Hadrons as laboratory for precision studies of the Electro-Weak sector and beyond

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International School of Nuclear Physics, 44th Course
« From quarks and gluons to hadrons and nuclei »
Erice, September 19 - 23, 2023

Outline :

1. Introduction and Motivation
2. Why hadronic physics matters?
3. Precision Hadronic Physics : selected examples
   1. Cabibbo angle anomaly
   2. Search for a light scalar mixing with the Higgs
4. Conclusion and outlook

1. Introduction and Motivation

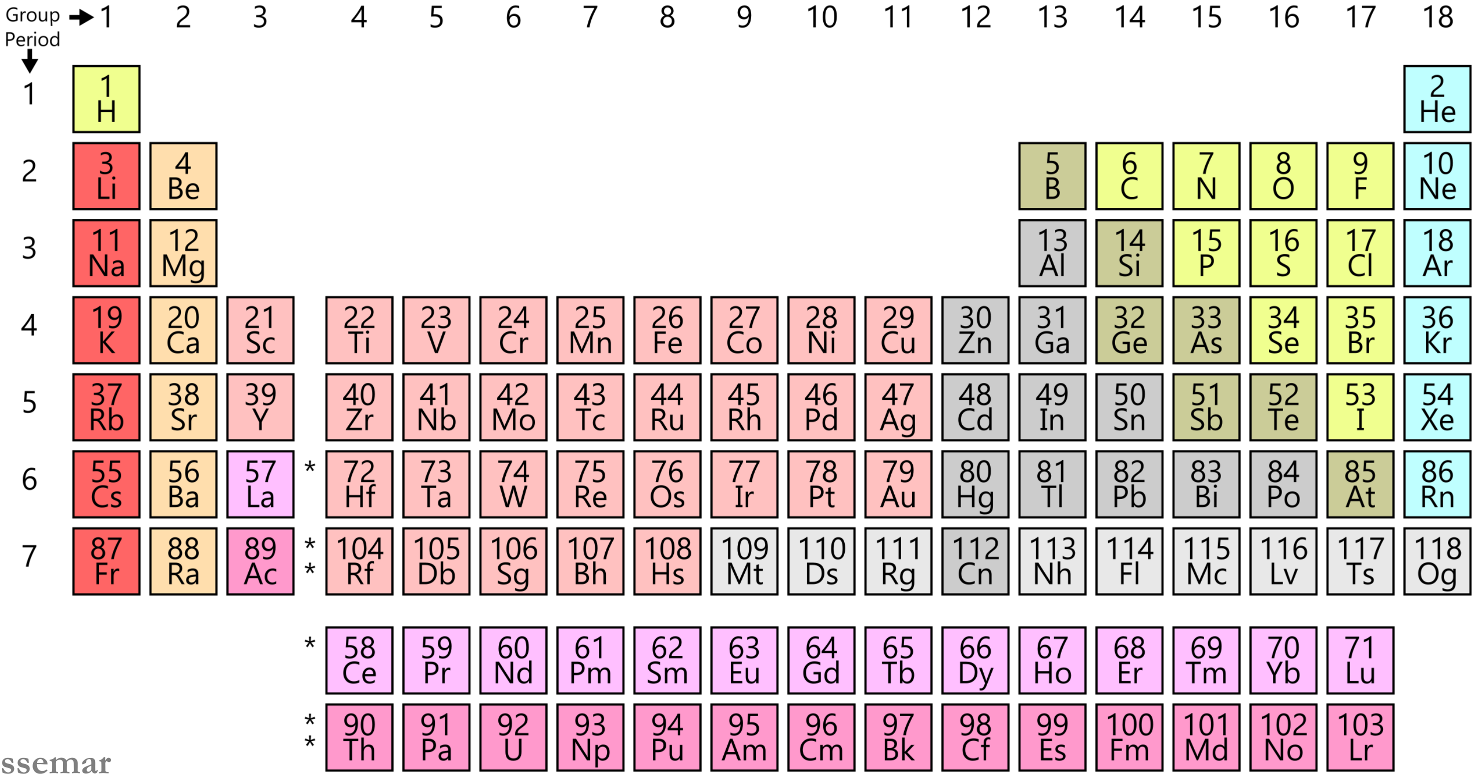
1.1 The Standard Model

- Particle and Nuclear Physics
  - extract fundamental parameters of Nature on the smallest scale
  - test our understanding of Laws of Nature

1.1 Precise test of the Standard Model

- Particle and Nuclear Physics
  - extract fundamental parameters of Nature at Quantum Level
  - test our understanding of Laws of Nature

- In Chemistry our knowledge summarized by Mendeleev table of chemical elements



Group → Period ↓	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																	2 He
2	3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La *	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac **	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
			*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
			**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

1.1 The Standard Model

- Particle and Nuclear Physics
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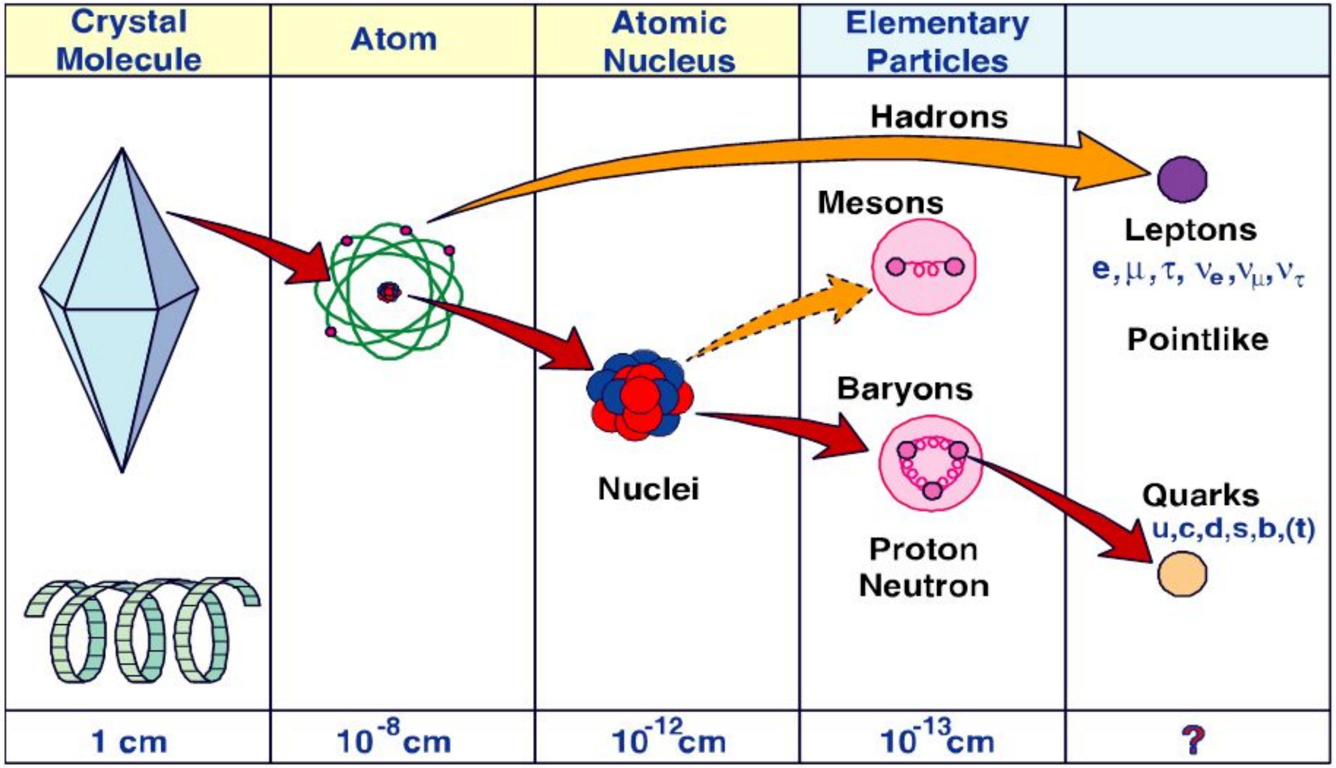
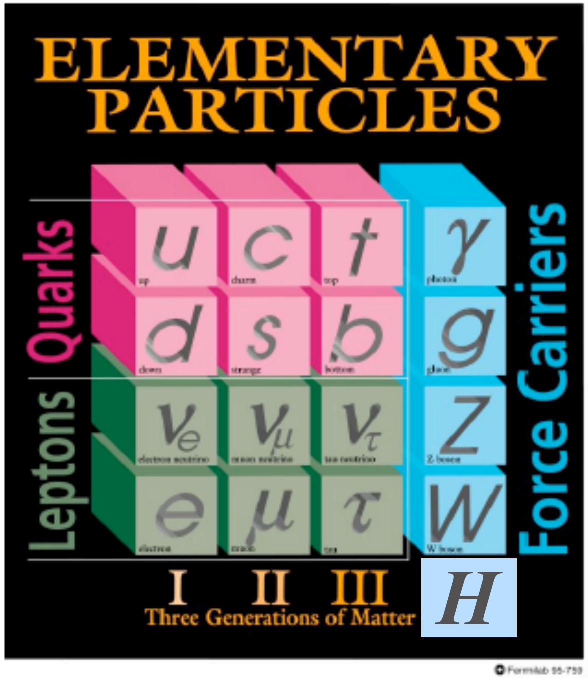
  - test our understanding of Laws of Nature

- In particle physics a simpler table made of leptons and quarks



1.1 The Standard Model

- In particle physics a simpler table made of leptons and quarks: the degrees of freedom



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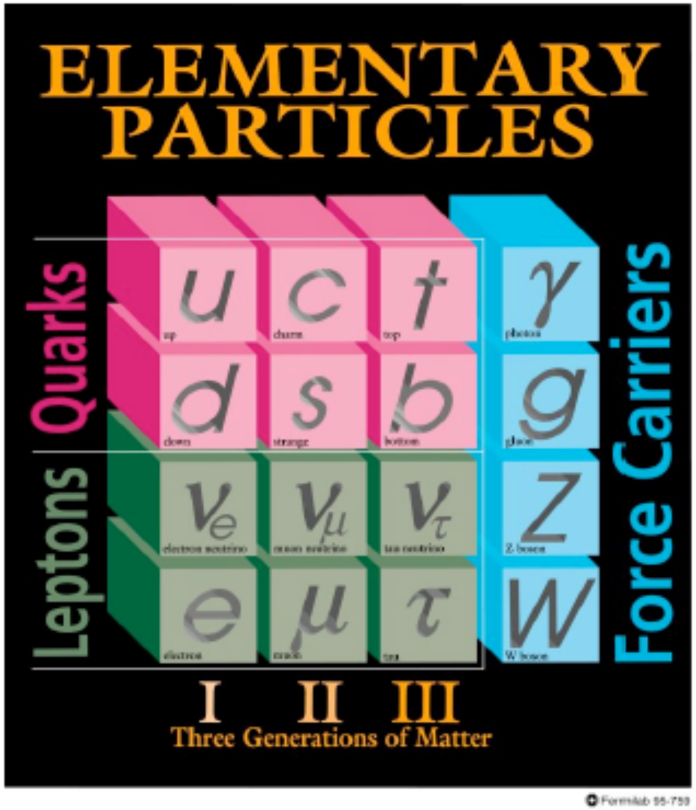
$$\begin{array}{ccccc} \text{Charge} & & & & \text{Int.} \\ \begin{matrix}0\\-1\end{matrix} & \begin{pmatrix}\nu_e\\ e\end{pmatrix} & \begin{pmatrix}\nu_\mu\\ \mu\end{pmatrix} & \begin{pmatrix}\nu_\tau\\ \tau\end{pmatrix} & \begin{matrix}w\\ w,e\end{matrix} \\ \begin{matrix}+2/3\\-1/3\end{matrix} & \begin{pmatrix}u\\ d\end{pmatrix} & \begin{pmatrix}c\\ s\end{pmatrix} & \begin{pmatrix}t\\ b\end{pmatrix} & w,e,s \end{array}$$

- 3 forces: electromagnetic, weak and strong forces

1.1 The Standard Model



Governed by gauge symmetry principle

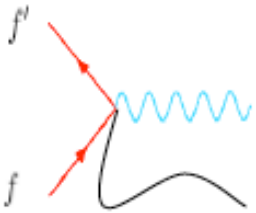
$$\overset{\nearrow}{SU(3)_C} \times \underbrace{SU(2)_{I_w} \times U(1)_Y}$$

