The Spacelike Electromagnetic Form Factors of Lambda and Sigma in Quark-Diquark Faddeev Equation

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International School of Nuclear Physics 44th Course

— From quarks and gluons to hadrons and nuclei

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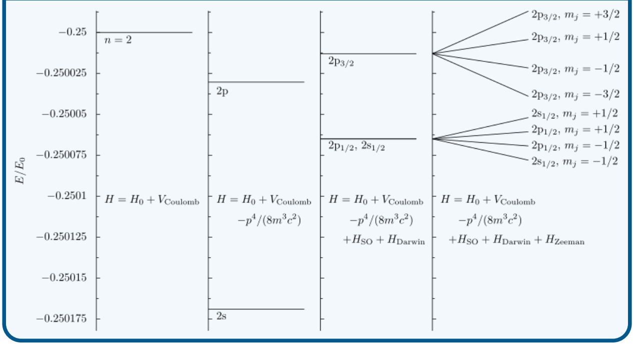
Hyperfine structures in quantum mechanics

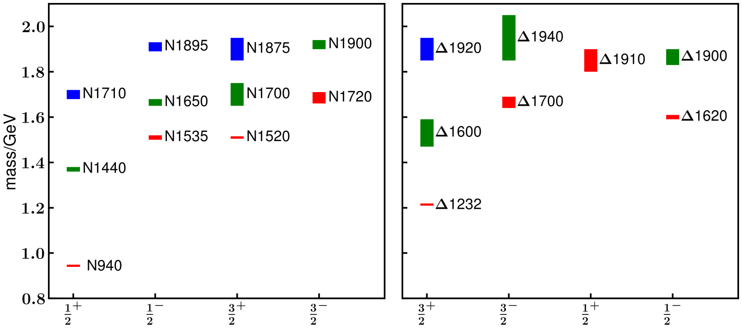


Some Possible Ways to Study Composite Systems

In the development of modern physics, some ways to study composite systems and interactions:

① Excite them.
Many interaction details are encoded in the excited states. (Gernot, Markus and Ulrike's talk)

Mass spectrum in N and Δ baryon



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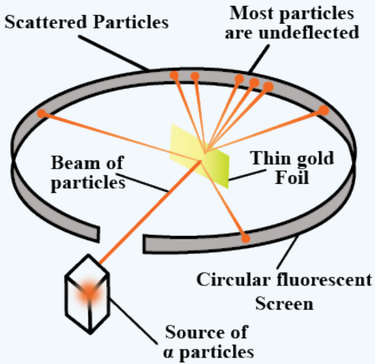
Rutherford α scattering $\longrightarrow$ cross section $\longrightarrow$ structure of atom



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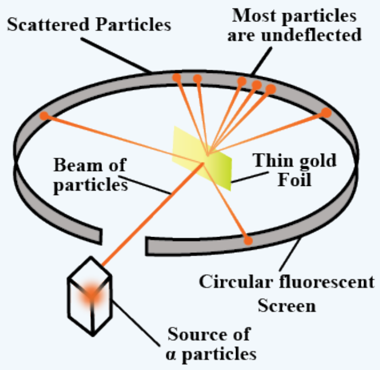
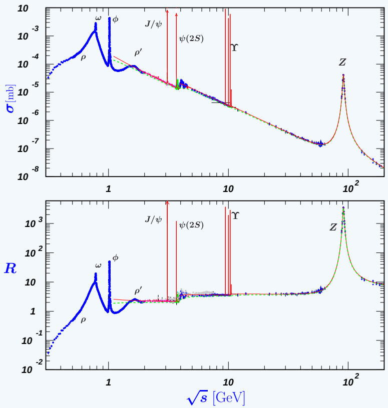


$e^+ + e^-$ annihilation $\rightarrow$ cross section plot $\rightarrow$ existence of quarks



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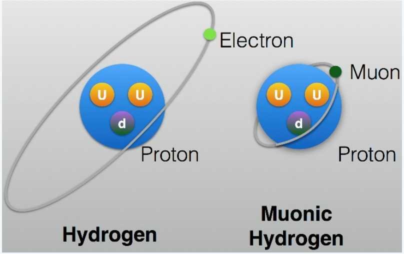
(Haiyan's talk)

