Chiral and deconfinement transitions from Dyson-Schwinger equations

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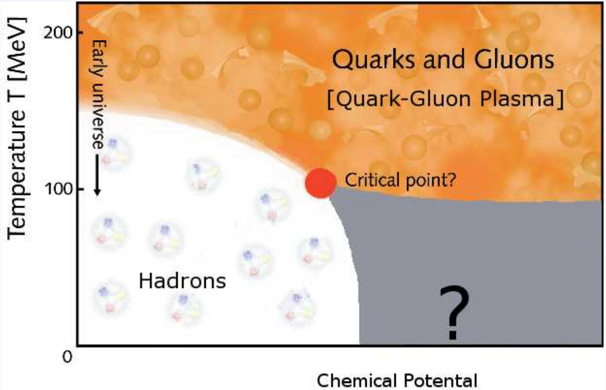
Work together with Jens Mueller and Axel Maas

C.F., PRL **103** (2009) 052003.
C.F. and J. A. Mueller, PRD **80** (2009) 074029.
C.F., A. Maas and J. A. Mueller in preparation
J. A. Mueller, C.F. and D. Nickel in preparation

Content

Content

QCD phase transitions



- Chiral limit ($m \to 0$): order parameter chiral condensate
- Static quarks ($m \to \infty$): order parameter Polyakov-loop

Lattice QCD vs. DSE/FRG: Complementary!

- Lattice simulations
  - Ab initio
  - Gauge invariant

- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  - Analytic solutions at small momenta
  - Space-Time-Continuum
  - Chiral symmetry: light quarks and mesons
  - Chemical potential: no sign problem

QCD in covariant gauge

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \, \exp\left\{ - \int_0^{1/T} dt \int d^3x \left(\bar{\Psi} \, (i\not{D} - m) \, \Psi \right.\right.$$
$$\left.\left. - \frac{1}{4} \left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Landau gauge ($\xi = 0$) propagators in momentum space, $q = (\vec{q}, \omega_q)$:

$$D^{\text{Gluon}}_{\mu\nu}(q) = \frac{Z_T(q)}{q^2} P^T_{\mu\nu}(q) + \frac{Z_L(q)}{q^2} P^L_{\mu\nu}(q)$$

$$S^{\text{Quark}}(q) = \frac{1}{-i \, \vec{\gamma}\vec{q} \, A(q) - i \, \gamma_4 \omega_n \, C(q) + B(q)}$$

The Goal:

Gauge invariant information from gauge fixed functional approach

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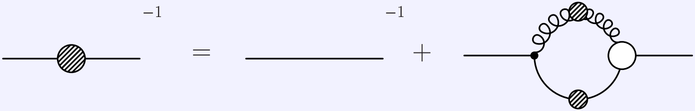
The ordinary chiral condensate



- spatial directions: periodic boundary conditions
  temporal direction: antiperiodic boundary condition
- Order parameter for chiral transition:

$$\langle \bar{\psi}\psi \rangle = Z_2 N_c T \sum_{n_p} \int \frac{d^3p}{(2\pi)^3} \, Tr_D \; S(p_{\vec{p},\omega_p})$$

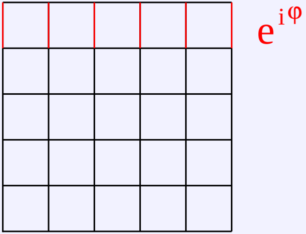
The dual condensate I

Consider general $U(1)$-valued boundary conditions in temporal direction for quark fields ψ:

$$\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$$

Matsubara frequencies: $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:



$$\langle\overline{\psi}\psi\rangle_\varphi \sim \sum \frac{exp[i\,\varphi\, n]}{m^l}$$ Closed Loops

E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).

