Phase transitions and the Renormalization Group

Welcome!

Kai Hebeler S2|11 107

kai.hebeler@physik.tu-darmstadt.de

Course website (notes, announcements, exercises):

https://theorie.ikp.physik.tu-darmstadt.de/strongint/teaching_RG_phasetransitions_22.html

questions:

- only physics students?
- completed theoretical physics I-IV ?
- field theory knowledge? not required for this course!
- lecture 3+1, 4-5 exercise sessions planned on wednesdays over the course of the semester, homework problems, discussion at indicated dates
- lectures/exercises always 90 mins
- breaks after 45 min
- Oral exams/presentation for those who need CPs
- questions from your side?

Please provide feedback of points that can be improved (too fast/slow? requests for exercise problems, more special topics)

Please ask questions!! Use opportunity of having a course with only ~ 15-20 people. Discussions are always welcome!

Recommended literature

* Lectures on phase transitions and the Renormalization Group,
  Nigel Goldenfeld,
  very readable book, discusses most topics covered in this course, highly recommended!

* Introduction to Renormalization Group methods in physics
  Creswick, Farach, Poole
  discussion of RG methods in various contexts

* The theory of critical phenomena
  Binney, Dowrick, Fisher, Newman

Phenomena in statistical physics can be (very roughly) divided into two categories

1.) constituents of system can be regarded as noninteracting, thermodynamic properties can be determined to good approximation based on single-particle properties of consituents

<u>examples</u>: - specific heat of gases and solids
- blackbody radiation
- electron theory of metals
- Bose-Einstein condensation

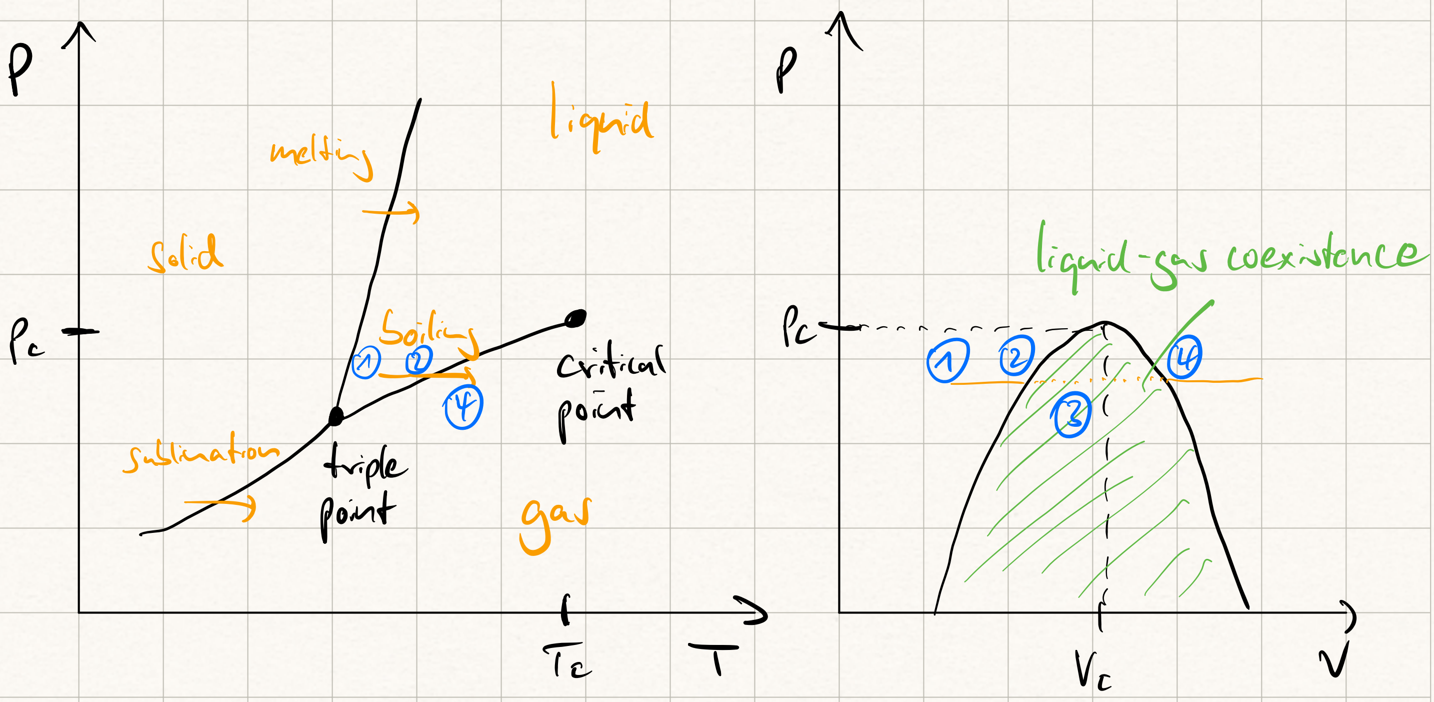
↑ usually studied in theo IV

2.) interactions between constituents play a key role, correlations and fluctuations need to be treated with care, thermodynamic quantities exhibit singularities and discontinuities
→ phase transitions