Muonic hydrogen → proton charge radius



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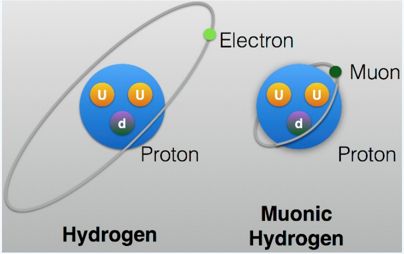
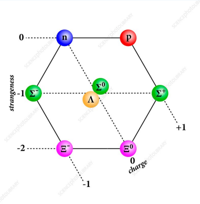
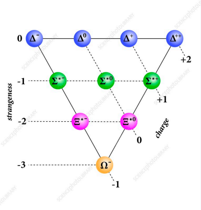
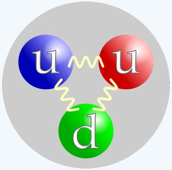


$N\Delta$ baryons ↔ hyperons







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④ The electromagnetic transition form factors for the only baryon octet transition $\Sigma \to \Lambda$.

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Quark-diquark Faddeev equation

The quark-diquark amplitude for baryon

$$\psi_i^a(p,P) = \left[\Gamma^a(l,p_d)D^a(p_d)\right]\left[\Phi_i^a(p,P)u(P)\right]\,, \quad i = 1,2,3\,,$$

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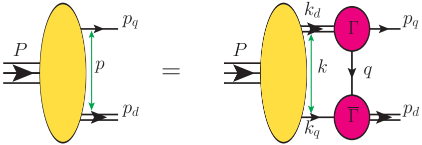

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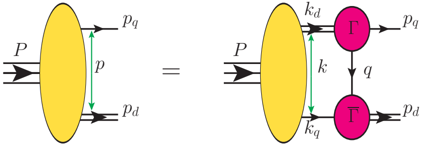
ingredients

quark propagators, diquark propagators, diquark amplitudes.



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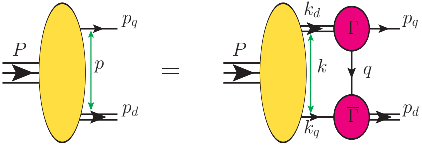

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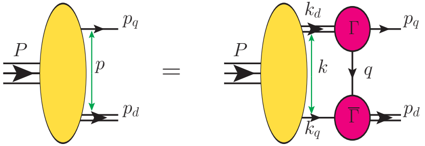
- ✔ QCD kindred quark and diquark propagators.



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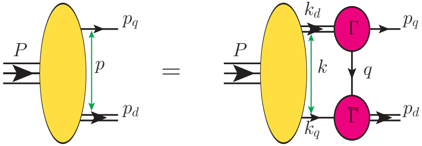
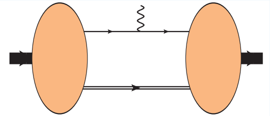
- ✔ QCD kindred quark and diquark propagators.
- ✔ analytic form, easy to perform analytic continuation.



Electromagnetic current in quark-diquark Faddeev equation

quark - photon



① quark-photon vertex

Electromagnetic current in quark-diquark Faddeev equation

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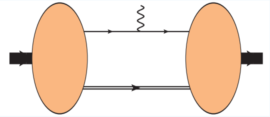
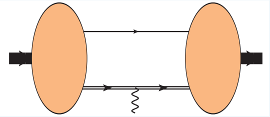


diquark - photon



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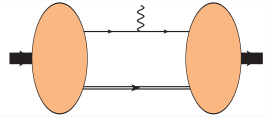

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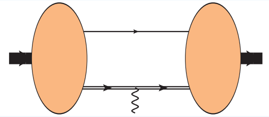
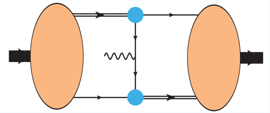


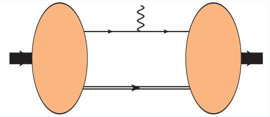
exchange quark - photon



1. quark-photon vertex
2. diquark-photon vertex

Electromagnetic current in quark-diquark Faddeev equation

quark - photon



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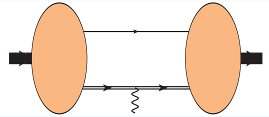

