Modelling the PHASES of QCD

Wolfram Weise
Technische Universität München

- **QCD SYMMETRIES** and
  SYMMETRY BREAKING PATTERNS

- Dynamical **entanglement** of
  CHIRAL and **DECONFINEMENT** transitions

- Role of the **AXIAL** U(1) **ANOMALY**

- **PHASE DIAGRAM** at finite **BARYON DENSITY**,
  NUCLEAR MATTER, **CRITICAL POINT**,
  and all that . . .

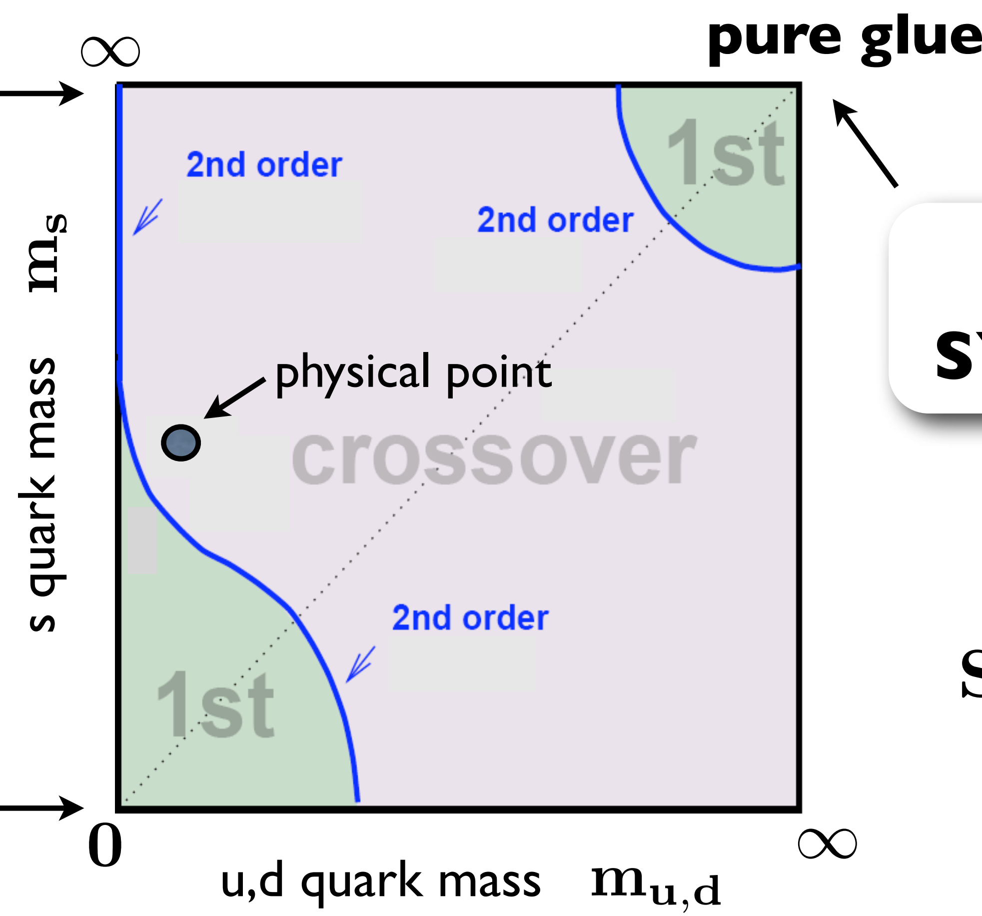
TWO SYMMETRIES that govern LOW-ENERGY QCD

$SU(2)_R \times SU(2)_L$

CHIRAL SYMMETRY

$SU(3)_R \times SU(3)_L$

exact for massless quarks



"Columbia plot"

$Z(3)$

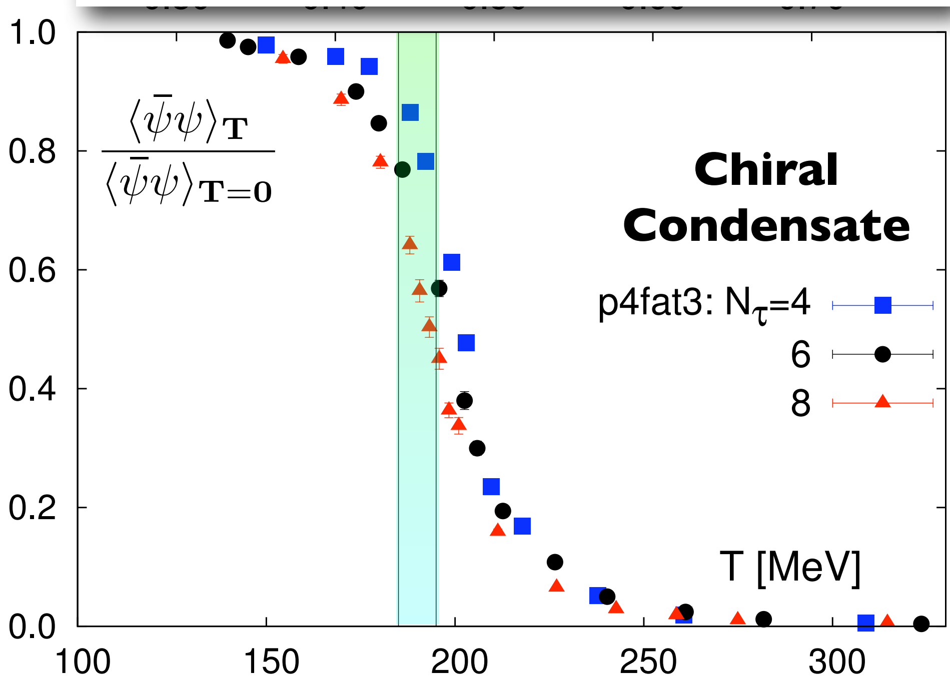
SYMMETRY

Center of gauge group $SU(N_c = 3)$

exact for infinitely heavy quarks

LATTICE QCD THERMODYNAMICS: CHIRAL and DECONFINEMENT TRANSITIONS

spontaneously broken **Chiral Symmetry**

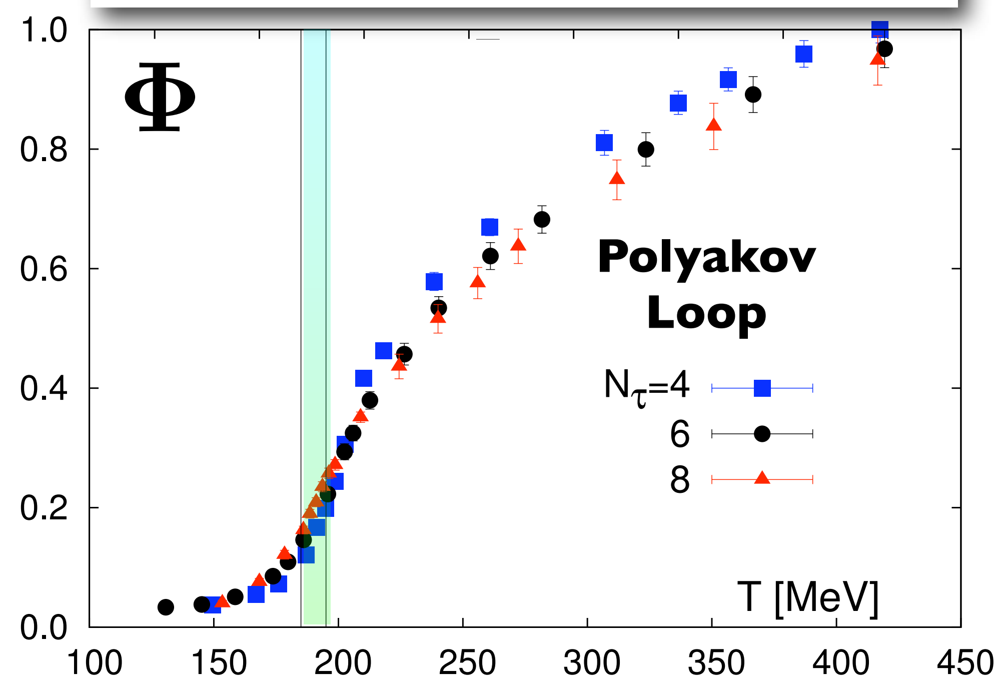


Lattice QCD (2+1 flavours)
almost physical quark masses

M. Cheng et al.
Bielefeld/BNL-Riken/Columbia
Phys. Rev. D77 (2008) 014511

F. Karsch et al. arXiv:0711.0661 [hep-lat]

spontaneously broken **Z(3) Symmetry**



- **crossover transitions**
- **no critical temperature** in strict sense
- **chiral** and **deconfinement transitions** seem to **coincide**

POLYAKOV LOOP dynamics ⟷ **Confinement**

Pisarsky (2000)

Synthesis

Fukushima (2004)

PNJL MODEL

Ratti, Thaler, W.W. (2005)

NAMBU & JONA-LASINIO model ⟷ Spontaneous **Chiral Symmetry** Breaking

Nambu, Jona-Lasinio (1961)

- **Action** :

Fermion (quark) effective **Hamiltonian**

Polyakov loop effective **potential**

$$\mathcal{S}(\psi,\psi^\dagger,\phi) = \int_0^{\beta=1/\mathbf{T}} \mathbf{d}\tau \int_{\mathbf{V}} \mathbf{d^3x} \left[\psi^\dagger \partial_\tau \psi + \mathcal{H}(\psi,\psi^\dagger,\phi)\right] - \frac{\mathbf{V}}{\mathbf{T}}\,\mathcal{U}(\phi,\mathbf{T})$$

- identify dominant **collective degrees of freedom** (→ **order parameters**)
- **quarks** as **quasiparticles** with dynamically generated masses

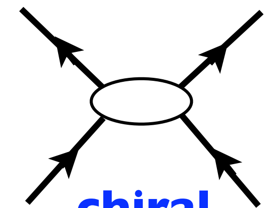
Sketch of (non-local) **PNJL MODEL**

- **Action** :

$$\mathcal{S}(\psi,\psi^\dagger,\phi) = \int_0^{\beta=1/\mathbf{T}} \mathbf{d}\tau \int_{\mathbf{V}} \mathbf{d^3x}\left[\psi^\dagger\partial_\tau\,\psi + \mathcal{H}(\psi,\psi^\dagger,\phi)\right] - \frac{\mathbf{V}}{\mathbf{T}}\,\mathcal{U}(\phi,\mathbf{T})$$

- **Fermionic Hamiltonian density (NJL)** :

$$\mathcal{H} = -i\psi^\dagger(\vec{\alpha}\cdot\vec{\nabla} + \gamma_4\,\mathbf{m_0} - \phi)\,\psi + \mathcal{V}(\psi,\psi^\dagger)$$



Non-local fermion interaction

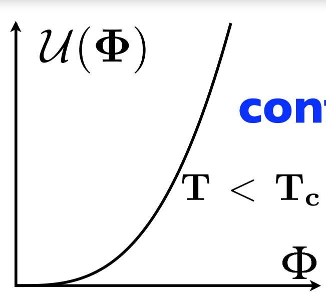
chiral invariant

- **Temporal background gauge field** $\phi = \phi_3\,\lambda_3 + \phi_8\,\lambda_8 \ \in \mathbf{SU(3)}$

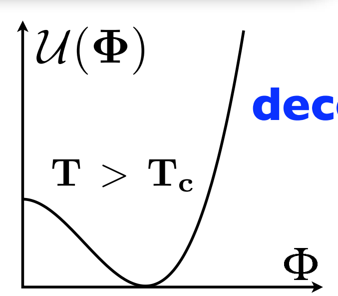
$$\mathbf{\Phi} = \frac{\mathbf{1}}{\mathbf{N_c}}\mathbf{Tr}\left[\exp\left(\mathbf{i}\int_0^{\mathbf{1/T}} \mathbf{d}\tau\,\mathbf{A_4}\right)\right] \equiv \frac{\mathbf{1}}{\mathbf{3}}\mathbf{Tr}\exp(\mathbf{i}\phi/\mathbf{T})$$