Strong force

Unified Electro-weak interactions

Introduce massless gauge bosons
(force carriers)

↓

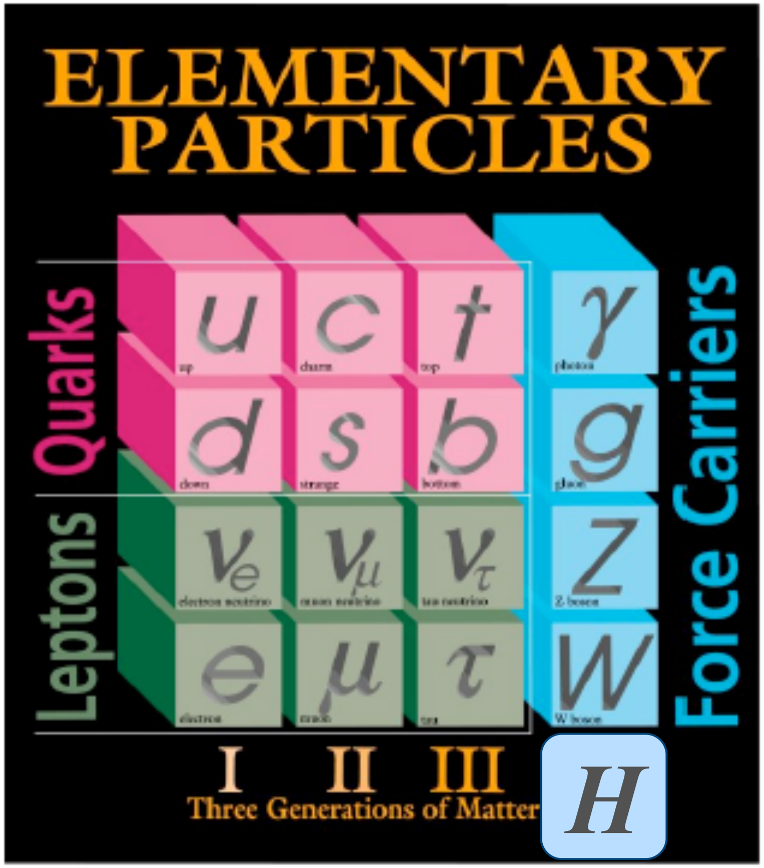


A_μ

e, g_{strong} , g_{weak}

$$\mathcal{L}_I(x) \sim J_\mu(x)\, A^\mu(x)$$

1.1 The Standard Model



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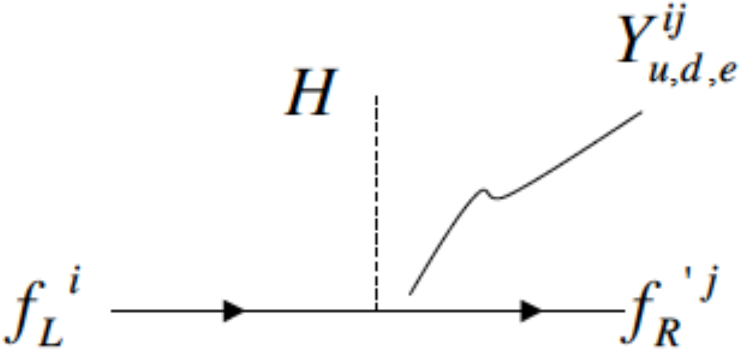
Yukawa interaction

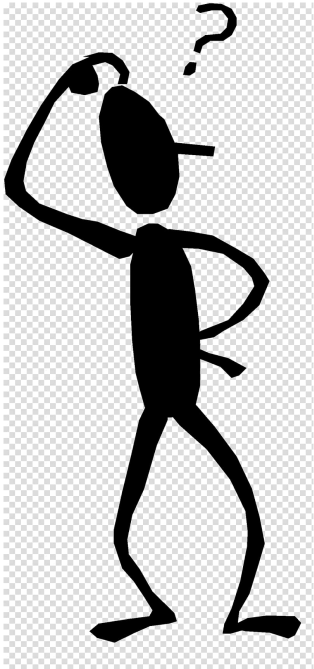
(matter-Higgs)



Massive fermions after EWSB

The mediators of weak interaction (W, Z) become massive through the Higgs Mechanism ⇒ one scalar particle remains in the spectrum: H

1.2 Challenges



- Searching physics beyond the Standard Model:
  - Are there new forces besides the 3 gauge group?
  - Are there new particles?
  - A more profound understanding of the origin of this table?
  - Origin of matter/anti-matter asymmetry
  - Origin of dark matter

- One type of new physics already discovered: neutrino masses

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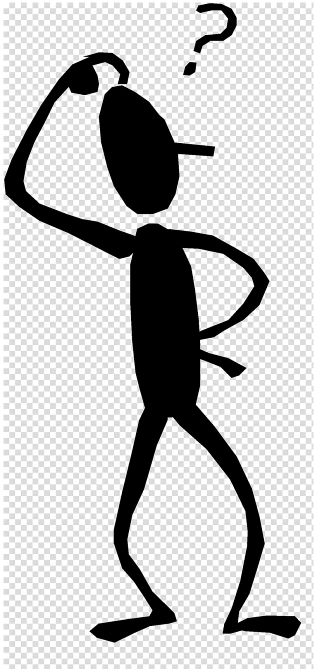

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- In this quest it is essential to have a *robust understanding* of *Hadronic Physics*

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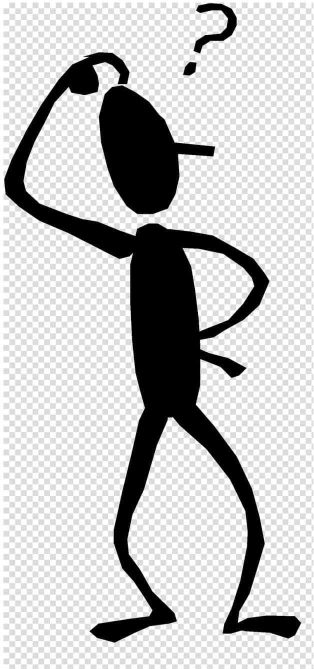

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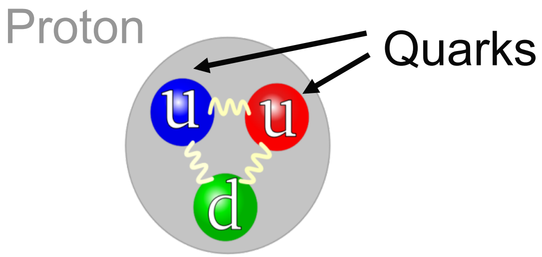
- In this quest it is essential to have a *robust understanding* of *Hadronic Physics*

- This is true for quarks and leptons and even for neutrinos!

2. Why hadronic physics matters?

2.1 Quark masses

- Let us consider the proton: it is not a fundamental particle, but a bound state of 3 quarks



Contrary to naïve expectation, most of its mass comes from *strong force*

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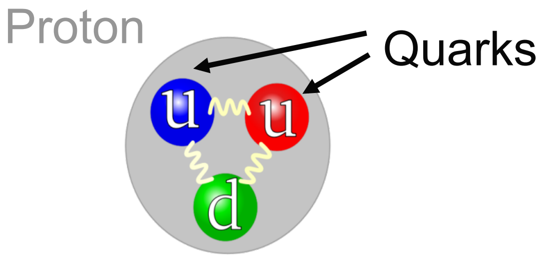

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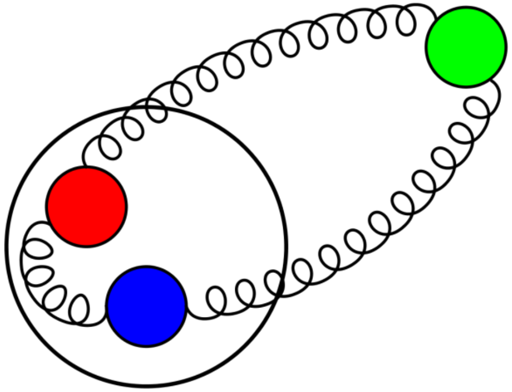
- How can we access the quark masses?

Strong interaction

- Problem: quarks and gluons are not free particles: they are bound inside hadrons



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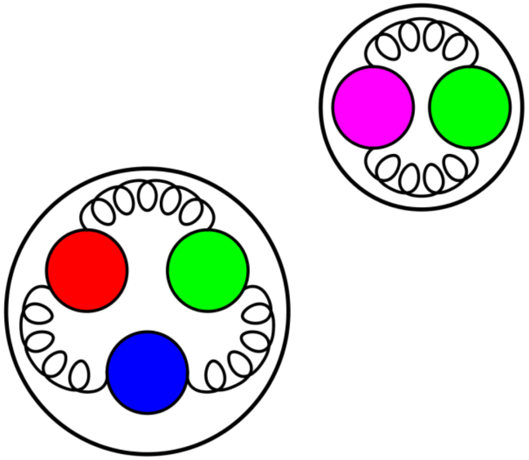

- Two properties:
  - Confinement
  - Asymptotic freedom : The interaction decreases at high energies Nobel Prize in 2004 for Frank Wilczek and David Gross and David Politzer

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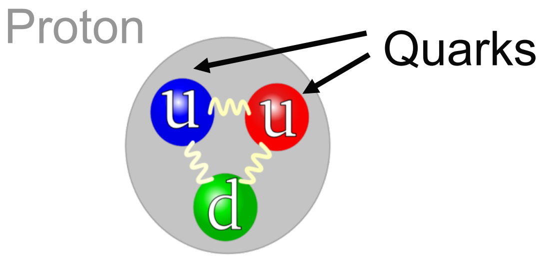

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Only 1% of its mass comes from the quark masses (Coupling of the quarks to the Higgs boson)

- How can we access the quark masses?
- In principle a theory → Quantum ChromoDynamics

→

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k}$$

Formulation of QCD

- SU(3)$_C$ QCD invariant Lagrangian

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- Different parts to describe the interactions

$$\mathcal{L}_{QCD} = -\frac{1}{4}\left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right)\left(\partial_\mu G_\nu^a - \partial_\nu G_\mu^a\right) + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu \partial_\mu - m_k \right) q_k$$

$$+ g_S G_a^\mu \sum_{k=1}^{N_F} \bar{q}_k \gamma_\mu \left(\frac{\lambda_a}{2} \right) q_k$$

$$- \frac{g_S}{2} f^{abc} \left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right) G_\mu^b G_\nu^c - \frac{g_S^2}{4} f^{abc} f_{ade} G_b^\mu G_c^\nu G_\mu^d G_\nu^e$$

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Kinetic terms

$$+ g_S G_a^\mu \sum_{k=1}^{N_F} \bar{q}_k \gamma_\mu \left(\frac{\lambda_a}{2} \right) q_k$$

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$$\Rightarrow \boxed{\mathcal{L}_{QCD} = -\frac{1}{4} G_a^{\mu\nu} G^a_{\mu\nu} + \sum_{k=1}^{N_F} \bar{q}_k \left(i\gamma^\mu D_\mu - m_k \right) q_k}$$

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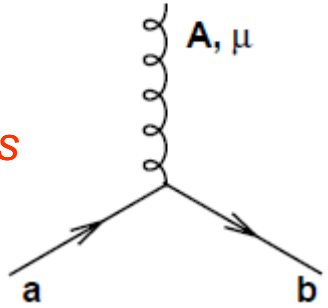
Interaction quarks gluon



$$-\frac{g_S}{2} f^{abc} \left(\partial^\mu G_a^\nu - \partial^\nu G_a^\mu\right) G^b_\mu G^c_\nu - \frac{g_S^2}{4} f^{abc} f_{ade} G_b^\mu G_c^\nu G^d_\mu G^e_\nu$$

