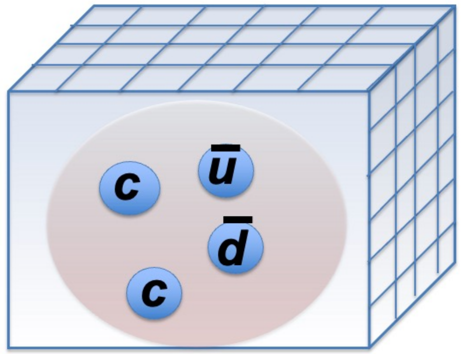
Exotic hadrons from lattice QCD



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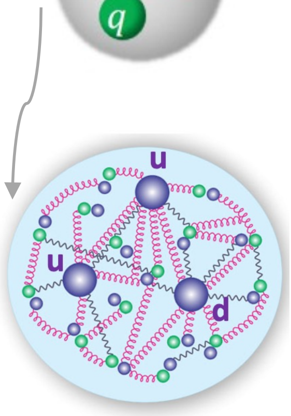
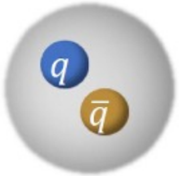
International School of Nuclear Physics
From quarks and gluons to hadrons and nuclei
Erice, September, 2023

Conventional and exotic hadrons

baryon



meson



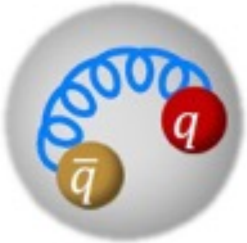
exotic

Minimal valence content

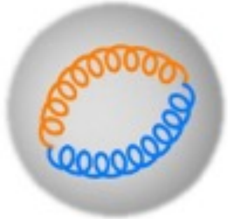
tetraquark

pentaquark

hybrid meson



glueball



+

cypto exotic

(not within quark model of qqq, $\underline{q}$q)

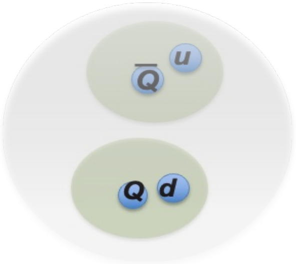
Plots: F.K. Guo

Exotic hadrons

exp. talk by Yuan

Simplistic argument: for a given V:

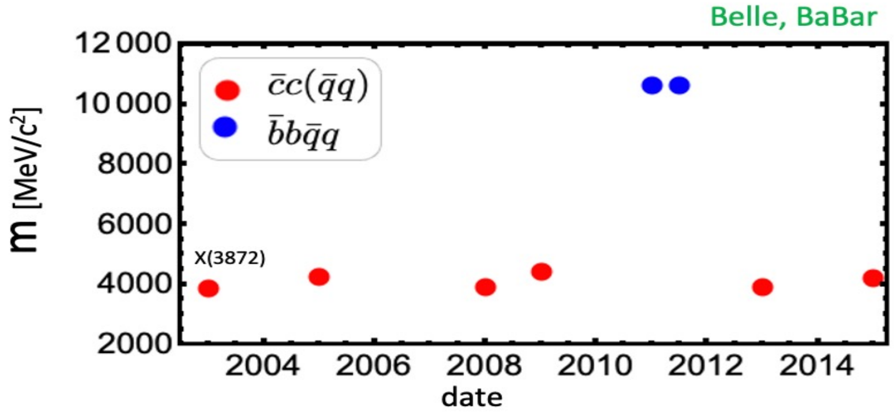
heavier particles are easier to bind

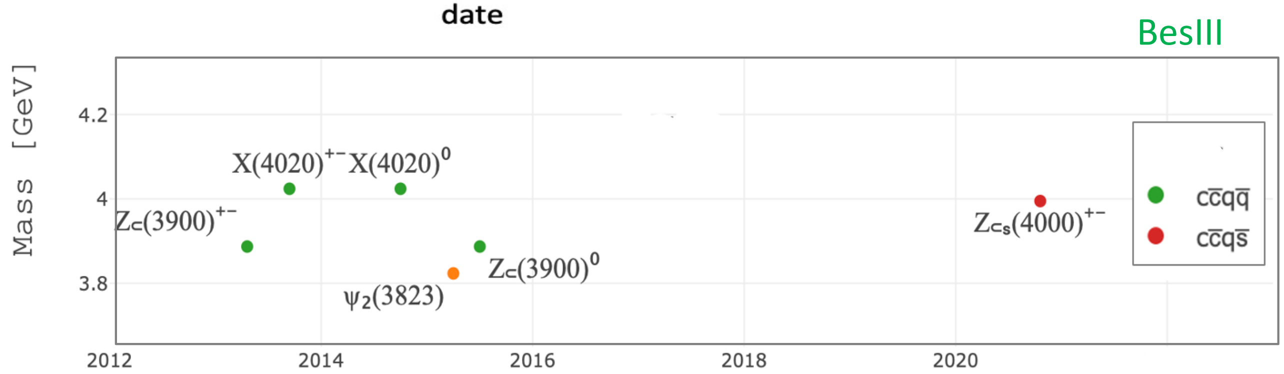


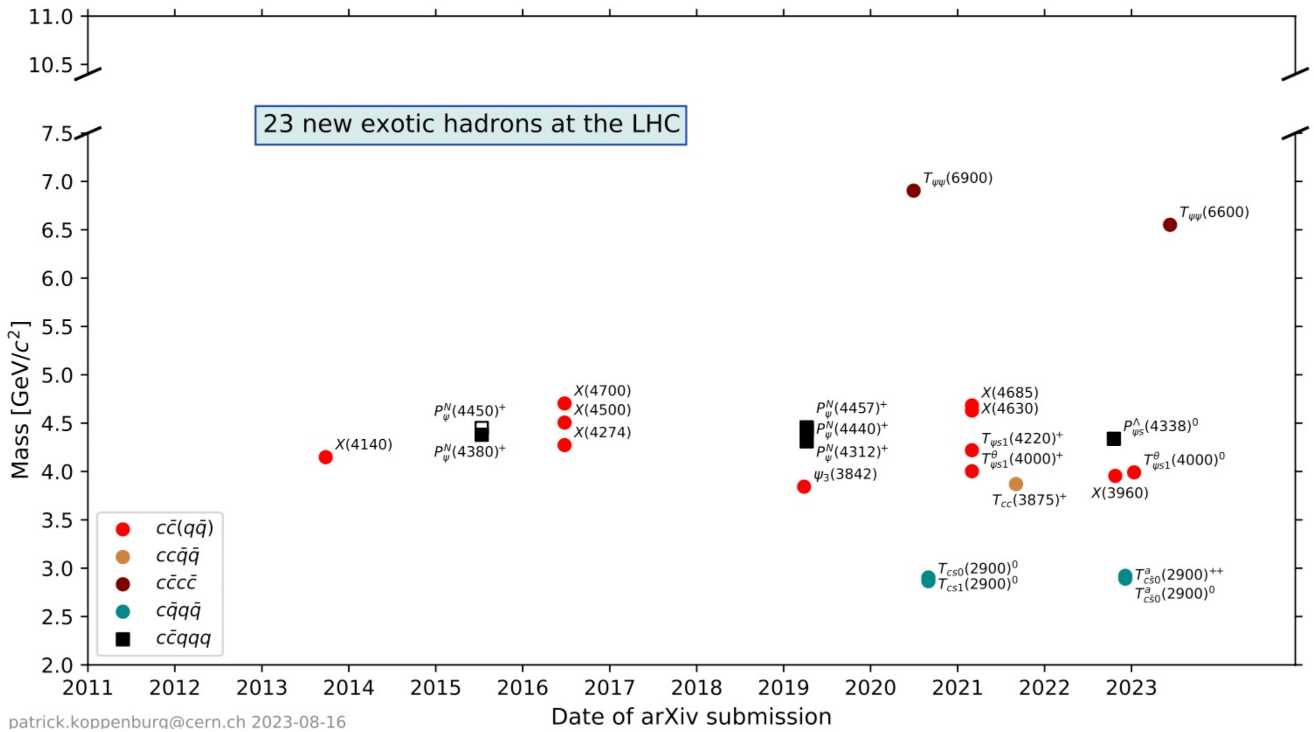
$$\hat{H} = \frac{\hat{p}^2}{2m_r} + V$$

https://www.nikhef.nl/~pkoppenb/particles.html

https://qwg.ph.nat.tum.de/exoticshub/





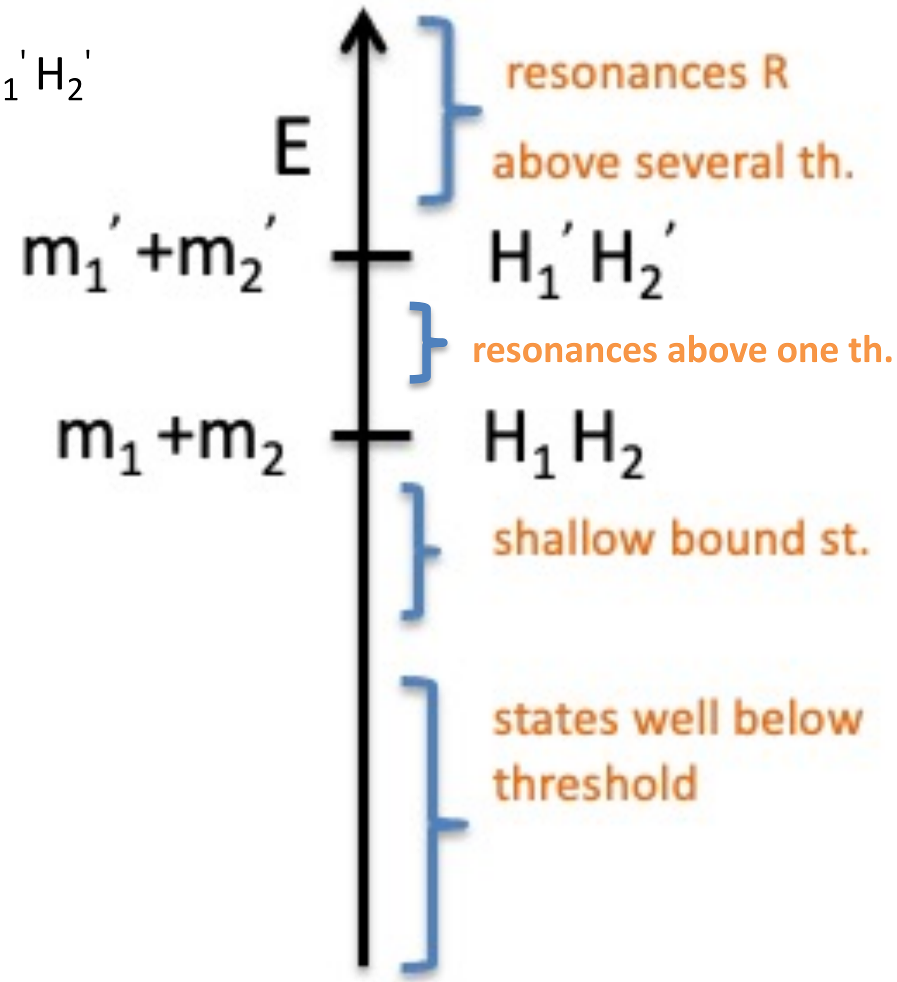


lattice QCD: strong, ~~EW~~

Outline

$R \to H_1 H_2 ,\ H_1' H_2'$

$R \to H_1 H_2$



(4) hadrons from static potentials

(3) hadrons from coupled-channel scat.

(2) hadrons from one-channel scattering

(1) hadrons well below threshold

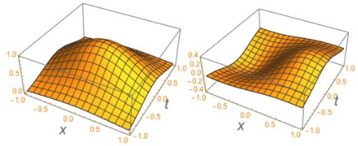
- **all examples: exotic hadrons**

- this is NOT a review of all existing results !
- some overlap with lecture by Max Hansen

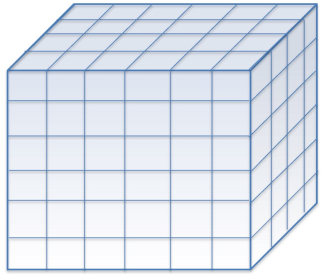
QCD: $\mathcal{L}_{QCD} = \frac{1}{4} G_a^{\mu\nu} G_a^{\mu\nu} + \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$ $g_s \not\ll 1$ at hadronic energy scale

Lattice QCD

$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\, C\, e^{-S_{QCD}/\hbar}$$



...



Main quantity extracted: finite-volume eigen-energies E_n $\hat{H}|n\rangle = E_n |n\rangle$

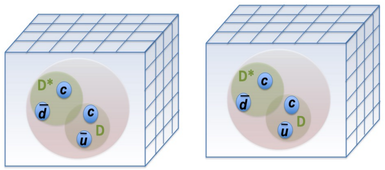
often "non-precision" studies:

single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

$\sum_n |n\rangle\langle n|$ $e^{-iE_n t_M}$ Eucledian time

$$C_{ij}^{2pt}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^+(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\ e^{-E_n t_E} \langle n | \mathcal{O}_j^+ | 0 \rangle$$

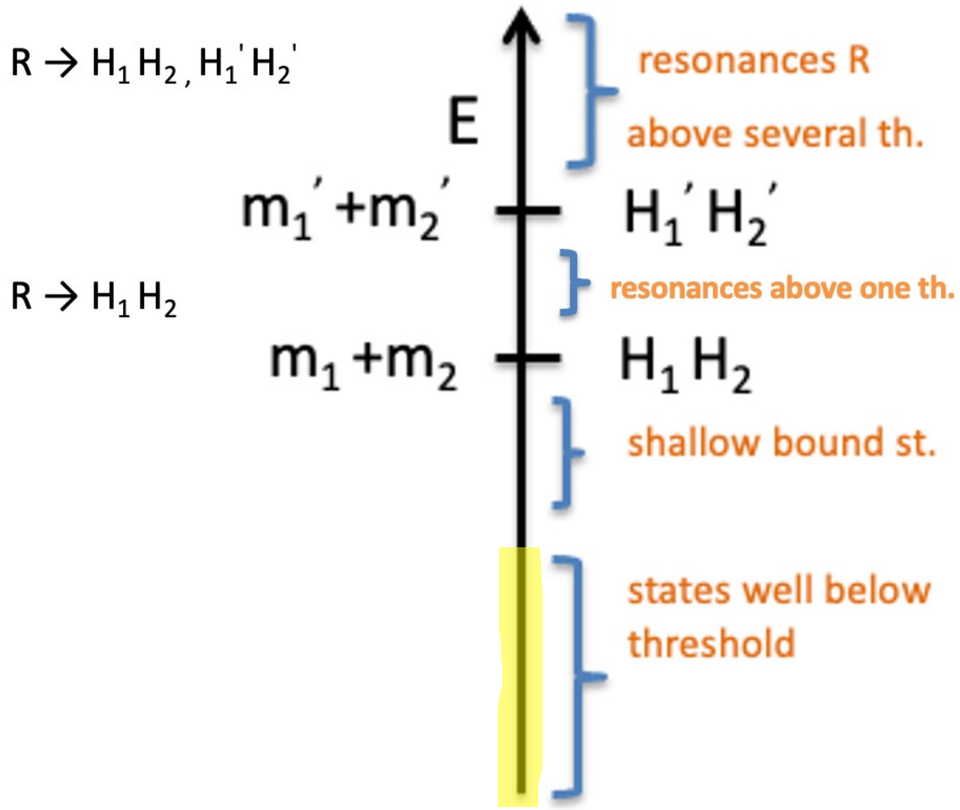
t, J^P, I t=0, J^P, I



$$\mathcal{O} = \mathcal{O}(q, G)$$

All results in this talk will be based on E_n :

