Coulomb gauge and Schwinger-Dyson equations

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(supported by DFG contracts no. Re856/6-1 & Re856/6-2)

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P. W. and H. Reinhardt, arXiv:0812.1989, 0808.2436, PRD75:045021

Outline

Coulomb gauge QCD is very physical and Gauß' law rules (as I'll try to show), but this comes at the expense of covariance: it is also very technical (as I'll try not to show)…

- Temporal zero modes, total charge and physical degrees of freedom
- Slavnov–Taylor identities (STids)
- Ghost Dyson–Schwinger equation (DSe)
- Summary and outlook

Charge!

Consider (continuum) Yang–Mills theory:

$$Z = \int \mathcal{D}\Phi \exp\{\imath \mathcal{S}_{YM}\}, \;\; \mathcal{S}_{YM} = \frac{1}{2}\int dx \left[E^2 - B^2\right]$$

in terms of spatial ($\vec{A}$) and temporal (A_0) gauge fields.

Importantly, E is linear in A_0.

Charge!

To fix to Coulomb gauge, $\vec{\nabla}\cdot\vec{A}=0$, use Faddeev–Popov:

$$1=\int\mathcal{D}\theta\delta\left(\vec{\nabla}\cdot\vec{A}^{a}\right)\mathrm{Det}\left[-\vec{\nabla}\cdot\vec{D}\right].$$

Since $-\vec{\nabla}\cdot\vec{D}$ only involves spatial operators, we still have spatially independent (time-dependent) gauge transforms and there are temporal zero-modes (not to mention the usual Gribov copies)!

Replace Det in the identity with $\overline{\mathrm{Det}}$, the determinant with the zero-modes removed.

Charge!

Now we convert to first order formalism by introducing an auxiliary field $\vec{\pi}$:

$$\exp\left\{\frac{\imath}{2}\int E^2\right\} \to \int \mathcal{D}\pi \exp\left\{\frac{\imath}{2}\int\left(\pi^2 - 2\vec{\pi}\cdot\vec{E}\right)\right\},$$

then split $\vec{\pi}$ into transverse ($\vec{\nabla}\cdot\vec{\pi}^{\perp a} = 0$) and longitudinal ($\vec{\nabla}\phi$) parts. The action is linear in E ($\sim A_0$) so integrate out to leave a δ-functional constraint (Gauß' law):

$$Z = \int \mathcal{D}\Phi\delta\left(\vec{\nabla}\cdot\vec{A}^a\right)\delta\left(\vec{\nabla}\cdot\vec{\pi}^{\perp a}\right)\overline{\mathrm{Det}}\left[-\vec{\nabla}\cdot\vec{D}\right]\times$$
$$\delta\left(\vec{\nabla}\cdot\vec{D}^{ab}\phi^b + g\hat{\rho}^a\right)\exp\left\{\imath\mathcal{S}'\right\},$$

$\hat{\rho}^a = f^{abc}\vec{A}^b\cdot\vec{\pi}^{\perp c} \to$ color charge. Now for $\phi\ldots$

Charge!

Integrating ϕ, and noting the temporal zero modes

$$\delta\left(\vec{\nabla}\cdot\vec{D}^{ab}\phi^b+g\hat{\rho}^a\right)\to\delta\left(\int d\vec{x}\hat{\rho}^a\right)\overline{\mathrm{Det}}\left[-\vec{\nabla}\cdot\vec{D}\right]^{-1}\delta\left(\phi+\ldots\right)$$

gives us then

$$Z=\int\mathcal{D}\Phi\delta\left(\vec{\nabla}\cdot\vec{A}^a\right)\delta\left(\vec{\nabla}\cdot\vec{\pi}^{\perp a}\right)\delta\left(\int d\vec{x}\hat{\rho}^a\right)\exp\left\{\imath\mathcal{S}\right\}$$

with the final effective action

$$\begin{aligned}\mathcal{S} &= \int dx\left[\vec{\pi}^{\perp}\cdot\partial_0\vec{A}-\frac{B^2}{2}-\frac{\pi^{\perp 2}}{2}\right.\\ &\quad\left.+\frac{g^2}{2}\hat{\rho}\left(-\vec{\nabla}\cdot\vec{D}\right)^{-1}\nabla^2\left(-\vec{\nabla}\cdot\vec{D}\right)^{-1}\hat{\rho}\right].\end{aligned}$$

No charge!

Thanks to Gauß' law, our system has the following properties:

- two (physical) transverse degrees of freedom ($\vec{A}$, $\vec{\pi}^{\perp}$) with a conserved and vanishing total charge ($\int d\vec{x}\hat{\rho} = 0$)
- ghosts and temporal zero modes have gone away and there are no energy divergences
  — the zero modes of the Faddeev–Popov operator are an expression of the Gribov problem. The temporal zero modes give rise to the total charge, what about the spatial zero modes (genuine Gribov copies)?
- the gauge is temporally fixed

However, this is formal and non-local. . .