The dual condensate II

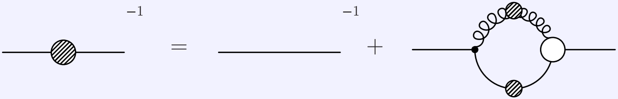
Then define dual condensate Σ_n:

$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi n} \langle\overline{\psi}\psi\rangle_\varphi$$

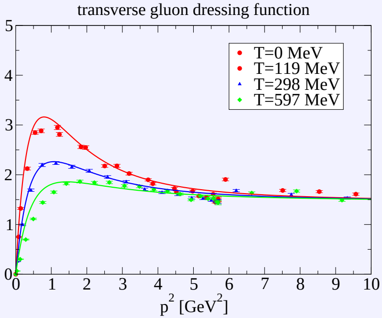
- $n = 1$ projects out loops with $n(l) = 1$: dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop: order parameter for center symmetry breaking
- Σ_1 is accessible with functional methods

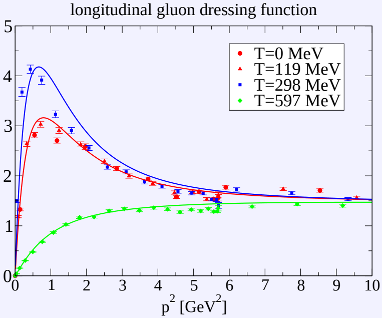
  C.F., PRL **103** (2009) 052003

C. Gattringer, PRL **97**, 032003 (2006)
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** 094007 (2008).
F. Synatschke, A. Wipf and K. Langfeld, PRD **77**, 114018 (2008).
J. Braun, L. Haas, F. Marhauser, J. M. Pawlowski, arXiv:0908.0008 [hep-ph]



- *T*-dependent gluon propagator from lattice data

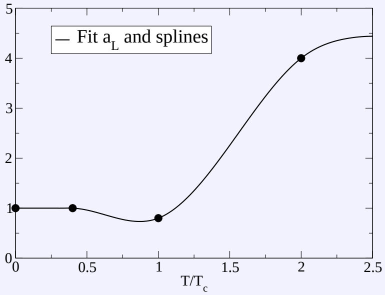




Cucchieri, Maas, Mendes, PRD75 (2007)

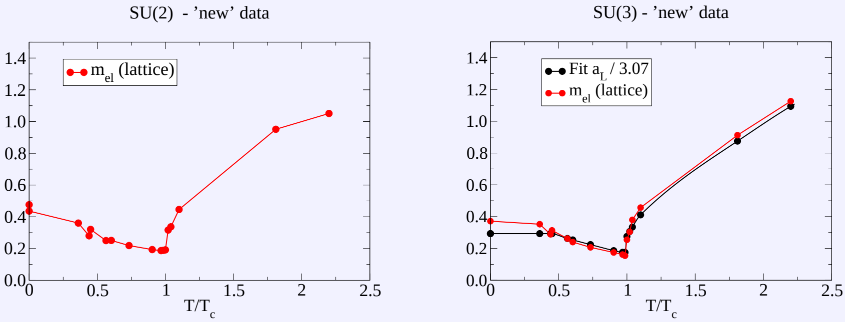
T-dependent gluon screening mass

SU(2) - 'old' data



$$Z_{T,L}(q,T) = \frac{q^2\Lambda^2}{(q^2+\Lambda^2)^2}\left\{\left(\frac{c}{q^2+\Lambda^2 a_{T,L}(T)}\right)^2 + \frac{q^2}{\Lambda^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2+1]}{4\pi}\right)^\gamma\right\}$$

T-dependent gluon screening mass II

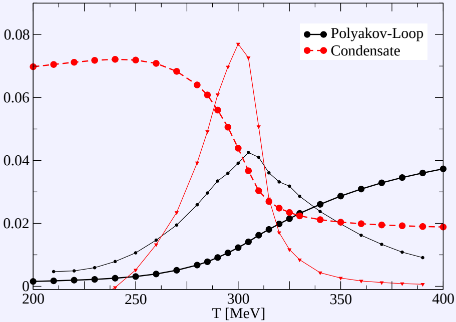


C.F., Maas, Mueller in preparation

$$Z_{T,L}(q,T) = \frac{q^2\Lambda^2}{(q^2+\Lambda^2)^2}\left\{\left(\frac{c}{q^2+\Lambda^2 a_{T,L}(T)}\right)^2 + \frac{q^2}{\Lambda^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2+1]}{4\pi}\right)^\gamma\right\}$$

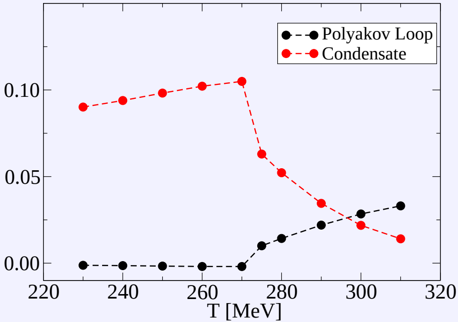
Transition temperatures for finite quark masses

SU(2) - 'old' data



C.F. and J. A. Mueller, PRD **80** (2009) 074029.