- for strongly stable state well below threshold : $E_n(P=0) = m$
- resonances (Luscher's relation) $E_n^{cm} \to T(E_n^{cm})$
- static potentials: $E_n \to V(r)$



Exotic hadrons well below threshold

(or studied as if located well below threshold)

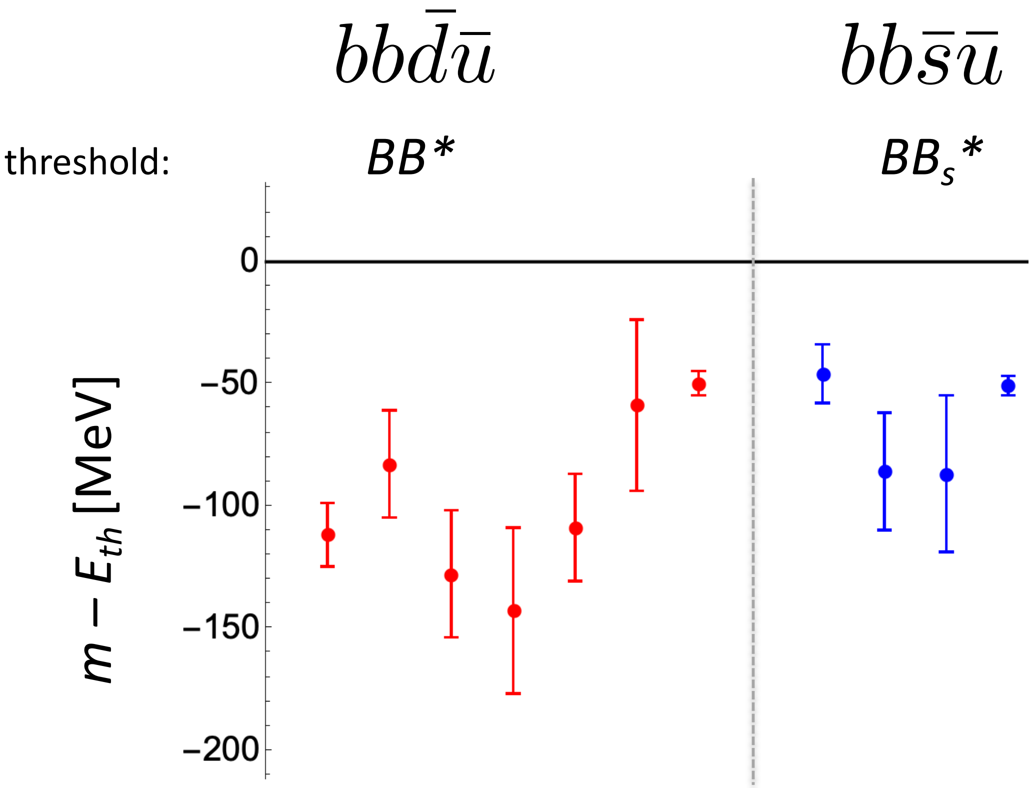
$$E_n(P=0) = m$$

Doubly bottom tetraquarks

not found in exp, difficult to find

$I=0,\ J^P=1^+$

$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$



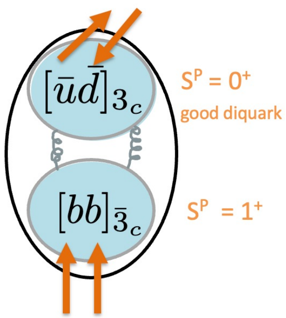
$$O = (\bar{u}\gamma_5 b)\ (\bar{d}\gamma_i b) + .. = BB^*$$
$$[b\Gamma_1 b]_{\bar{3}_c}[\bar{u}\Gamma_2\bar{d}]_{3_c}$$
$$\dots$$

from left to right (lattice QCD)

Hudspith, Mohler, 2303.17295
HALQCD, 2306.03565 (cosidering coupling with B*B*)
Leskovec, Meinel, Pflaumer, Wagner, 1904.04197
Junnarkar, Mathur, Padmanth, 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021 PosLat)
Bicudo, Wagner et al. 1612.02758, static potentials
Brown, Orginost, 1210.1953, static potentials

Hudspith, Mohler, 2303.17295
Meinel, Pflaumer, Wagner, 2205.13982
Junnarkar, Mathur, Padmanth 1810.12285
Frances, Colquhoun, Hudspith, Maltman (2021, PosLat)

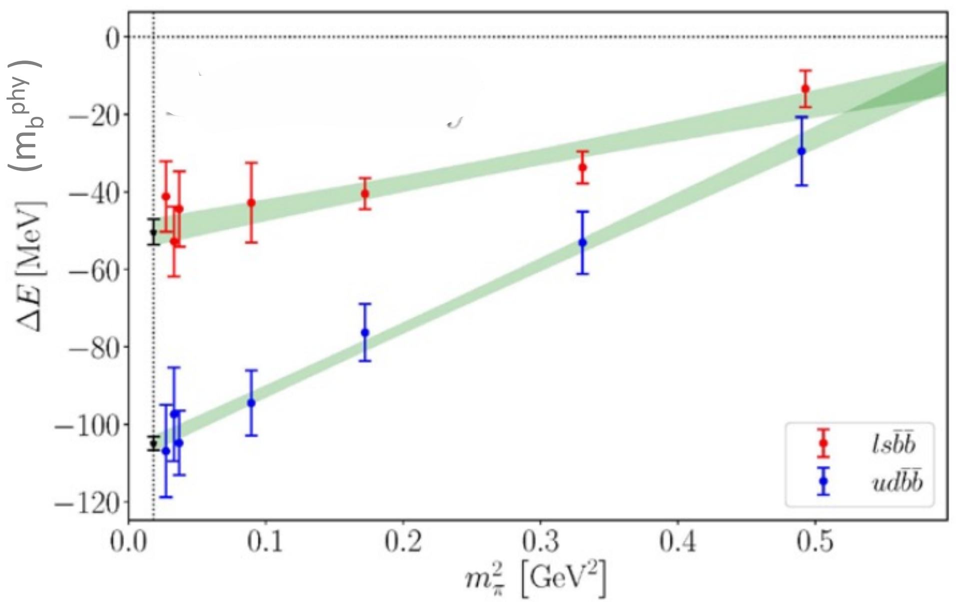
likely dominant
(B and B* to close
in BB* molecule
with binding ~0.1 GeV)



Doubly bottom tetraquarks

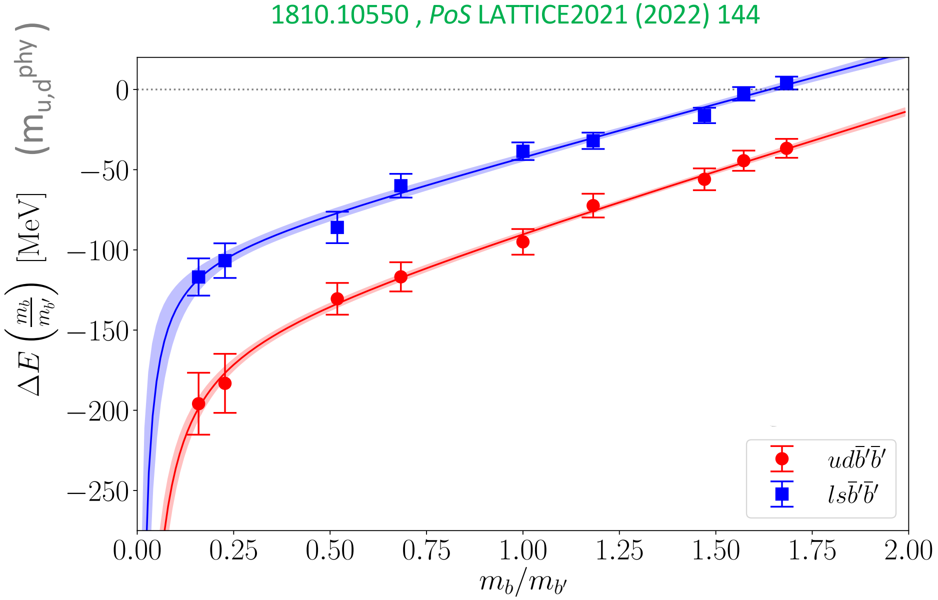
$bb\bar{d}\bar{u}$ $bb\bar{s}\bar{u}$ $I=0,\ J^P=1^+$

lattice: dependence on m_b and $m_{u,d}$



$m_{u,d}$ increases →

Colquhoun, Francis, Hudspith, Maltman, Lewis
1810.10550 , *PoS* LATTICE2021 (2022) 144



$m_{b'}$ decreases →

Other $QQ'\bar{q}\bar{q}'$ and J^P : $bc\bar{q}\bar{q}'$, $cc\bar{q}\bar{q}'$

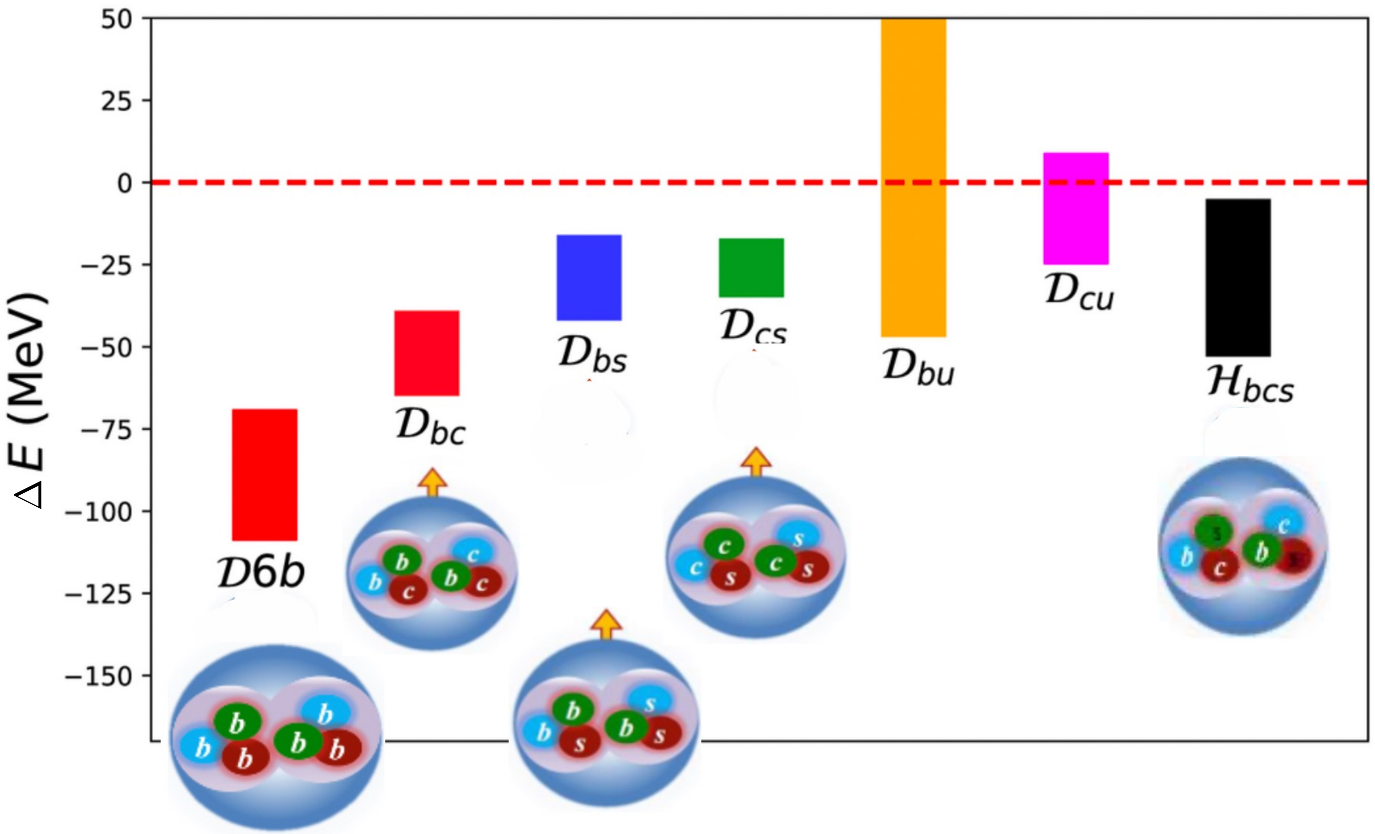
Theoretically expected near or above threshold

States near or above threshold have to be identified from scattering T(E): next Section

Di-baryons with heavy quarks

$$O = qqq\ qqq$$

binding energy $\Delta E = m - m_{B1} - m_{B2}$



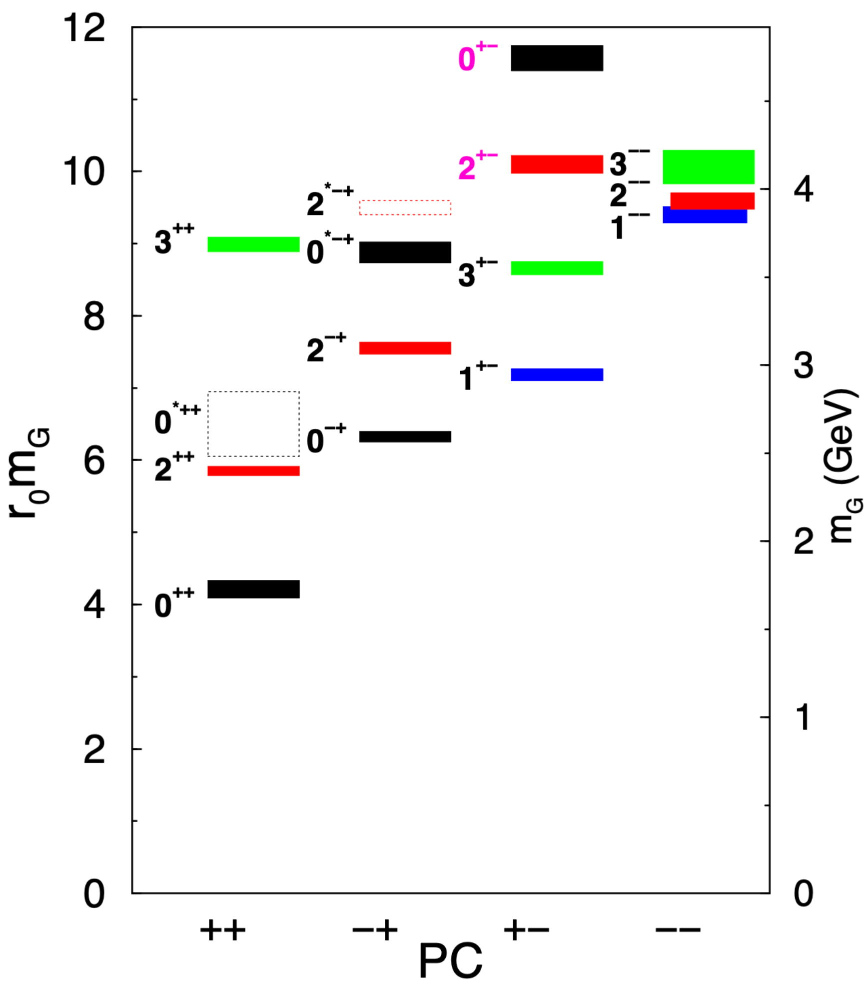
Junnarkar Mathur
1906.06054, PRL

Mathur, Padmanath, Chakraborty
2205.02862

Junnarkar, Mathur, 2206.02942, PRL

Gluebals (no dynamical quarks)