Slavnov–Taylor identities

To solve Dyson–Schwinger equations, we need a truncation scheme for vertices. This is done via Slavnov–Taylor identities. Go back to the original (local) action and fix to Coulomb gauge: $\vec{\nabla}\cdot\vec{A}=0\ldots$ Ghost term looks like

$$\mathcal{S}_{FP}=\int dx\left[-\overline{c}^{a}\vec{\nabla}\cdot\vec{D}^{ab}c^{b}\right]$$

Because $-\vec{\nabla}\cdot\vec{D}$ only involves spatial operators, we have invariance under a Gauß-BRST transform – a *time-dependent* BRS transform (N.B. $\theta\rightarrow\theta(t,\vec{x})$):

$$\theta_x^a=c_x^a\delta\lambda_t.$$

D. Zwanziger, Nucl. Phys. B518 (1998) 237.

STids: 2-point

After some work, for the 2-point proper functions...

$$
\begin{aligned}
k_0\Gamma_{00}(k_0,\vec{k}) &= \imath\frac{k_i}{\vec{k}^2}\Gamma_{0Ai}(k_0,\vec{k})\Gamma_{\overline{c}c}(q_0+k_0,\vec{k}) \\
k_0\Gamma_{A0k}(k_0,\vec{k}) &= \imath\frac{k_i}{\vec{k}^2}\Gamma_{AAki}(k_0,\vec{k})\Gamma_{\overline{c}c}(q_0+k_0,\vec{k})
\end{aligned}
$$

- analogue of Landau gauge transversality
- gluon polarization is not transverse (even at tree-level)
- inverse ghost propagator independent of energy
- A_0-leg Green's functions known in terms of others – (local) elimination of A_0-field (Gauß' law)!

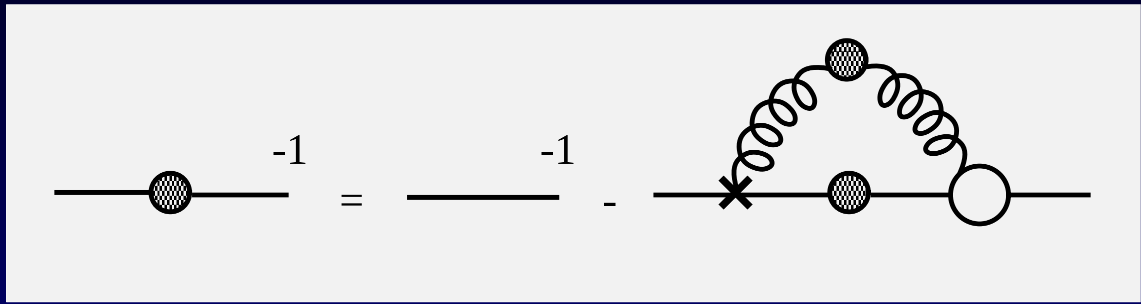
STids: n-point

After lots more work, for the 3 & 4-point proper functions...

- STids form closed sets from which A_0-leg Green's functions known in terms of others
  – (local) elimination of A_0-field (Gauß' law)!
- $\Rightarrow$ It is possible that the Gauß-BRST charge can be identified with the physical charge, à la Kugo–Ojima (unlike, as it appears, in Landau gauge)
- $g^2 W_{00}$ is RG invariant

K. -I. Kondo, arXiv:0907.3249…

Ghost DSe



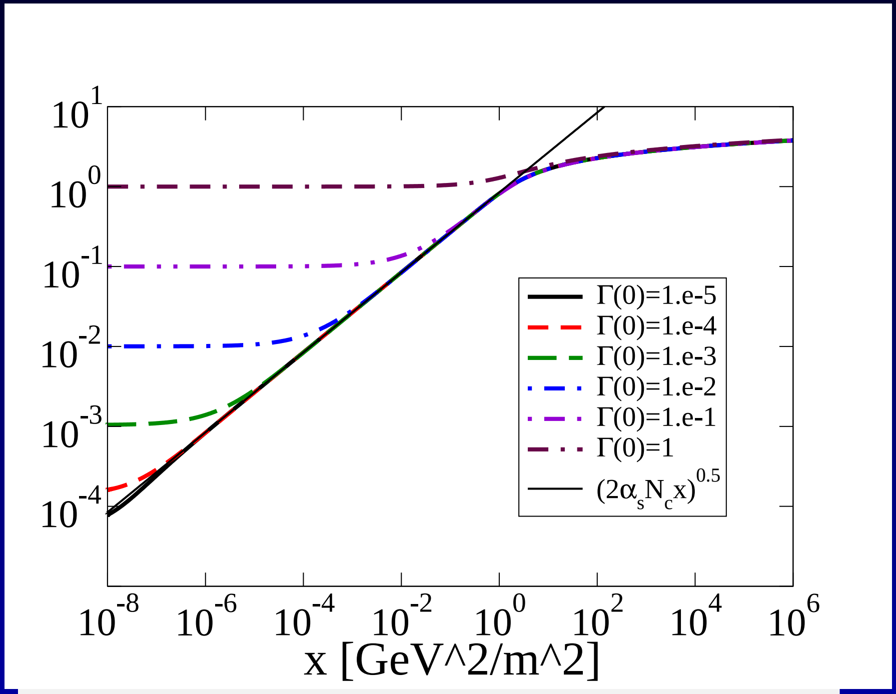
- tree-level vertex
- lattice gluon input ($m \approx 2\sqrt{\sigma} \approx 0.88$GeV):

