Astrophysics implications of dense matter phase diagram

Armen Sedrakian [1]

[1]Institute for Theoretical Physics, University of Frankfurt, Germany

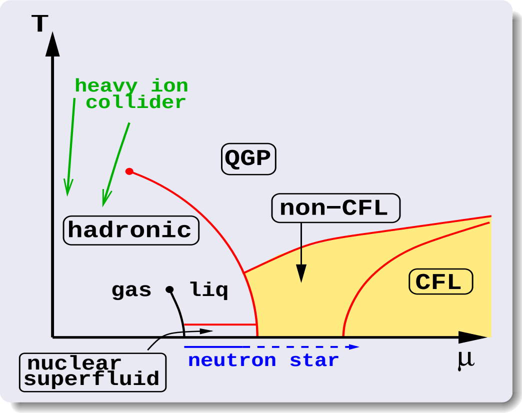
Hirschegg 2010

Januray 20, Hirschegg

Phase diagram of dense matter

Phases:

- High-temperature QGP phase is probed in heavy ion colliders
- Low-density low-temperature nucleonic matter in nuclei and low-densities of neutron stars
- Low-temperature high density phase of dense matter may be in the quark state (compact stars)

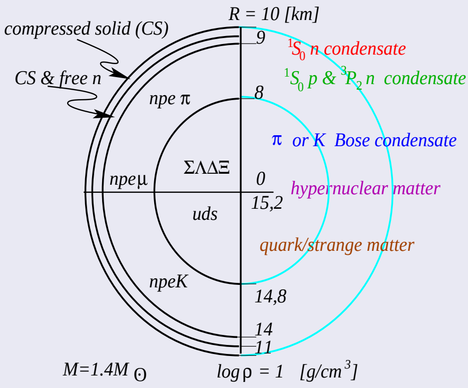


Learning about the interiors of neutrons stars

Key ideas: Quark matter and color superconductivity may have observational effect on neutron stars, e.g.
(i) gravitation wave radiation,
(ii) surface X-ray emission,
(iii) timing anomalies in the radio emission

This talk is focused on:

- Searches for the most favorable phase of color superconductor under charge, color neutrality and β-equilibrium
- Neutrino cooling of neutron stars with quark cores
- Gravitational radiation from strained superfluid

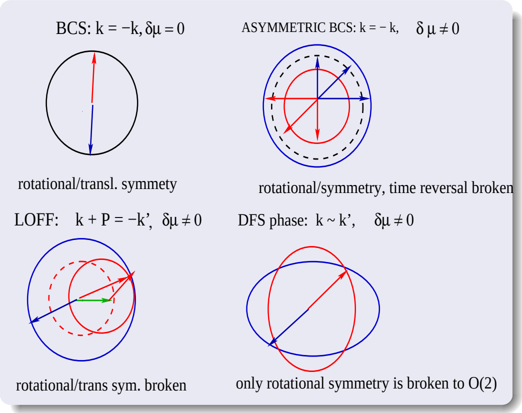


A. S., Prog. Part. Nucl. Phys. **58**, 168 (2007)

Stressed pairing

-Initially isospin symmetric matter acquires d-quark excess via the inverse β-decay $e + u \to d + \nu$, this implies shift in the Fermi spheres of by amount $\mu_e = \mu_d - \mu_u$: - Appearance of strange quarks will have the same effect of shifting of Fermi-surfaces

- Standard BCS requires the numbers to be equal, coherence is optimal among the fermions bound in a Cooper pair
- Asymmetric BCS, shifted Fermi surfaces, coherence is destroyed
- LOFF phase, Finite momentum of the condensate, restores coherence. Simplest ansatz $\Delta(\vec{r}) = \Delta \exp(i\vec{r}\cdot\vec{q})$.