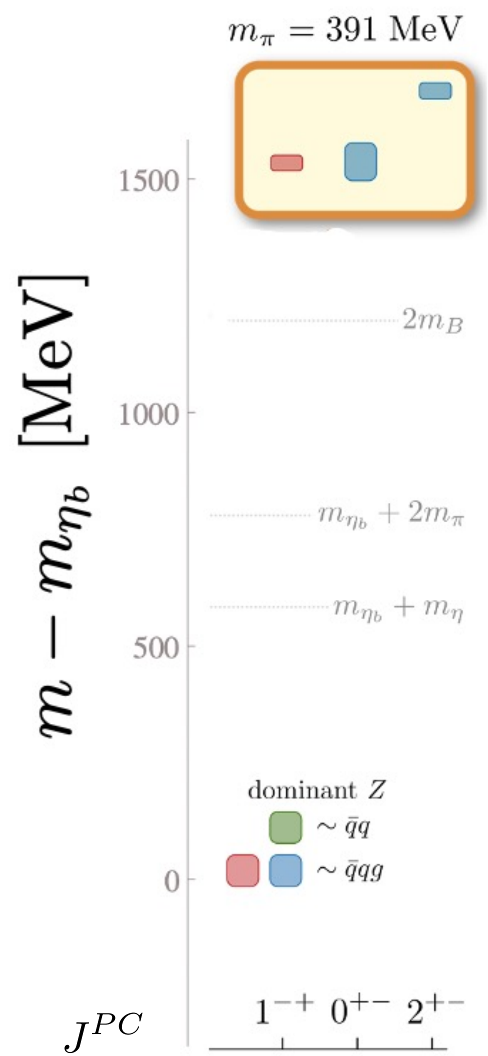
$GG.. \not\to (\bar{q}q)(\bar{q}q), \ldots$



Morningstar & Peardon 1999

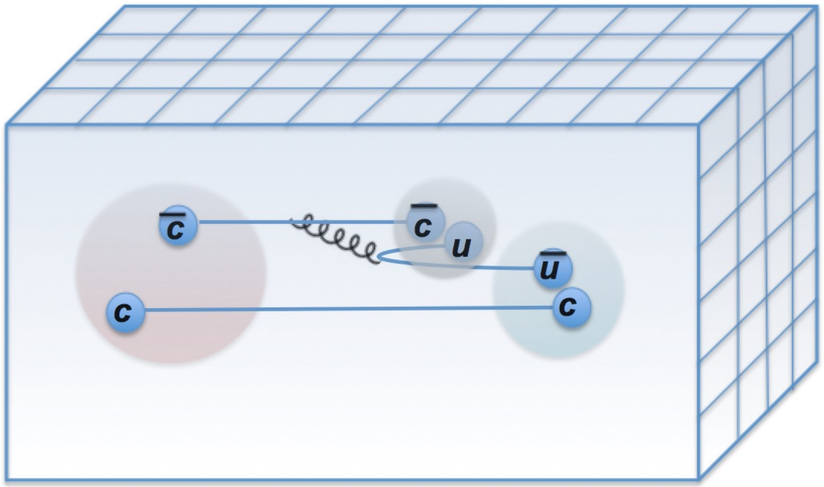
Hybrids (omitting strong decays)

$\bar{b}Gb$



Ryan & Wilson (HadSpec) 2008.02656, JHEP

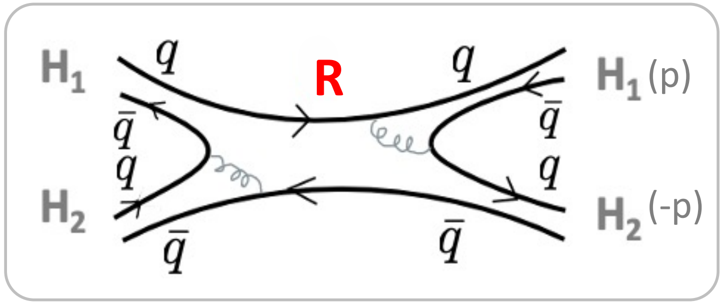
Incorporating strong decays and threshold effects



Resonances $R \to H_1 H_2$, bound states near threshold

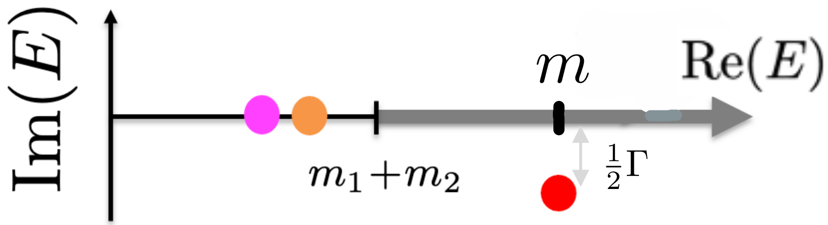
T here is M in Hansen's talk

scattering amplitude T(E)



$$S(E) = e^{2i\delta(E)} = 1 + 2i\,\frac{2p}{E}\,T(E) \to \boxed{T \propto \frac{1}{p\cot\delta - ip}}$$

E:= $\sqrt{m_1^2+p^2}+\sqrt{m_2^2+p^2}$

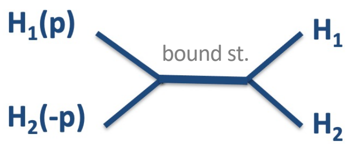


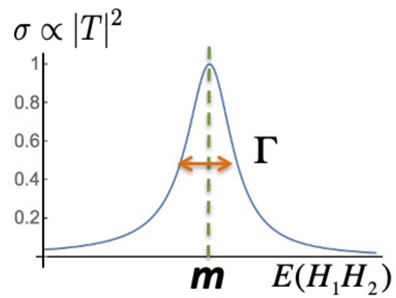
Virtual bound st.	Bound st.	Resonance
p = - i |p|, sheet II	p = i |p|, sheet I	sheet II

talk by Pilloni $p^2<0$

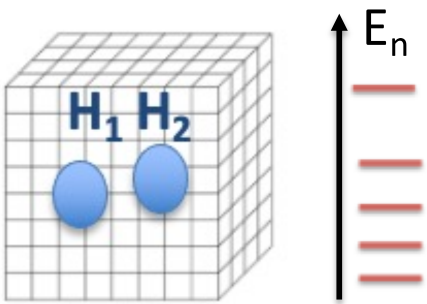
$$T(E) \propto \frac{1}{E^2-m^2}$$

$$T(E) \propto \frac{1}{E^2-m^2+iE\Gamma}$$





Scattering amplitude T(E) from lattice QCD



$$\sigma(E) \propto |T(E)|^2$$

real E → for real E → for complex E

$$E \to T(E) \to T(E^c)$$

analytic relation:
Luscher 1991
generalizations by many authors

analytic contin.
to complex E

Relation between E and δ(E) , T(E): 1D quantum mechanics

$$S(E) = e^{2i\delta(E)} = 1 + 2i\,\frac{2p}{E}\,T(E)$$

$$E \to \delta(E), T(E)$$

$E=p^2/2m$

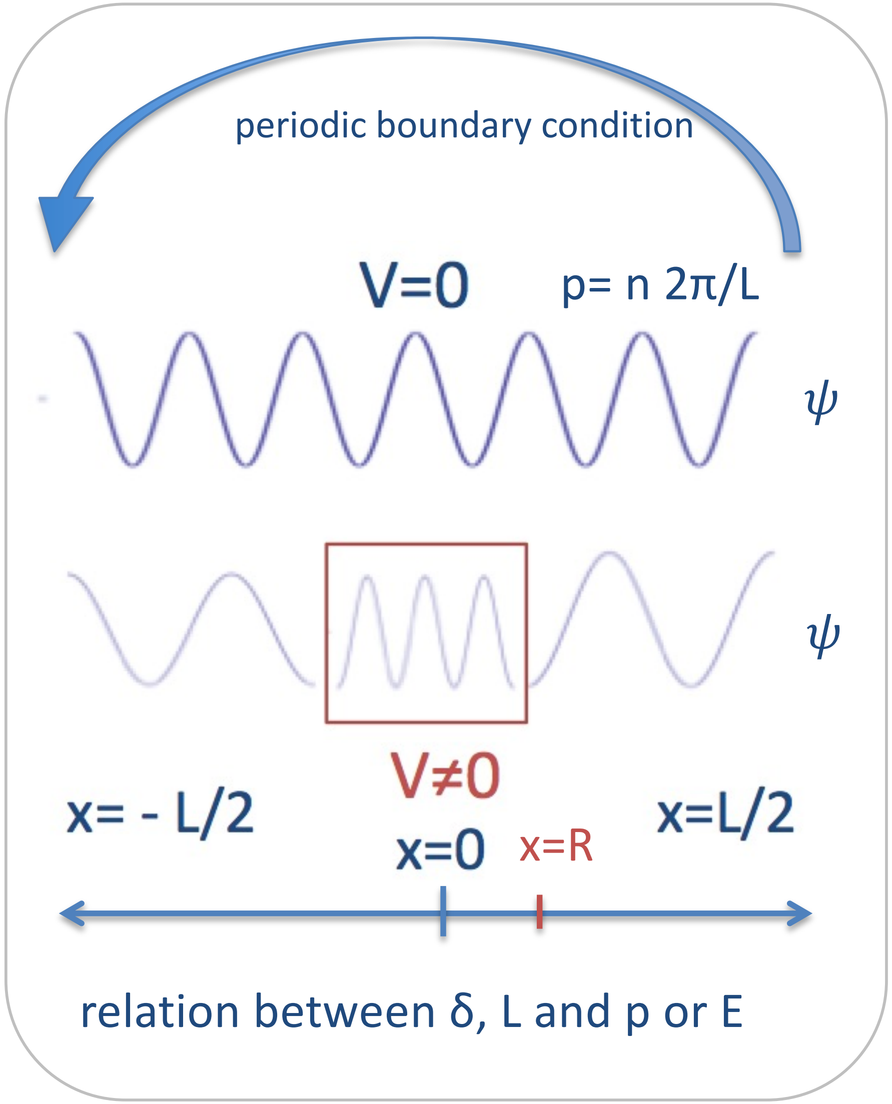
derivation of relation

$$\psi(x) = A\cos(p|x|+\delta) = \begin{cases} A\cos(px+\delta) & x>R \\ A\cos(-px+\delta) & x<-R \end{cases}$$

- this form already ensures
  $\psi(L/2) = \psi(-L/2)$
- the other BC:
  $\psi'(L/2) = \psi'(-L/2)$
  this requires
  $Ap\sin(p(\tfrac{L}{2})+\delta) = -Ap\sin(-p(-\tfrac{L}{2})+\delta)$
  $\rightarrow \psi'(L/2)=0$, $\sin(p\tfrac{L}{2}+\delta)=0$

$p\frac{L}{2}+\delta = m\pi$ $\quad \boxed{p = m\frac{2\pi}{L} - \frac{2}{L}\delta}$

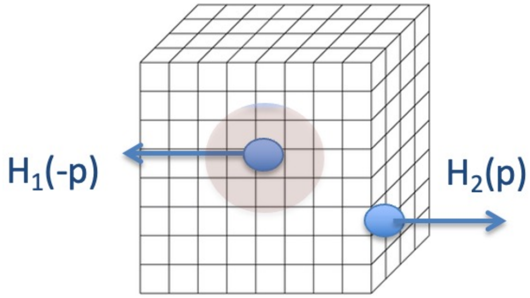
relation between δ, L



Relation between E and δ(E) , T(E): QFT

talk by Hansen

E = $\sqrt{m_1^2+p^2}+\sqrt{m_2^2+p^2}$



$$E \to \delta(E), T(E)$$

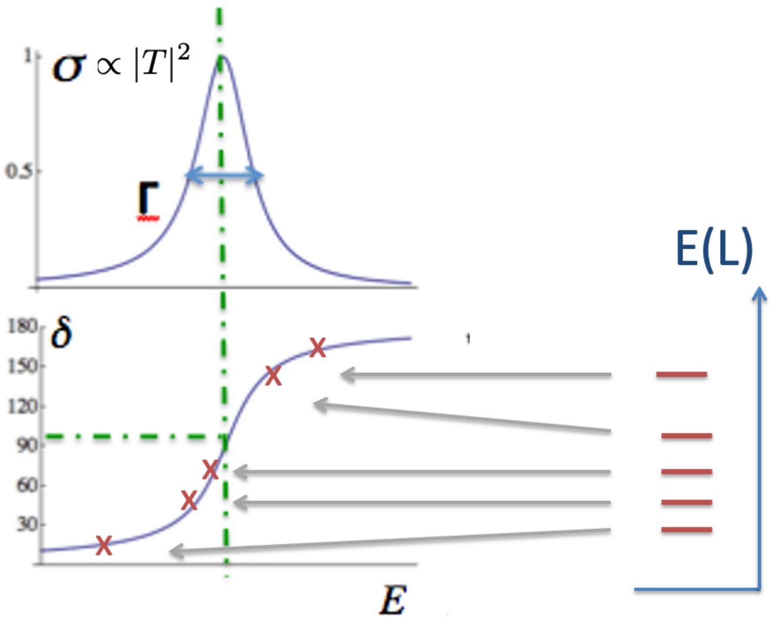
$$T \propto \frac{1}{\mathcal{K}^{-1} - \frac{2}{E}\; i\; p} \propto \frac{1}{p\cot\delta - ip}$$

Luscher's relation:

general : $$\det[\mathcal{K}^{-1}(E) + F(E,\vec{P},L)] = 0$$

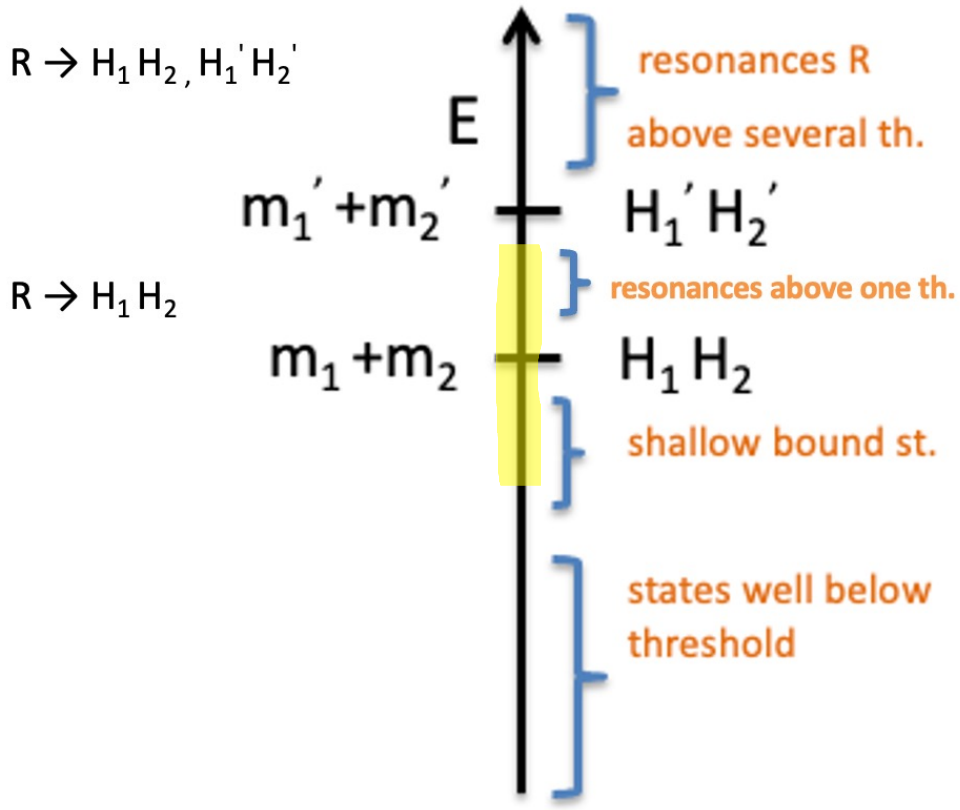
$F(P,L) \equiv$ Matrix of known geometric functions

one channel : $$\frac{2}{E}\; p\cot\delta(E) = \mathcal{K}(E)^{-1} = -F(E,\vec{P},L)$$