SU(3) - 'new' data

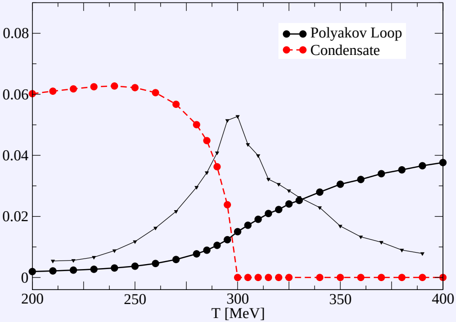


C.F., Maas, Mueller in preparation

- New data: better temperature-resolution
- SU(2): $T \approx 305$ MeV SU(3): $T \approx 270$ MeV

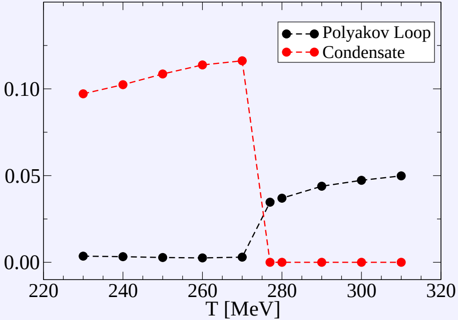
Transition temperatures in chiral limit

SU(2) - ’old’ data



C.F. and J. A. Mueller, PRD **80** (2009) 074029.

SU(3) - ’new’ data



C.F., Maas, Mueller in preparation

- similar transition temperatures
- increasing chiral condensate due to electric screening masses

Content

Framework

Idea: Fit spectral representation to quark propagator

F. Karsch and M. Kitazawa, PRD **80**, 056001 (2009).
F. Karsch and M. Kitazawa, PLB **658**, 45 (2007).

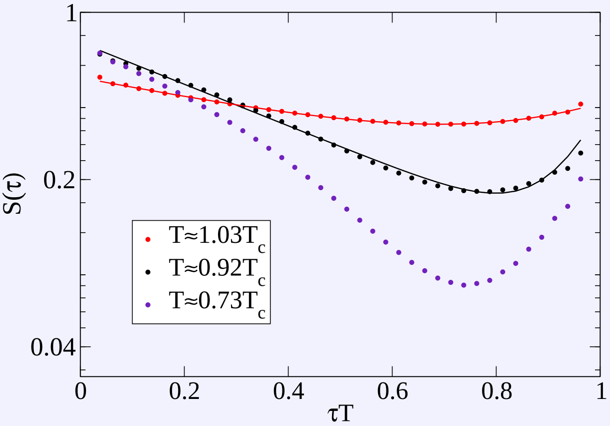
$$S(\omega_p, \vec{p}) = \int d\omega' \, \frac{\rho(\omega', \vec{p})}{\omega_p - \omega'}$$

Use ansatz for spectral function:

$$\rho(\omega) = Z_1 \; \delta(\omega - E_1) + Z_2 \; \delta(\omega + E_2)$$

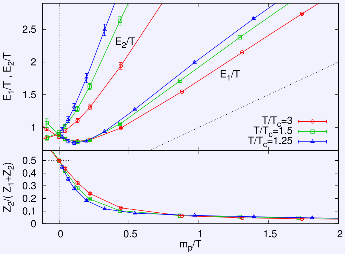
Pseudoparticles: Quark and Plasmino

Results I



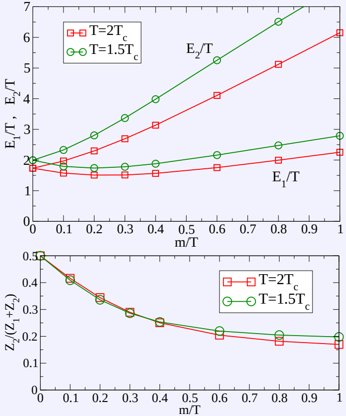
- $T > T_c$: Two pole ansatz works: quark and plasmino
- $T < T_c$: Positivity violations; two pole fit does not work at all

Lattice



Karsch and Kitazawa, PRD **80**, 056001 (2009).