Formulation of QCD

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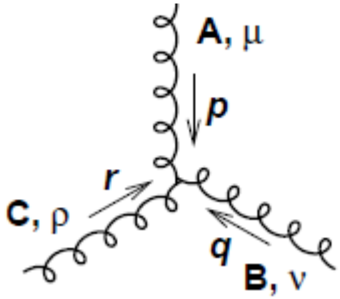
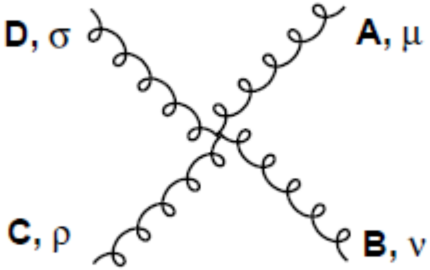


Interaction gluon gluon



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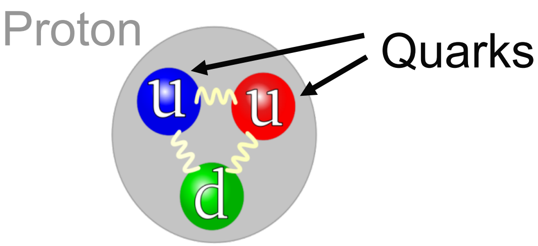
- ➢ One single universal coupling : $\boxed{\alpha_S(\mu) = \frac{g_S^2(\mu)}{4\pi}}$ *strong coupling constant*

- ➢ It is not a constant, depends on the *energy !*

Strong interaction

- Problem: quarks and gluons are bound inside hadrons



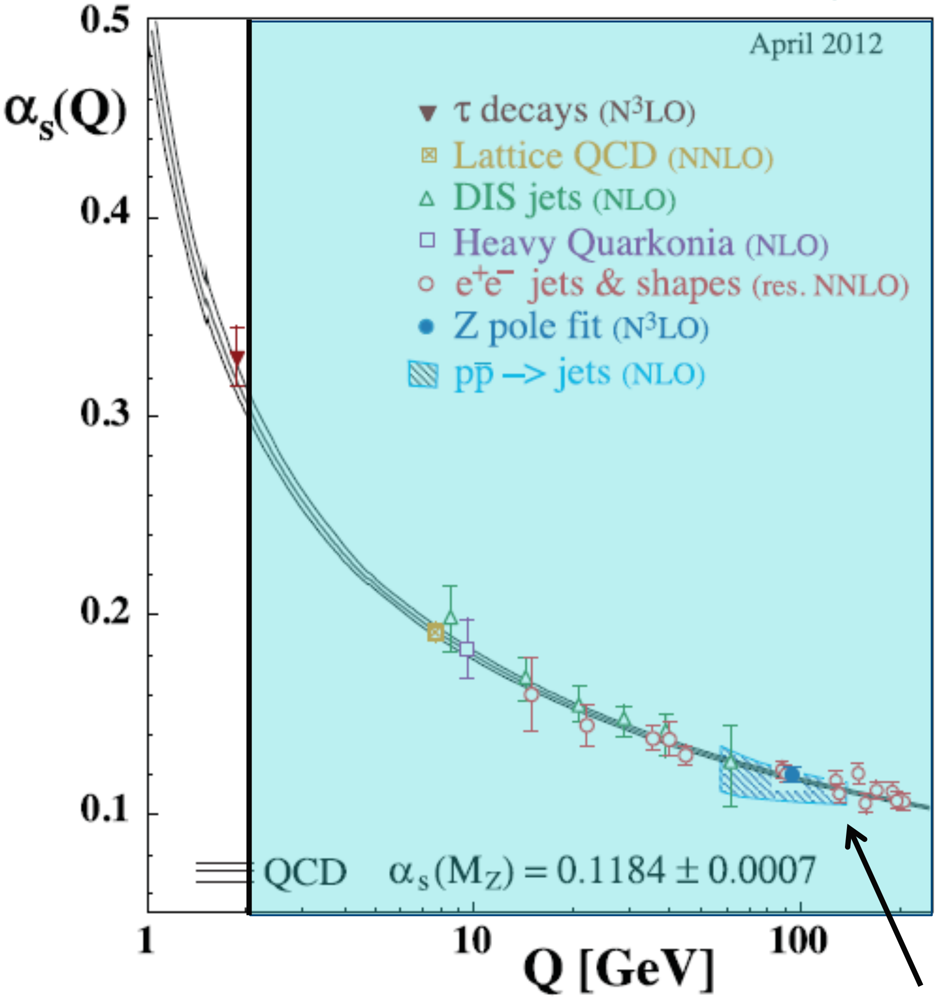
- High energies, short distance:
  α_S small ⇨ Asymptotic freedom

  Perturbative QCD
  Theory “easy” to solve

  Order-by-order *expansion* in $\frac{\alpha_S(\mu)}{\pi}$

$$\sigma = \sigma_0 + \underbrace{\frac{\alpha_S}{\pi}\sigma_1}_{\text{small}} + \underbrace{\left(\frac{\alpha_S}{\pi}\right)^2\sigma_2}_{\text{smaller}} + \left(\frac{\alpha_S}{\pi}\right)^3\sigma_3 + \dots$$

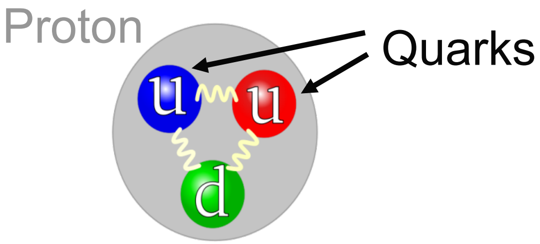
PDG’12



Asymptotic freedom

Strong interaction

- Looking for new physics in hadronic processes ⇨ not direct access to quarks due to confinement



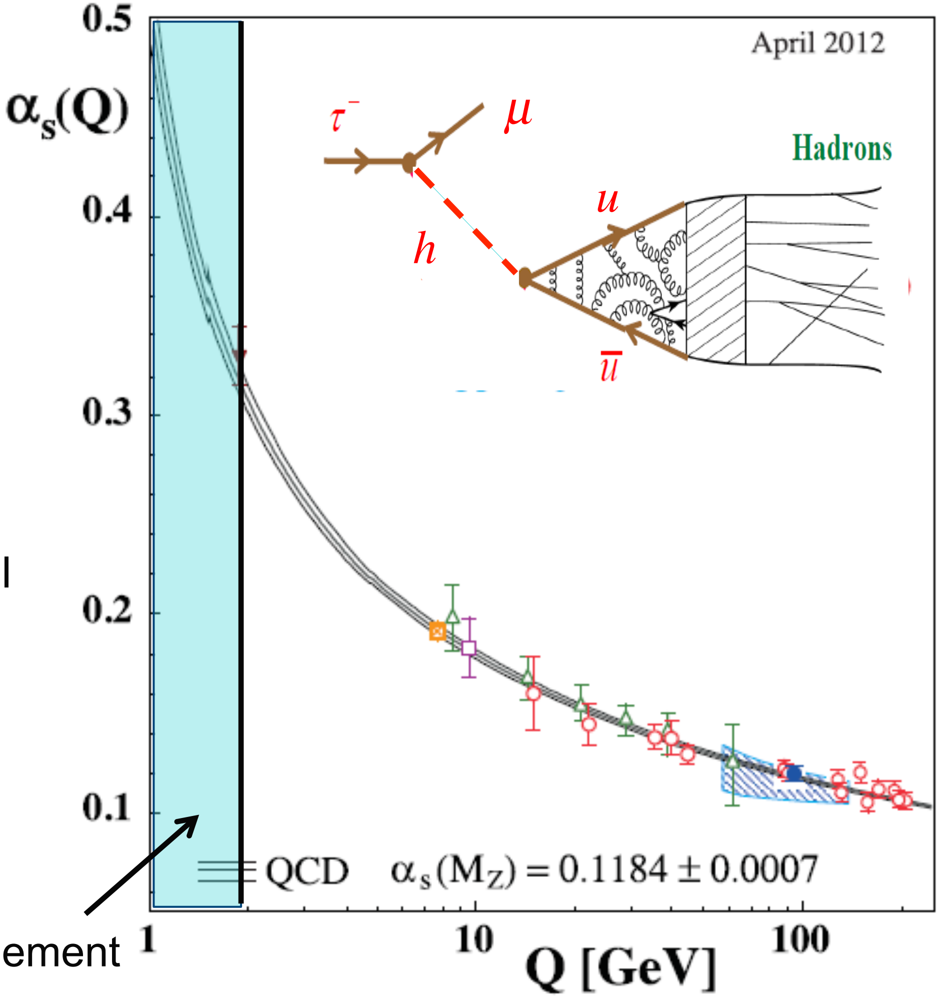
- Low energy (Q <~1 GeV), long distance: *α_S becomes large !*

⇨ *Non-perturbative QCD*

A perturbative expansion in the usual sense fails

⇨ Use of alternative approaches, expansions…

PDG'12



Confinement

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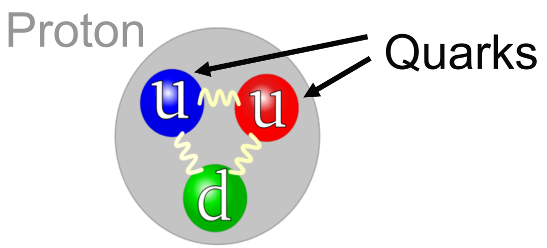
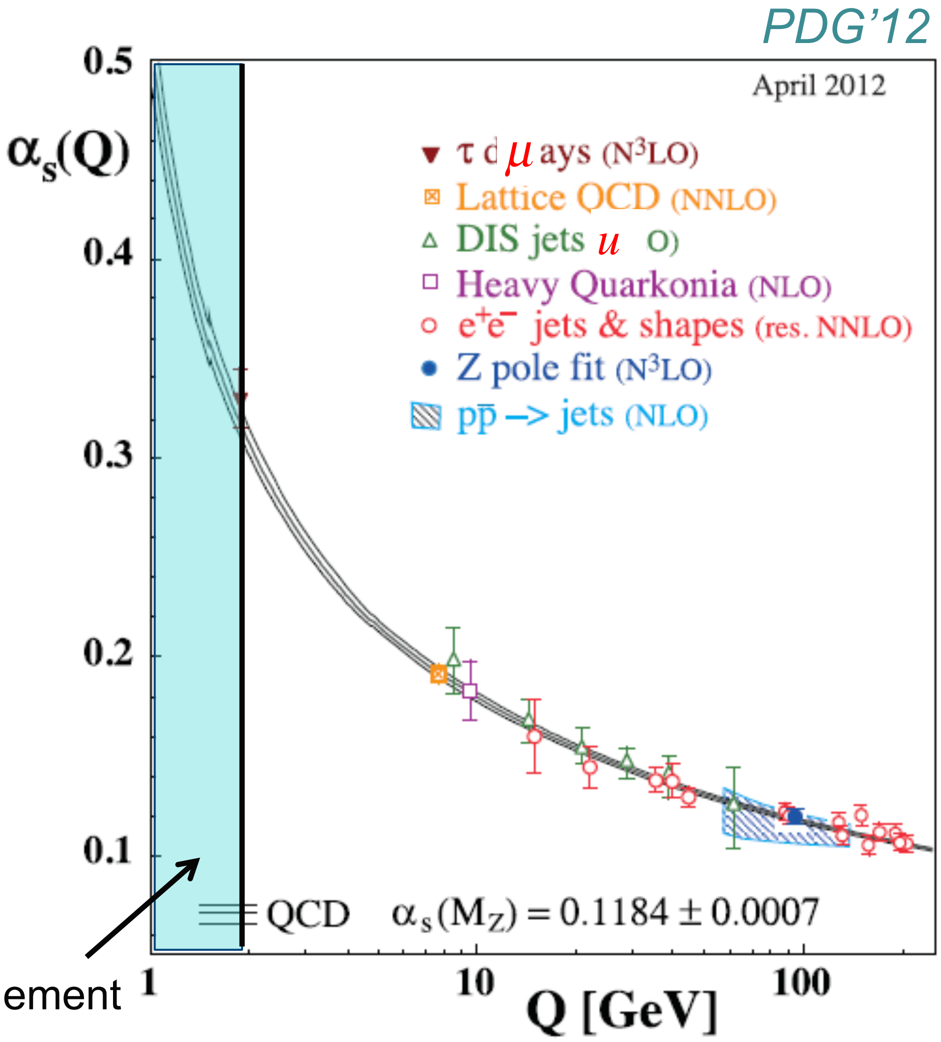
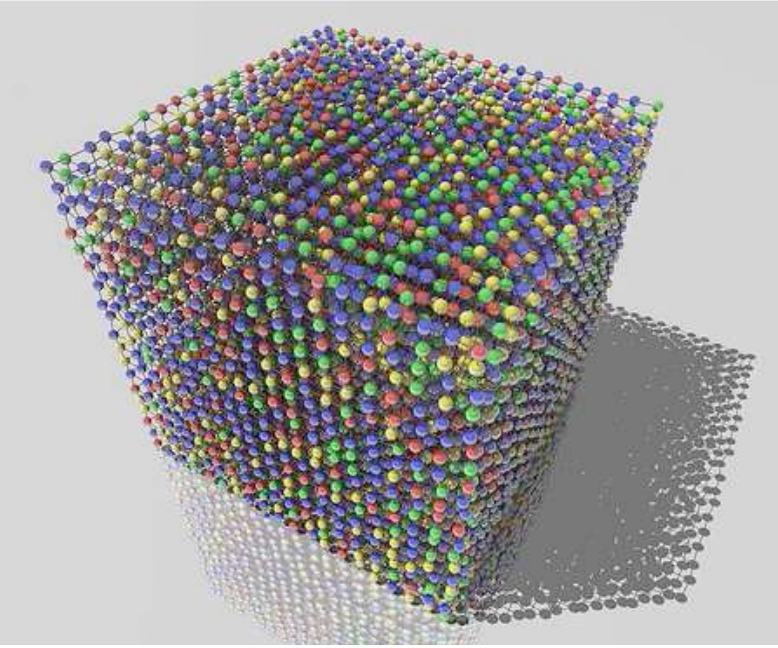