Polyakov loop

- **Effective potential** :



confinement



deconfinement

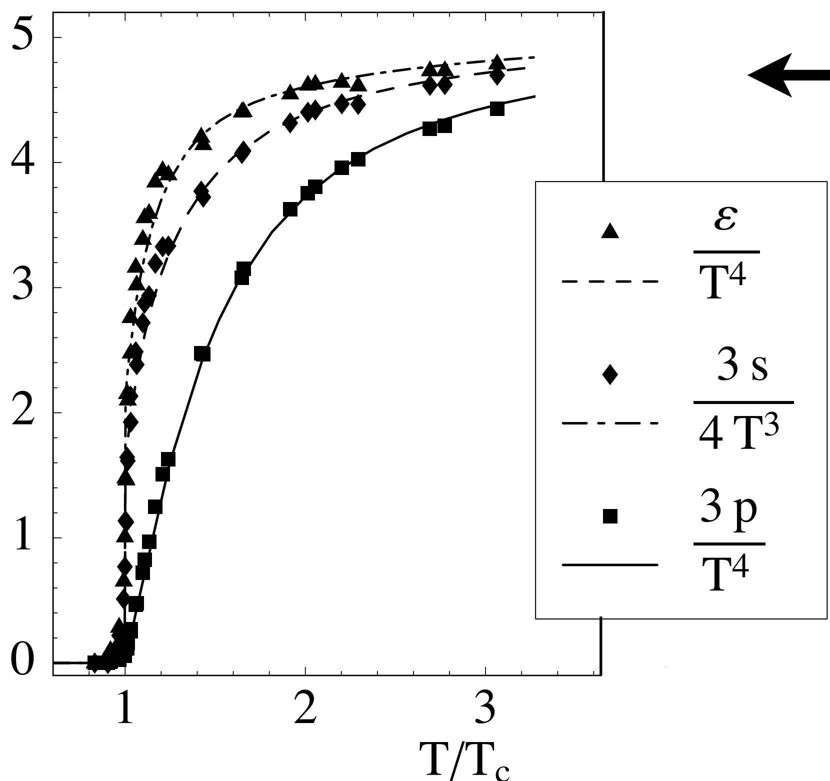
Polyakov Loop Effective Potential from "PURE GLUE" **Lattice Thermodynamics**

- Minimization of $\mathcal{U}(\Phi(T), T) = -p(T)$

R. Pisarsky (2000)
K. Fukushima (2004)

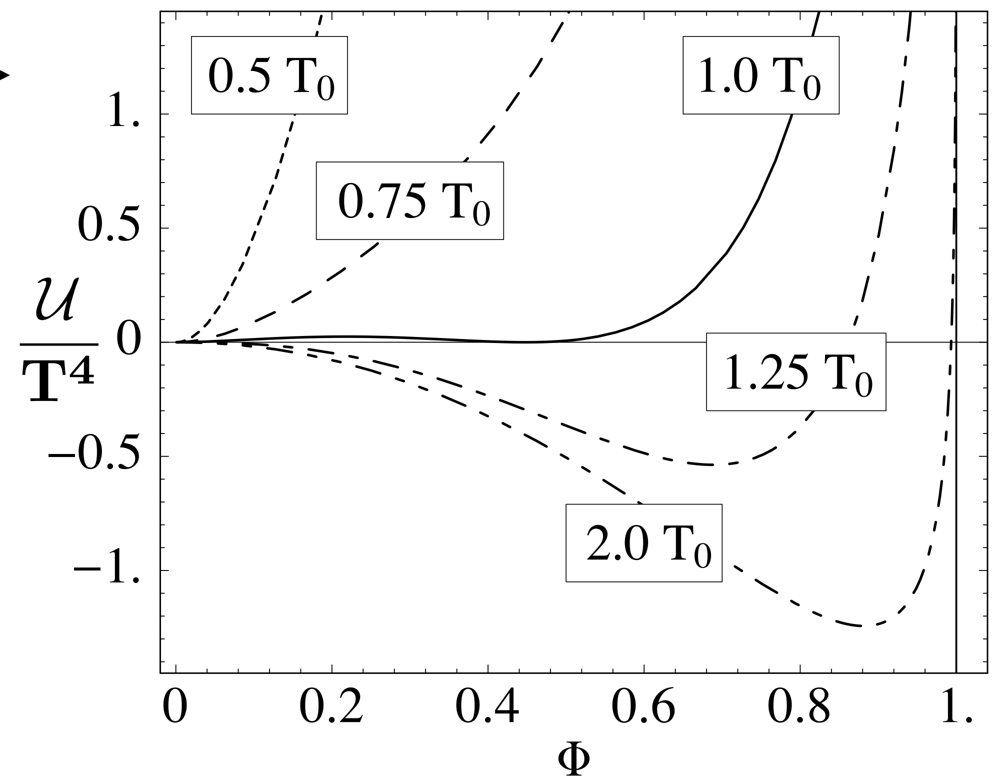
$$\mathcal{U}(\Phi, T) = -\frac{1}{2} a(T)\, \Phi^* \Phi - b(T)\, \ln[1 - 6\, \Phi^* \Phi + 4(\Phi^{*3} + \Phi^3) - 3(\Phi^* \Phi)^2]$$

energy density, entropy density, pressure



lattice results:
O. Kaczmarek et al. PLB 543 (2002) 41

Polyakov loop effective potential



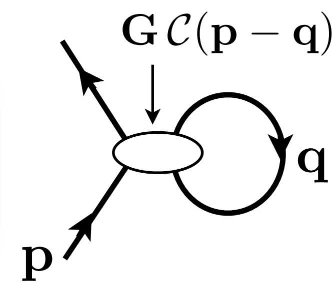
S. Rößner, C. Ratti, W.W. PRD 75 (2007) 034007

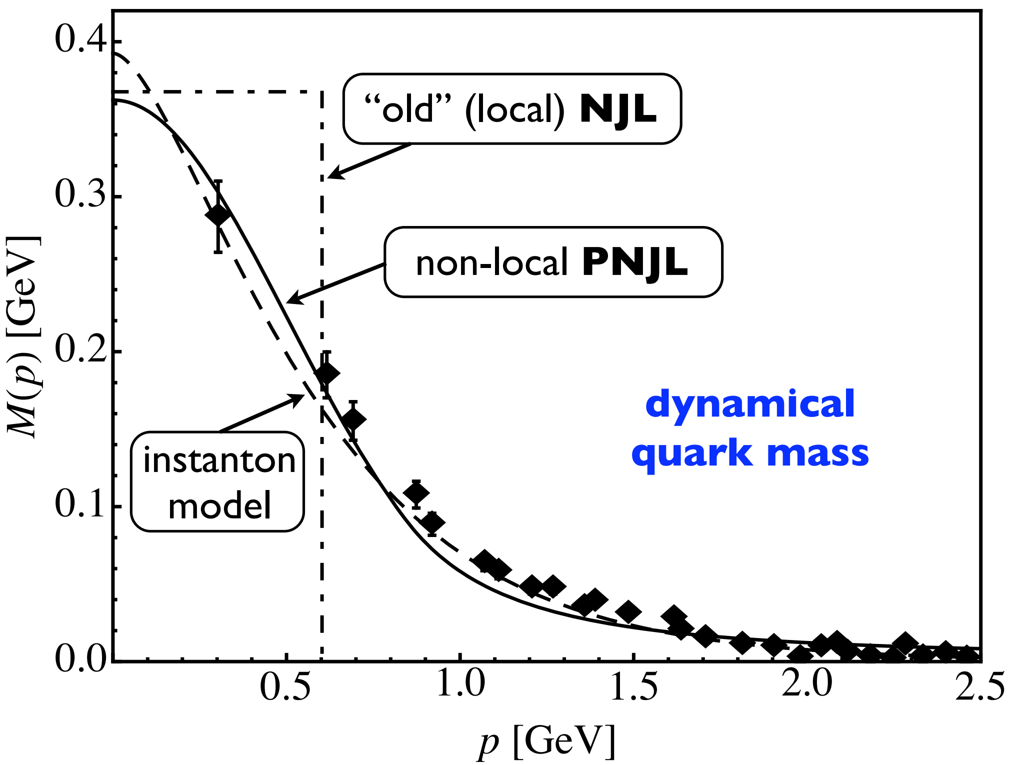
first order phase transition $T_c(pure\ gauge) \equiv T_0 \simeq 270\,\text{MeV}$

Non-Local PNJL Model: **GAP EQUATION**

- **momentum dependent, dynamical quark mass**

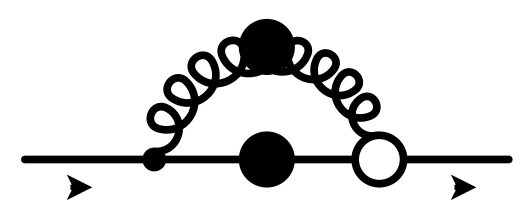
$$\mathbf{M(p) = m_0 + 4N_fN_c\,G \int \frac{d^4q}{(2\pi)^4}\mathcal{C}(p-q)\frac{M(q)}{q^2+M^2(q)}}$$





T. Hell, S. Rößner, M. Cristoforetti, W.W.
Phys. Rev. D79 (2009) 014022, and preprint

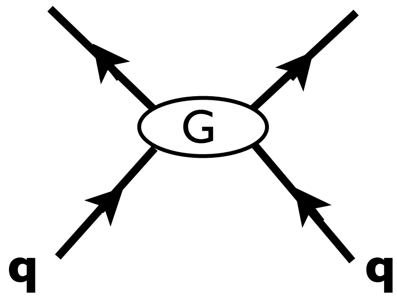
- correlation length $\mathbf{d \simeq 0.35\ fm}$ (typical **instanton** size)
- coupling strength $\mathbf{\sqrt{G} \simeq 1\ fm}$
- consistent with self-energy from **Dyson-Schwinger** calculations (Landau gauge)

$-i\Sigma(p) =$ 

C.D. Roberts, S.M. Schmidt, et al.
Ch. Schäfer et al.

THREE - FLAVOUR non-local PNJL MODEL

- includes:

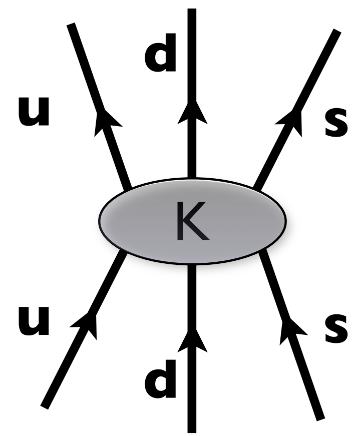


chiral $\mathbf{U(3)_R \times U(3)_L}$ invariant

- plus

Kobayashi-Maskawa-
´t Hooft interaction

$$\mathbf{K[\det \bar{\psi}(1-\gamma_5)\psi + \det \bar{\psi}(1+\gamma_5)\psi]}$$



breaks **axial** $\mathbf{U(1)_A}$

- **input**:

$$\mathbf{G \simeq 1\ fm^2} \qquad \mathbf{K \simeq -(0.63\ fm)^5}$$

$m_{u,d}$	m_s
3.0 MeV	70 MeV

T. Hell, S. Rößner, M. Cristoforetti, W.W. : Phys. Rev. D79 (2009) 014022 and preprint

THREE - FLAVOUR non-local PNJL MODEL

(contd.)

- Chiral low-energy theorems and Current Algebra relations o.k.

  e.g.: Gell-Mann, Oakes, Renner relation

$$\mathbf{m_\pi^2\, f_\pi^2 = -m_q}\, \langle\bar{\psi}\psi\rangle + \mathcal{O}(m_q^2)$$

- **output**:

$\langle\bar{u}u\rangle = \langle\bar{d}d\rangle$	$\langle\bar{s}s\rangle$
$-(0.304\,\mathrm{GeV})^3$	$-(0.323\,\mathrm{GeV})^3$

m_π	m_K	m_η	$m_{\eta'}$	f_π	f_K
139 MeV	495 MeV	547 MeV	964 MeV	92.8 MeV	110.1 MeV