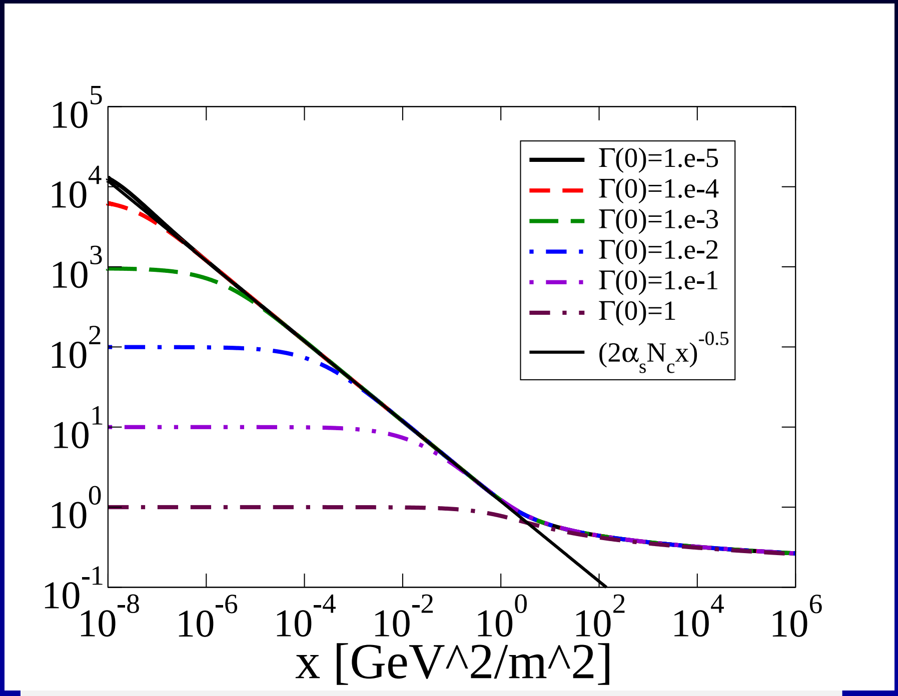
$$W_{AA}(p) \sim \frac{1}{(p_0^2 - \vec{p}^2)} \frac{\vec{p}^2}{\sqrt{\vec{p}^4 + m^4}}$$

- no gluon anomalous dimension, $W_{AA}(0) = 1/m^2$ finite!
- regularize by subtracting at $\vec{p}^2 = 0 \Rightarrow \Gamma_{\bar{c}c}(0)$ is an independent input.

G. Burgio, M. Quandt, H. Reinhardt, PRL 102, 032002.

Ghost DSe





Infrared powerlaw for $\Gamma_{\bar{c}c}(0) = 0$

$$\Gamma_{\bar{c}c}(\vec{p}^2 \to 0) \sim \frac{\sqrt{\alpha_s N_c}}{\sqrt{2\sigma}} |\vec{p}|$$

Ghost DSe

- DSes are (functional) differential equations
  – more than one solution is possible with different $\Gamma_{\bar{c}c}(0)$
- IR finite gluon can still give divergent ghost dressing (unlike Landau gauge)
- For the powerlaw solution

$$g^2 W_{00} \sim \frac{1}{\vec{p}^2 \Gamma_{\bar{c}c}^2 \overline{\Gamma}_{AA}} \sim \frac{\sigma}{\vec{p}^4} \ ?$$

Summary and outlook

- Coulomb gauge is a physical choice
  - Gauß' law dominates
  - two transverse degrees of freedom
  - vanishing and conserved total charge
- STids come from Gauß-BRST
- ghost DSe (toy version)

To do:

- investigate the physical/Gauß-BRST charge further
- solve the other DSes: is $g^2 W_{00} \sim \sigma/\vec{p}^4$? (in progress)
- find the physical input for $\Gamma_{\bar{c}c}(0)$