Luscher 1991

generalization by many authors



Exotic hadrons from one-channel scattering

Scalar charmed meson

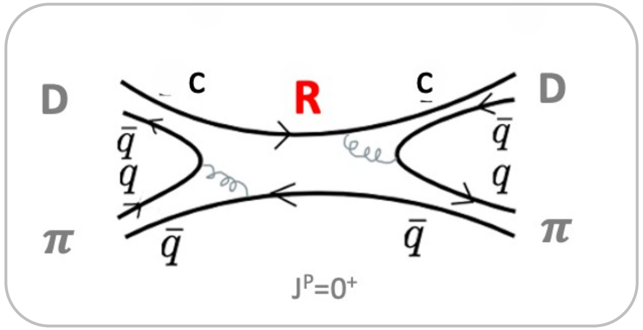
$c\bar{d}$, $c\bar{d}q\bar{q}$

not explicity exotic;

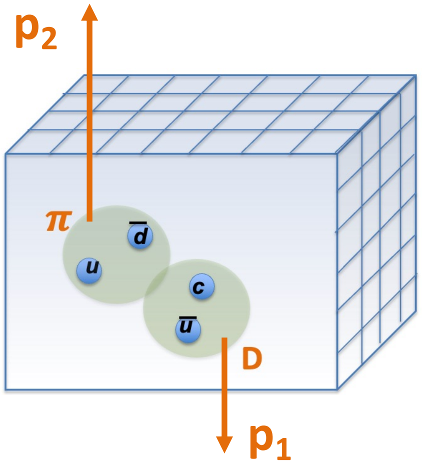
it's low mass indicates non-conventional states in this sector

HadSpec, Gayer et al, 2102.04973

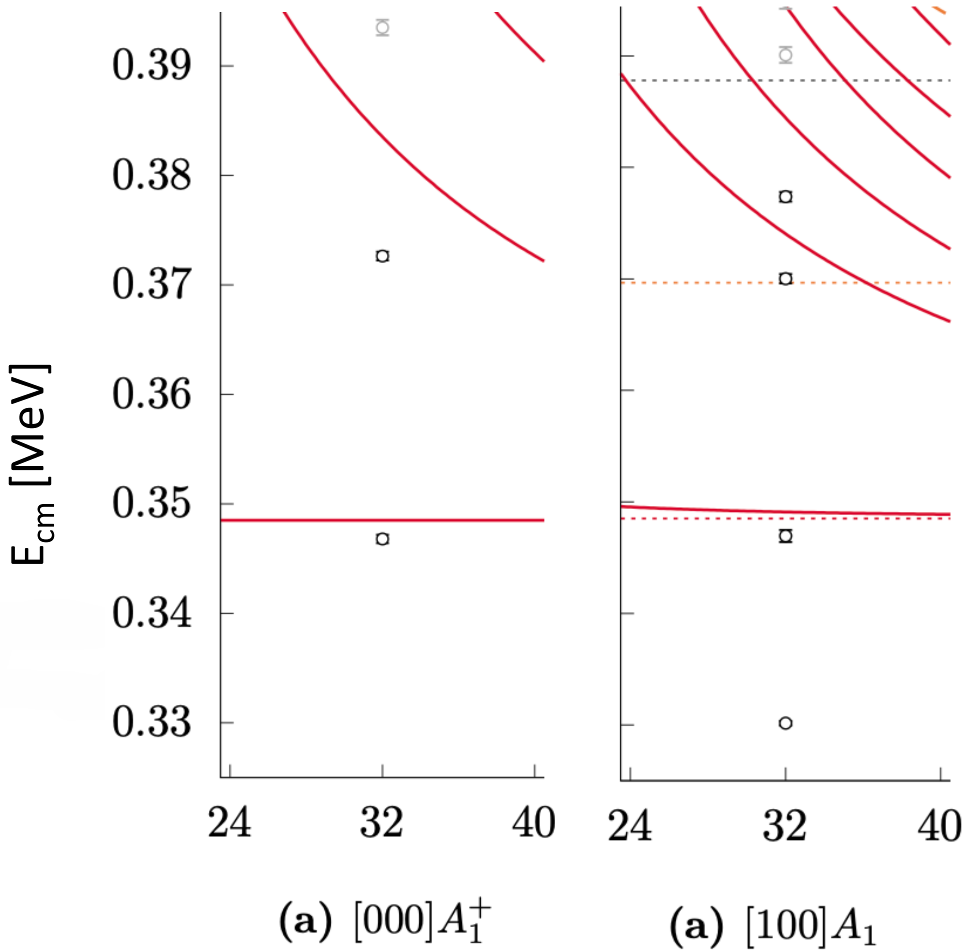
$m_\pi \approx 240$ MeV



$$O \sim (\bar{u}\Gamma_1 c)_{\vec{p}_1} (\bar{d}\Gamma_2 u)_{\vec{p}_2} + ..$$
$$\sim D(\vec{p}_1) \quad \pi(\vec{p}_2)$$



red lines: $E^{\text{non-int.}} = \sqrt{m_D^2 + p_1{}^2} + \sqrt{m_\pi^2 + p_2{}^2}$ $\quad \vec{p}_i = \vec{n}_i \frac{2\pi}{L}$

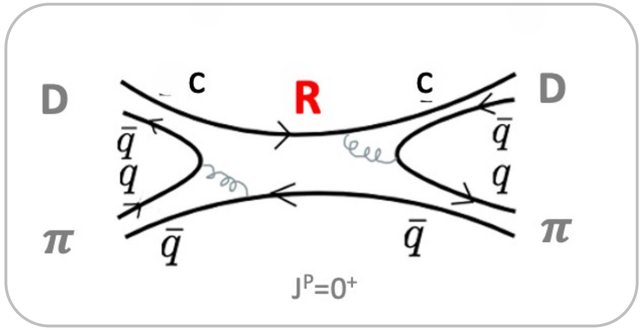


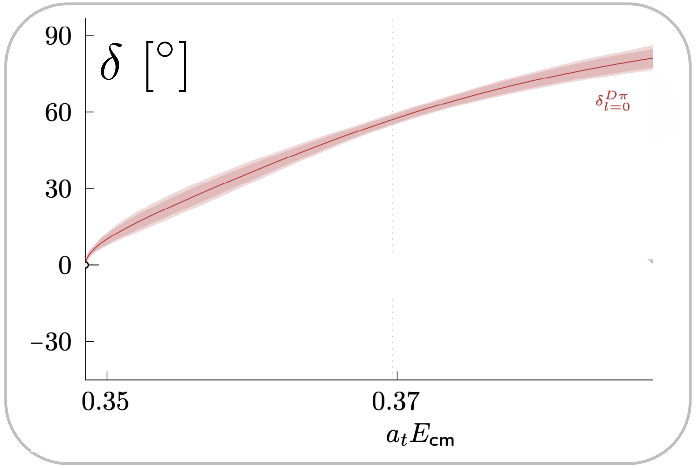
\+ other total momenta and irreps

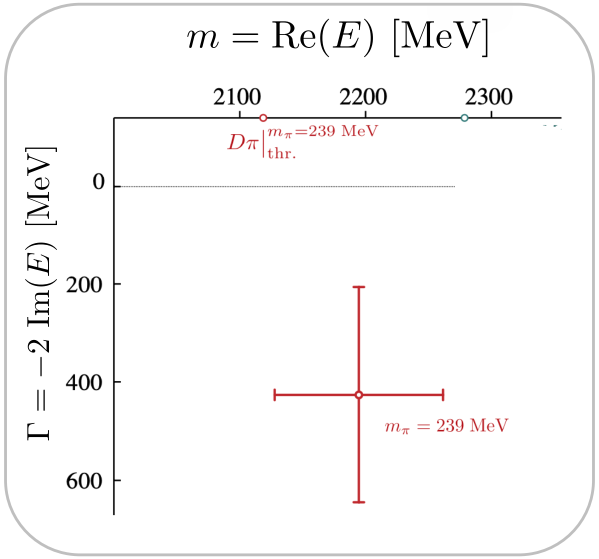
Scalar charmed meson, cont'

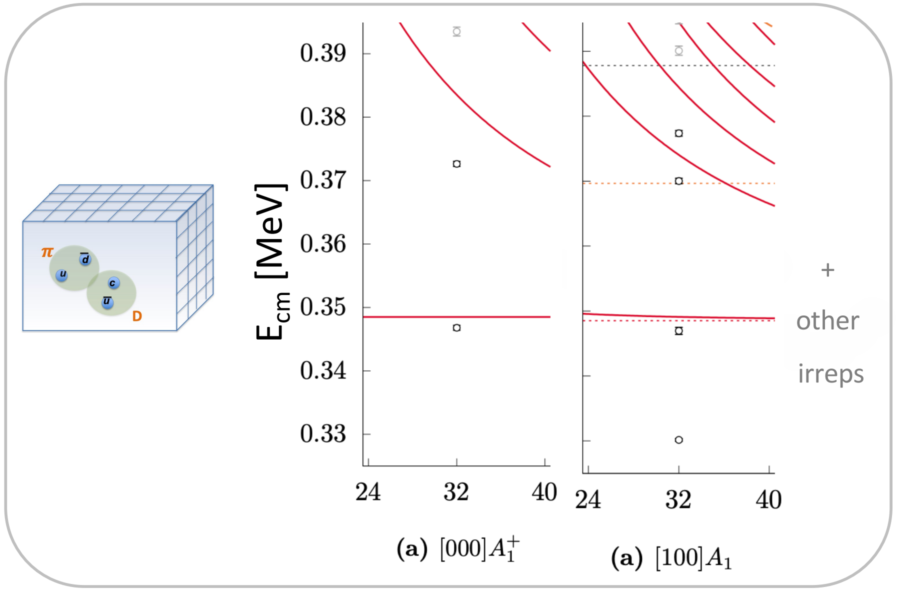
HadSpec, Gayer et al, 2102.04973

$m_\pi \approx 240$ MeV



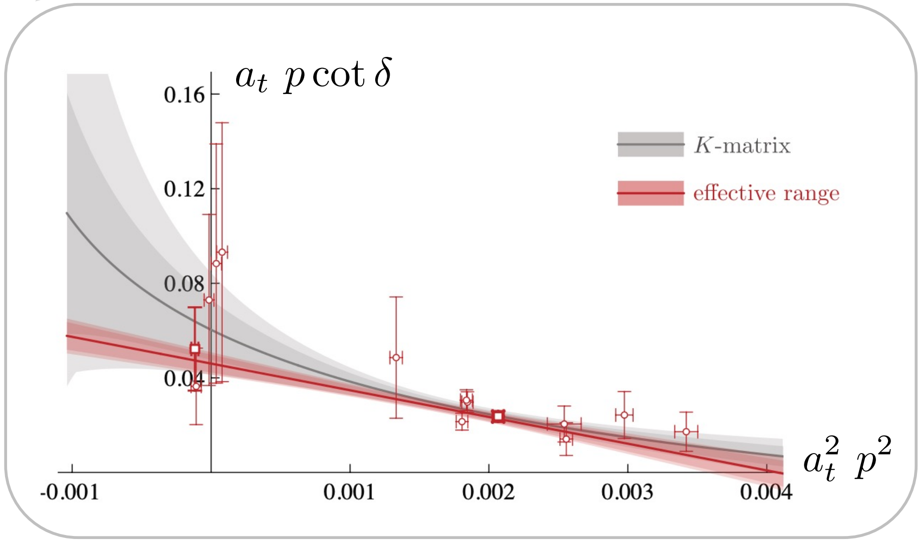






Luscher's relation

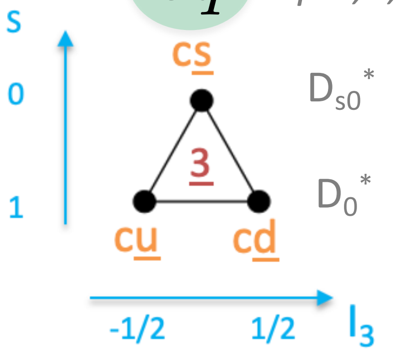
$$\frac{2}{E}\, p \cot\delta(E) = -F(E, \vec{P}, L)$$



Scalar heavy-light mesons $J^P = 0^+$

Conventional quark model

$c\bar{q}$ q=u,d,s



new paradigm supported by:

- lattice
- ChPT+HQET, UChPT
- reanalysis of exp data
- states circled by blue feature in the spectrum

Scattering on the lattice

~~SU(3)F~~

S=1 Mohler et al, 1308.3175, PRL
Lang et al, 1403.8103, PRD
RQCD, 1706.01247, PRD
HadSpec 2008.06432, JHEP

S=0 Mohler et al. 1208.4059, PRD
HadSpec, 1607.07093, JHEP
HadSpec 2102.04973, JHEP

S=-1 HadSpec, 2008.06432, JHEP

$SU(3)_F$: Gregory et al, 2106.15391
attraction in 6, repulsion in 15

New paradigm

Lutz et al, 2003 PLB, 2209.10601
Du et al, 1712.07957, PRD
Albaladejo et al, 1610.06727

$c\bar{q}$ + $c\bar{q}\ q\bar{q}$ q=u,d,s n=u,d

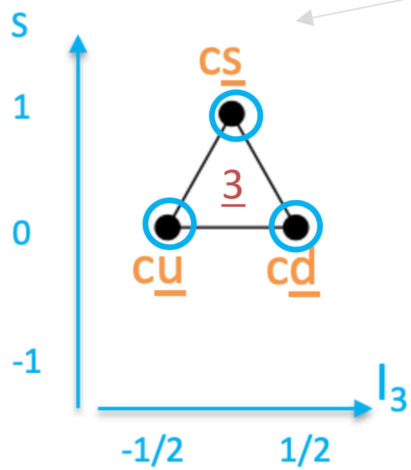
$3 \otimes 8 = 3 \oplus 6 \oplus 15$ $SU(3)_F$

most attractive (3), attractive (6), repulsive (15)

$D_{s0}(2317)$: 70-100% DK molecule 2.3 GeV

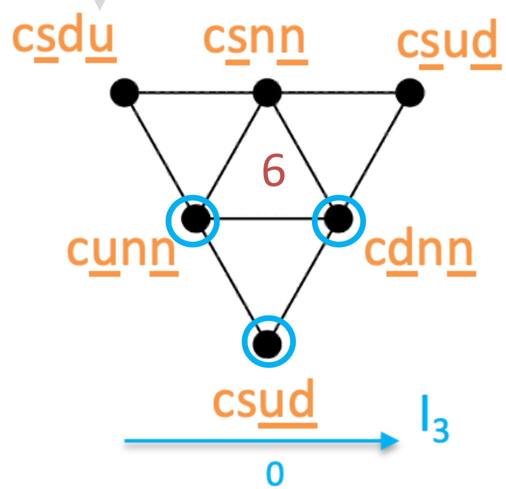
lat: 2.1-2.2 GeV (pole)

PDG: 2.3 GeV (BW)



mix

~~SU(3)F~~



mixes with 15

2.4-2.5 GeV

eanalysis of lat 1607.07093 by Albaladejo 1610.06727

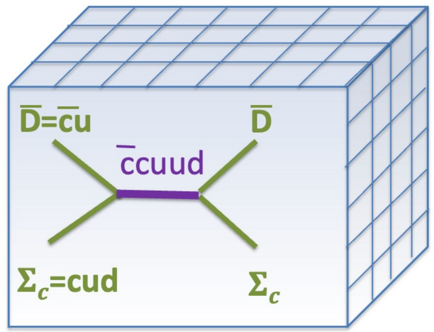
virtual bound state
HadSpec 2008.06432
partner of X(2900) [LHCb] ?