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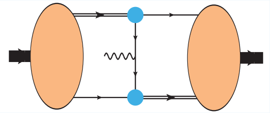
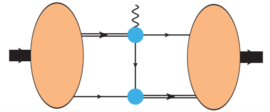


diquark amplitude - photon



① quark-photon vertex

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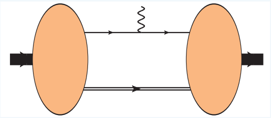
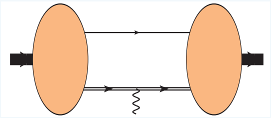
Electromagnetic current in quark-diquark Faddeev equation

quark - photon



diquark - photon



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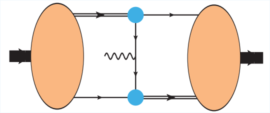

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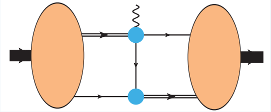
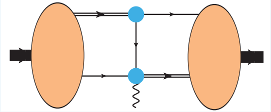


conjugated diquark amplitude - photon



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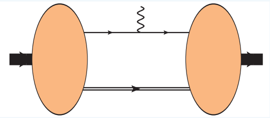
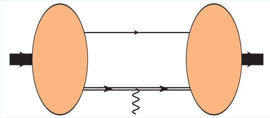
Electromagnetic current in quark-diquark Faddeev equation

quark - photon



diquark - photon



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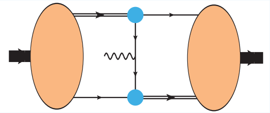

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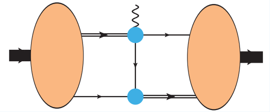

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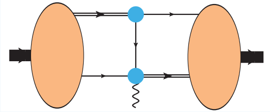

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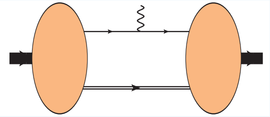
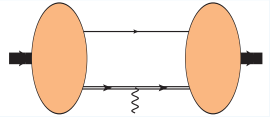
- A great simplification in Faddeev equation.

Electromagnetic current in quark-diquark Faddeev equation

quark - photon



diquark - photon



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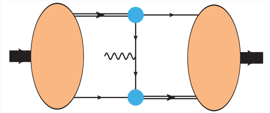

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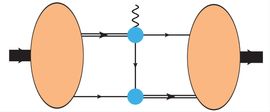

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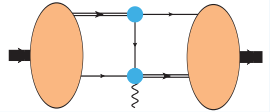

① quark-photon vertex
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- A great simplification in Faddeev equation.
- A great difficulty in electromagnetic current too!

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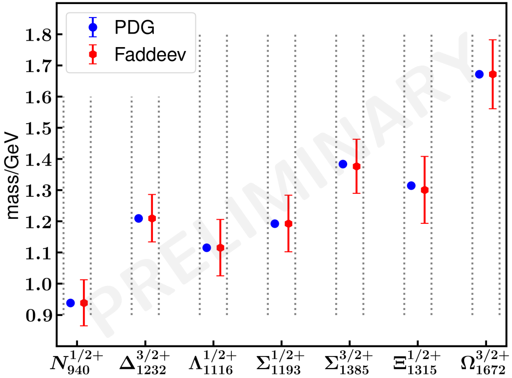
Mass spectrum