- ➢ Non-perturbative methods:
  - Numerical simulations on the lattice

Lattice QCD

- Principle: Discretization of the space time and solve QCD on the lattice numerically
  - All quark and gluon fields of QCD on a 4D-lattice
  - Field configurations by Monte Carlo sampling



- Important subtleties due to the discretization, should come back to the continuum, formulation of the fermions on the lattice…

See talks by *M. Hansen, S. Prelovsek*

Strong interaction

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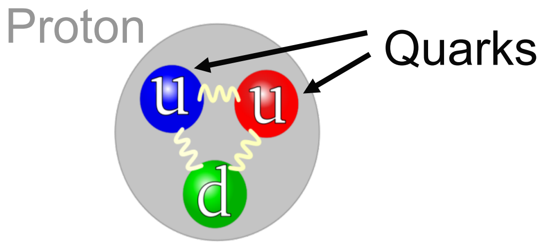
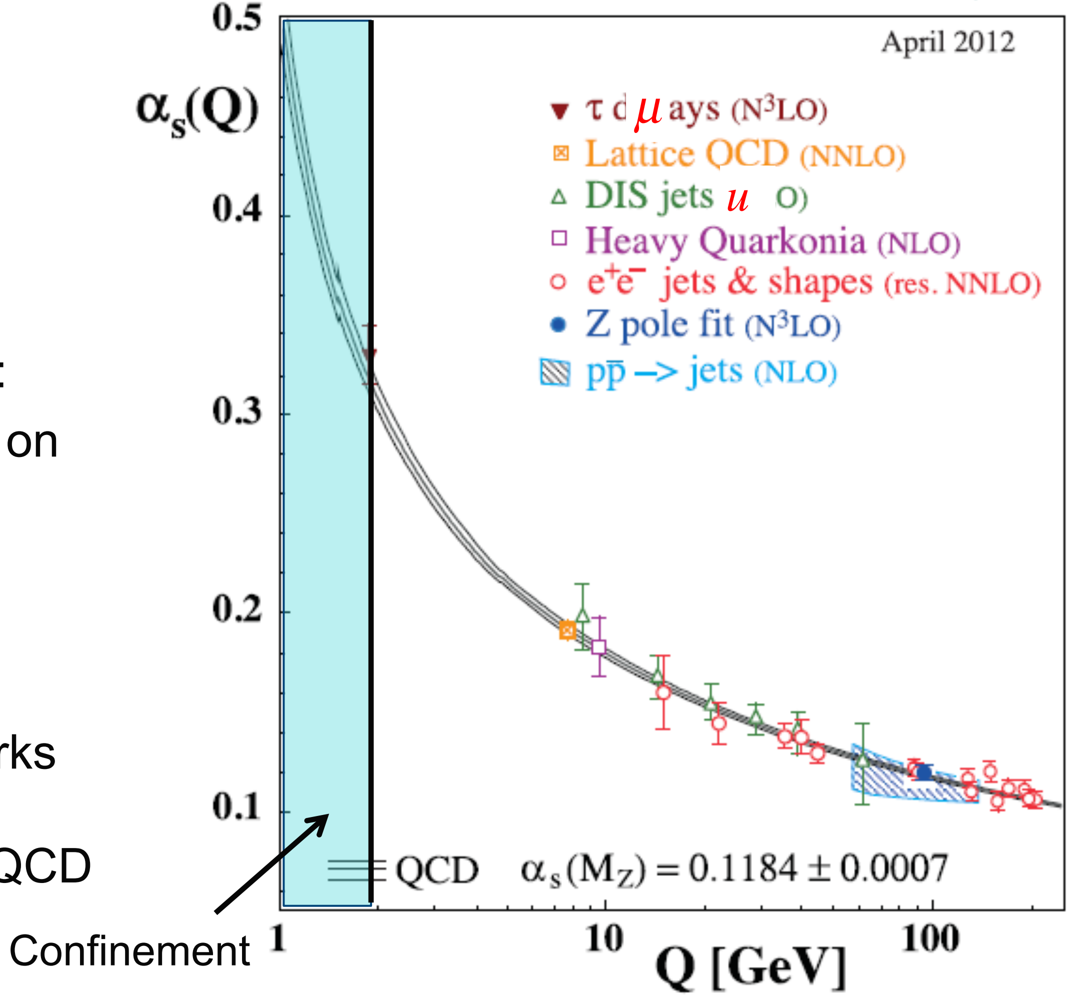


➢ Non-perturbative methods:
  - Numerical simulations on the lattice
  - Analytical methods:
    Effective field theory
    Ex: ChPT for light quarks
    Dispersion relations
    Synergies with lattice QCD

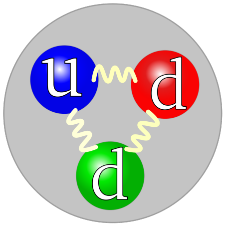
⇨ *Hadronic Physics*

PDG'12



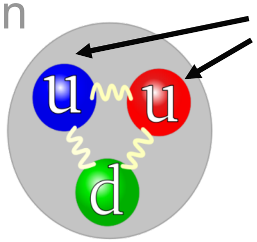
2.1 Quark masses

Neutron



vs.

Proton



Quarks

$$\boxed{M_n = 939.57 \text{ MeV}}$$

$$\boxed{M_p = 938.27 \text{ MeV}}$$

- Strong force: If $m_u \sim m_d$: $M_n \sim M_p$ *isospin symmetry* *Heisenberg'60*

  Countless experiments have shown that strong force obeys isospin symmetry Results are the same if we interchange neutrons and protons (or up and down quarks)

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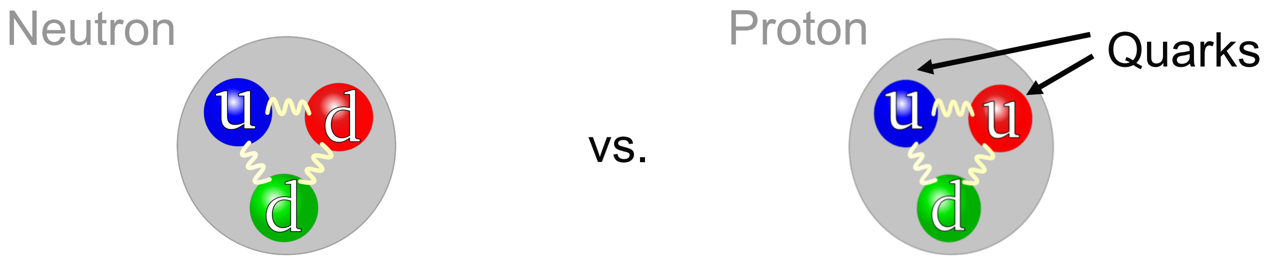

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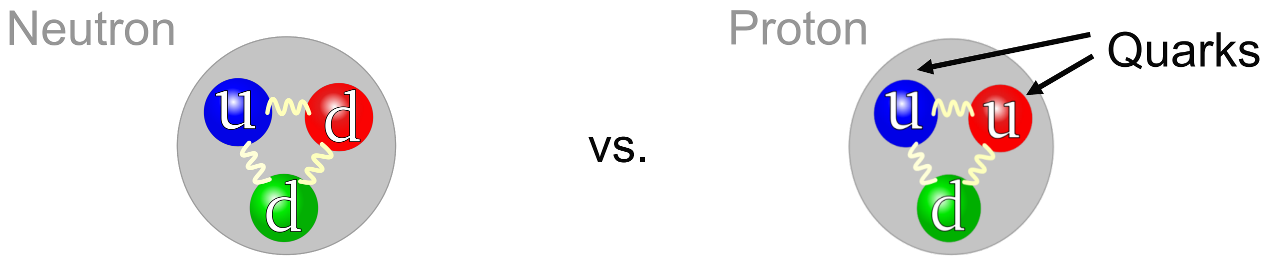

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  Results are the same if we interchange neutrons and protons

- Electromagnetic energy: one obvious difference between a neutron and a proton is their electric charges:

$Q_P = 1$ and $Q_n = 0$ Since $E_e \propto \frac{Q^2}{R}$ ⇒ $\boxed{M_p > M_n}$ **?**

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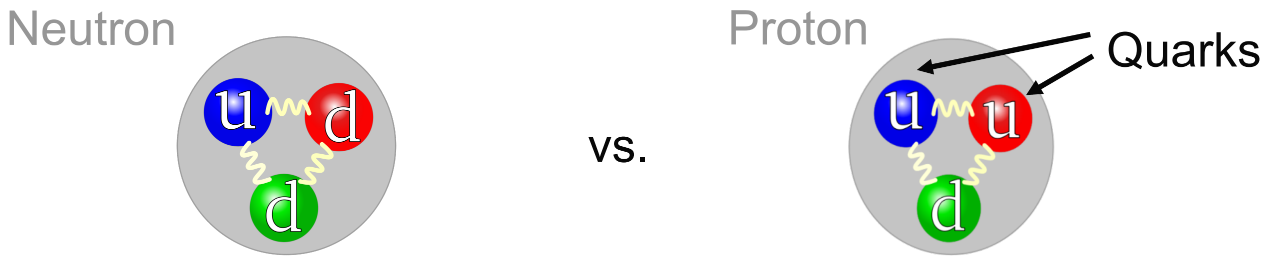

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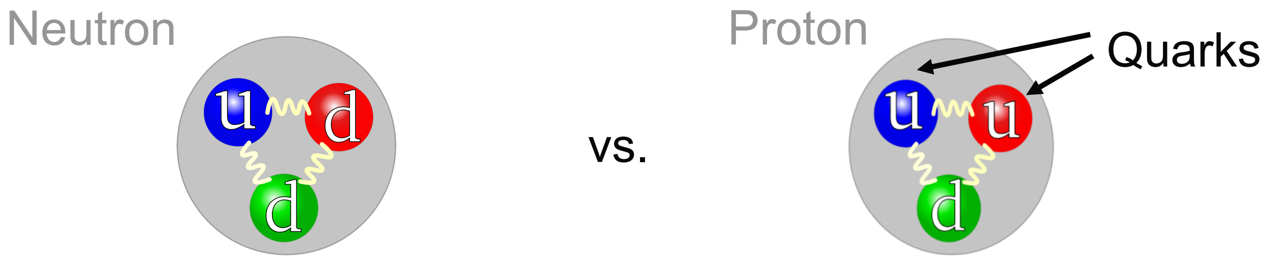
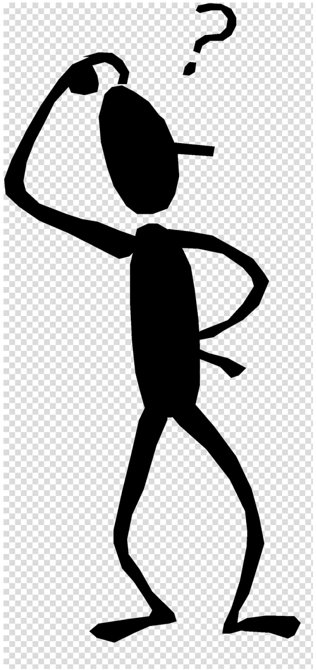
- Electromagnetic energy: one obvious difference between a neutron and a proton is their electric charges:

$Q_p = 1$ and $Q_n = 0$ Since $E_e \propto \frac{Q^2}{R}$ ⇒ $\boxed{M_p > M_n}$ **?**

⇒ *Terrible consequences :* Proton would decay into neutrons and there will be no chemistry and we would not be there in this room!