Effective models of QCD

Pairing ansatz:

$$\Delta \propto \langle \psi^T(x) C\gamma_5 \tau_2 \lambda_2 \psi(x)\rangle.$$

$$\ln \mathcal{Z}^{\Delta}_{\rm MF} = \frac{3}{8}\frac{\Lambda^2}{g^2}\beta \sum_n \Big(|\Delta^+_n|^2 + 3\sum_{n',n\neq n'} \Delta^+_n \Delta^+_{n'}\Big)$$
$$+ \frac{1}{2}\sum_{c,n}\int \frac{d^3k}{(2\pi)^3}\Big\{\beta\left[E^+_c(\Delta^c_n) - E^-_c(\Delta^c_n)\right] - 2\ln f\left[-E^+_c(\Delta^c_n)\right] - 2\ln f\left[-E^-_c(\Delta^c_n)\right]\Big\}, \quad (35)$$

where $f(\omega) = [1+\exp(\beta\omega)]^{-1}$ is the Fermi distribution. The contribution of the blue quarks is

$$\ln \mathcal{Z}^0_{\rm MF} = 2\int \frac{d^3k}{(2\pi)^3}\sum_{e,f}\Big\{-\ln f\left[-\xi^+_e(k,\mu_{b,f})\right] - \ln f\left[-\xi^-_e(k,\mu_{b,f})\right]\Big\}. \quad (36)$$

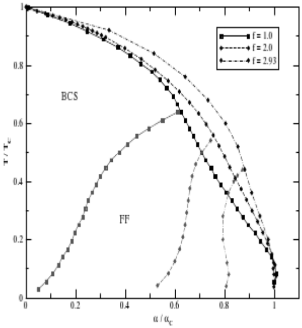
The thermodynamic potential is obtained from the logarithm of the partition function as

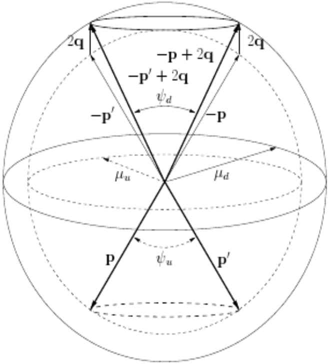
$$\Omega_{\rm MF} = -\frac{1}{\beta}\ln \mathcal{Z}_{\rm MF}. \quad (37)$$

The stationary point(s) of the thermodynamic potential determine the equilibrium values of the order parameters

$$\frac{\partial \Omega_{\rm MF}}{\partial \Delta^c_1} = 0, \quad \frac{\partial \Omega_{\rm MF}}{\partial \Delta^c_2} = 0, \quad \frac{\partial \Omega_{\rm MF}}{\partial |Q|} = 0. \quad (38)$$

FFLO phase in quark matter

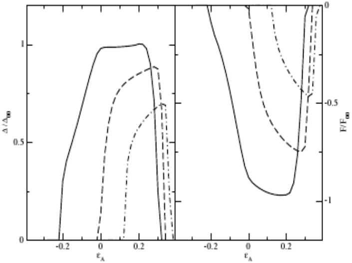


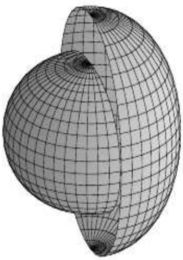


- Compact stars start at $T \sim T_c \sim$ 60 MeV and $\alpha \simeq 0$ and evolve to $T \to 0$ and $\alpha = 0.9$.
- They will entre the FF phase within short period of time

Pairing induced Fermi-surface deformations in QM

- Pomeranchuk-type instability of Fermi surfaces induced by asymmetric pairing
- Modelling the surfaces: $\mu_f = \sum_l \epsilon_l P_l \cos\theta$, where $f = u, d$ refers to flavor
- Parameters ϵ_l from minimum principle





more details in PRL **88** 252503; Phys. Rev. D **67** 085024.