P_c

$\bar{c}cuud \rightarrow (\bar{c}u)(cud), \ldots$

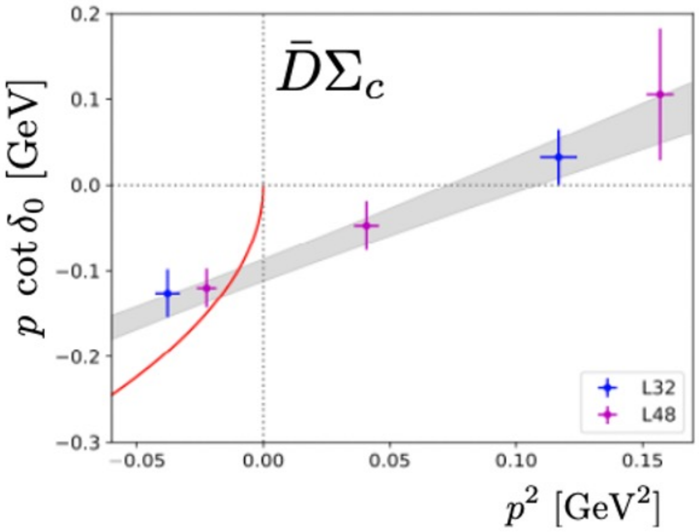
$\not\rightarrow (\bar{c}c)(uud)$

caution: coupling to charmonium+proton omitted

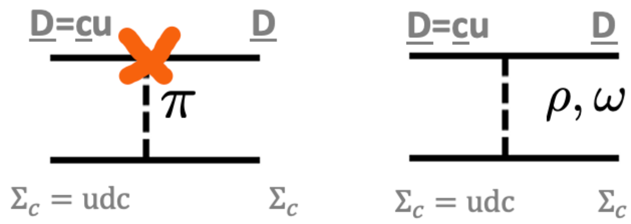


H. Xiang et al., 2210.08555 $m_\pi \approx 294$ MeV

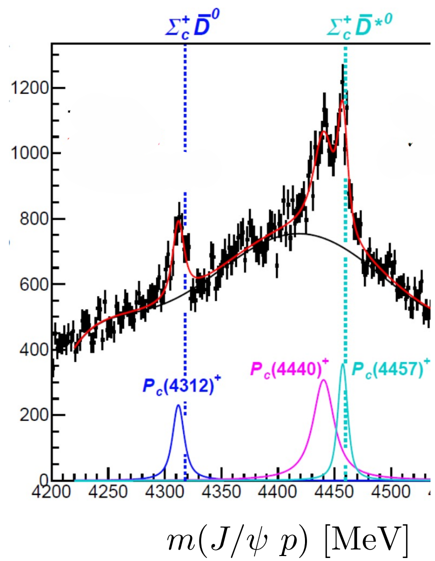
$\underline{D}\Sigma_c$ in s-wave $J^P=1/2^-$



Wu, Molina, Oset, Zou, PRL 2010



LHCb 2019



$$T \propto \frac{1}{p\cot\delta - ip}\ , \quad p\cot\delta = \frac{1}{a_0} + \frac{1}{2} r_0\ p^2$$

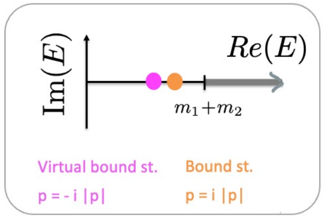
$$\frac{1}{a_0} + \frac{1}{2} r_0 p^2 - ip = 0 \ \rightarrow \ p_b = i|p_b|$$

$$m_{P_c} = \sqrt{m_D^2 + p_b^2} + \sqrt{m_{\Sigma_c}^2 + p_b^2}$$

$$m_{P_c} - (m_D + m_{\Sigma_c}) = -6 \pm 3 \text{ MeV}$$

pole in T:

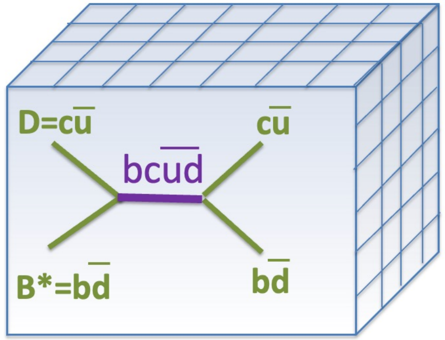
real bound st.



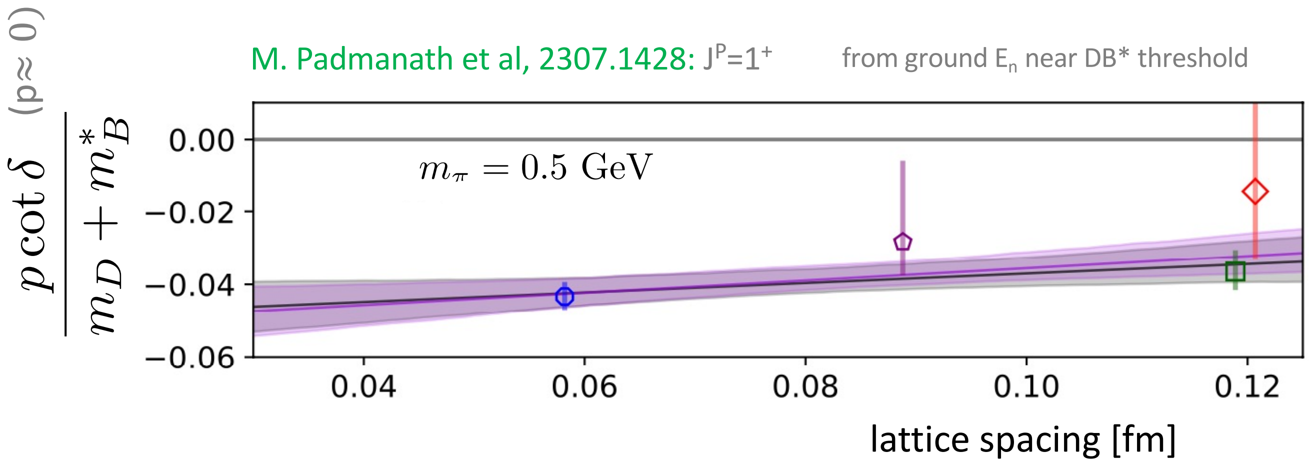
T_{bc} : next exciting discovery from exp ?

$$I=0,\ J^P=1^+,0^+ \qquad bc\bar{u}\bar{d}$$

$$O \sim (\bar{u}b)(\bar{d}c),\ \ [bc][\bar{u}\bar{d}]$$



M. Padmanath et al, 2307.1428: $J^P=1^+$ from ground E_n near DB* threshold



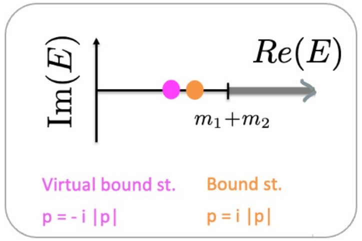
$$T \propto \frac{1}{p\cot\delta - ip}\ ,\quad p\cot\delta \overset{\text{approx}}{=} \frac{1}{a_0} \qquad a_0<0$$

$$\frac{1}{a_0} - ip = 0 \ \rightarrow\ p_b = -i\frac{1}{a_0} = i\left|\frac{1}{a_0}\right|$$

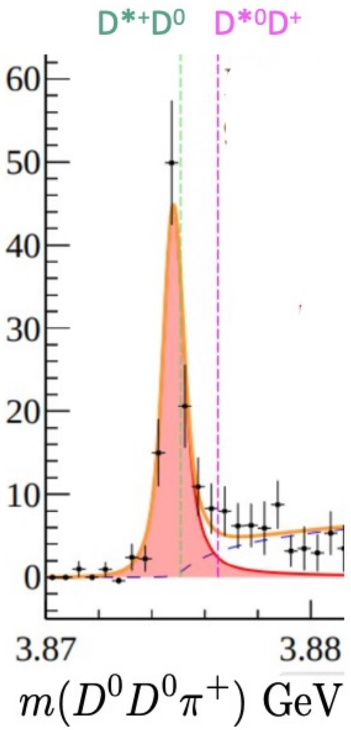
$$m_{T_{bc}} - (m_D + m_{B^*}) = -43(^{+6}_{-7})(^{+14}_{-24})\ \text{MeV}$$

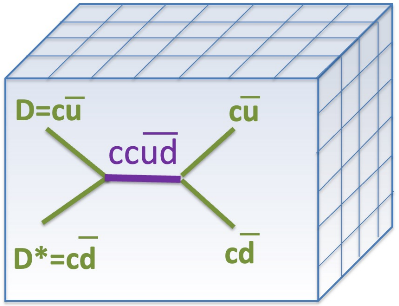
pole in T : real bound st.

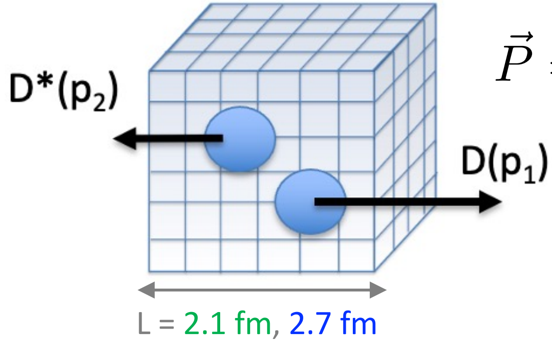
after continuum extrap. and chiral extrap. from $m_\pi = 0.5 - 1$ GeV



T_{cc}

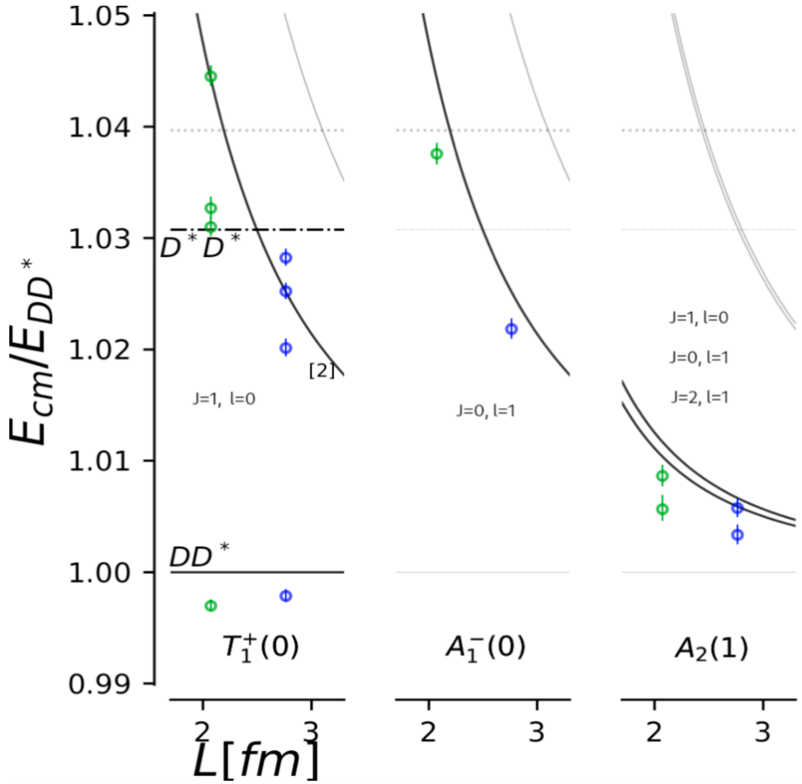






$\vec{P} = \vec{p}_1 + \vec{p}_2$

$\vec{P} = 0 \qquad \vec{P} = \frac{2\pi}{L}\,(0,0,1)$



Padmanath & SP, PRL2022, $m_\pi \approx 280$ MeV

$E_{DD^*} \equiv m_D + m_{D^*}$

E < $E^{non.int.}$ (lines) :

attractive interaction between D and D*

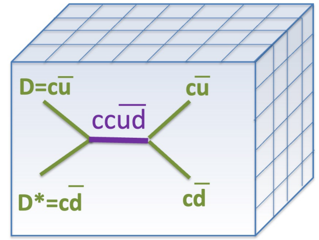
T_{cc}

dependence on $m_{u/d}$

$cc\bar{d}\bar{u}$

I=0, $J^P=1^+$

$$T \propto \frac{1}{p\cot\delta - ip}$$

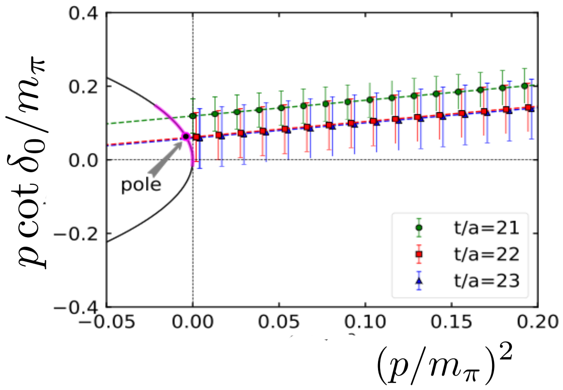


D* is stable at these m_π

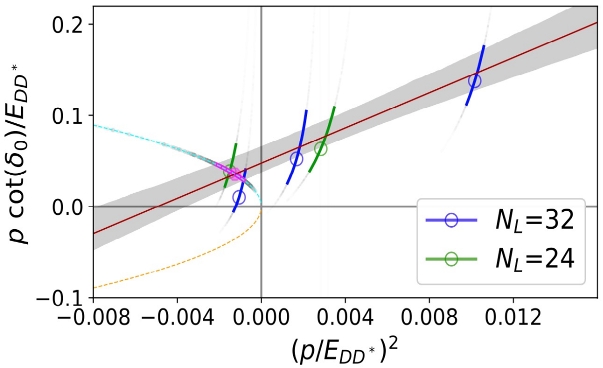
$T(E) \propto \frac{1}{E^2-m^2}$ for E~m

LHCb

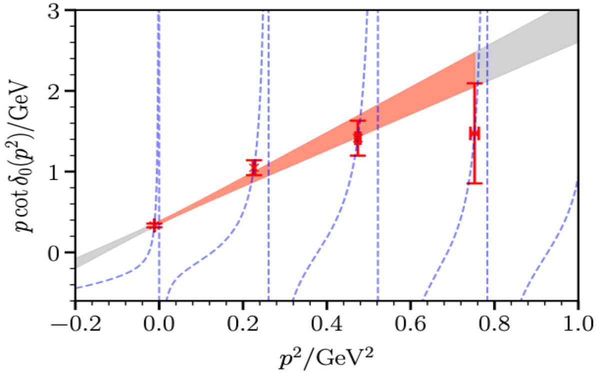
HALQCD method, 2302.04505, $m_\pi \approx 146$ MeV



Padmanath, SP: 2202.10110, PRL, $m_\pi \approx 280$ MeV



CLQCD 2206.06185, PLB, $m_\pi \approx 348$ MeV

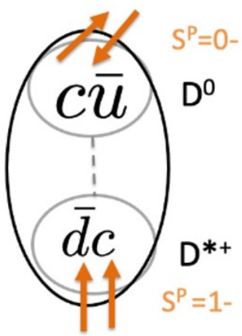


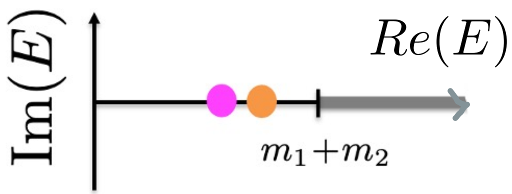
- 0.36(4) MeV - 0.045(77) MeV $-9.9(^{+3.6}_{-7.2})$ MeV : binding energy δm

bound st. virtual bound st. pole

$\pi, \rho, \pi\pi$?