2.1 Quark masses





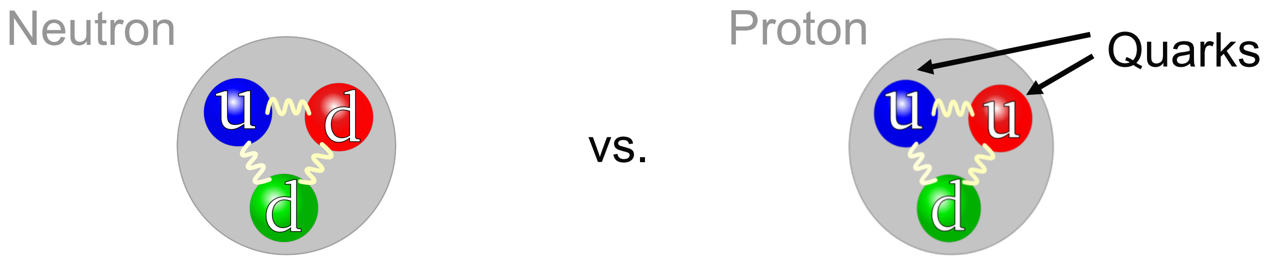
- Strong force: If $m_u \sim m_d$: $M_n \sim M_p$ *isospin symmetry*

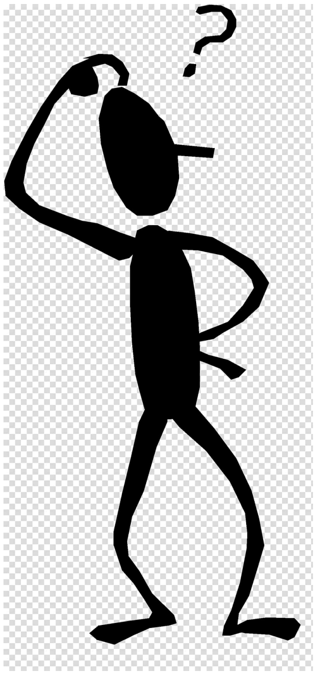
  Heisenberg'60

- Electromagnetic energy: $\boxed{M_p > M_n}$

- This is not the case: *Why?*

2.1 Quark masses





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  Heisenberg'60

- Electromagnetic energy: $\boxed{M_p > M_n}$

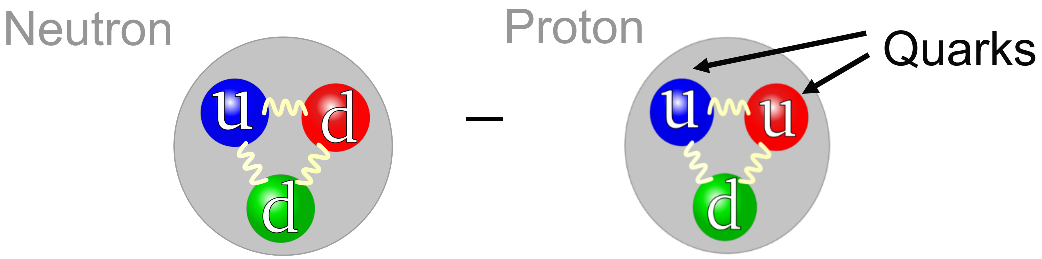
- This is not the case: *Why?*

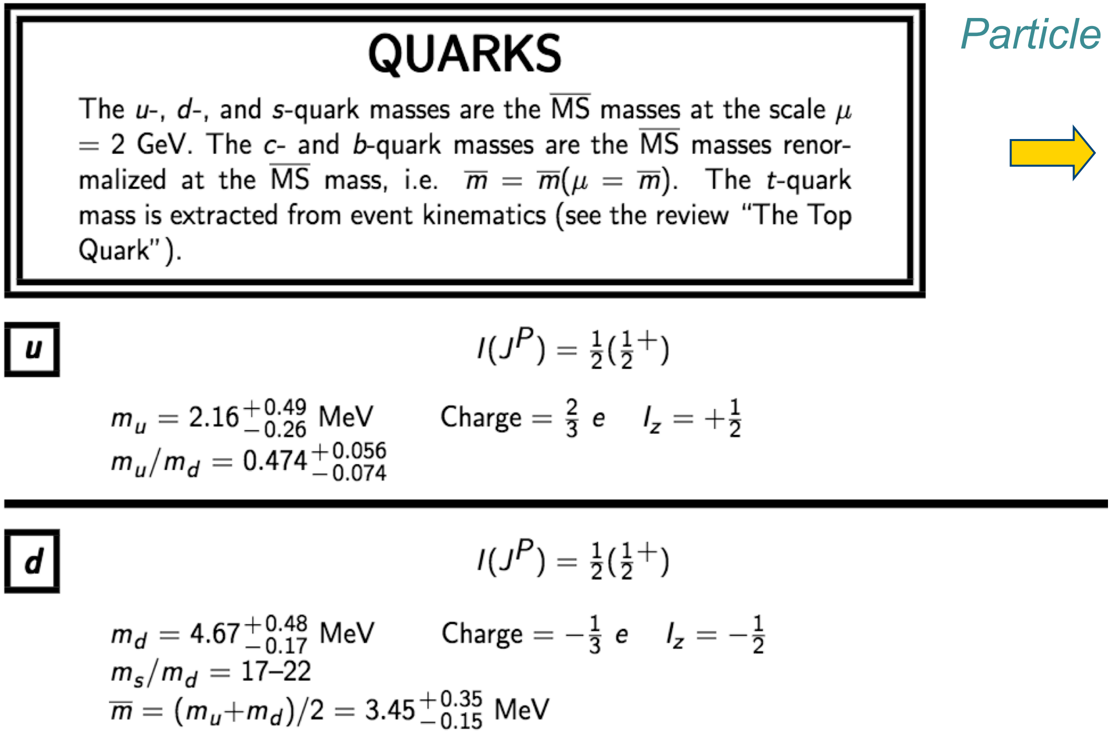
- Another small effect in addition to e.m. force:

  different fundamental quark masses $\boxed{m_d \neq m_u}$

  ⇒ Different coupling to Higgs field

2.1 Quark masses





QUARKS

The u-, d-, and s-quark masses are the $\overline{\text{MS}}$ masses at the scale $\mu = 2$ GeV. The c- and b-quark masses are the $\overline{\text{MS}}$ masses renormalized at the $\overline{\text{MS}}$ mass, i.e. $\overline{m} = \overline{m}(\mu = \overline{m})$. The t-quark mass is extracted from event kinematics (see the review "The Top Quark").

u $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

$m_u = 2.16^{+0.49}_{-0.26}$ MeV Charge $= \frac{2}{3}\ e$ $I_z = +\frac{1}{2}$

$m_u/m_d = 0.474^{+0.056}_{-0.074}$

d $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

$m_d = 4.67^{+0.48}_{-0.17}$ MeV Charge $= -\frac{1}{3}\ e$ $I_z = -\frac{1}{2}$

$m_s/m_d = 17–22$

$\overline{m} = (m_u+m_d)/2 = 3.45^{+0.35}_{-0.15}$ MeV

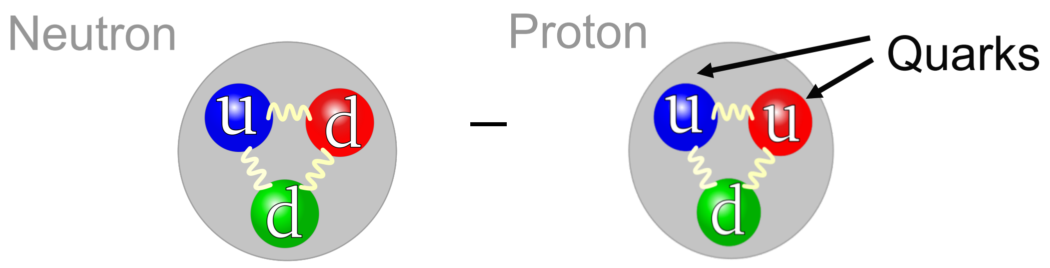
Particle Data Group'22

$$\Rightarrow \quad \boxed{m_d - m_u = 4.7 - 2.2 = 2.5 \text{ MeV}}$$

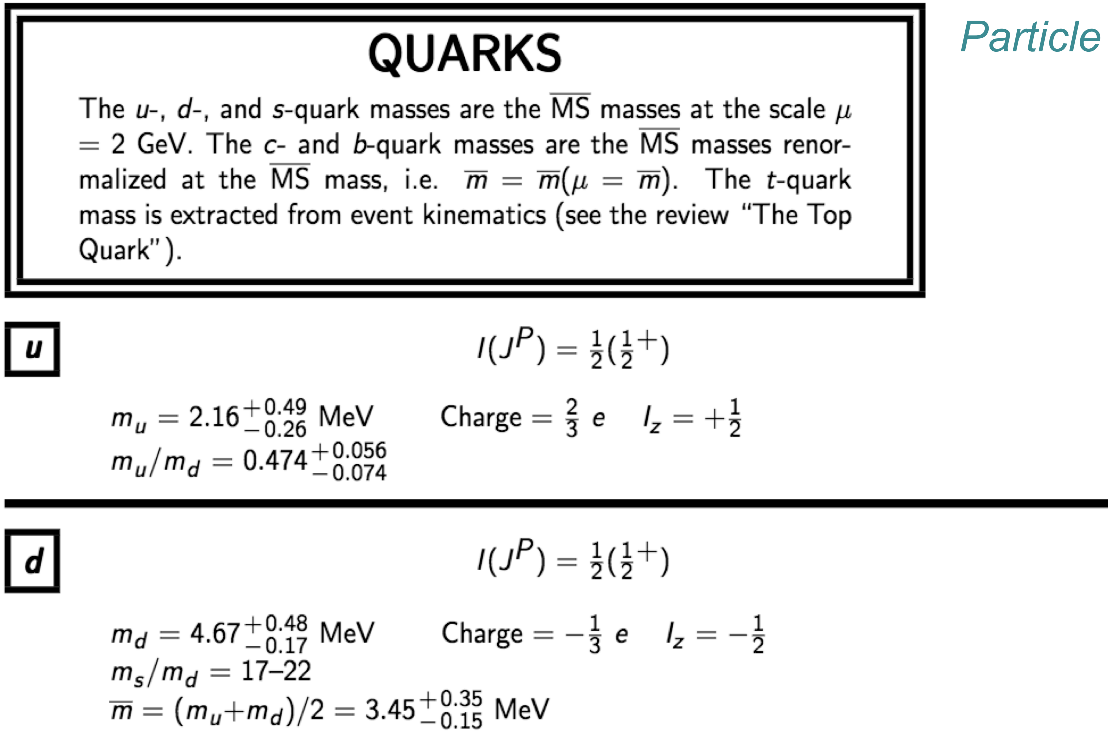
Quark mass difference more important than e.m. effect

Neutrons can decay in protons!