$\eta - \eta'$ mixing angle $\theta_{\eta'} \simeq \mathbf{-29°}$

T. Hell, S. Rößner, M. Cristoforetti, W.W.: preprint (2009)

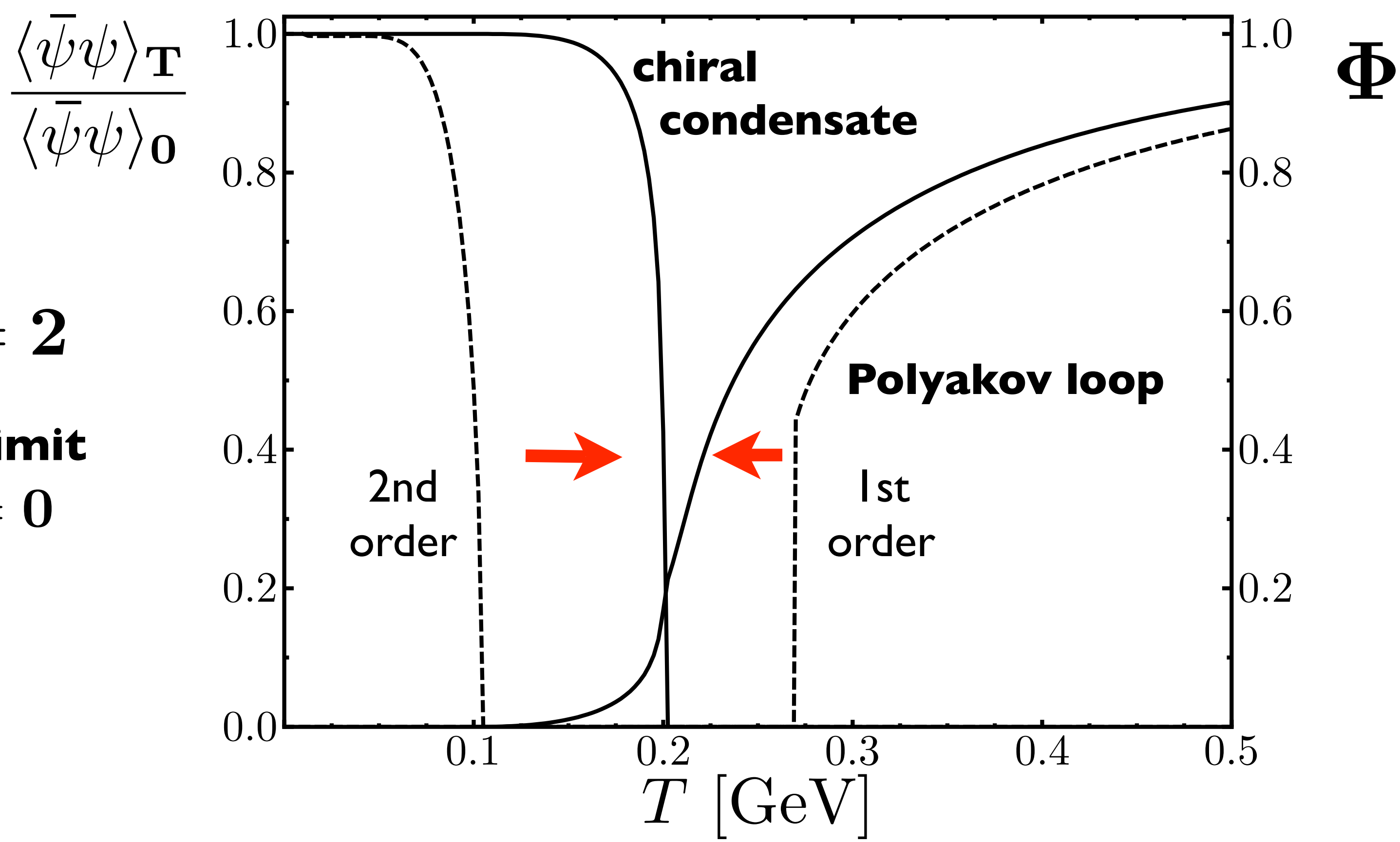
Entanglement of **CONFINEMENT** and **SPONTANEOUS CHIRAL SYMMETRY BREAKING**

Thermodynamics of the **PNJL model**

$\mathbf{N_f = 2}$

chiral limit

$\mathbf{m_q = 0}$



S. Rössner, C. Ratti, W.W.
Phys. Rev. D 75 (2007) 034007

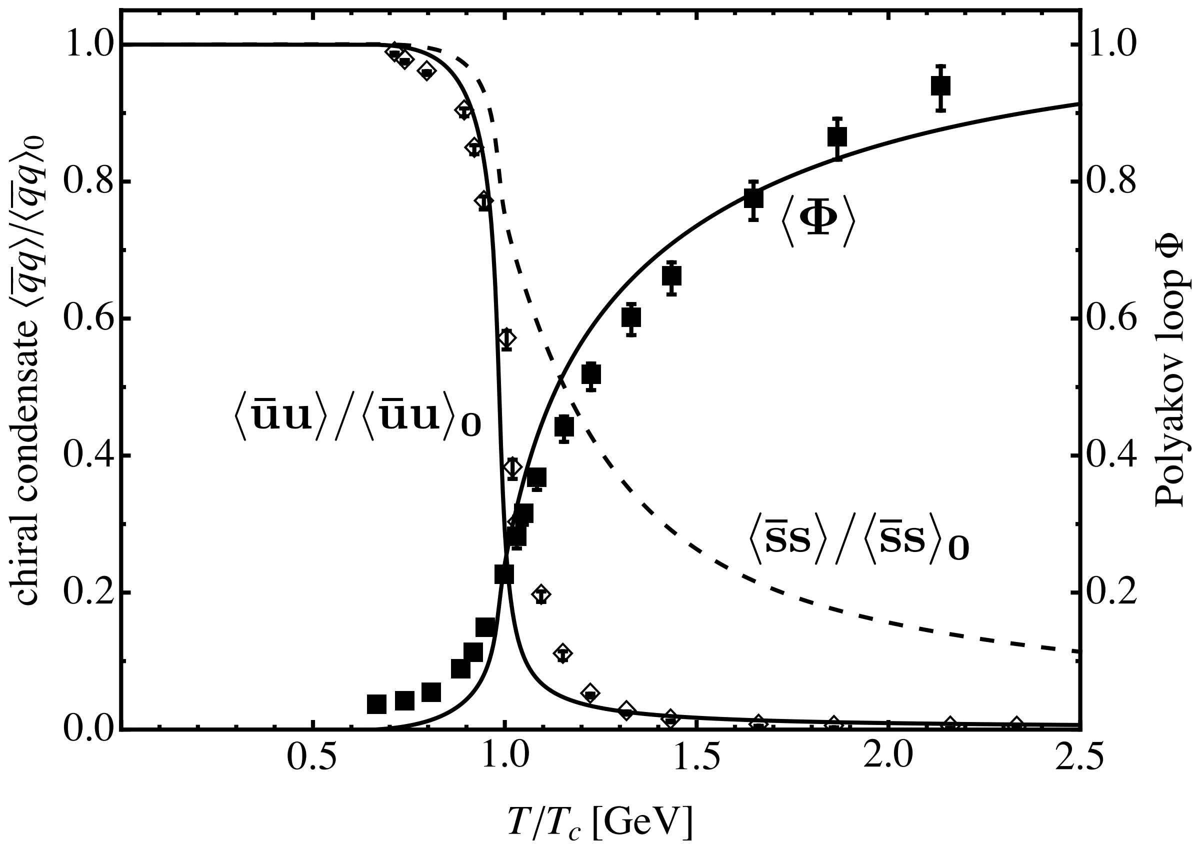
T. Hell, S. Rössner, M. Cristoforetti, W.W.
Phys. Rev. D 79 (2009) 014022

Entanglement of **CONFINEMENT** and **SPONTANEOUS CHIRAL SYMMETRY BREAKING**

$$\mathbf{N_f = 2 + 1}$$

Thermodynamics of the **PNJL model**

in comparison with **Lattice QCD** (with almost physical quark masses)



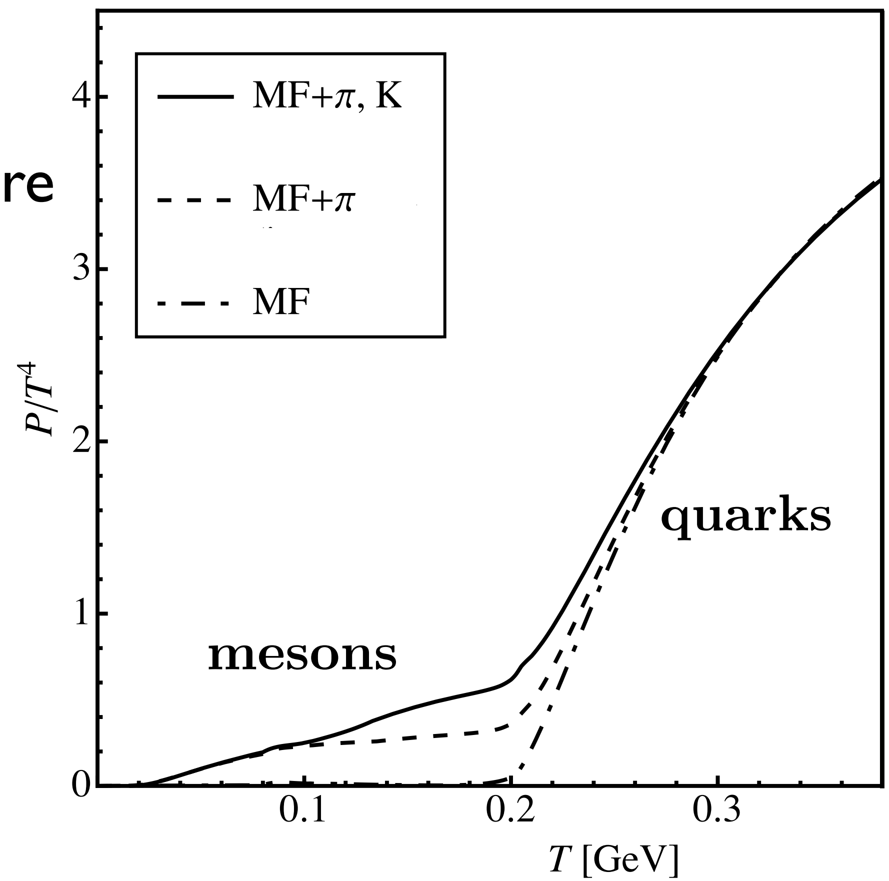
PNJL: T. Hell, M. Cristoforetti, S. Rössner, W.W. (2009)

Lattice: M. Cheng et al. (Bielefeld/BNL/Columbia) Phys. Rev. D77 (2008) 014511

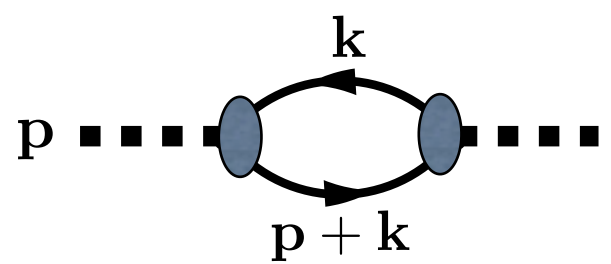
Beyond Mean Field: **Mesonic Excitations**

- contribution of mesonic quark-antiquark modes to pressure

pressure



$$\mathbf{N_f = 2 + 1}$$



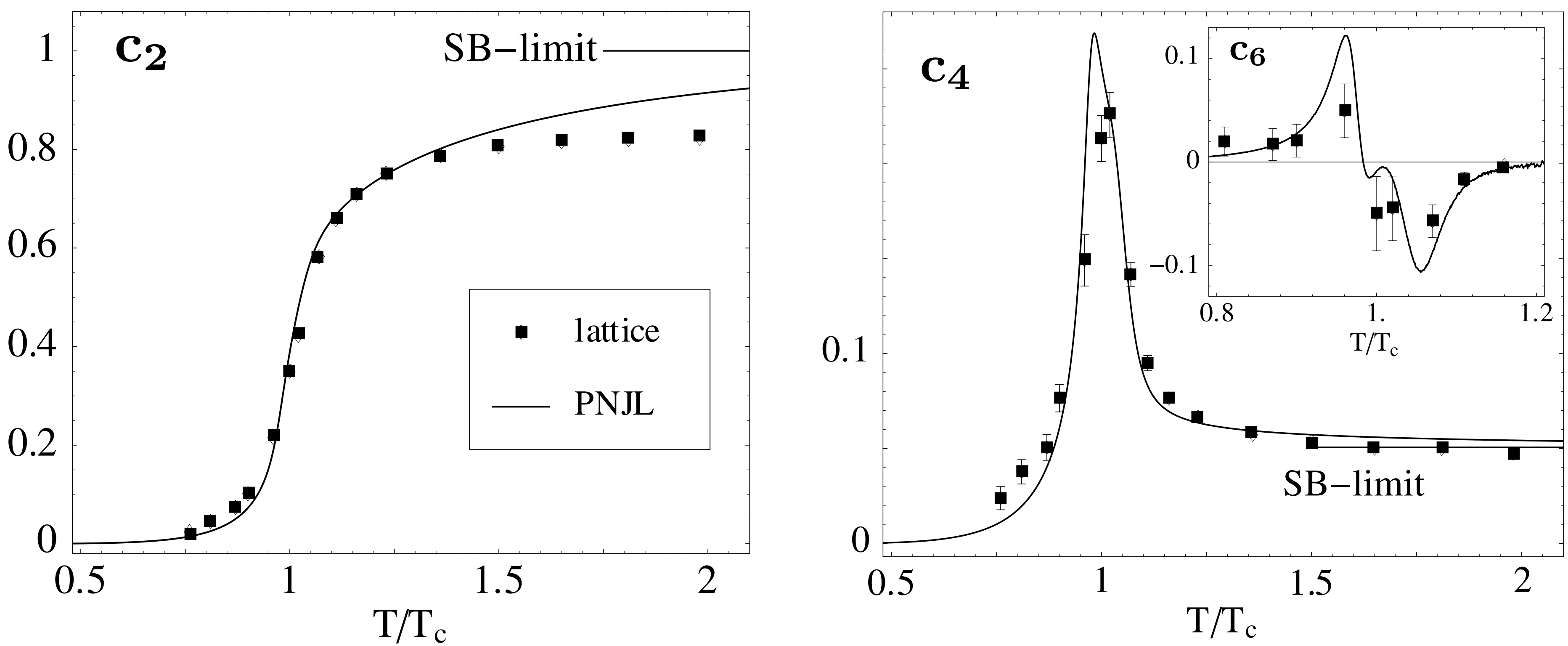
T. Hell, S. Rössner,
M. Cristoforetti, W.W.
(2009)

$$P_{\text{meson}}(T) = -\sum_{M=\pi,K,\ldots} \frac{d_M}{2} T \sum_{m\in\mathbb{Z}} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln\left[1 - G\Pi_M(\nu_m, \vec{p})\right]$$