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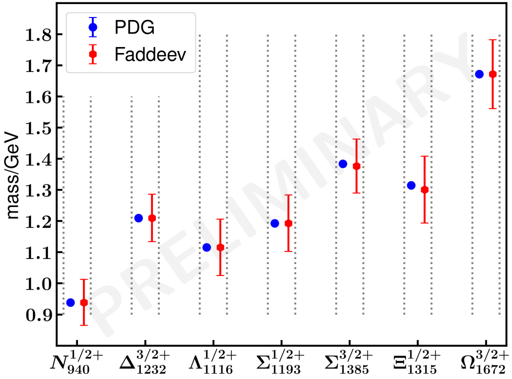
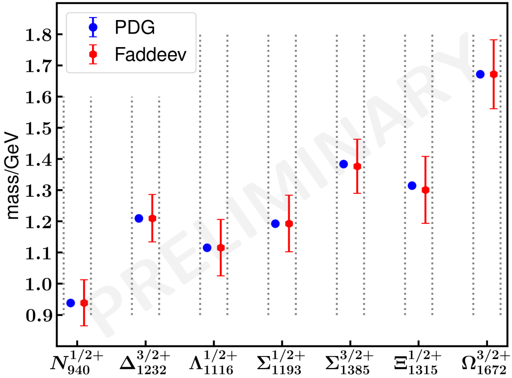


- The masses are consistent with the experimental values.

Mass spectrum



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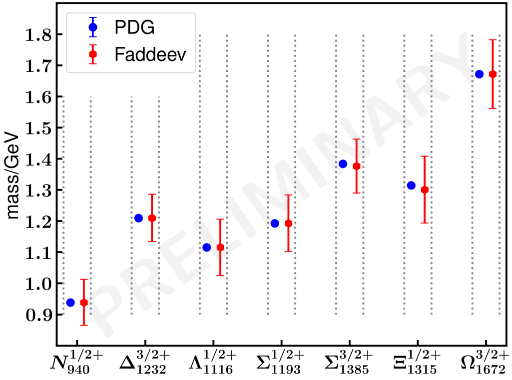
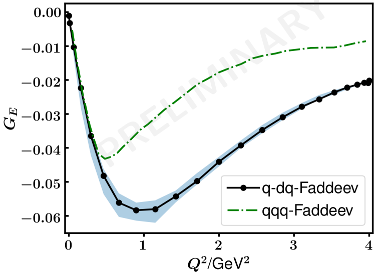


- The masses are consistent with the experimental values.
- In the isospin symmetry assumption, it is the diquark correlations lead to the mass split between Λ and Σ.
- The masses are sensitive to the masses of diquarks. But EMFFs is not sensitive to them.

Elastic EMFFs of Λ

H.Alepuz, C.Fischer, Eur.Phys.J.,A52,34



- The electric charge radius $\langle r_E^2 \rangle_{\text{q-dq}} \sim \langle r_E^2 \rangle_{\text{qqq}}$.

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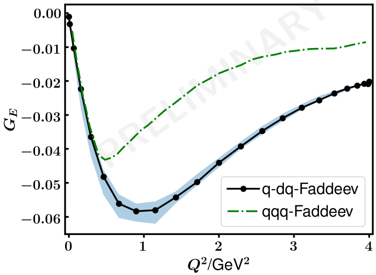

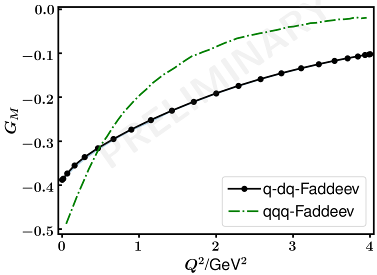

- The electric charge radius $\langle r_E^2 \rangle_{\text{q-dq}} \sim \langle r_E^2 \rangle_{\text{qqq}}$.
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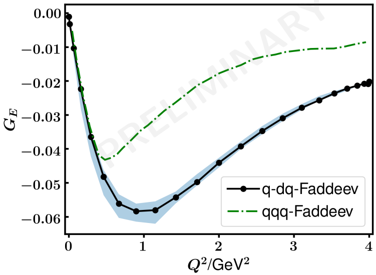

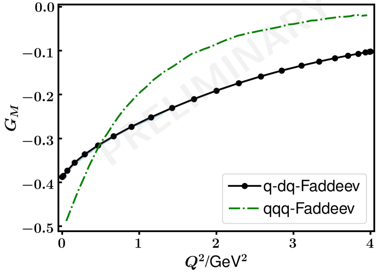

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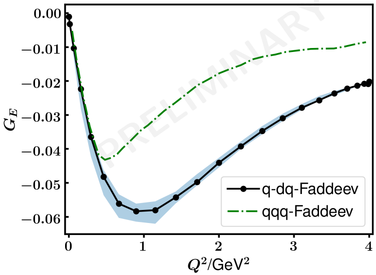