2.1 Quark masses



Particle Data Group'22



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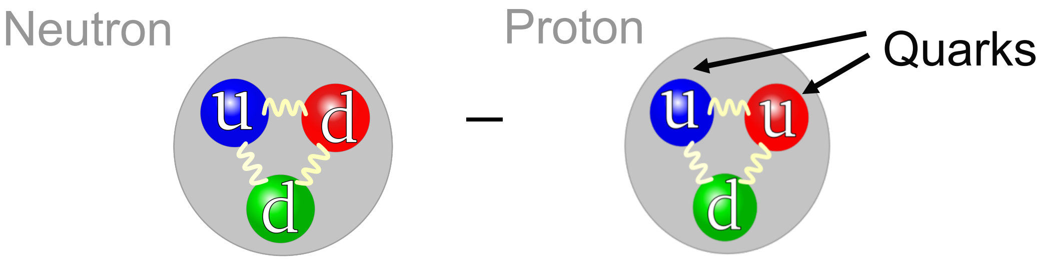
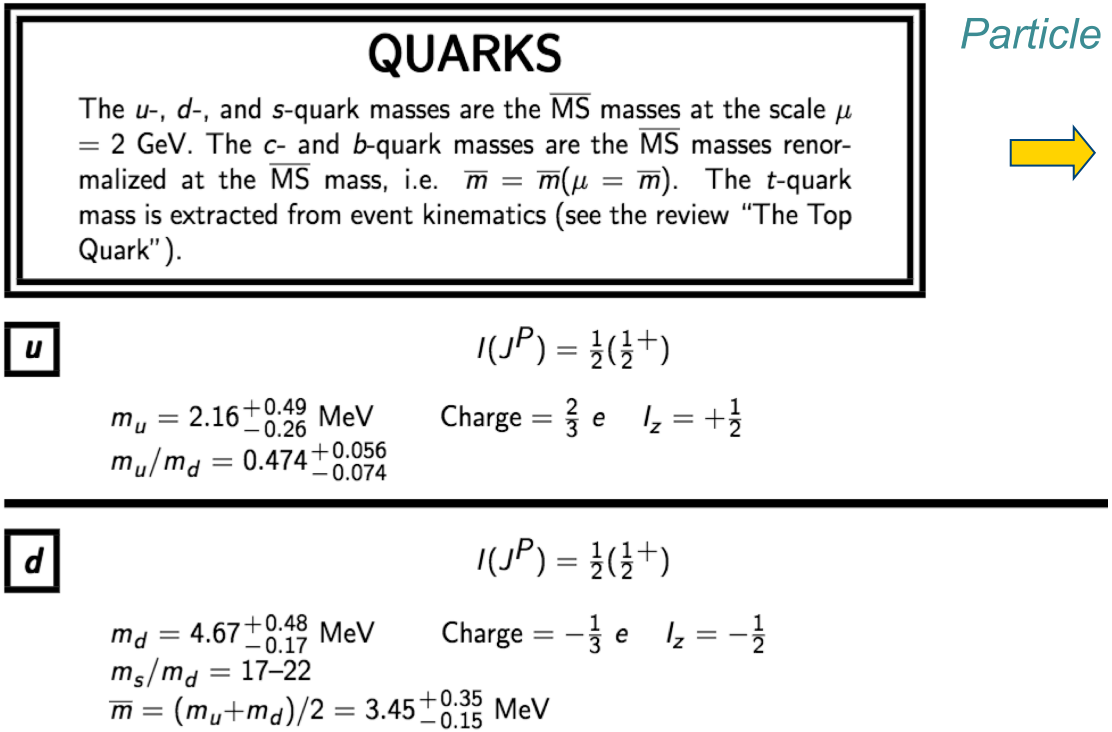
Quark mass difference more important than e.m. effect

Neutrons can decay in protons!

Neutron lifetime experiments

2.1 Quark masses





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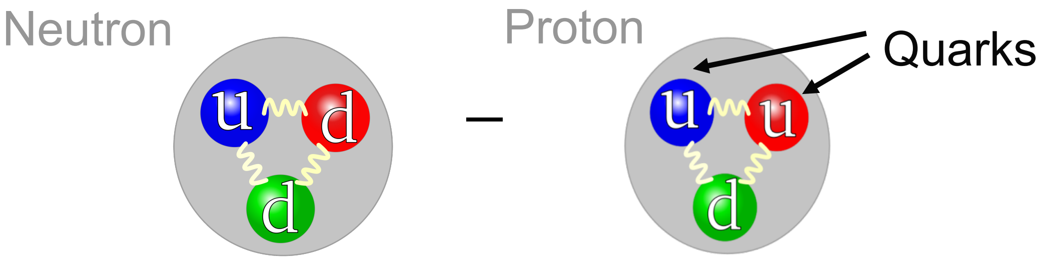
Particle Data Group'22

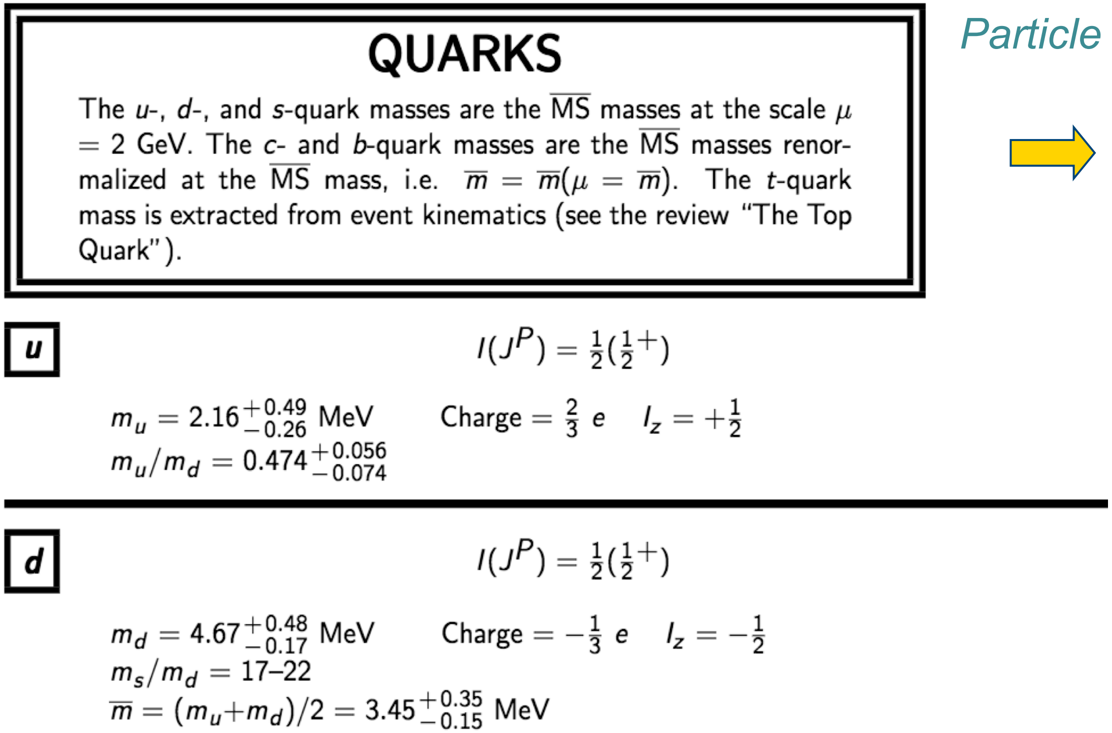
⇨ $\boxed{m_d - m_u = 4.7 - 2.2 = 2.5 \text{ MeV}}$

To determine these fundamental parameters need to know how to disentangle them from *QCD*

⇨ treat *strong interactions*

2.1 Quark masses





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Particle Data Group'22

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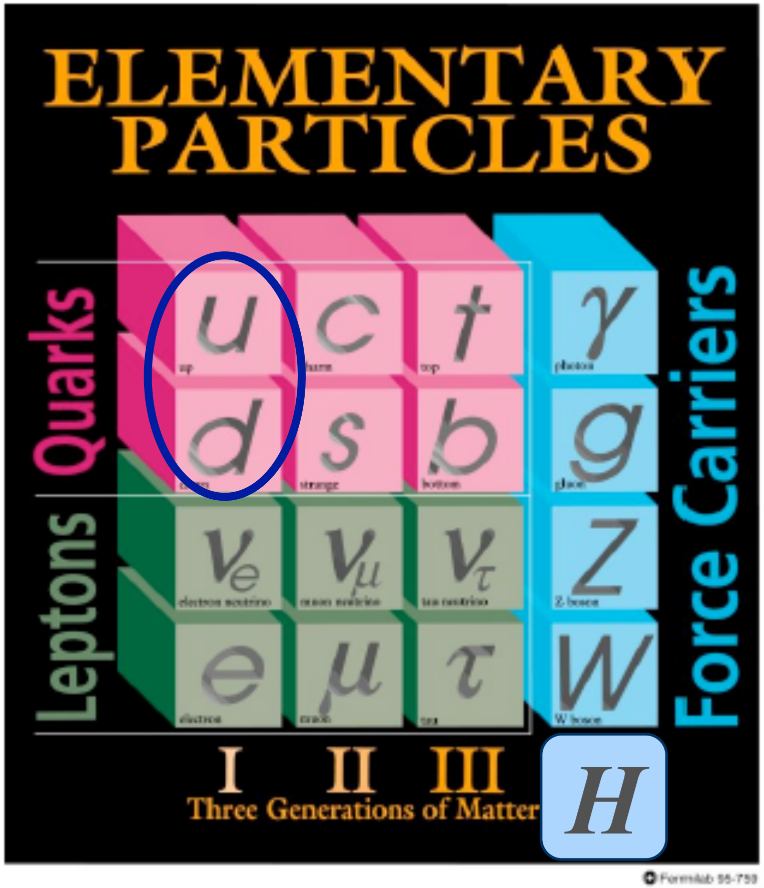
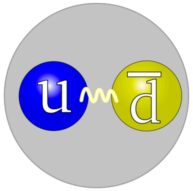
Can also be determined from η → 3π

2.2 Oscillations of Kaons

- Let us consider simplest hadrons: the mesons. They are quark-anti-quark bound states. They interact with strong, electromagnetic and weak forces

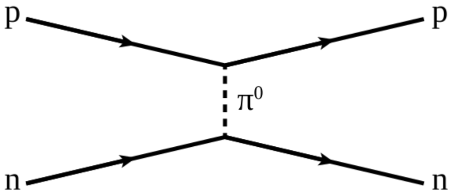




  - The simplest one is the pion:

$$\pi^+ : u\bar{d} \quad , \quad \pi^0 : u\bar{u} \text{ or } d\bar{d}$$

$$\pi^- : \bar{u}d$$



The pions mediate strong force in nuclei
It is ubiquitous in hadronic collisions

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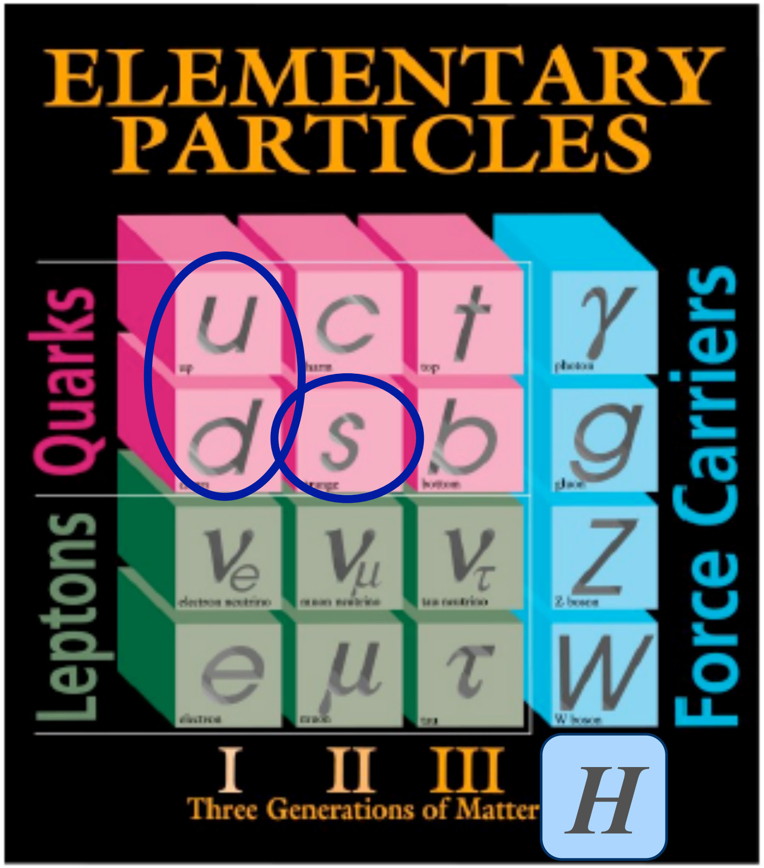