Non-zero QUARK CHEMICAL POTENTIAL

- Taylor expansion of pressure:

$$\mathbf{P(T,\mu) = T^4 \sum_n c_n(T) \left(\frac{\mu}{T}\right)^n}$$



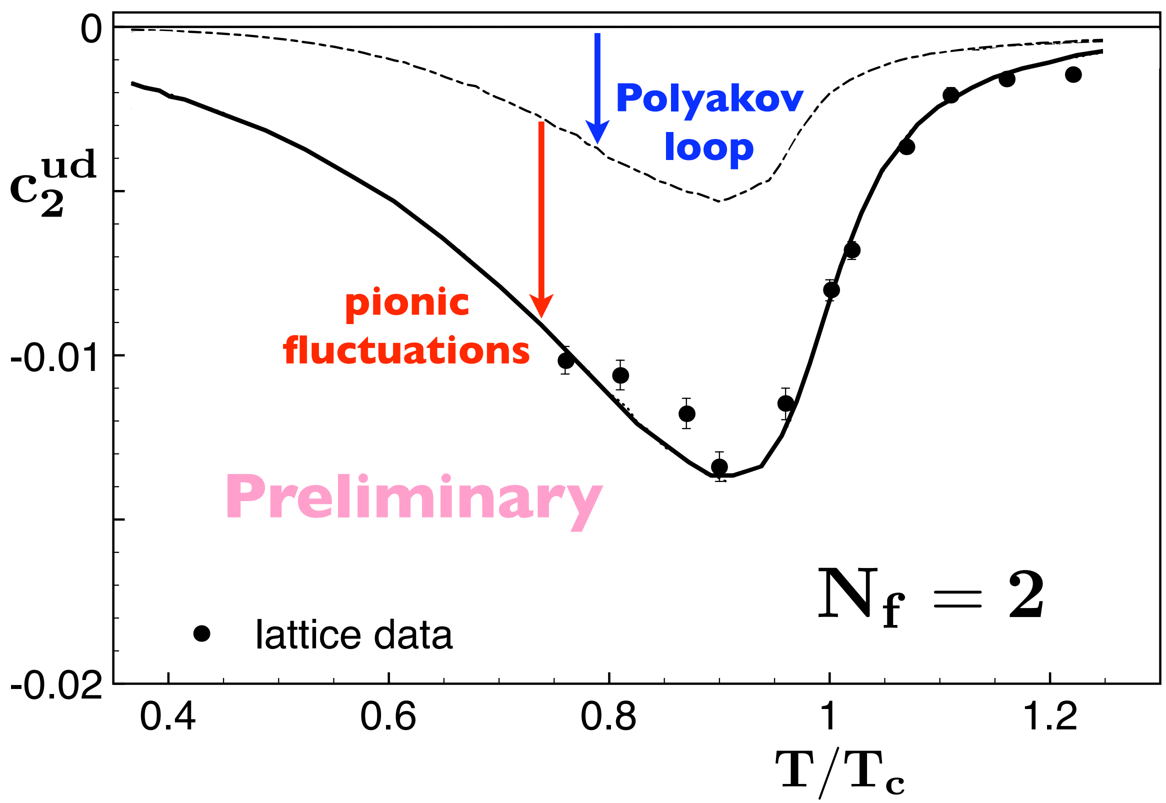
$\mathbf{N_f = 2}$

S. Rößner, C. Ratti, W.W.
Phys. Rev. D 75 (2007) 034007

Lattice:
C.R. Allton et al.
Phys. Rev. D 71 (2005) 054508

Flavour non-diagonal Susceptibilities

- Example: $$\mathbf{c_2^{ud}} \sim \frac{\partial^2 \mathbf{P}}{\partial \mu_\mathbf{u} \partial \mu_\mathbf{d}}$$



Monte Carlo simulations with PNJL model

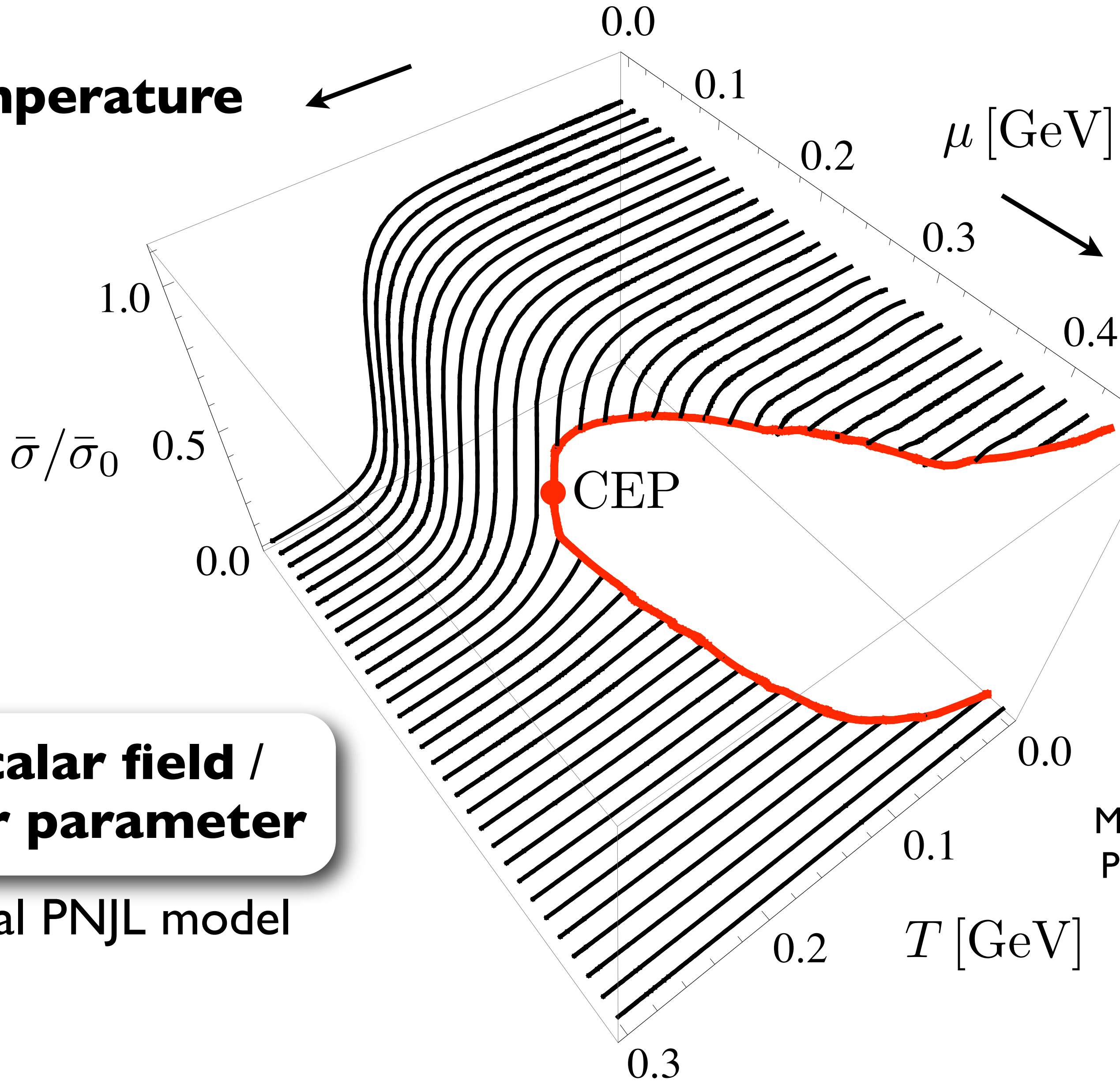
M. Cristoforetti, T. Hell, B. Klein, W.W. (2009)

- Sensitivity to effects beyond mean field (e.g. **pionic fluctuations**)

PHASE DIAGRAM and CRITICAL POINT

temperature ←

$\mathbf{N_f = 2}$



→ **quark chemical potential**

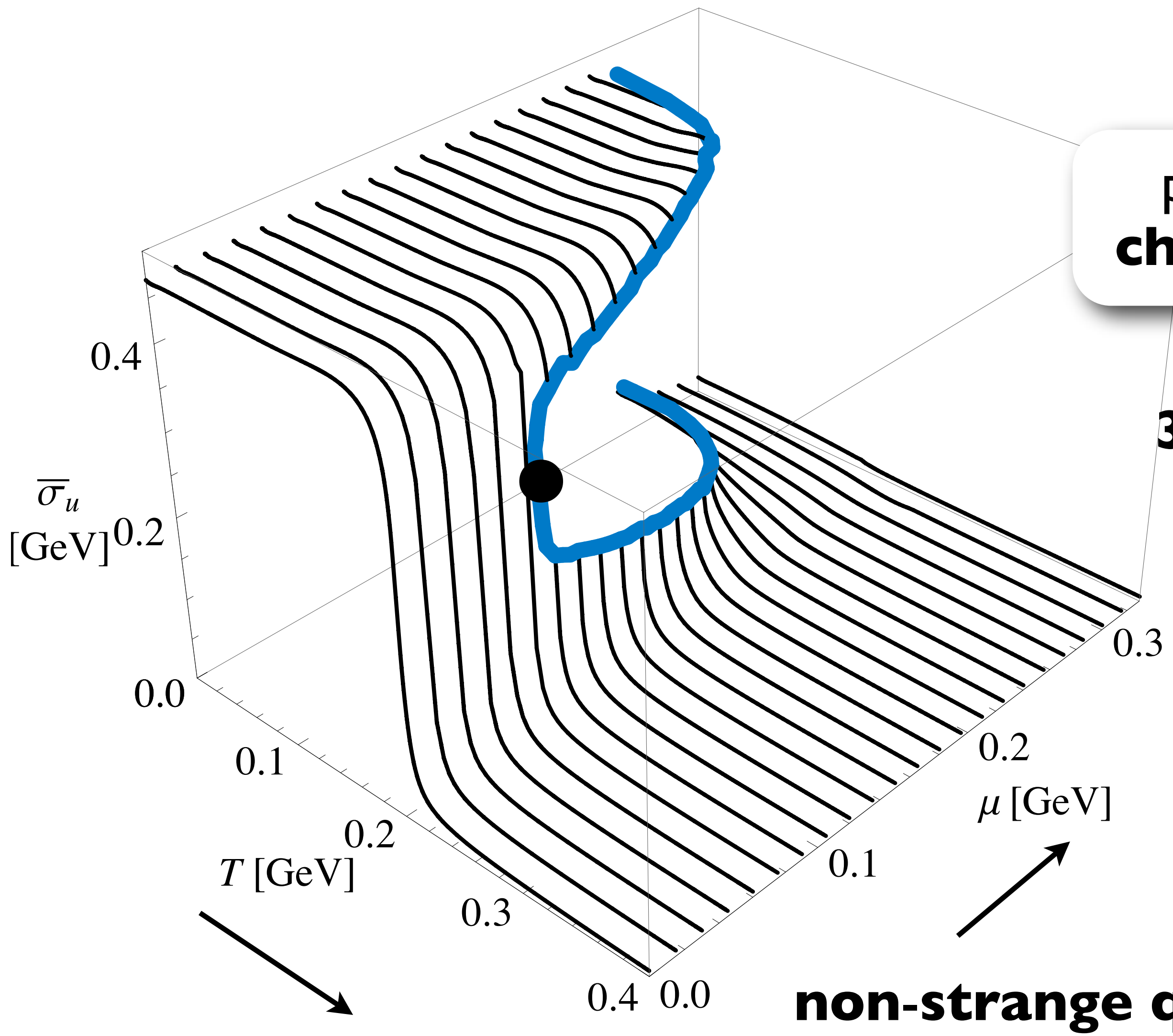
profile of **scalar field / chiral order parameter**