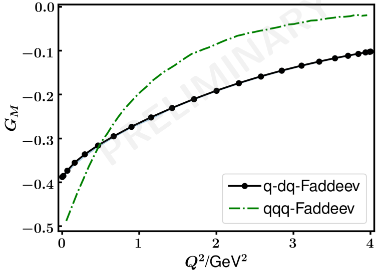

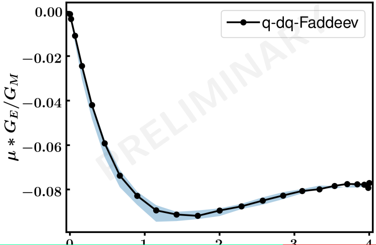

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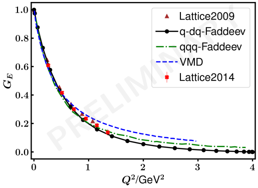

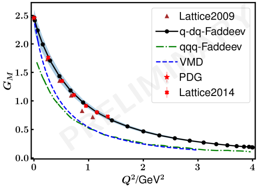

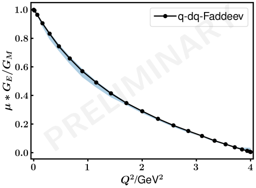

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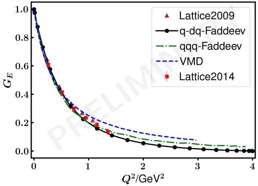

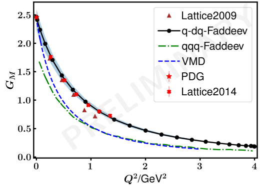

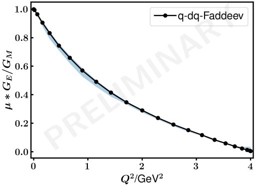

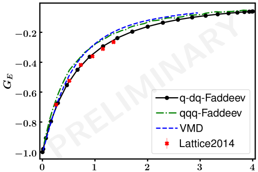

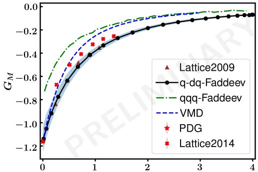

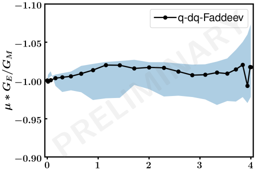

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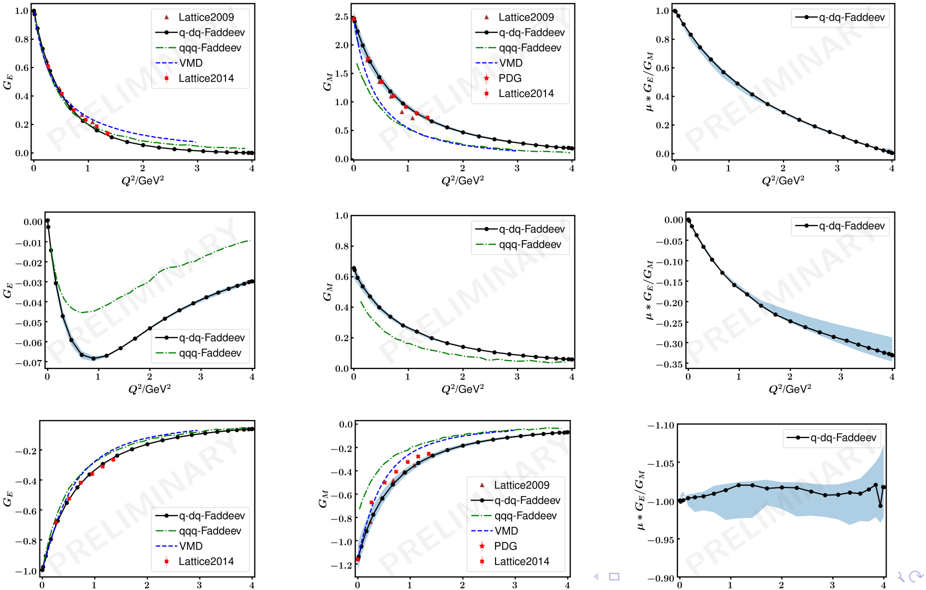