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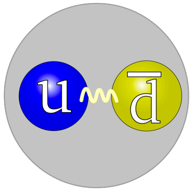

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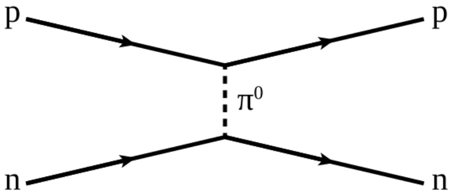
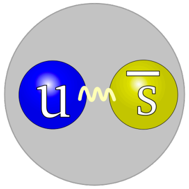




- The ones containing a s quark are the kaons

$$K^+ : u\bar{s}, \quad K^0 : d\bar{s}, \bar{K}^0 : s\bar{d}$$

$$K^- : \bar{u}s$$

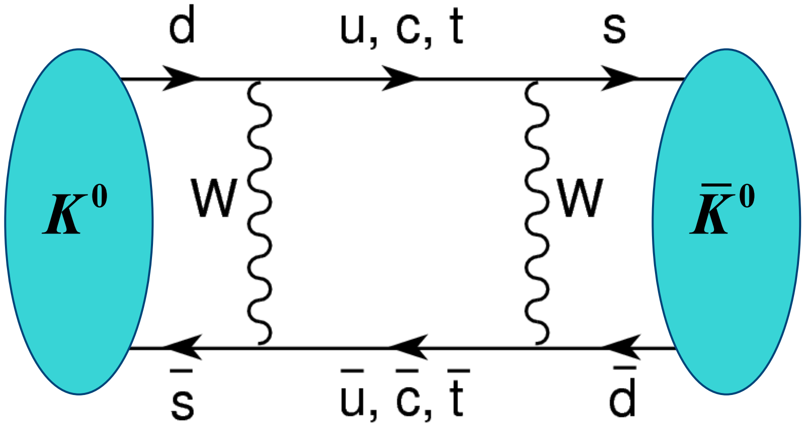
Discovered in *cosmic ray experiments*

2.2 Oscillations of Kaons

- Discovered in 1964 by Christenson, Cronin, Fitch and Turlay

  ⇨ Nobel Prize in 1980 for Cronin and Fitch

- Start with a K^0 ⇨ after some time it transforms into a $\bar{K}^0$



through weak interaction
Short distance effect

- The rate of this oscillation is suppressed but measurable in the Standard Model

  ⇨ goes through *weak interactions* ~ G_F $G_F \simeq 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$

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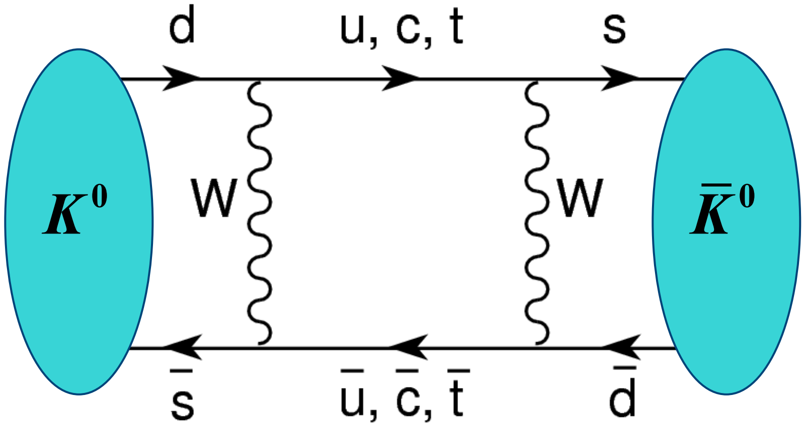


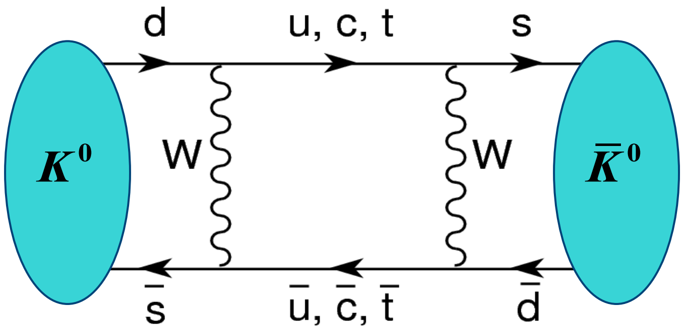
through weak interaction
Short distance effect

- The rate of this oscillation is very suppressed in the Standard Model

  ⇨ goes through *weak interactions* ~ G_F

- How can we understand the oscillation rate?

2.2 Oscillations of Kaons



- Process described using the bag parameter B_K
  Fundamental hadronic quantity proportional to matrix element
  ⇨ determined using *lattice QCD*

$$\langle \bar{K}^0 | \mathbf{H} | K^0 \rangle \sim \sum_{ij} \lambda_i \lambda_j \, S(r_i, r_j) \, \eta_{ij} \quad \langle O_{\Delta S=2} \rangle$$

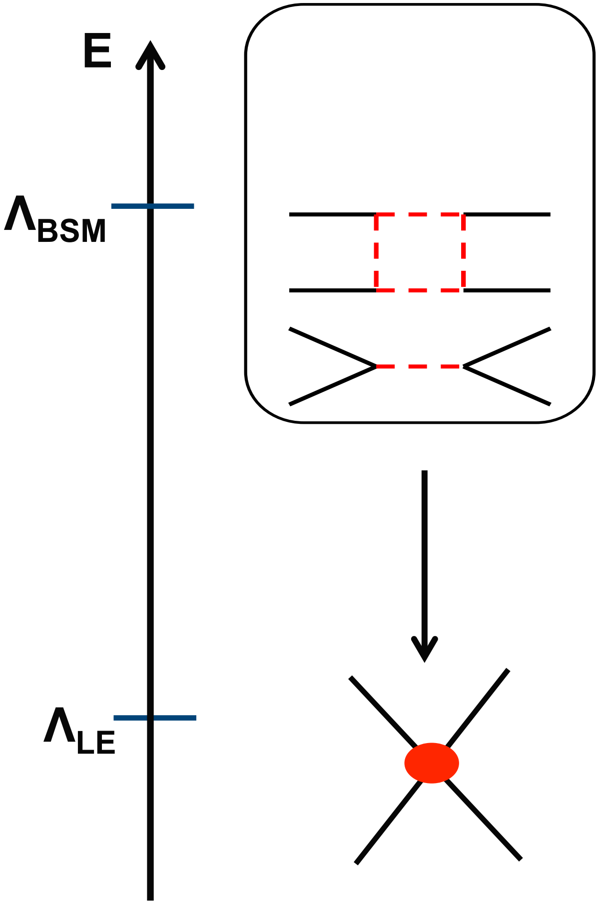
$$\langle O_{\Delta S=2} \rangle = \alpha_s(\mu)^{-2/9} \langle \bar{K}^0 | (\bar{s}_L \gamma^\alpha d_L)(\bar{s}_L \gamma_\alpha d_L) | K^0 \rangle \equiv \left(\frac{4}{3} M_K^2 f_K^2 \right) \hat{B}_K$$

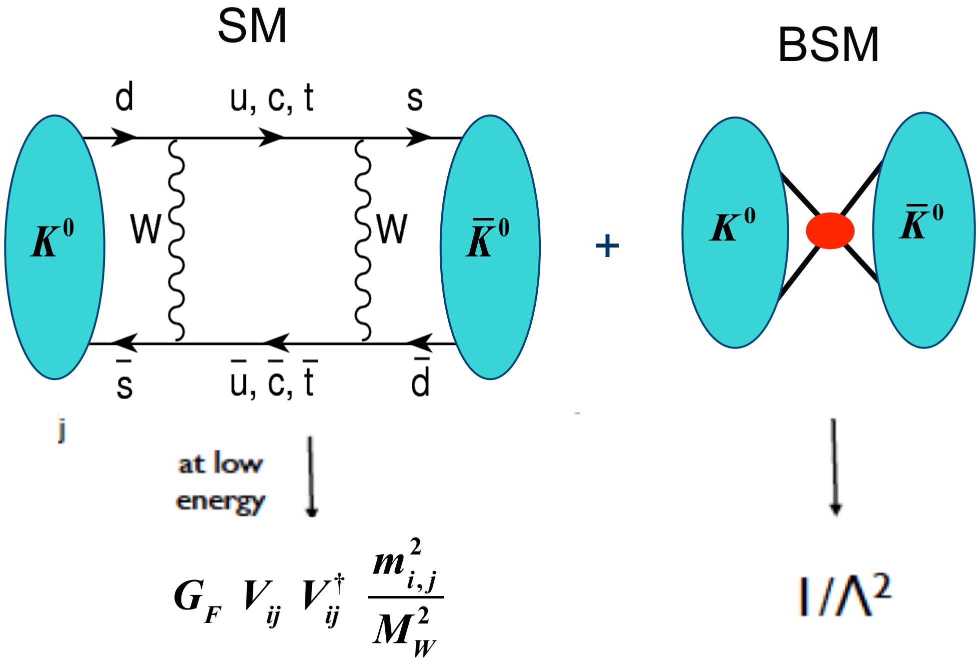
$$\lambda_i \equiv V_{id} V_{is}^* \quad ; \quad r_i \equiv m_i^2 / M_W^2 \quad (i = u, c, t)$$

2.2 Oscillations of Kaons

- Since process is suppressed in the Standard Model:

  ⇨ very sensitive to *new physics*: new degrees of freedom and symmetries





$$G_F \; V_{ij} \; V_{ij}^{\dagger} \; \frac{m_{i,j}^2}{M_W^2}$$

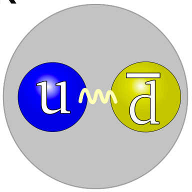
$$1/\Lambda^2$$

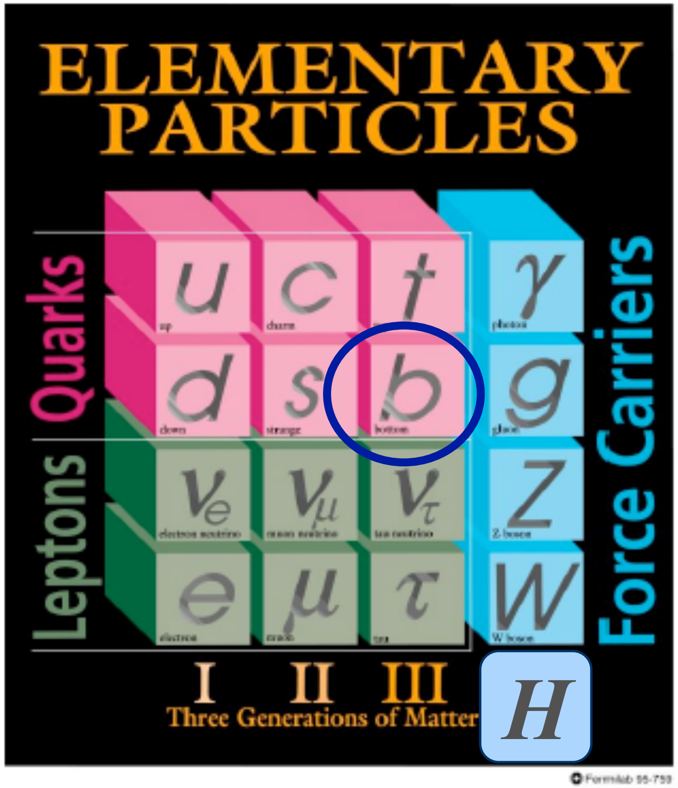
- If measured with very good precision provided the SM contribution is known