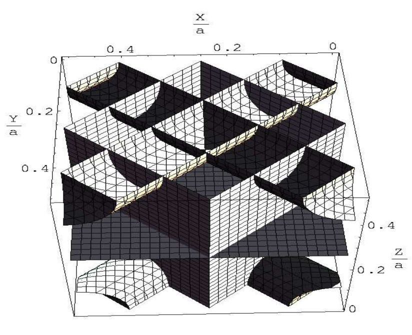
Shear modulus of CCS phase

Important: The superconducting phase has a nonzero shear modulus, i. e. it can support quadrupole and higher order deformations “mountains”

The key quantities are:

- Breaking strain
  $10^{-5} \leq \sigma \leq 10^{-2}$
- Shear modulus

Color superconducting phases can be solid with shear moduli by many orders exceeding that of the crusts.

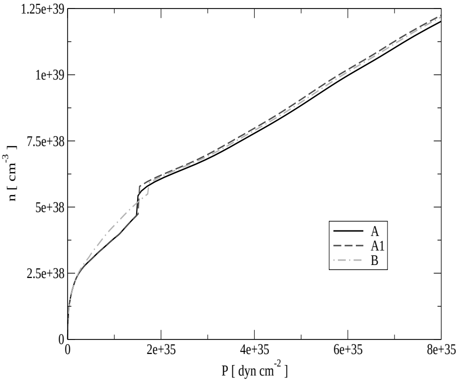


bcc superconducting lattice; figure courtesy J. Bowers

$$\mu = 2.47 \text{ MeV fm}^{-3} \left(\frac{\Delta}{10 \text{ MeV}}\right)^2 \left(\frac{\mu_q}{400 \text{ MeV}}\right)^2, \quad (1)$$

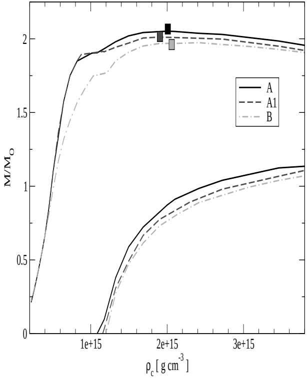
(K. Rajagopal and R. Sharma: arxiv:hep-ph/0606066)

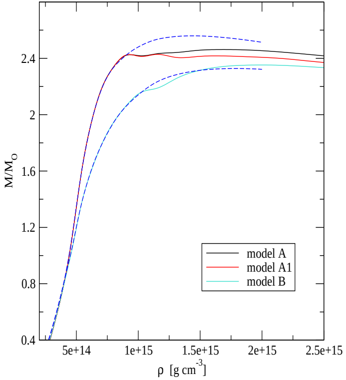
Equations of state



- The nuclear equation of state is taken from covariant BHF theory with two parameterizations (both stiff)
- The two quark equation of states differ by pressure normalization in the vacuum (slight vertical shift)

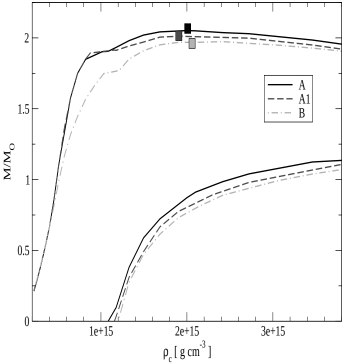
Stellar configurations

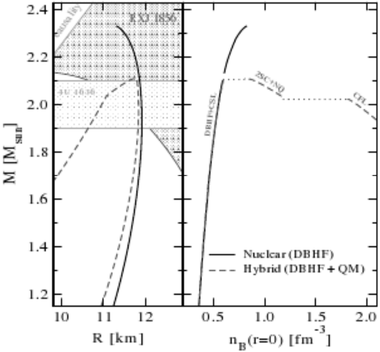




- Maximal masses are large $\sim 2M_{\odot}$. Quark core masses are $\leq 0.8M_{\odot}$.
- N. Ippolito et al Phys. Rev. D **77** (2008) 023008, B. Knippel, et al Phys. Rev. D **79**, 083007 (2009).