Elastic EMFFs of $\Sigma^+, \Sigma^0, \Sigma^-$

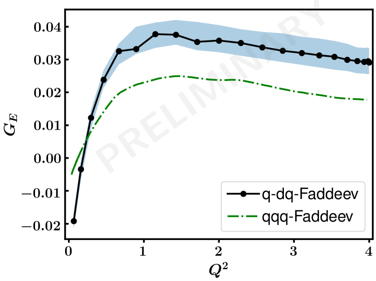
$$\langle r_E^2 \rangle = -6 \frac{d}{dQ^2} \frac{G_E(Q^2)}{G_E(0)}\bigg|_{Q^2=0}, \quad \langle r_M^2 \rangle = -6 \frac{d}{dQ^2} \frac{G_M(Q^2)}{G_M(0)}\bigg|_{Q^2=0}.$$

	Λ	Σ^+	Σ^0	Σ^-
$\langle r_E^2 \rangle$	0.036(14)	0.469(9)	0.068(9)	0.353(26)
$\langle r_E^2 \rangle_{\text{PDG}}$	-	-	-	0.61(15)
$\langle r_M^2 \rangle$	0.120(76)	0.374(41)	0.201(169)	0.459(122)
μ	-0.390(3)	2.422(180)	0.630(48)	-1.145(106)
μ_{PDG}	-0.613(4)	2.458(10)	-	-1.160(25)

- Our quark-diquark description of Σ is quite well.
- All the theoretical predictions agree qualitatively.(Z.Li,J.Xie,Commu.Theo.Phys.,73,055201;P.Shanahan,et.al,Phys.Rev.,D90,034502 H.Lin,K.Orginos,Phys.Rev.,D79,074507.)
- The ratios of EMFFs for Σ^+ and Σ^- behave differently.
- This difference is as a result of diquark correlations (notice the tendency from $\Sigma^+ \to \Sigma^0 \to \Sigma^-$).

Electromagnetic transition form factors of $\Sigma^0 \to \Lambda$

H.Alepuz, R.Alkofer, C.Fischer, Eur.Phys.J.,A54,41;



- The large numerical error near $Q^2 = 0$.
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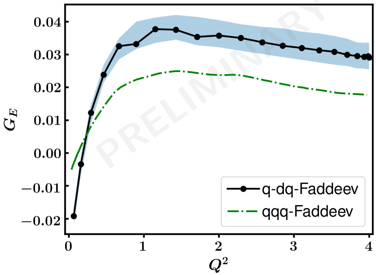

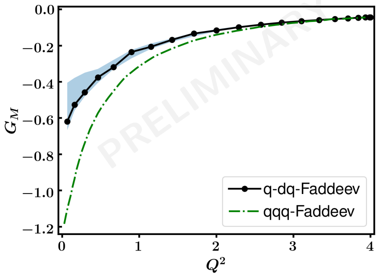

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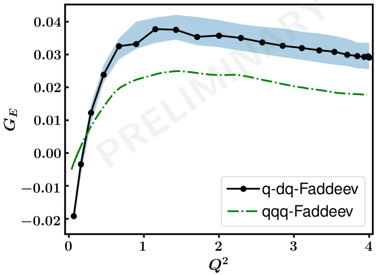

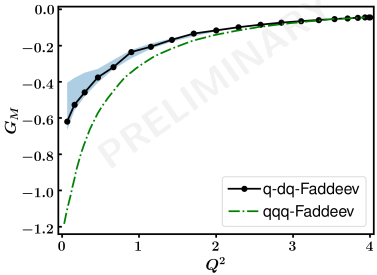

- Similar behaviors.
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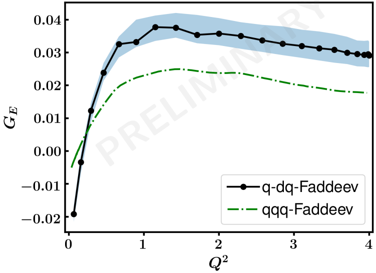