from non-local PNJL model

T. Hell, S. Rössner, M. Cristoforetti, W.W. Phys. Rev. D79 (2009)

PHASE DIAGRAM and CRITICAL POINT

$\mathbf{N_f = 2 + 1}$



profile of **scalar field / chiral order parameter**

from non-local **3-flavour PNJL** model

temperature

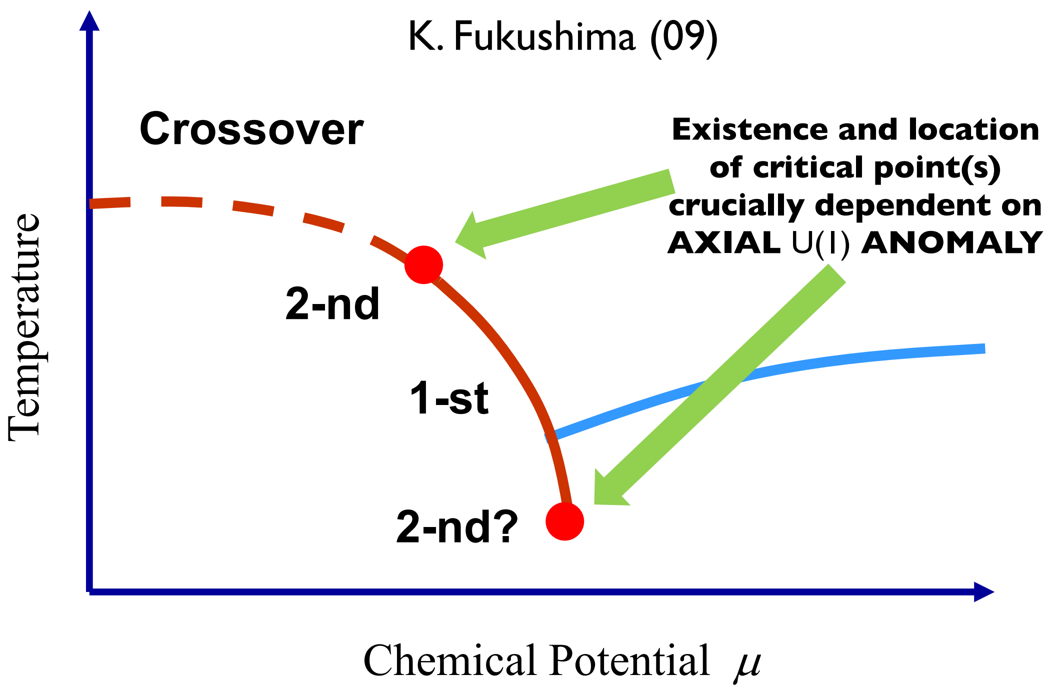
non-strange quark chemical potential

T. Hell, S. Rössner, M. Cristoforetti, W.W. preprint (2009)

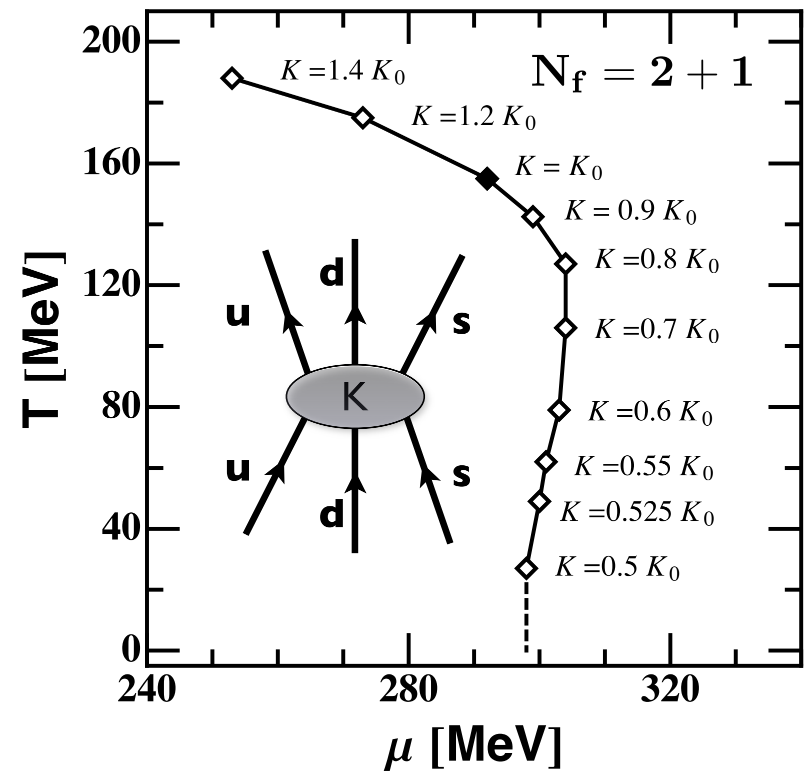
PHASE DIAGRAM (contd.)

- Issues: ⇨ **Critical Point**
  ⇨ Diquarks and **CSC Phase**

K. Fukushima (09)



Critical point:
Role of **axial anomaly**

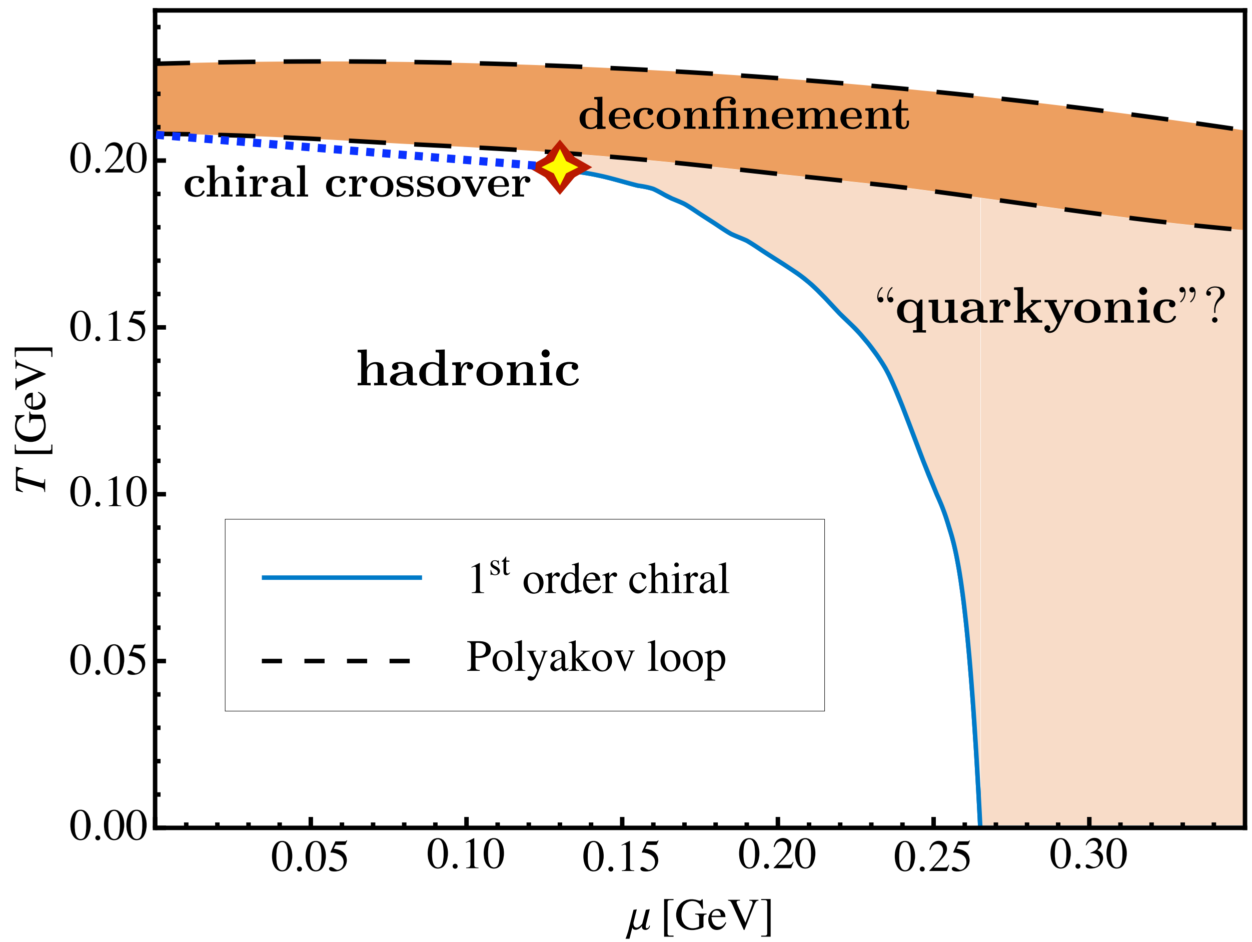


Yamamoto, Hatsuda, Baym (2007) K. Fukushima (2008) N. Bratovich, T. Hell, S. Rößner, W.W. (2009)

- Location (and existence) of **critical point**(s) depends sensitively on **no. of flavours**, **quark masses**, **axial** $\mathrm{U}(1)_A$ **anomaly**, etc. ...

PHASE DIAGRAM (contd.)

from non-local **3-flavour PNJL** model

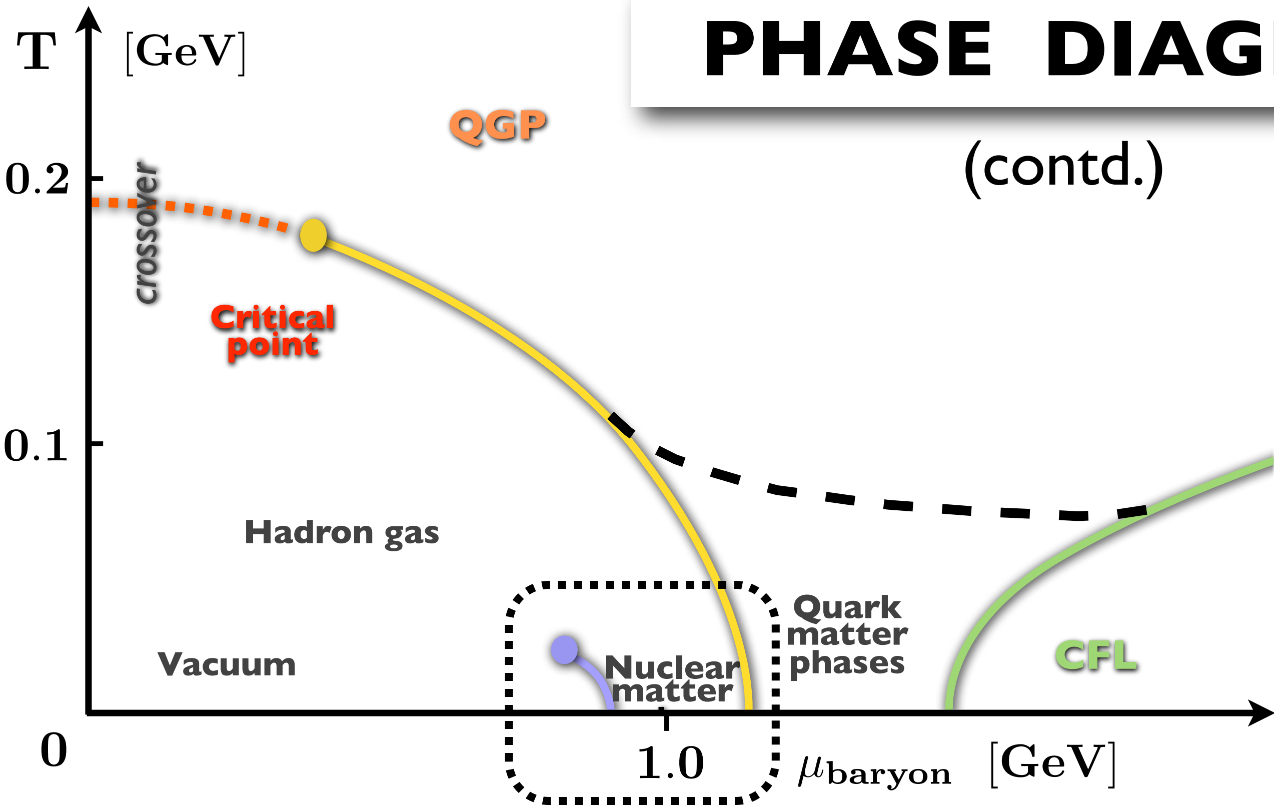


T. Hell, S. Rössner, M. Cristoforetti, W.W. preprint (2009)

- warning: critical chemical potential **too low** at T = 0
- "wrong" degrees of freedom at low temperature, non-zero density ?

PHASE DIAGRAM

(contdd.)