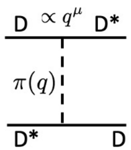
$m_{u/d}$ $V(r) \propto -\frac{e^{-m_{ex} r}}{r}$





Virtual bound st. Bound st.

p = - i |p| p = i |p|



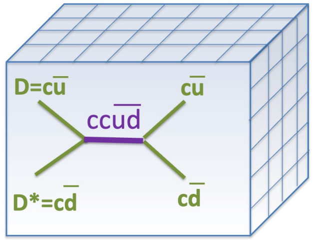
Disclaimer:

- the extaction of pole omits possible effect from the left hand cut
- investigated by Du, F.K. Guo, Nefediev et al. 2303.09441, talk by Nefediev
- under ongoing investigation, keep tuned

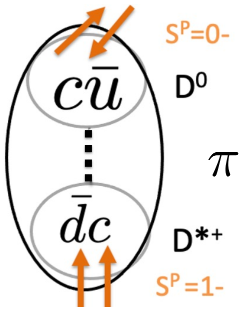
T_{cc}

$cc\bar{d}\bar{u}$ I=0, $J^P=1^+$

D* is stable at these m_π



Exchange of which particles drives the attraction (within molecular picture)?



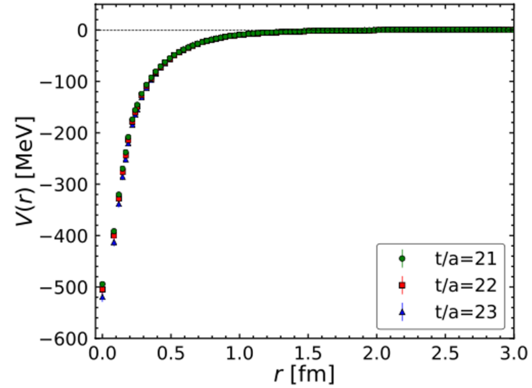
$\pi, \rho, \pi\pi$?

HALQCD, 2302.04505, $m_\pi \approx$146 MeV

~~π~~, $\rho, \pi\pi$?

$V(r) \simeq -\frac{e^{-2m_\pi r}}{r^2}$ r > 1 fm

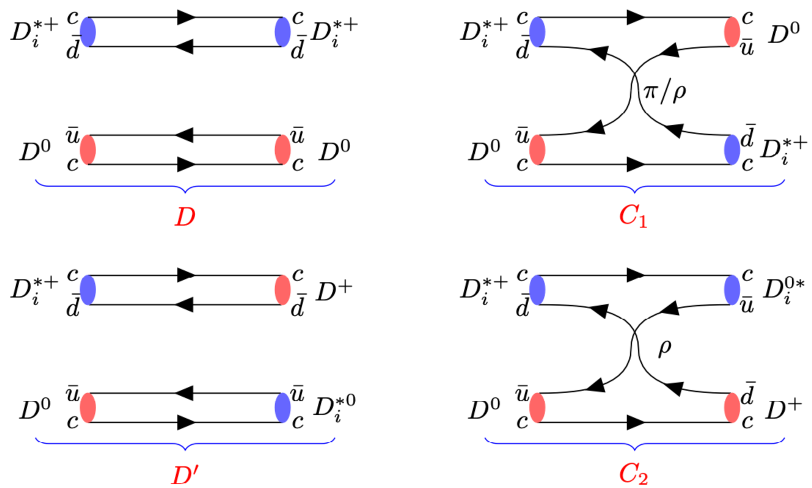
HALQCD method



CLQCD 2206.06185, PLB,
$m_\pi \approx$ 348 MeV

~~π~~, $\rho, \pi\pi$?

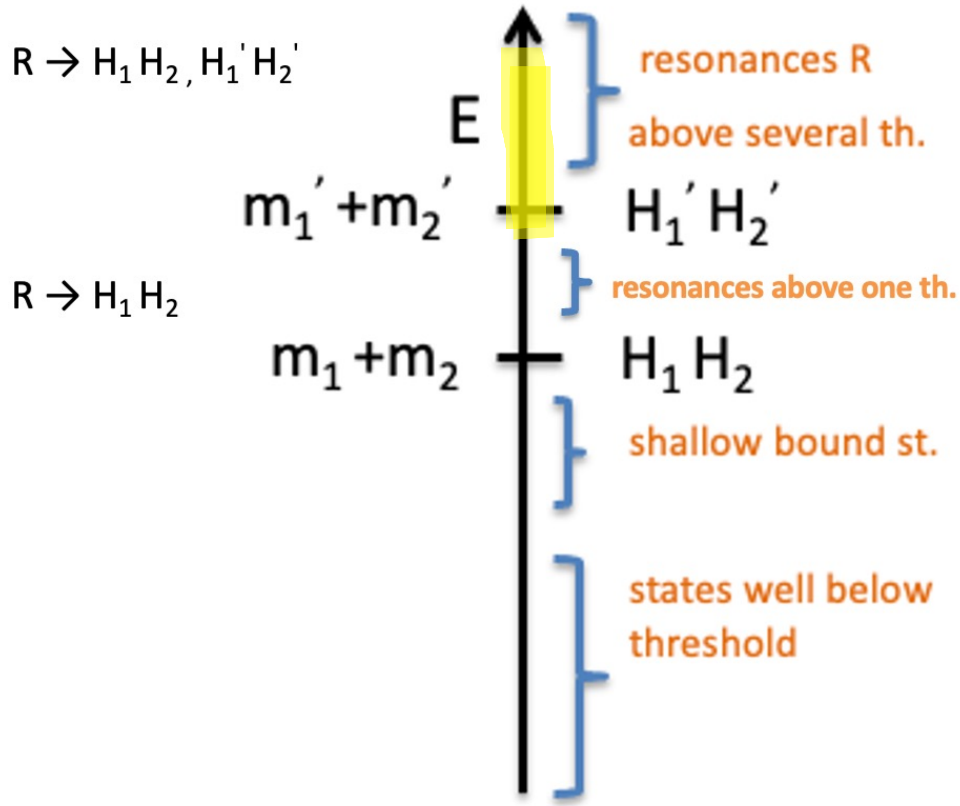
not excluded



I=0 attractive, I=1: repulsive

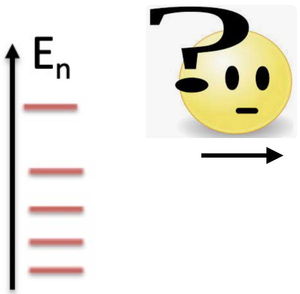
$C^{(I)}(p,t) = D - C_1(\pi/\rho) + (-)^{I+1}(D' - C_2(\rho))$

C_2 drives attraction in I=0 channel



Hadrons from coupled-channel scattering

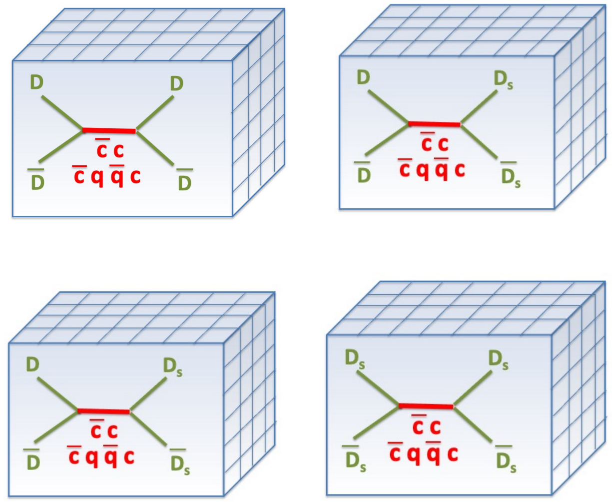
Relation between E and $T_{ij}(E)$: QFT



$$O_{I=0} \simeq \begin{array}{l} D\bar{D} \\ D_s\bar{D}_s \\ \bar{c}c \end{array}$$

$$T \propto \frac{1}{\mathcal{K}^{-1} - \frac{2}{E}\, i\, p} =$$

2x2 matrices



a: DD

b: D$_s$D$_s$

$$\det[\mathcal{K}^{-1}(E) + F(E, \vec{P}, L)] = 0$$

F=known matrix

generalized Luscher's relation

[Hansen & Sharpe 2012, ...]

talk by Hansen

$$\det\left[\begin{pmatrix} \mathcal{K}^{(E)}_{a\to a} & \mathcal{K}^{(E)}_{a\to b} \\ \mathcal{K}^{(E)}_{b\to a} & \mathcal{K}^{(E)}_{b\to b} \end{pmatrix}^{-1} + \begin{pmatrix} F_a & 0 \\ 0 & F_b \end{pmatrix}\right] = 0$$

at given E: $f(\mathcal{K}_{a\to a}(E), \mathcal{K}_{b\to b}(E), \mathcal{K}_{a\to b}(E)) = 0$

strategy:

- parametrize energy dependence of K matrix
- perform global fit to all eigen-energies
- applied for many light-meson resonances by HadSpec

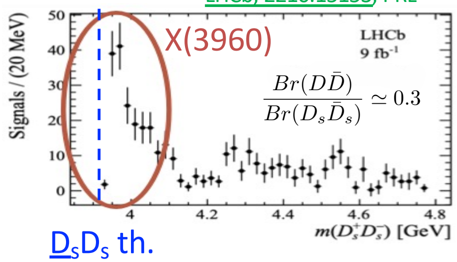
Charmonium(like) resonances and bound states

$\bar{c}c$, $\bar{c}q\bar{q}c$ q=u,d,s $I=0$

$\bar{D}_s D_s$ J[P]=0[+] state

$$T_{ij}(E_{cm}) \sim \frac{c_i\ c_j}{E_{cm}^2 - m^2}$$

$$\frac{|c^2_{D\bar{D}}|}{|c^2_{D_s\bar{D}_s}|} \overset{\text{lat}}{=} 0.02\ ^{+0.02}_{-0.01}$$

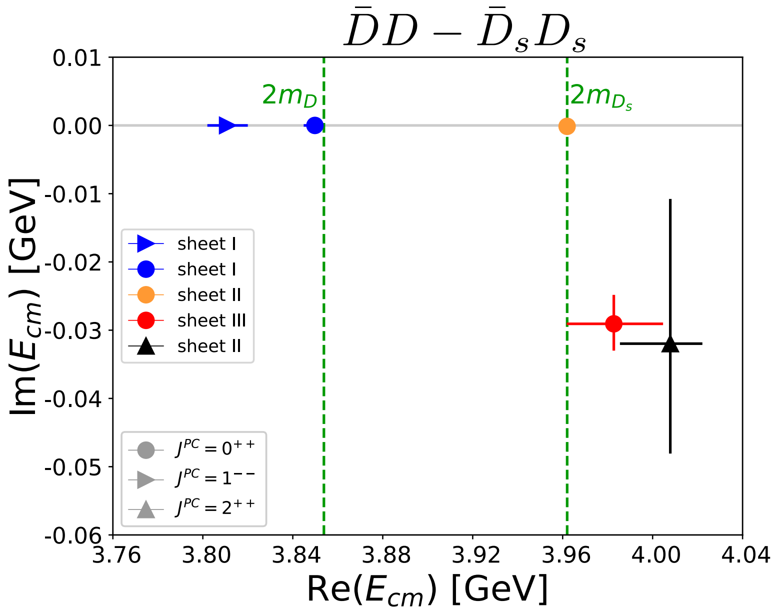


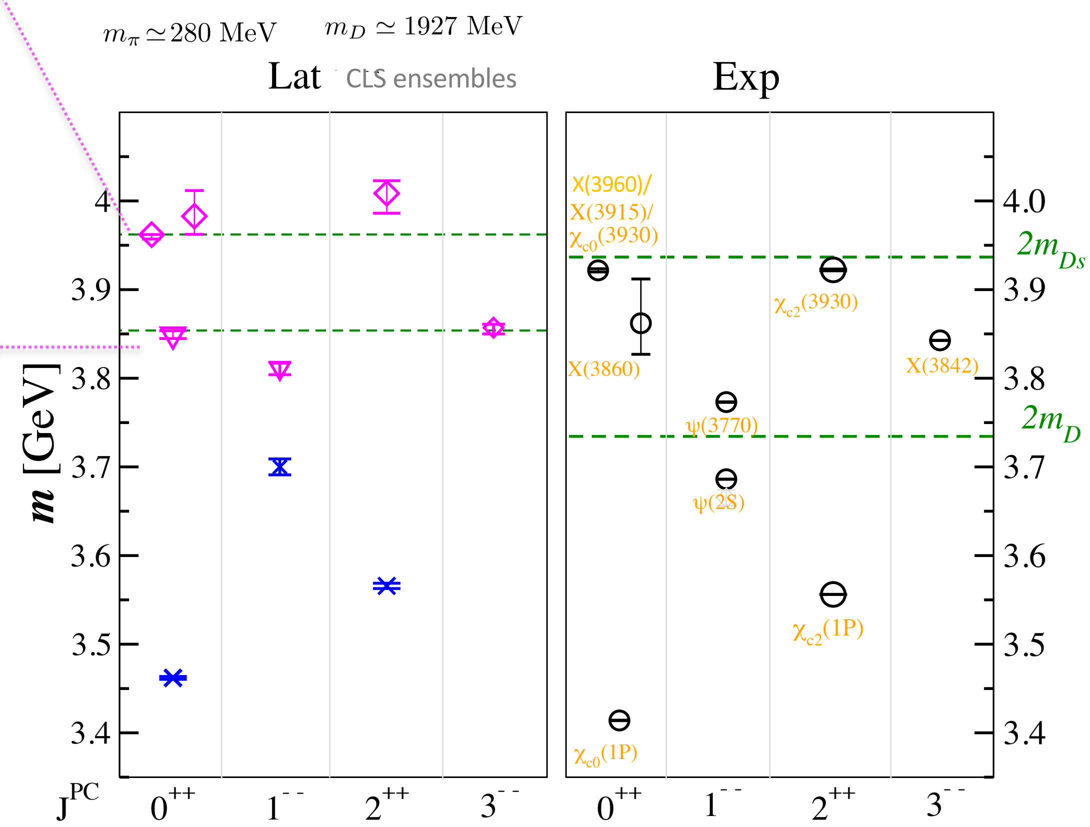
$\bar{D}D$ J[P]=0[+] state

predicted in models [Oset et al, 0612179 PRD, Guo et al 2101.01021]

seen in re-analysis of exp. [Danilkin et al 2111.15033, Ji, F.K. Guo et al., 2212.00613]

+ expected conventional charmonia





S.P. , Collins, Padmanath, Mohler, Piemonte
2011.02541 JHEP, 1905.03506 PRD

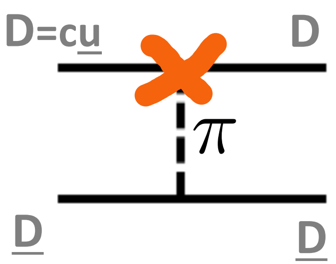
Likely interpretation of some near-threshold states: "molecules" attracted by V exchange

a number of pheno studies
Oset et al, 0612179 PRD,
Guo et al, 2101.01021,...