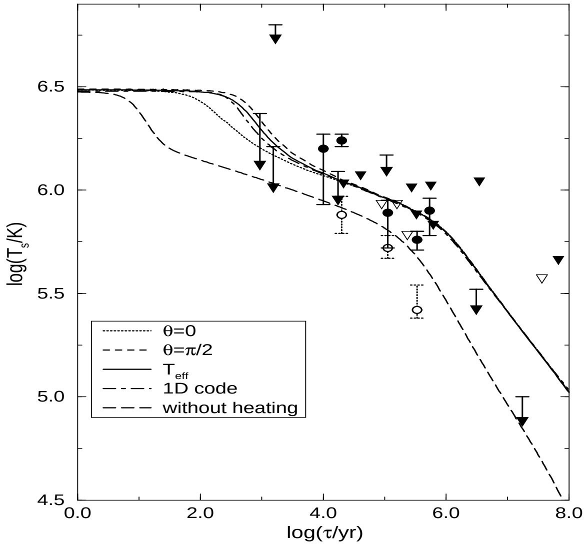
Stellar configurations





- Unstable 2SC and CFL configuration vs our stable configurations

Neutrinos in superconducting quark matter



Transport equations

- The $S^{>,<}$ propagators obey in non-equilibrium the KB equation

$$i\left\{\mathrm{Re}\, S^{-1}(q,x), S^{>,<}(q,x)\right\}_{P.B.} + i\left\{\mathrm{Re}\; S(q,x), \Omega^{>,<}(q,x)\right\}_{P.B.} = S^{>,<}(q,x)\Omega^{>,<}(q,x) + \Omega^{>,<}(q,x)S^{>,<}(q,x),$$

- ν-quasiparticle propagators:

$$S_0^{<}(q,x) = \frac{i\pi\gamma^{\mu}q_{\mu}}{\omega_{\nu}(\vec{q})}\Big[\delta\left(q_0 - \omega_{\nu}(\vec{q})\right) f_{\nu}(q,x) - \delta\left(q_0 + \omega_{\nu}(\vec{q})\right)\left(1 - f_{\bar{\nu}}(-q,x)\right)\Big]. \qquad (2)$$

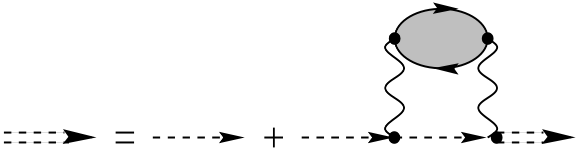
- ν and $\bar{\nu}$ - Boltzmann equations with KB collision integrals

$$\left[\partial_t + \vec{\partial}_q\, \omega_{\nu}(\vec{q})\vec{\partial}_x\right] f_{\nu}(\vec{q},x) = \int_0^{\infty} \frac{dq_0}{2\pi}\mathrm{Tr}\left[\Omega^{<}(q,x)S_0^{>}(q,x) - \Omega^{>}(q,x)S_0^{<}(q,x)\right],$$

Self-energies

- ν and $\bar{\nu}$-self-energies (second order in weak force)

$$-i\Omega^{>,<}(q_1,x) = \int \frac{d^4q}{(2\pi)^4}\frac{d^4q_2}{(2\pi)^4}(2\pi)^4\delta^4(q_1-q_2-q)$$
$$i\Gamma^{\mu}_{L\,q}\, iS_0^{<}(q_2,x)\, i\Gamma^{\dagger\,\lambda}_{L\,q}\, i\Pi^{>,<}_{\mu\lambda}(q,x), \qquad (3)$$

- 
- the problem is to compute the polarization tensor!

Neutrinos in a color superconductors

energy loss per unit time and volume

$$\epsilon_{\nu\bar{\nu}} = \frac{d}{dt}\int \frac{d^3q}{(2\pi)^3}\left[f_\nu(\vec{q}) + f_{\bar{\nu}}(\vec{q})\right]\omega_\nu(\vec{q}) \quad (4)$$