- Question:

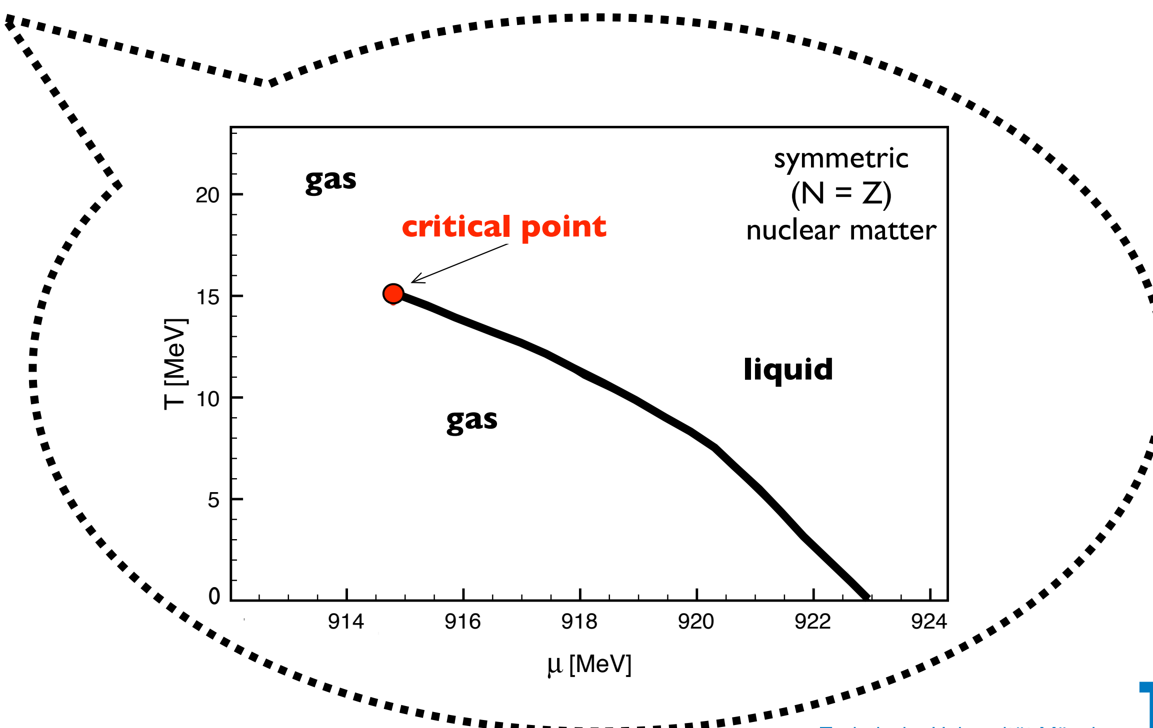
first order chiral transition
boundary all the way down to T = 0

?

Nuclear Liquid-Gas phase diagram

from

Chiral Thermodynamics



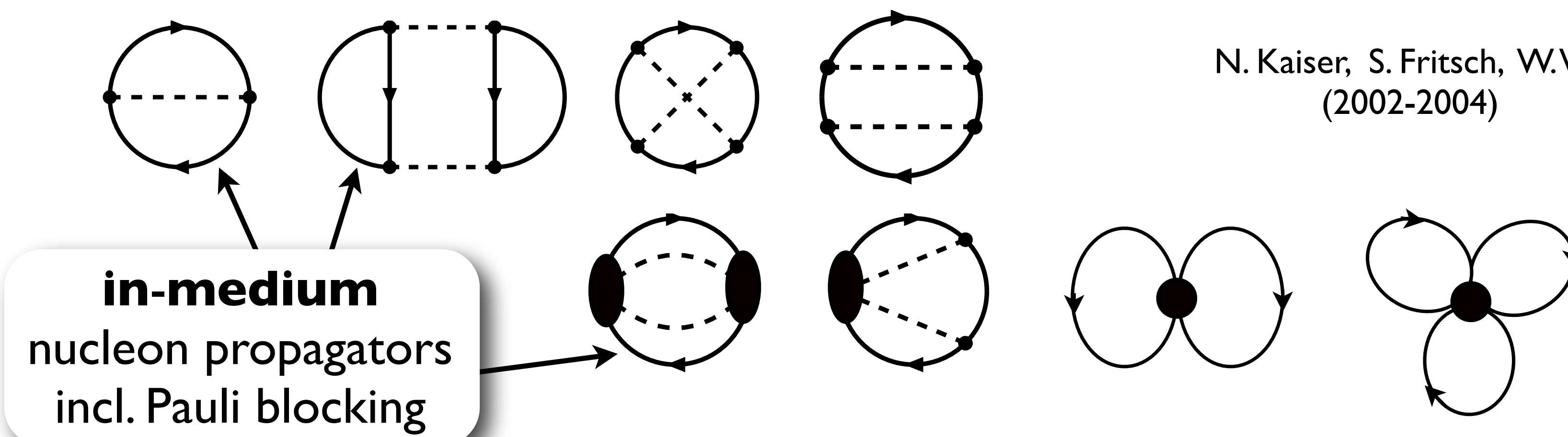
From **NUCLEI** to **COMPRESSED BARYONIC MATTER**

- Framework: **E**ffective **F**ield **T**heory implementing the **chiral symmetry breaking pattern** of Low-Energy QCD:

In-medium chiral perturbation theory

- Active degrees of freedom in the hadronic phase: **pions**, **nucleons**, **delta**-isobars

- Compute **Free Energy Density** (3-loop order)



N. Kaiser, S. Fritsch, W.W.
(2002-2004)

NUCLEAR THERMODYNAMICS

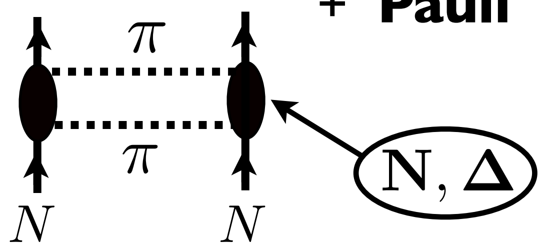
NUCLEAR CHIRAL (PION) DYNAMICS

BINDING & SATURATION:

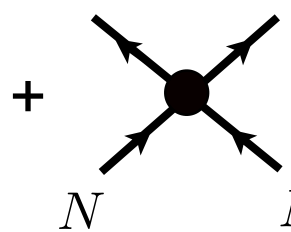
Yukawa

+ Van der Waals

+ Pauli



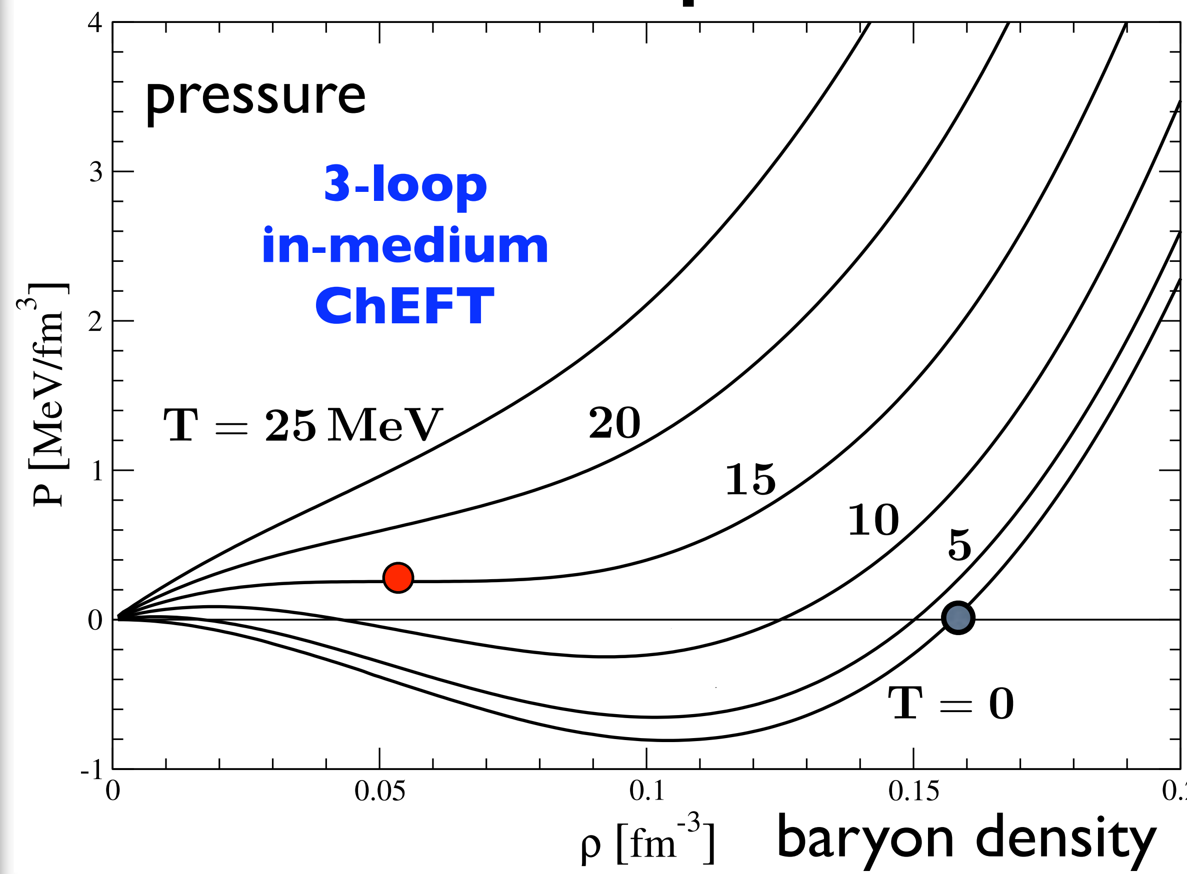
$$V(r) \sim -\frac{e^{-2m_\pi r}}{r^6} P(m_\pi r)$$



+ 3-body forces

contact terms

nuclear matter: equation of state



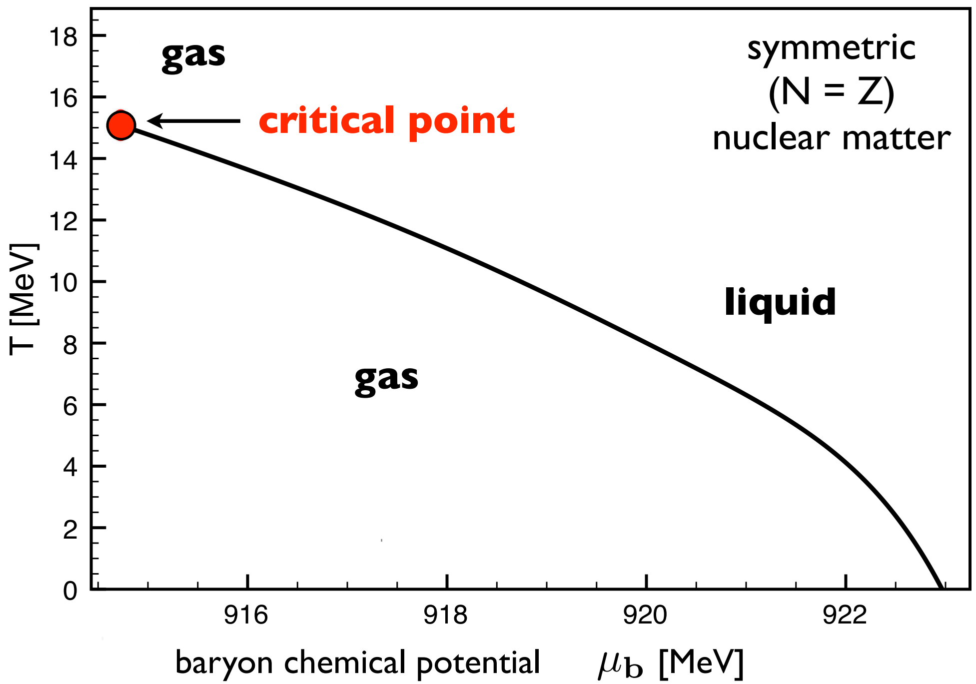
Liquid - Gas Transition at

Critical Temperature T_c = 15 MeV

(empirical: T_c = 16 - 18 MeV)

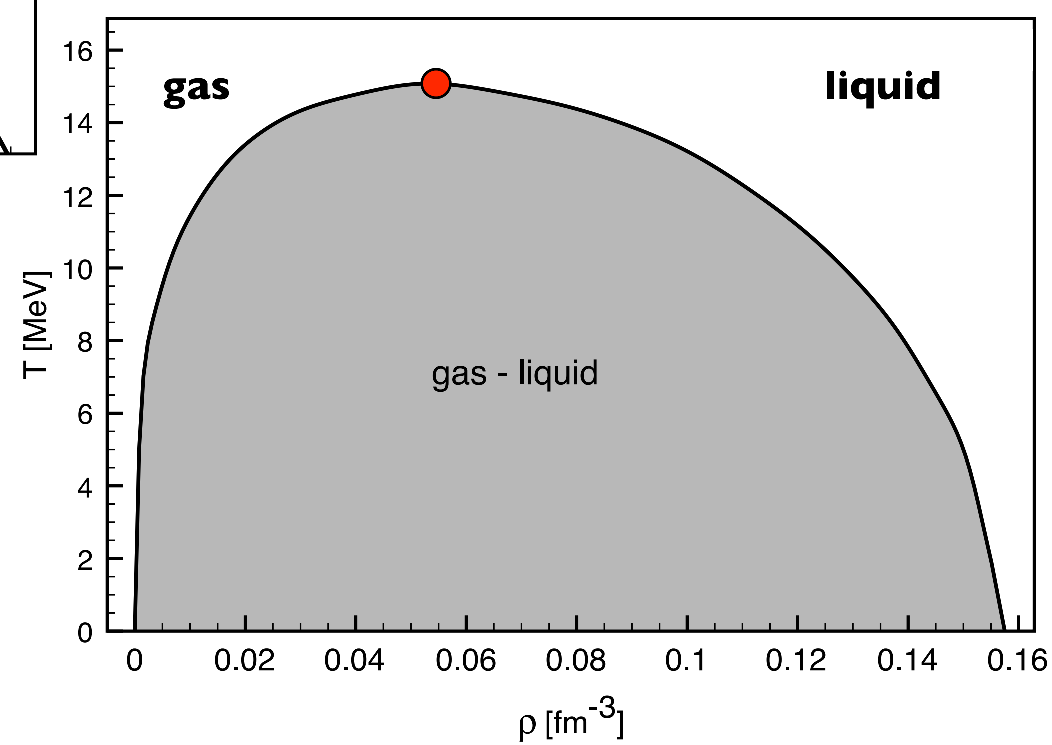
S. Fritsch, N. Kaiser, W.W.: Nucl. Phys. A 750 (2005) 259