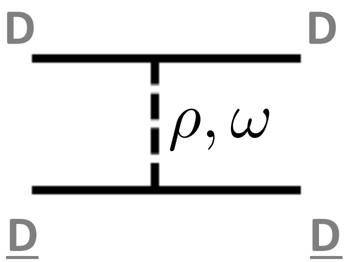
$\bar{c}q\bar{q}c$ $I=0$ $J^P=0^+$

now support also from lattice

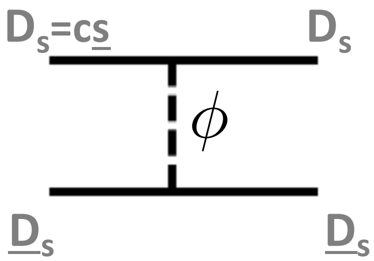
D_sD_s th.

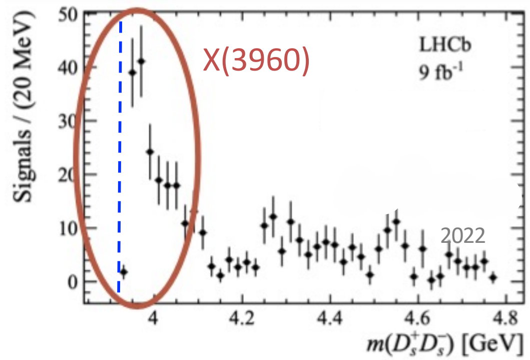


$\bar{D}D$



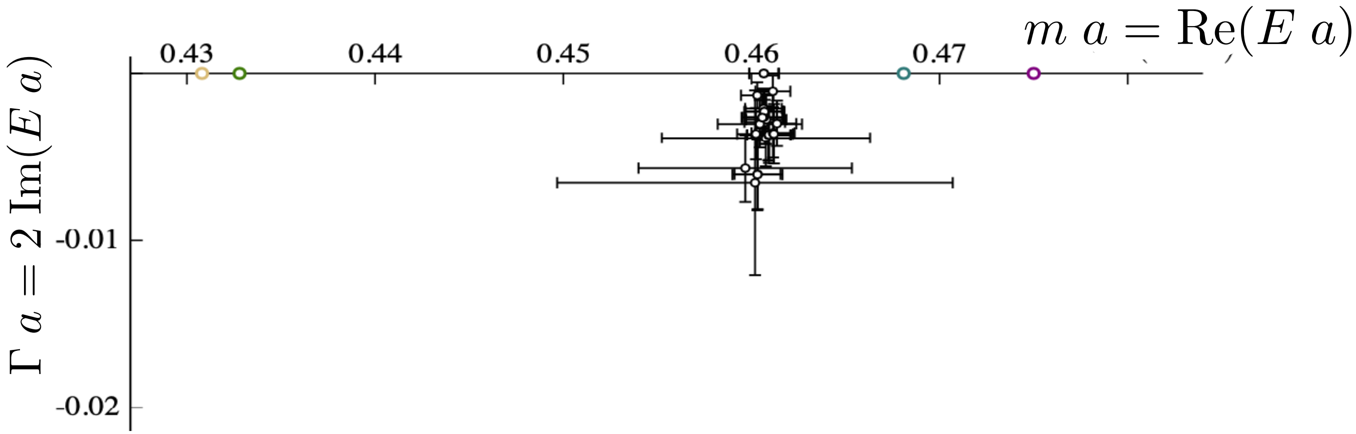
$\bar{D}_sD_s$



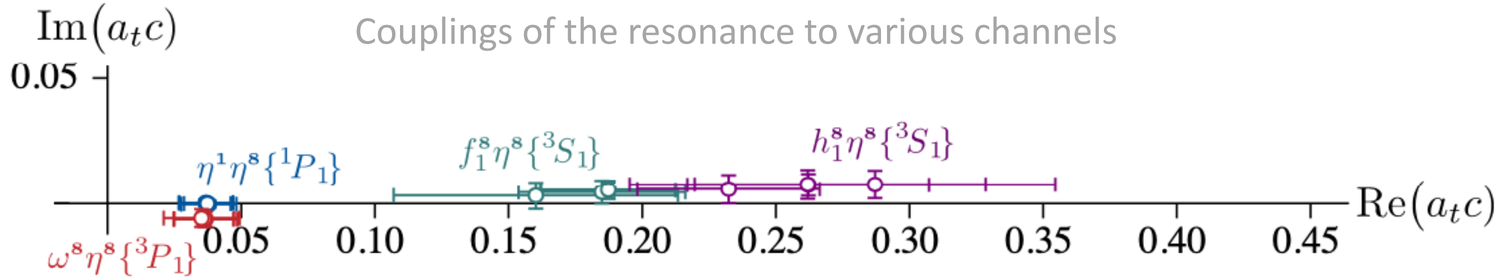


light hybrid meson π_1 from lattice

$\bar{d}Gu$ $\quad J^{PC} = 1^{-+}$



Couplings of the resonance to various channels



$$T_{ij} \sim \frac{c_i\ c_j}{E_p^2 - E^2}$$

Woss et al. (HadSpec)

2009.10034

$m_u = m_d = m_s$, $m_\pi \approx 700\ MeV$

pheno

analysis

$\rho\ \pi$ $\quad \eta'\ \pi$ $\quad f_1\ \pi$ $\quad b_1\ \pi$

dominant coupling

physical world

resemblence to experimental $\pi_1(1564)$: COMPASS+JPAC Rodas 1810.04171 [PRL]

$\pi_1(1564)$ in COMPASS+JPAC replaces two older resonances $\pi_1(1400)$ and $\pi_1(1600)$

Exotic hadrons from static potentials

heavy: b

light: G, u,d,s

- Born-Oppenheimer approach with static heavy particles renders V(r)
- Motion of heavy particles under this V(r) is given by $-\frac{\hbar^2}{2m_r}\nabla^2\psi + V\psi = E\psi$

Bottomonia and bottomonium hybrids

I=0, varius J^P

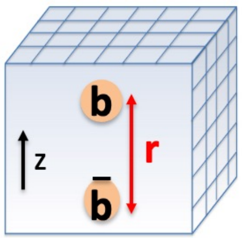
$\bar{b}\, b$

$\bar{b}Gb$

omit strong decays

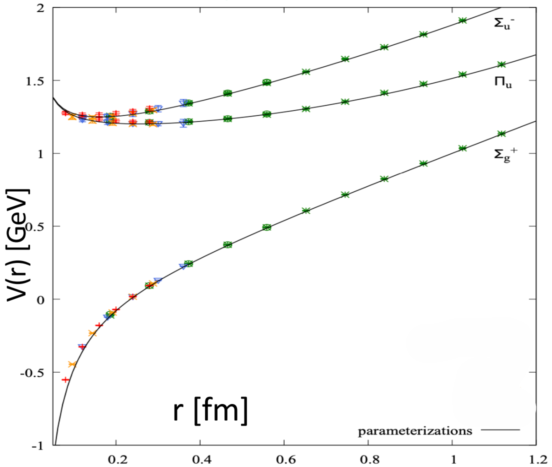
quenched

E = V(r) + const.

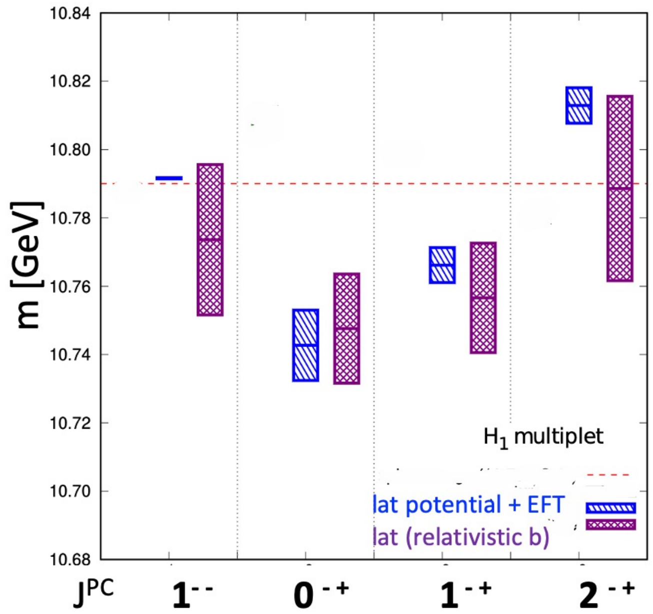


Juge, Kuti, Morningstar, 1997, 1998 →

Schlosser & Wagner, 2111.00741



$\bar{b}Gb$ Segovia, Tarrus; Brambilla @ MITP 2022

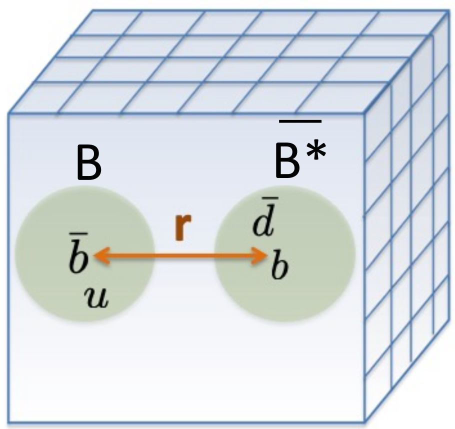


Ryan & Willson 2020

see also talk by Szczepaniak

- Born-Oppenheimer approach with static heavy particles renders V(r)
- Motion of heavy particles under this V(r) is given by $-\frac{\hbar^2}{2m_r}\nabla^2\psi + V\psi = E\psi$

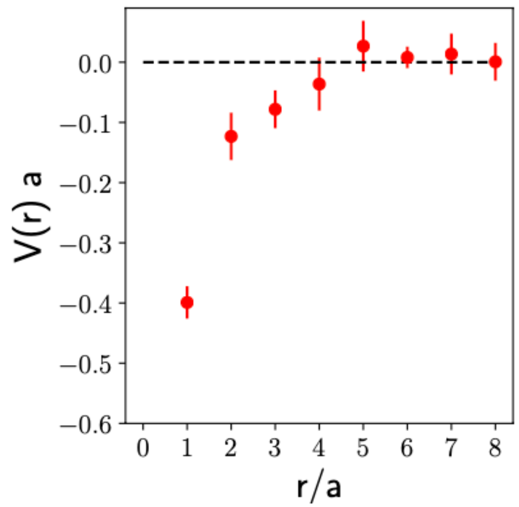
$\bar{b}b\bar{d}u$ $I=1,\ J^P=1^+$



$\bar{b}b\bar{d}u \to B\bar{B}^*$
$\Upsilon\pi$

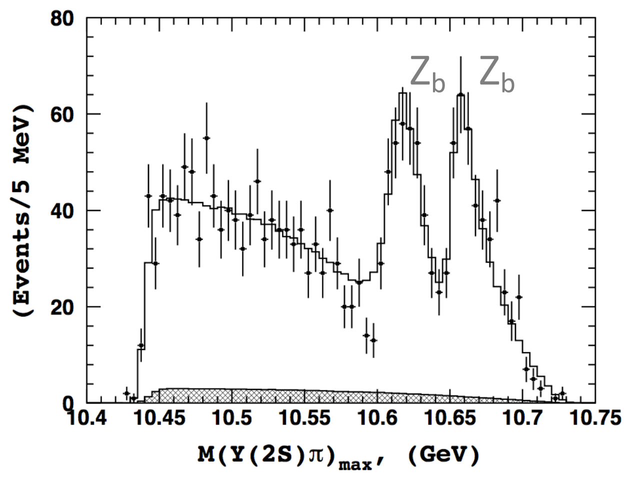
S.P., Bahtiyar, Petkovic, PLB 2019

Sadl, S.P., PRD 2021

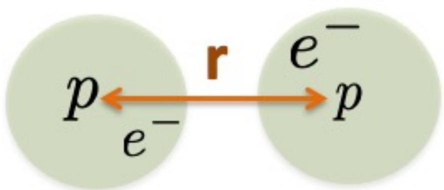


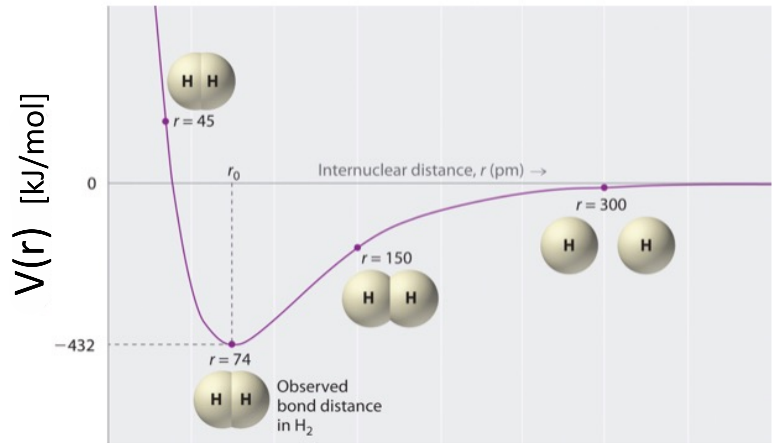
many other channels: Wagner et al.

Belle 2011 PRL 2011



H_2





More in recent reviews

hadron spectrum from lattice:

N. Brambilla et al. 1907.07583, Phys. Rept

M. Mai, U. Meissner, C. Urbach, 2206.01477

N. Brambilla, 2111.10788

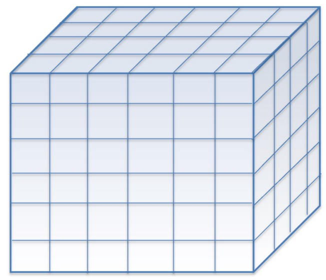
P. Bicudo, 2212.07793

.....