<u>examples</u>: - condensation of gases
- coexistence of phases, latent heat
- ferromagnetism
- superconductivity
- critical phenomena

↑ subject of this course

Consider liquid-gas phase transition (constant pressure)



① start in liquid phase, apply heat
⇒ temperature rises and system expands

② At some T liquid begins to boil, temperature remains constant while adding heat, volume V still increases (latent heat)

③ System is in state of coexistence of liquid and gas phases, fraction of gas increases until all liquid has turned into gas (at ④)

④ Further heating leads to temperature increase

- critical point is a bifurcation point in phase diagram 1 phase → 2 phases

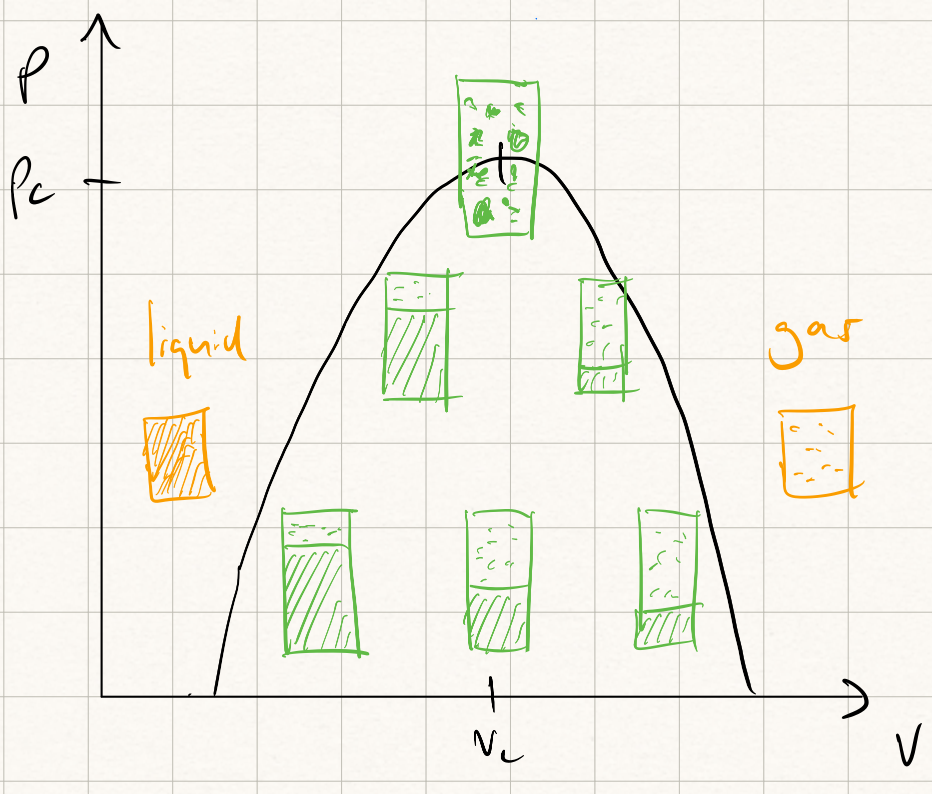
- around T_c the volume difference between liquid and gas becomes smaller, i.e. the width of the coexistence region decrases in pV diagram:

$$V_g - V_l \sim |T - T_c|^{\beta}$$

← critical exponents

similarly: $c_V \sim |T - T_c|^{-\alpha}$

$$p_g - p_l \sim |V_g - V_l|^{\delta}$$



- the correlation length ξ describes the spatial extent of fluctuations in a physical quantity, at the critical point $\xi \to \infty$ and density fluctuations are correlated over all length scales, i.e. droplet sizes of all scales are present in the system $\Rightarrow$ critical opalescence

- very interesting and rich physical phenomena are happening around the critical point, due to $\xi \to \infty$ many microscopic details are "washed out" and many different systems show identical critical behaviour (universality)

- theoretical description of physics around critical point very challenging triggered the development of powerful new methods, e.g. the Renormalization Group,

- RG methods are based on very general ideas and can be applied to many different areas like chaos, fractals, networks, quantum field theory, many-body theory (all systems that involve degrees of freedom at various length scales)

→ Kenneth Wilson, Nobel prize 1982
for development and application of RG
to critical phenomena

later methods heavily used also in
elementary particle physics

Outline

1. Review of relevant parts of statistical phys.

ensembles, partition function, thermodynamic potentials, density matrix, ergodicity

2. Phase transitions

definition of phases and phase boundaries, Ehrenfest classification, phase transitions in one and two dimensions, spontaneous symmetry breaking, ergodicity breaking, correlation length

3. Mean field theory

critical phenomena in Ising model, liquid-gas transitions, Van der Waals equation of state, Landa theory of phase transitions, effective theories, universality, coarse graining, inclusion of correlations, breakdown of mean-field theory

4. Introduction to the Renormalization Group

block spins, coarse graining, fix points, relevant / marginal / irrelevant couplings, application of RG to 1d and 2d Ising model, anomalous dimension, role of microscopic length scales

5. Application of RG to different systems

topological phase transitions (Kosterlitz-Thouless), percolation, sol-gel transition, chaos, networks, nuclear physics, ...
still open, depending on time ...