PHASE DIAGRAM of NUCLEAR MATTER



- In-medium **chiral effective field theory** (3-loop in the free energy density)

S. Fritsch, N. Kaiser, W.W.: NPA 750 (2005) 259



- Pion-nucleon dynamics incl. delta isobars
- Short-distance NN contact terms
- Three-body forces

S. Fiorilla, N. Kaiser, W.W. (2009)

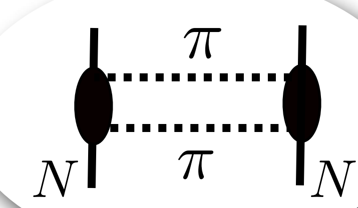
CHIRAL CONDENSATE at finite DENSITY

$$(\mathbf{T = 0})$$

sigma term

$$\mathbf{m_q \frac{\partial M_N}{\partial m_q}}$$

in-medium chiral effective field theory



$$\frac{\langle \bar{\mathbf{q}}\mathbf{q}\rangle_\rho}{\langle \bar{\mathbf{q}}\mathbf{q}\rangle_0} = \mathbf{1} - \frac{\rho}{\mathbf{f}_\pi^2}\left[\frac{\sigma_\mathbf{N}}{\mathbf{m}_\pi^2}\left(\mathbf{1} - \frac{\mathbf{3\,p_F^2}}{\mathbf{10\,M_N^2}} + \dots\right) + \frac{\partial}{\partial \mathbf{m}_\pi^2}\left(\frac{\mathbf{E_{int}(p_F)}}{\mathbf{A}}\right)\right]$$

(free) Fermi gas of nucleons

nuclear interactions (dependence on pion mass)

CHIRAL CONDENSATE: DENSITY DEPENDENCE

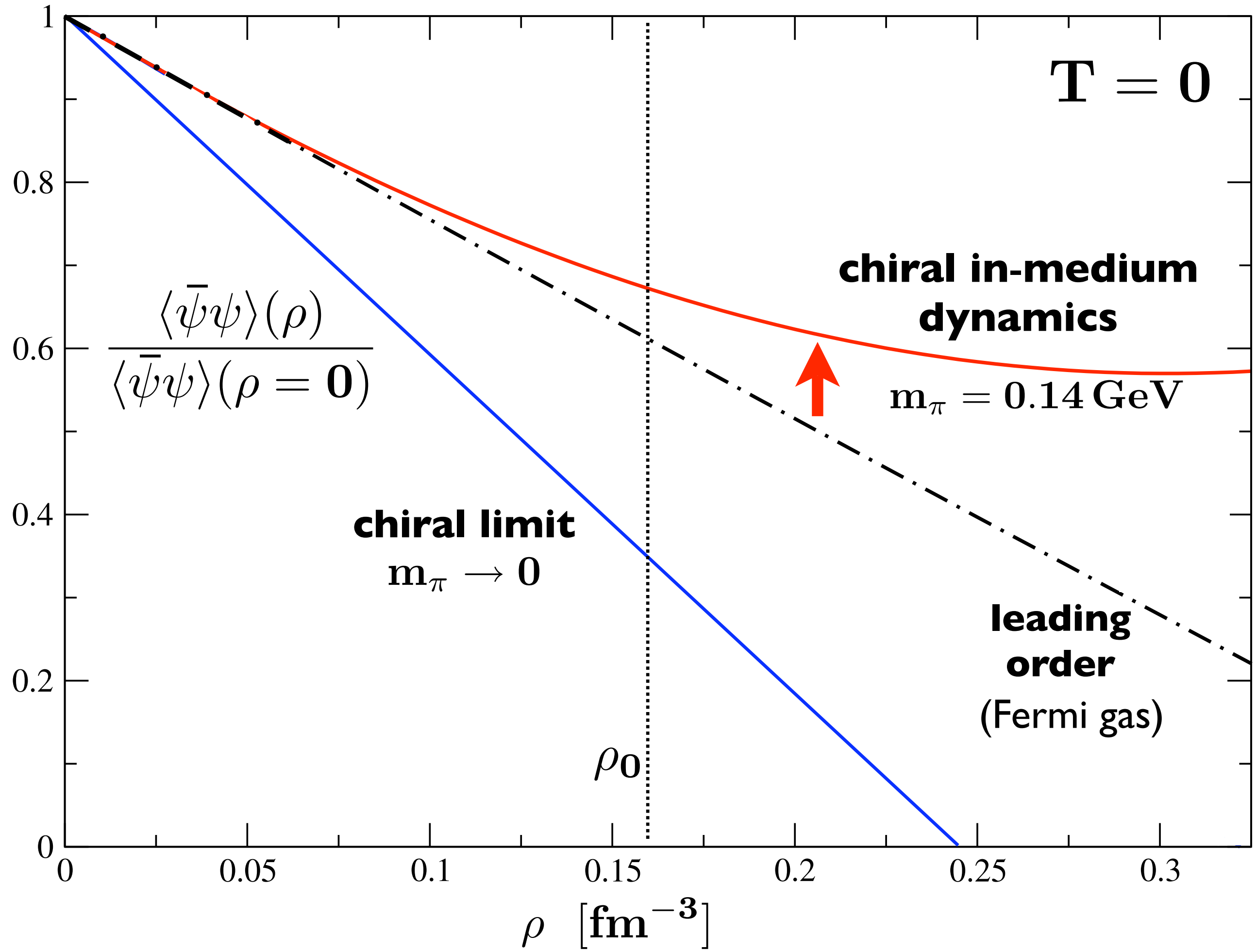
Symmetric Nuclear Matter

- **In-medium Chiral Effective Field Theory**

  (NLO 3-loop)

  constrained by **realistic nuclear equation of state**

  N. Kaiser, Ph. de Homont, W.W.
  Phys. Rev. C 77 (2008) 025204



- Substantial **change** of **symmetry breaking scenario** between chiral limit $m_q = 0$ and physical quark mass $m_q \sim 5\,\mathrm{MeV}$
- **Nuclear Physics** would be **very different** in the **chiral limit** !

Conclusions

- Role of $\mathrm{SU(N_f)_L \times SU(N_f)_R}$ and $\mathrm{Z(3)}$ Symmetries in QCD

- Entanglement of **CHIRAL** and **DECONFINEMENT** crossover transitions in QCD (at zero chemical potential)

  - transition temperatures (at zero chemical potential) **coincide** in **PNJL** models and on the **Lattice**

- **PHASE DIAGRAM** at low T, large **BARYON DENSITY**, **CRITICAL POINT**, and all that . . .

  - role of **axial** U(1) **anomaly**
  - **constraints** from **realistic nuclear EoS**

thanks to:

Nino Bratovic, Marco Cristoforetti, Salvatore Fiorilla, Thomas Hell, Bertram Klein, Norbert Kaiser, Claudia Ratti, Simon Rössner

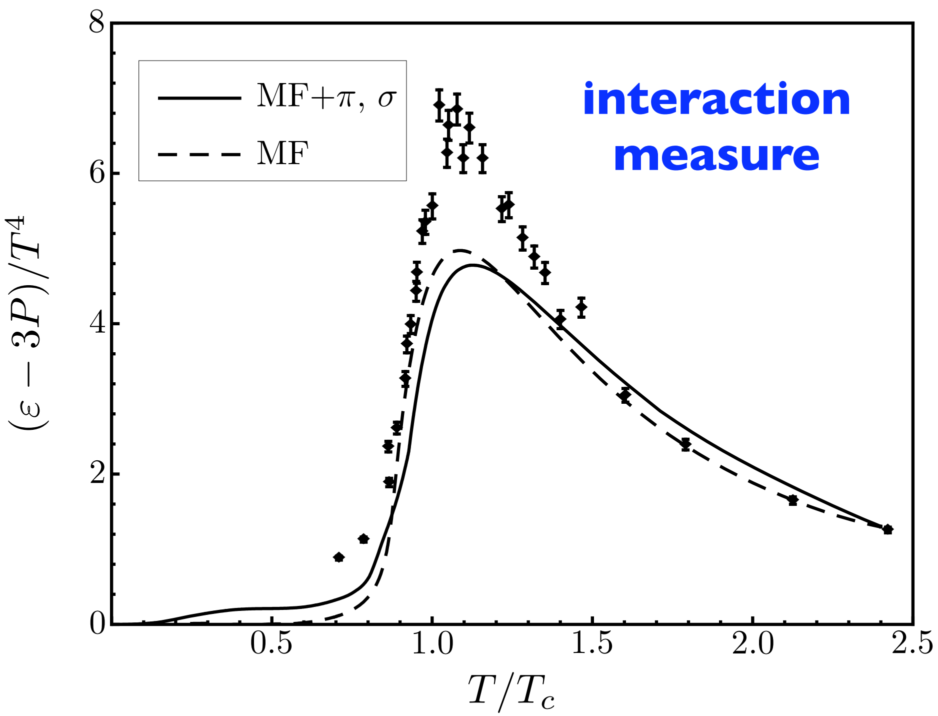
Supplementary Materials

PNJL model vs. **Lattice QCD Thermodynamics**

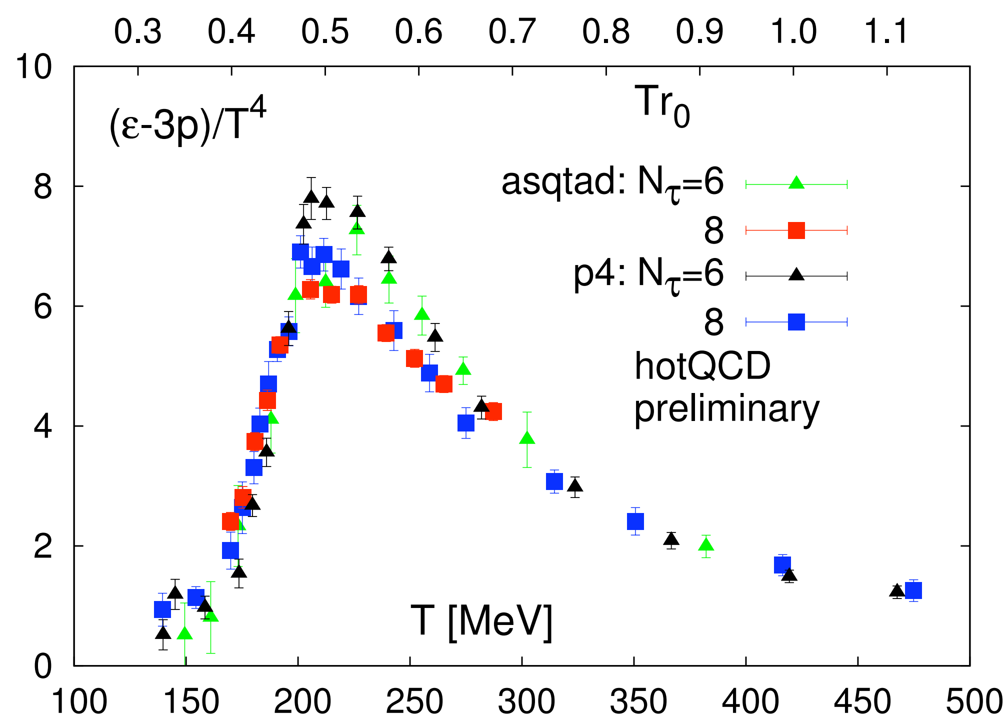
- **PRESSURE** and **ENERGY DENSITY** at zero chemical potential

$$\mathbf{p} = -\boldsymbol{\Omega}(\mathbf{T}, \mu = \mathbf{0})$$

$$\varepsilon = \mathbf{T}\frac{\partial \mathbf{p}(\mathbf{T}, \mu = \mathbf{0})}{\partial \mathbf{T}} - \mathbf{p}(\mathbf{T}, \mu = \mathbf{0})$$



