, \quad \underbrace{[r_0] = L^{-2}}_{\sim t}$$

generalizing dimensional analysis leads to:

$$\xi = r_0^{-\frac{1}{2}} \, f(r_0 a^2)$$

→ see also exercise sheet 3

function to be determined

What happens close to the critical point $t \to 0$?

assume $f(x) \sim x^\theta$ for $x \to 0$ (non-analytic behaviour)

$$\Rightarrow \xi \sim t^{-\frac{1}{2}+\theta} \, a^{2\theta}$$

this result is remarkable, even though $\frac{a}{\xi} \ll 1$, we cannot in general replace a function $\Phi\left(\frac{a}{\xi}\right)$ by $\Phi(0)$, otherwise a would not appear in any quantity

this implies the function $\Phi(x)$ needs to have a non-analytic behaviour for $x = \frac{a}{\xi} \to 0$

$\Rightarrow$ effects at very different length scales contribute to the physics of critical phenomena !