All presented results are extracted from E_n

(except from HALQCD Tcc)

$$\langle C \rangle = \int DG\, Dq\, D\bar{q}\; C\; e^{-S_{QCD}/\hbar}$$

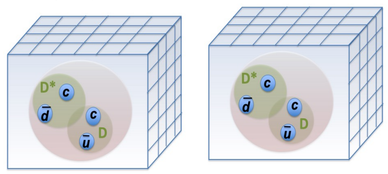


often "non-precision" studies:

single a, $m_{u/d} > m_{u/d}^{phy}$, $m_\pi > 140$ MeV

$$C_{ij}^{2pt}(t) = \langle 0 | \mathcal{O}_i(t)\; \mathcal{O}_j^{+}(0) | 0 \rangle = \sum_n \langle 0 | \mathcal{O}_i | n \rangle\; e^{-E_n t_E} \langle n | \mathcal{O}_j^{+} | 0 \rangle$$

t, J^P, I   t=0, J^P, I



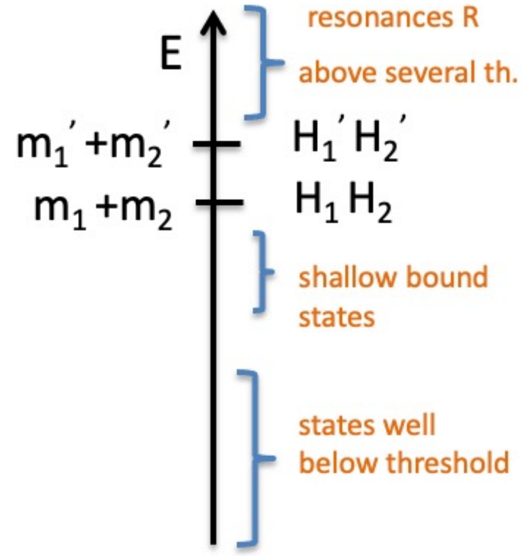
$$\mathcal{O} = \mathcal{O}(q, G)$$

- for strongly stable state well below threshold : $\boldsymbol{E_n(P=0)\; = \; m}$
- resonances (Luscher's relation) $\quad E_n^{cm} \to T(E_n^{cm})$
- static potentials: $\quad E_n \to V(r)$

Conclusions

Compliments to experimental colleagues for great results

Status on exotic hadrons from Lattice :



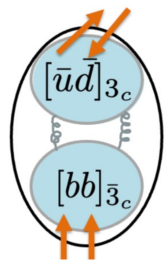
- exotic hadrons that are not resolved (yet)

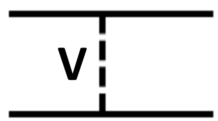
  strongly decay via many decay channels: $Z_c(4430)$, X(6900),...

- available: valuable results on exotic (and conventional) hadrons

  strongly stable ; strongly decaying to 1,2,3 channels

  support for specific binding mechanisms





...

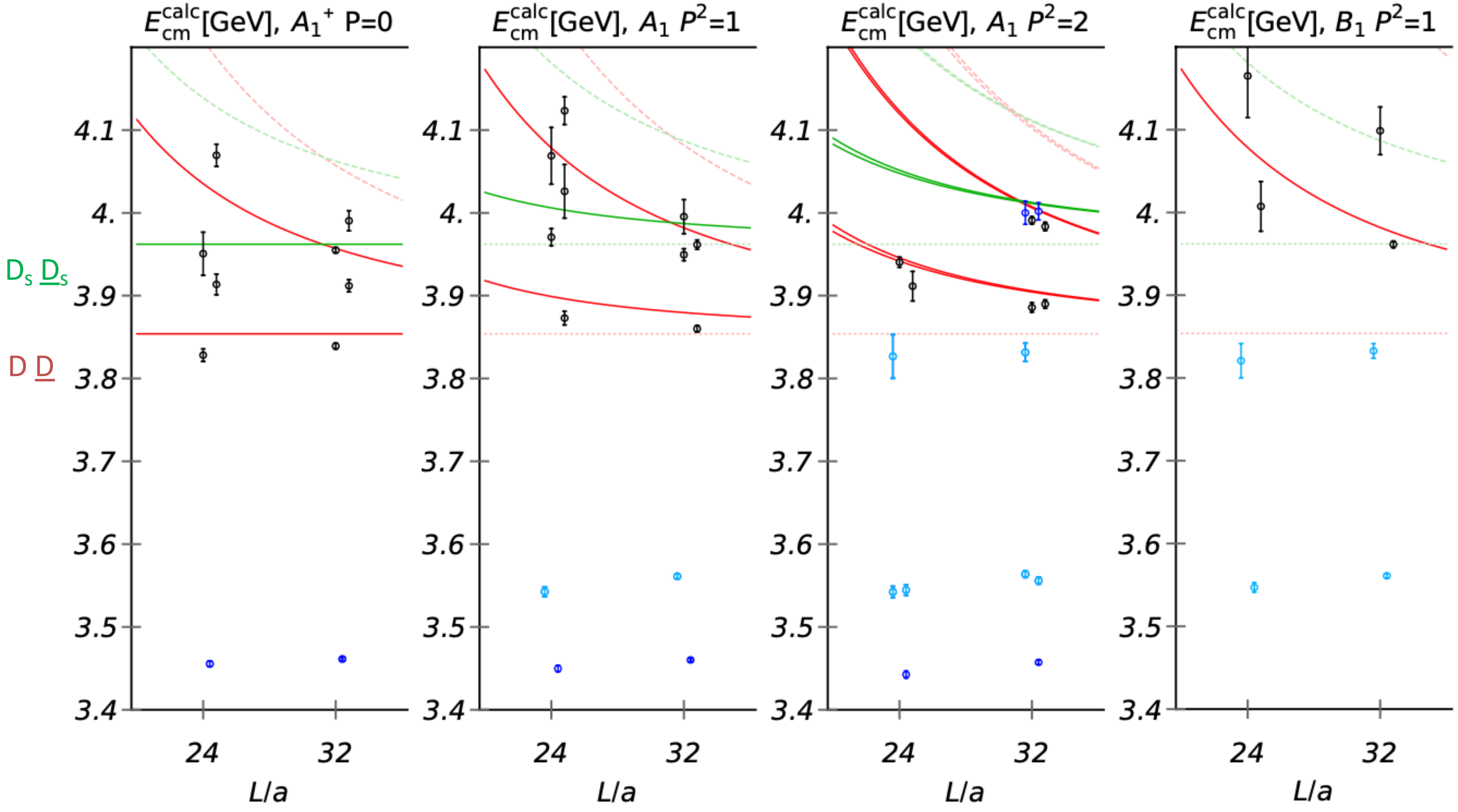
Pc, certain charmonium-like states

  one picture can not explain all exotic hadrons

  for each exotic hadron there is at least one viable picture

backup

Energies of eigen-states E_n in irreps that contain $J^{PC}=0^{++}, 2^{++}$



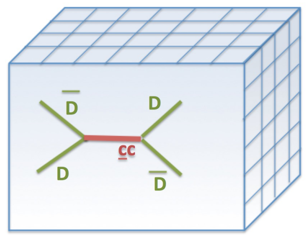
non-interacting energies denoted

D $\underline{D}$

D_s $\underline{D_s}$

$J^{PC}=0^{++}$

$J^{PC}=2^{++}, 2^{-+}$



$$E \not\to t(E)$$

$$S_{ij}(E_{cm}) = 1 + 2i\ \rho\ t_{ij}(E_{cm})$$

Extraction of matrix t(E) : NOT straightforward !

$$det[1 + i\ t(E_{cm})\ F(E_{cm})] = 0$$

known 2x2 matrix

$$t(E_{cm}) = \begin{vmatrix} t_{11}(E_{cm}) & t_{12}(E_{cm}) \\ t_{12}(E_{cm}) & t_{22}(E_{cm}) \end{vmatrix}$$

1: $\underline{D}$D, 2: $\underline{D}$sDs

one equation, three unknowns (at each E_{cm})

$$(t^{-1})_{ij} = \frac{2}{E_{cm}\ p_i^l p_i^l}\ (\tilde{K}^{-1})_{ij} - i\ \rho_i\ \delta_{ij} \qquad \frac{\tilde{K}^{-1}_{ij}(s)}{\sqrt{s}} = a_{ij} + b_{ij} s$$

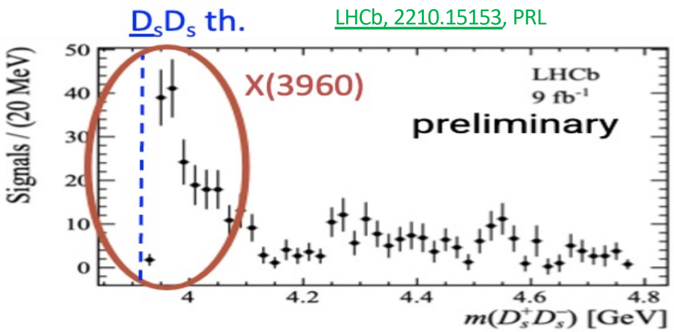
$$\rho_i \equiv 2p_i/E_{cm} \qquad s = E^2_{cm}$$

$J^{PC}=0^{++}$ $\bar{c}s\bar{s}c$

X(3915) , χ_{c0}(3930) , X(3960)

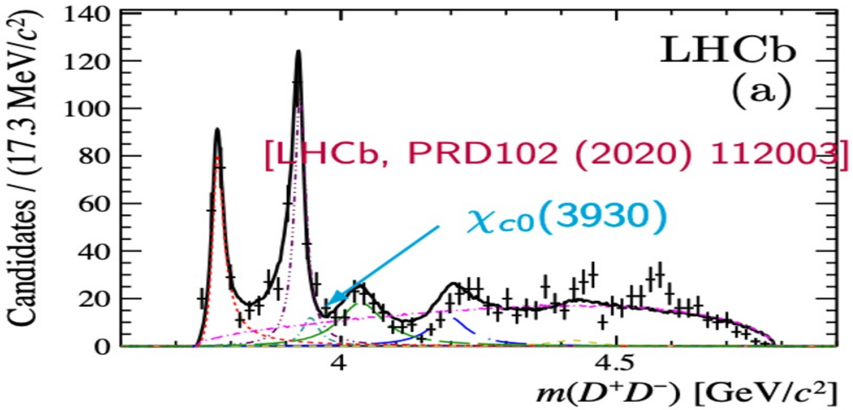
lat: $\frac{|c^2_{D\bar{D}}|}{|c^2_{D_s\bar{D}_s}|} = 0.02\ ^{+0.02}_{-0.01}$

all three likely the same state
currently named χ_{c0}(3914) in PDG

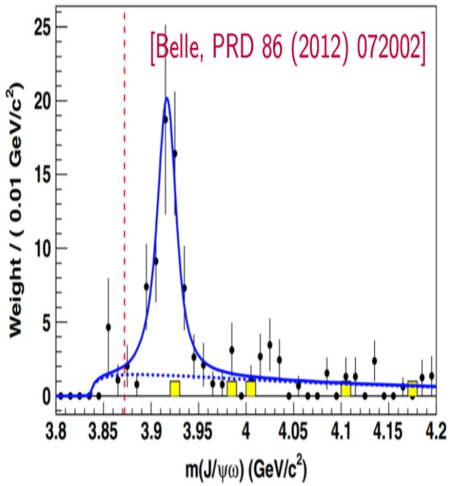


$X(3960) \to D_s\bar{D}_s$

exp: $\frac{Br(D\bar{D})}{Br(D_s\bar{D}_s)} \simeq 0.3$

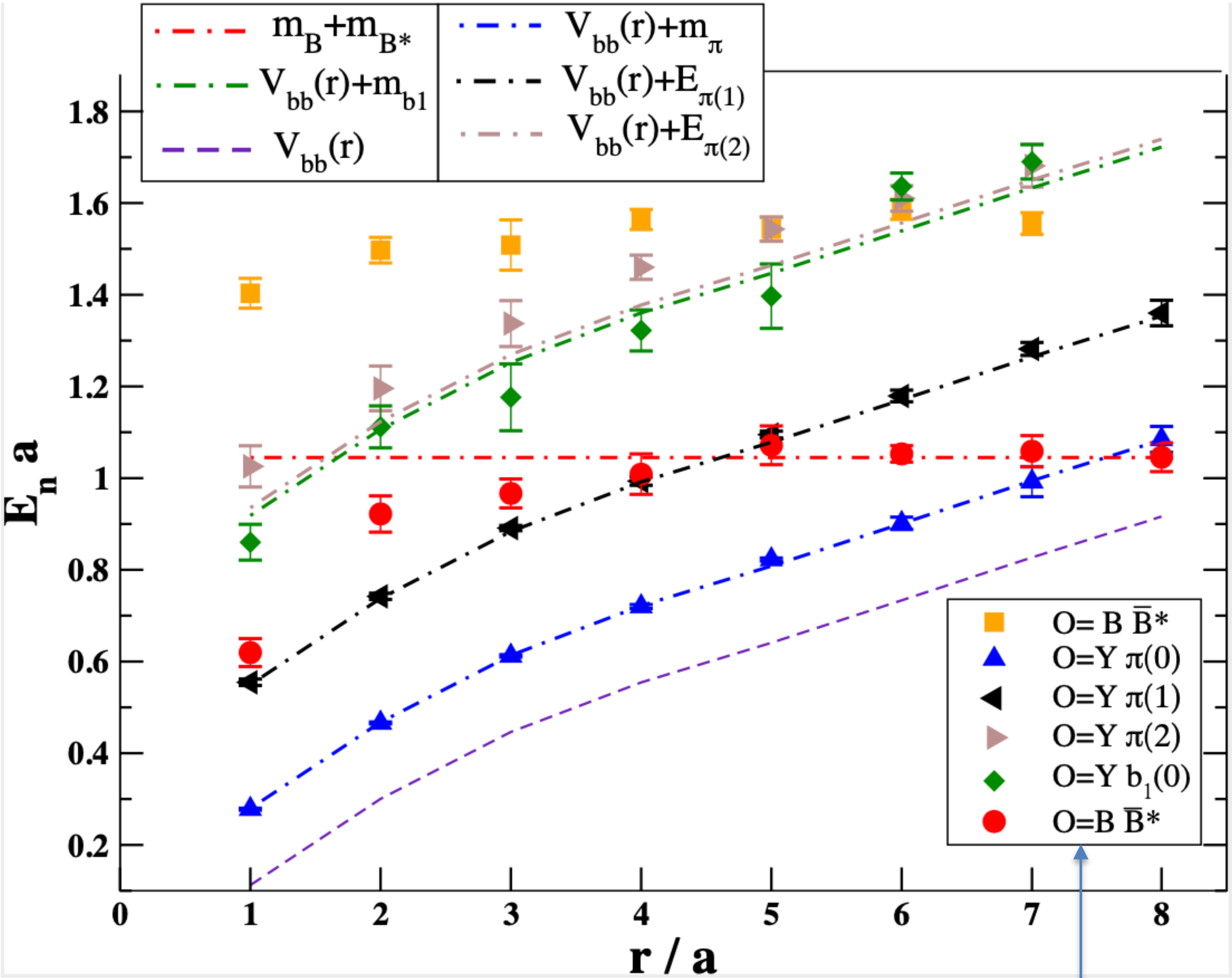


$\chi_{c0}(3930) \to D\bar{D}$



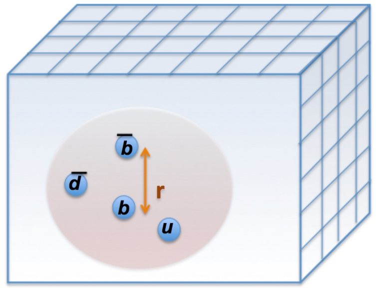
$X(3915) \to J/\psi\ \omega$

Eigen-energies $E_n(r)$: channel $S_h=1, J_l=0$ (CP=-1, ϵ=-1)



m_B+m_{B*}

dominant operator in each |n> according to $\langle O_i | n \rangle$



dot-dashed-lines:

$E_n^{non\text{-}int}$