T. Hell, S. Rößner, M. Cristoforetti, W.W.
Phys. Rev. D79 (2009) 014022

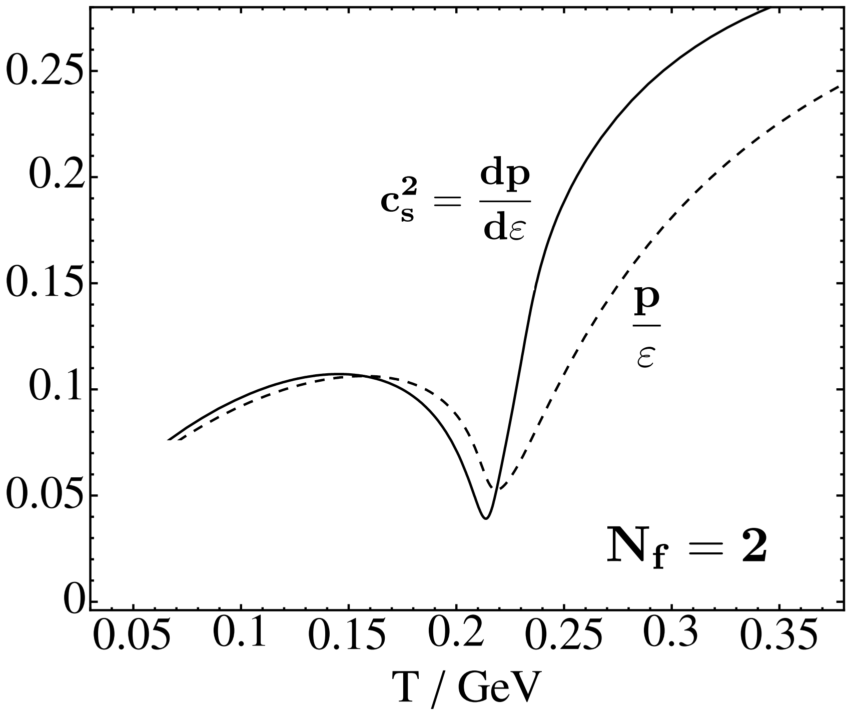


lattice data:
F. Karsch et al. : arXiv:0804.4148 [hep-lat]

Sound Velocity: PNJL and LATTICE QCD

- **PNJL**

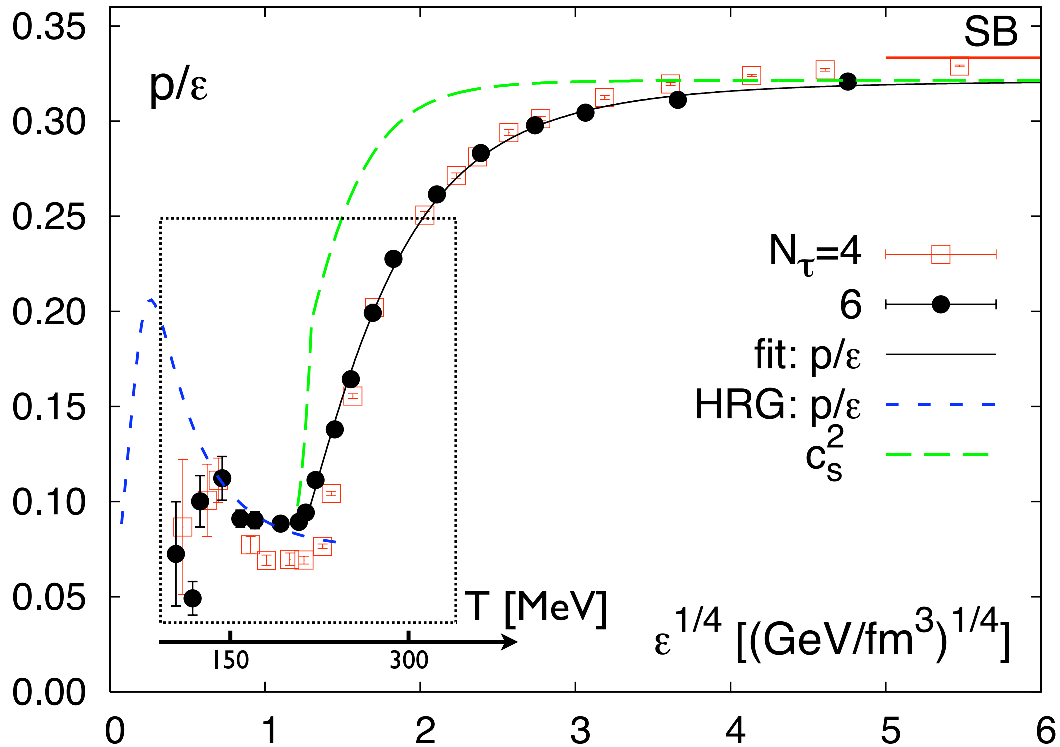
S. Rössner, Th. Hell, C. Ratti, W.W.
arXiv:0712.3152 [hep-ph]



- **Lattice** $N_f = 2 + 1$

C. Bernard et al.,
Phys. Rev. D 75 (2007) 094505

F. Karsch,
arXiv:0711.0565 [hep-lat]



- **PNJL** model works

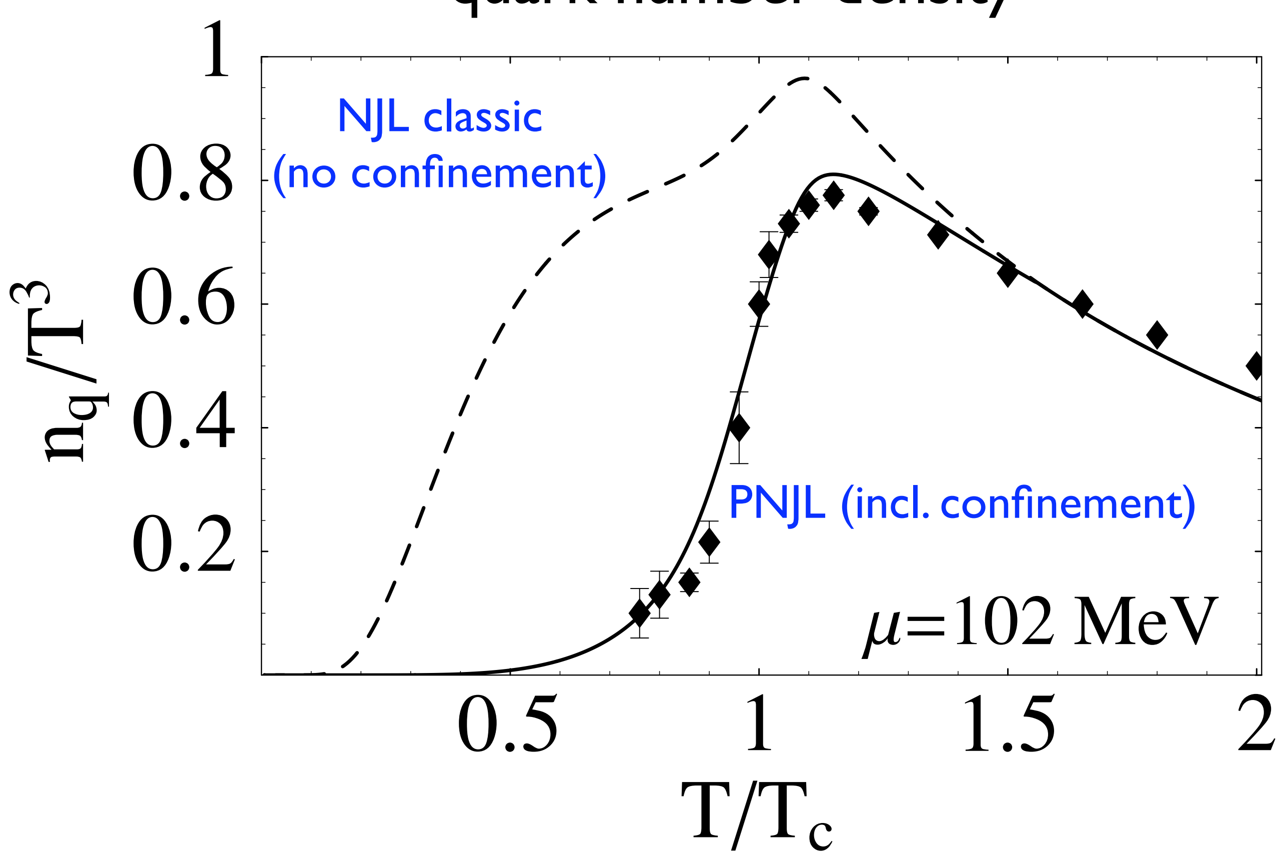
Active degrees of freedom around critical temperature $T > T_c$:

Quarks as **quasiparticles** interacting with **Polyakov** loop

Non-zero **QUARK CHEMICAL POTENTIAL**

- Role of CONFINEMENT (**POLYAKOV** loop dynamics)
  suppression of quark propagator in “forbidden” region

quark number density



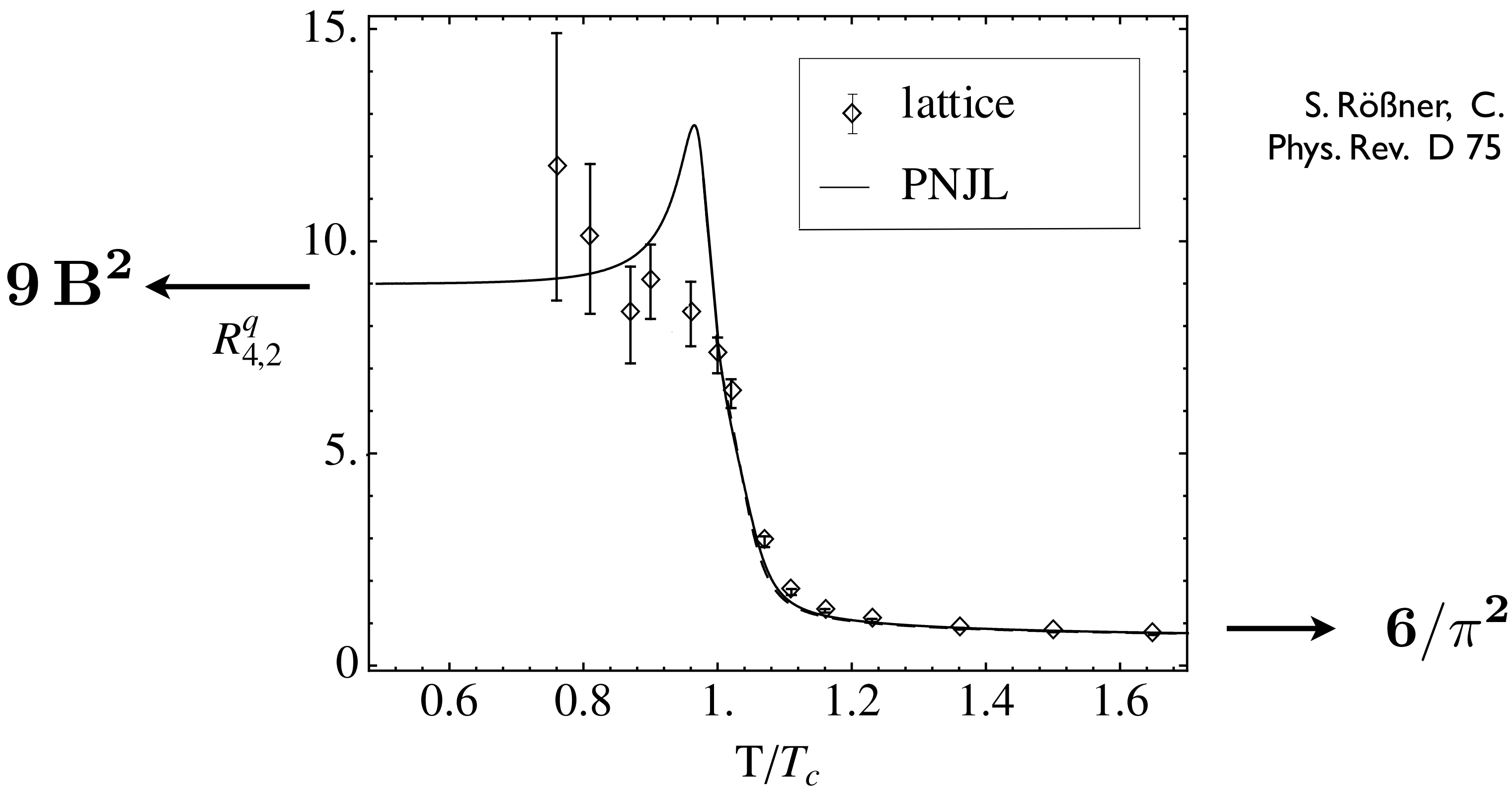
C. Ratti, M. Thaler, W.W.
PRD 73 (2006)

Lattice “data”:
Allton et al.
Phys. Rev. D 68 (2003)

Cumulant Ratios

- Example: quark number cumulant ratio $R^q_{4,2} = \frac{12\,c_4}{c_2}$

S. Ejiri, F. Karsch, K. Redlich
Phys. Lett. B 633 (2006) 275



S. Rößner, C. Ratti, W.W.
Phys. Rev. D 75 (2007) 034007

Lattice:
C.R. Allton et al.
Phys. Rev. D 71 (2005) 054508