DSE

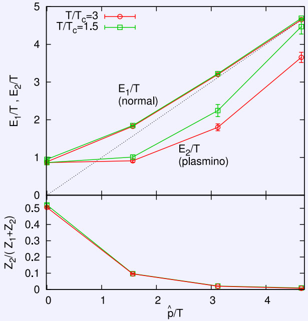


Mueller, C.F., Nickel in preparation

- Qualitative agreement with lattice results
- Large quark masses: plasmino disappears

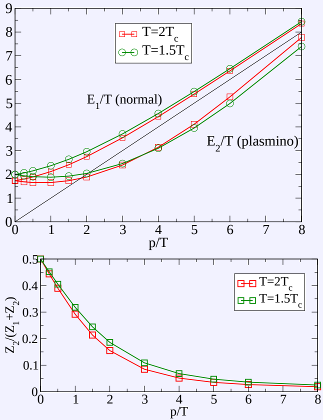
Results III: Dispersion Relation

Lattice



Karsch and Kitazawa, PRD **80**, 056001 (2009).

DSE



Mueller, C.F., Nickel in preparation

- Min for Plasmino at $p \neq 0$
- Plasmino enters spacelike region

Next steps and further references

To do:

- Unquenching
- Finite chemical potential
- Thermodynamic observables

Further results from functional methods:

- Gluon at $T = 0$:

  C.F., Maas, Pawlowski, Annals Phys. **324** (2009) 2408

- Chiral and deconfinement transition using the FRG:
  Braun and Gies, arXiv:0912.4168.
  Braun, arXiv:0908.1543.
  Braun, Haas, Marhauser and Pawlowski, arXiv:0908.0008.
  Braun, Eur. Phys. J. C **64** (2009) 459.
  Marhauser and Pawlowski, arXiv:0812.1144.
  Braun, Gies and Pawlowski, PLB in press; arXiv:0708.2413.
  Braun and Gies, PLB **645**, 53 (2007)
  Braun and Gies, JHEP **0606**, 024 (2006)
  Gies and Jaeckel, EPJ C **46**, 433 (2006)

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TECHNISCHE UNIVERSITÄT DARMSTADT

HELMHOLTZ GEMEINSCHAFT

GSI

HIC for FAIR

LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

EMMI

Helmholtz-Alliance: Extremes of density and temperature; cosmic matter in the laboratory

Gluon and Quark-Gluon-Vertex

Fit function for gluon:

$$Z_{T,L}(\vec{q},\omega_q,T) = \frac{q^2\Lambda^2}{(q^2+\Lambda^2)^2}\left\{\left(\frac{c}{q^2+\Lambda^2 a_{T,L}(T)}\right)^2 + \frac{q^2}{\Lambda^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2+1]}{4\pi}\right)^\gamma\right\}$$

Ansatz for Quark-Gluon-Vertex:

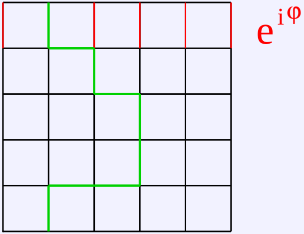
$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3\left(\delta_{4\nu}\gamma_4\frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j\frac{A(k)+A(p)}{2}\right)\times \\ \times\left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2}\left(\frac{\beta_0\alpha(\mu)\ln[q^2/\Lambda^2+1]}{4\pi}\right)^{2\delta}\right).$$

The dual condensate II

Relation of condensate to loops of link variables $U_\mu(x)$:

$$\langle\overline{\psi}\psi\rangle_\varphi = Tr\,[m + D_\varphi]^{-1} = \frac{1}{Vm}\sum_{l\in\mathcal{L}} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} Tr_c \prod_{(x,\mu)\in l} s(l)U_\mu(x)\,.$$

- geometric series of inverse staggered Dirac operator
- winding number $n(l)$ of loop l around temporal direction



E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).