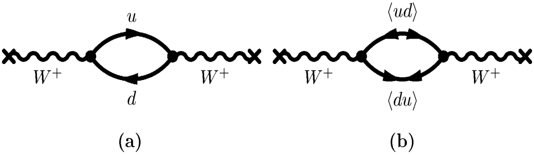
expressed through the collision integrals

$$\begin{aligned}\epsilon_{\nu\bar{\nu}} = & -2\left(\frac{G}{2\sqrt{2}}\right)^2 \sum_f \int \frac{d^3q_2}{(2\pi)^3 2\omega_\nu(\vec{q}_2)} \int \frac{d^3q_1}{(2\pi)^3 2\omega_\nu(\vec{q}_1)} \int \frac{d^4q}{(2\pi)^4} \\ & (2\pi)^4\delta^3(\vec{q}_1+\vec{q}_2-\vec{q})\delta(\omega_\nu(\vec{q}_1)+\omega_\nu(\vec{q}_2)-q_0)\ [\omega_\nu(\vec{q}_1)+\omega_\nu(\vec{q}_2)] \\ & g_B(q_0)\,[1-f_\nu(\omega_\nu(\vec{q}_1))]\,[1-f_{\bar{\nu}}(\omega_\nu(\vec{q}_2))]\,\Lambda^{\mu\lambda}(q_1,q_2)\mathrm{Im}\,\Pi^R_{\mu\lambda}(q).\end{aligned}$$

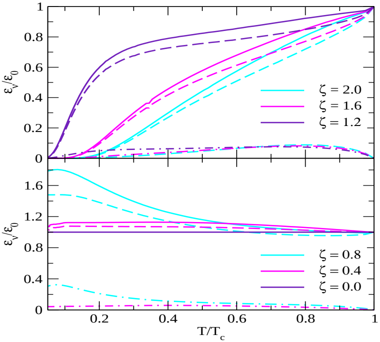
One loop results

Polarization tensors

$$\Pi_{\mu\lambda}(q) = -i\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left[(\Gamma_-)_\mu S(p)(\Gamma_+)_\lambda S(p+q)\right]$$
$$\Gamma_\pm(q) = \gamma_\mu(1-\gamma_5)\otimes\tau_\pm$$

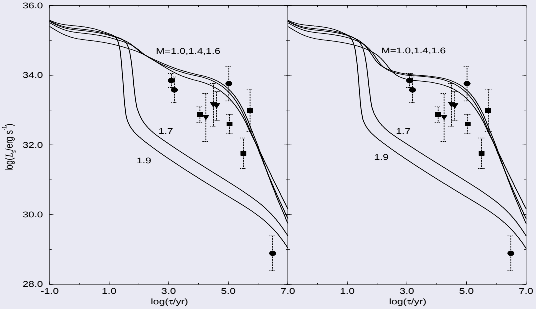


$$S_{f=u,d} = i\delta_{ab}\frac{\Lambda^+(p)}{p_0^2-\epsilon_p^2}(\not p-\mu_f\gamma_0), \quad F(p) = -i\epsilon_{ab3}\epsilon_{fg}\Delta\frac{\Lambda^+(p)}{p_0^2-\epsilon_p^2}\gamma_5 C$$

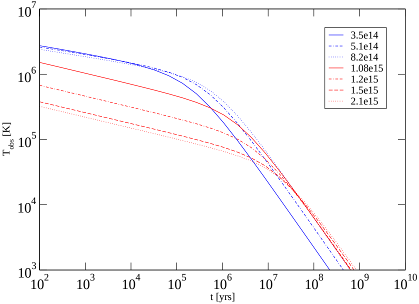


$\zeta = \Delta/\delta\mu$, where $\delta\mu = \mu_d - \mu_u = \mu_e$.

form P. Jaikumar, C. D. Roberts, and A. S., Phys. Rev. C **73** (2006) 042801.



Cooling of neutron stars with Urca and pion-condensed cores. Onset at high densities with the drop of temperature of high mass stars.



Cooling of hybrid color superconducting stars with constant $\zeta = 0.8$. Blue - baryonic stars, red - hybrid stars.