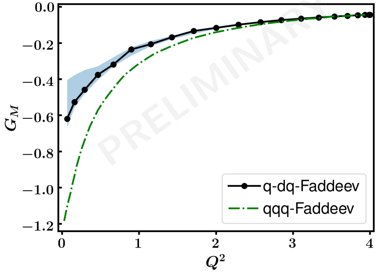

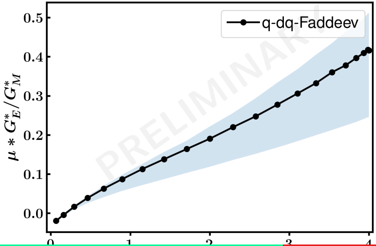

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THANK YOU!

Quark and diquark propagators

$$S(p) = -i\not{p}\sigma_v(p^2) + \sigma_s(p^2)\,, \quad \sigma_v = \frac{\bar{\sigma_v}}{\lambda^2}\,, \quad \sigma_s = \frac{\bar{\sigma_s}}{\lambda}\,,$$

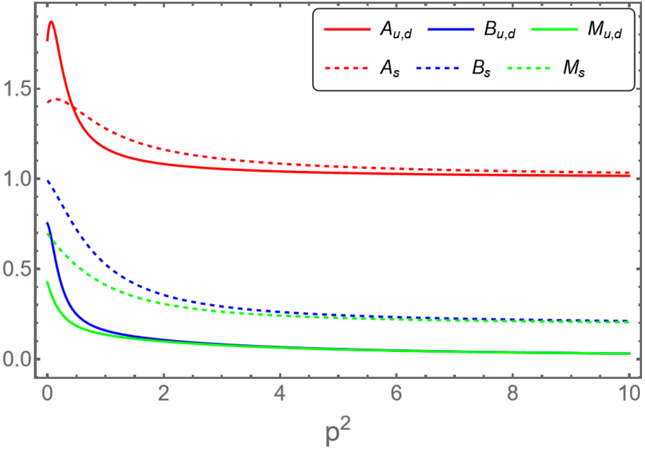
where

$$\bar{\sigma_v}(x) = \frac{1}{x+\bar{m}^2}\left[1 - \mathcal{F}\left(2(x+\bar{m}^2)\right)\right]\,, \quad \mathcal{F}(x) = \frac{1-\exp[-x]}{x}\,,$$

$$\bar{\sigma_s}(x) = 2\bar{m}\mathcal{F}\left(2(x+\bar{m}^2)\right) + \mathcal{F}(b_1 x)\mathcal{F}(b_3 x)\left[b_0 + b_2\mathcal{F}(\epsilon x)\right]\,, \quad x = \frac{p^2}{\lambda^2}\,,$$

$\lambda_{u,d}$	$\bar{m}_{u,d}$	$b_0^{u,d}$	$b_1^{u,d}$	$b_2^{u,d}$	$b_3^{u,d}$	ε
0.566	0.00897	0.131	2.90	0.603	0.185	0.0001
λ_s	$\bar{m}_s$	b_0^s	b_1^s	b_2^s	b_3^s	
0.817556	0.223	0.198323	1.19203	0.202049	1.19204	

Quark and diquark propagators



Diquarks

Diquark propagators:

$$D_{0^+}(k) = \frac{1}{M_{0^+}^2}\mathcal{F}(k^2/\omega_{0^+}^2)\,,$$
$$D_{1^+}^{\mu\nu}(k) = \left(g^{\mu\nu} + \frac{k^\mu k^\nu}{M_{1^+}^2}\right)\frac{1}{M_{1^+}^2}\mathcal{F}(k^2/\omega_{1^+}^2)\,,$$

$$\omega_{J^P}^2 = \frac{1}{2}m_{J^P}^2\,.$$

$$\Gamma_{0^+}^a(l,p_d) = ig_{0^+}\gamma_5 * C * \mathcal{F}(l^2)\lambda^a\,,$$
$$\Gamma_{1^+}^{a,\mu}(l,p_d) = ig_{1^+}\gamma^\mu * C * \mathcal{F}(l^2)\lambda^a\,,$$