Gravitation radiation

Given a deformation the characteristic strain amplitude:

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{\epsilon I_{zz} \nu^2}{r}, \tag{5}$$

$\epsilon = (I_{xx} - I_{yy})/I_{zz}$ is the equatorial ellipticity. Strain amplitude can be expressed in terms of the $m = 2$ mass quadrupole moment as

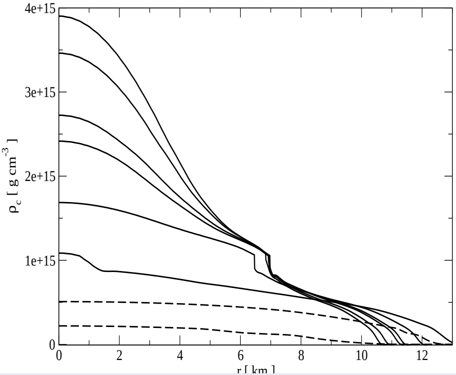
$$h_0 = \frac{16\pi^2 G}{c^4} \left(\frac{32\pi}{15}\right)^{1/2} \frac{Q_{22} \nu^2}{r}, \tag{6}$$

Quadrupole moment

$$Q_{22} = \int_0^{R_{\rm core}} \frac{dr r^3}{g(r)} \left[\frac{3}{2}(4 - U)t_{rr} + \frac{1}{3}(6 - U)t_\Lambda + \sqrt{\frac{3}{2}}\left(8 - 3U + \frac{1}{3}U^2 - \frac{r}{3}\frac{dU}{dr}\right) t_{r\perp}\right], \tag{7}$$

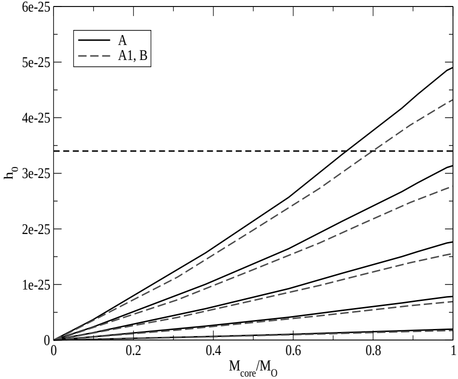
where $U = 2 + d\ln g(r)/d\ln r$ and t_{rr}, t_Λ and $t_{r\perp}$ are the coefficients of the expansion of the shear stress tensor in spherical harmonics.

Comparison with the previous work



- L.-M. Lin, Phys. Rev. D **76**, 081502(R) (2007, *incompressible models without nuclear crusts*
- Haskell et al, Phys. Rev. Lett. **99**, 231101 (2007), *incompressible quark matter plus* $n = 1$ *polytrope*
- B. Knippel, A. Sedrakian, Phys. Rev. D **79**, 083007 (2009), *microscopic equations of state*

Strain amplitudes



GW strain amplitudes for breaking strain 10^{-4}, Gaps from 10 to 50 MeV.
Dashed line Crab pulsars' upper limit from S5 run

h_0 can pin down the product $\sigma\Delta^2$, currently $\bar{\sigma}_{\max}\Delta^2 \sim 0.25$ MeV^2 (under the assumptions of the present model).

Summary, conclusions, and outlook

- Constructed hybrid configurations of CCS featuring compact stars
- The sequence contains entirely heavy mass (2 solar) objects with core masses 0.8 solar mass and radii up to 7 km. We think this is model independent (largely).
- If the core is maximally strained then the h_0 is detectable for realistic values $\sigma \sim 10^{-4}$ and $\Delta \sim 40$ MeV.
- The spread in the cooling curves can be explained by the gaplessness of the spectrum.

Questions for the feature

- Is there quark matter in the CCS state in compact stars?
- Is it strained and to which extent?
- What are the dynamic avenues for obtaining stressed cores?
- Other signatures of quark superconducting phases:

Thanks to

Bettina Knippel (ITP, Frankfurt-Main)
Nicola Ippolito, Marco Ruggieri (INFN, Bari)
Dirk Rischke (ITP, Frankfurt-Main)
Fridolin Weber (San-Diego)
Prashant Jaikumar (Cennei)
Craig Roberts (Argonne)