  ⇨ *stringent constraints* on new physics models

Oscillations of B mesons

- Similar tests with other mesons ⇒ Beauty mesons contain a b-quark





$$B^+ : u\bar{b} \;, \qquad B^0 : d\bar{b}$$

$$B^- : \bar{u}b \;, \qquad \bar{B}^0 : \bar{d}b$$

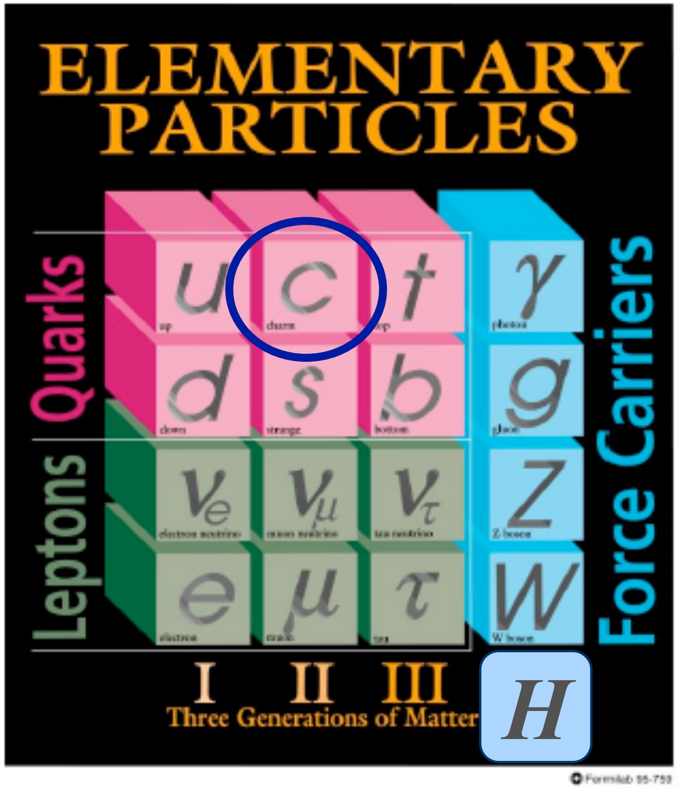
$$B_s^0 : s\bar{b} \;, \qquad \bar{B}_s^0 : \bar{s}b$$

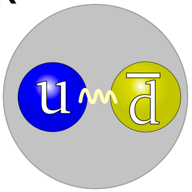
$$B_c^0 : c\bar{b} \;, \qquad B_c^0 : \bar{c}b$$

- B meson physics have been studied extensively at BaBar, Belle, CDF, D0@Tevatron and now Belle-II, LHCb, CMS and ATLAS@LHC

Oscillations of B mesons

- Similar tests with other mesons ⟹ Beauty mesons contain a b-quark





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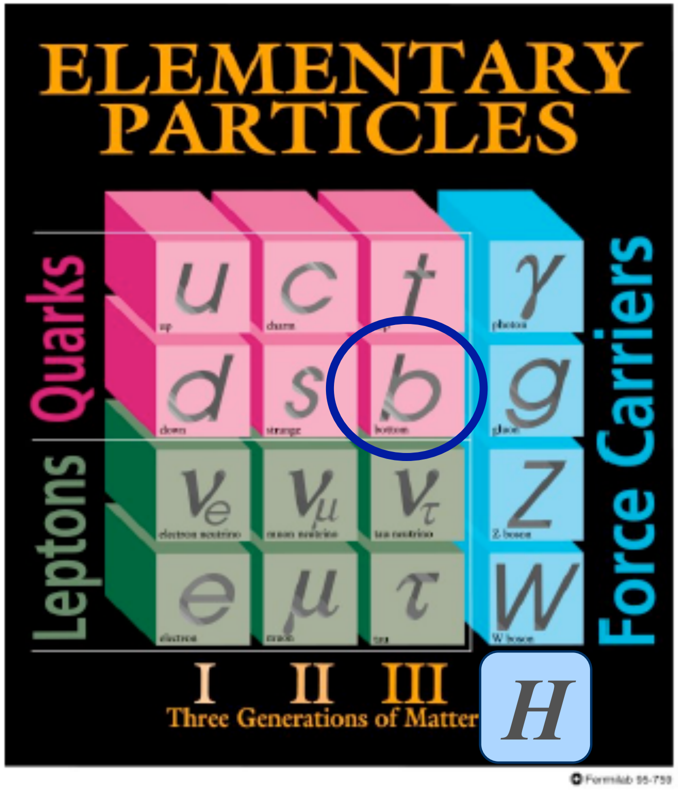
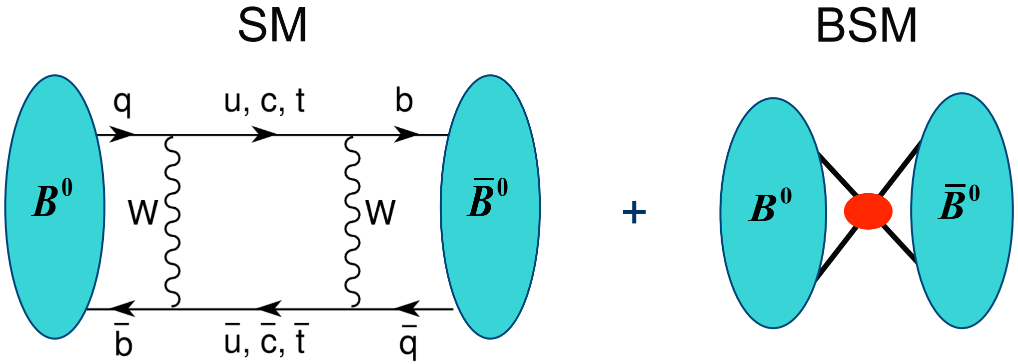
- Similar tests with D mesons

Oscillations of B mesons

- Similar tests with other mesons





- B-$\bar{\text{B}}$ measured by *BaBar* and *Belle'01*
- B_S-$\bar{B}_S$ mixing observed by *CDF'06* and *LHCb'11*

CP violation in B decays *LHCb'13*

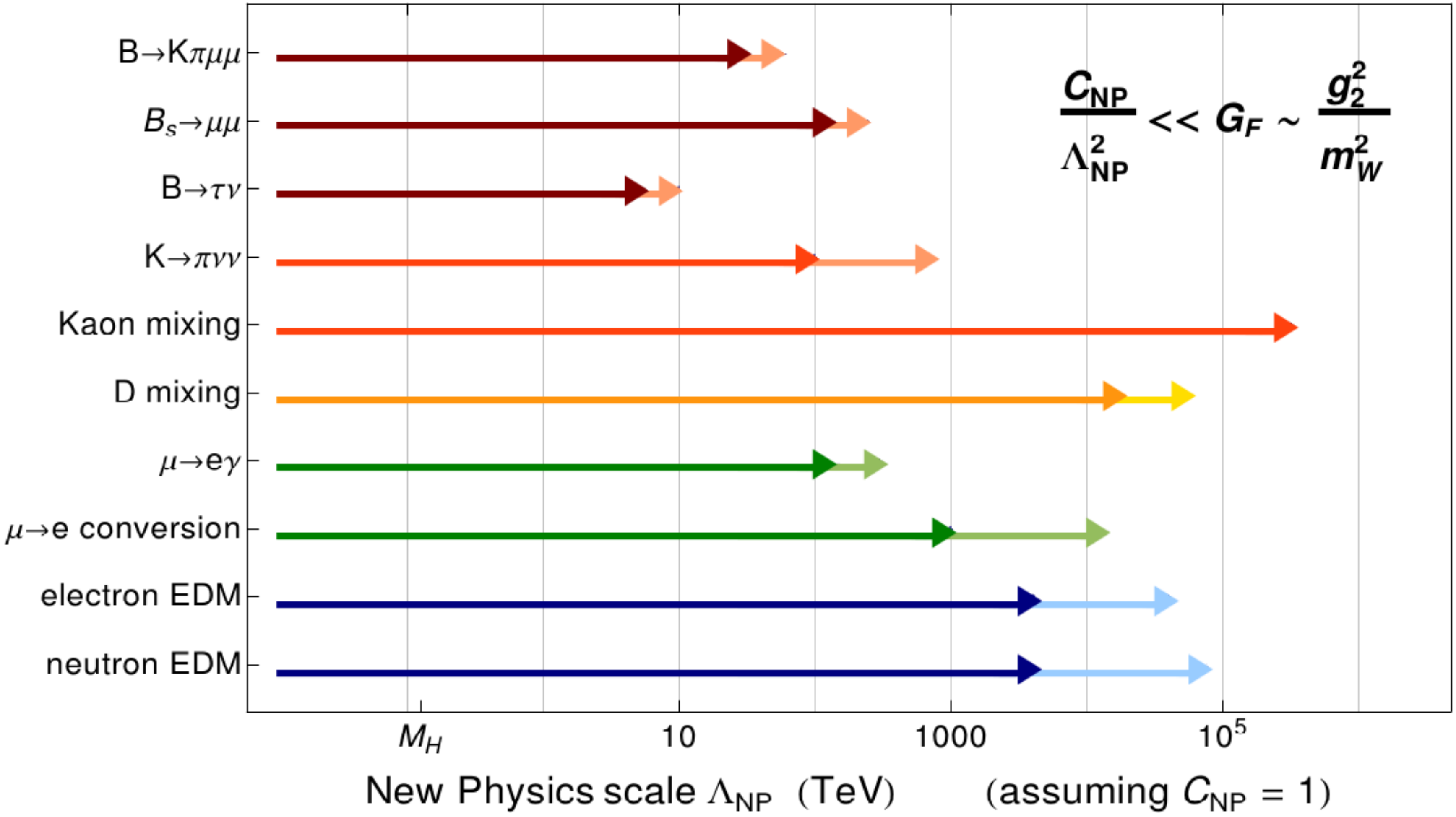
⇨ CP violation in D decays *LHCb'19 & '21*

- Stringent constraints on new physics models provided *hadronic* matrix elements known

New Physics and Flavour sector

- Very sensitive to New Physics

W. Altmannshofer



Anomalies in Flavour Physics

- Exciting discrepancies reported recently in B physics sector :

2 Accelerators Find Particles That May Break Known Laws of Physics

The LHC and the Belle experiment have found particle decay patterns that violate the Standard Model of particle physics, confirming earlier observations at the BaBar facility

By Clara Moskowitz | September 9, 2015 | Véalo en español

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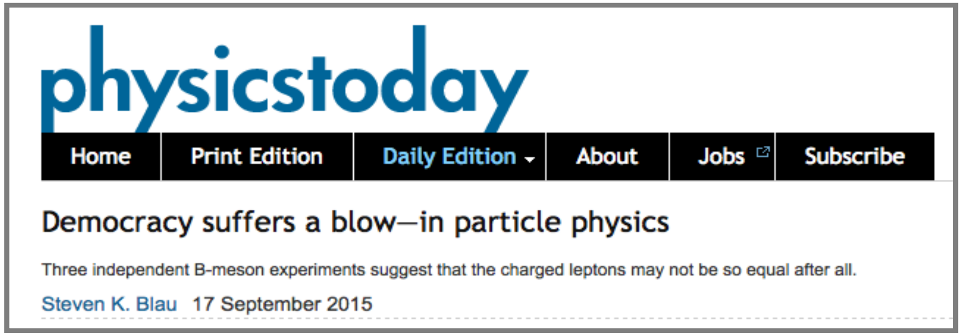
Tensions in the Standard Model



Democracy suffers a blow—in particle physics

Three independent B-meson experiments suggest that the charged leptons may not be so equal after all.

Steven K. Blau 17 September 2015

Went away in
December 2022 *LHCb'22*

Anomalies in Flavour Physics

- Cabibbo Angle Anomaly:

Description of the weak interactions :

$$\boxed{\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{D}_L \underset{\overset{\uparrow}{\text{Unitary matrix}}}{V_{CKM}} \gamma^\alpha U_L + \bar{e}_L \gamma^\alpha \nu_{e_L} + \bar{\mu}_L \gamma^\alpha \nu_{\mu_L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau_L} \right) + \text{h.c.}}$$

- Unitary 3x3 Matrix, parametrizes rotation between mass and weak interaction eigenstates in Standard Model

$$\underset{\text{Weak Eigenstates}}{\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}} = \underset{\text{CKM Matrix}}{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}} \underset{\text{Mass Eigenstates}}{\begin{pmatrix} d \\ s \\ b \end{pmatrix}}$$

Anomalies in Flavour Physics

- Cabibbo Angle Anomaly:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Weak Eigenstates CKM Matrix Mass Eigenstates

- Check the unitarity of the first row of the CKM matrix:

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} \overset{?}{=} 1}$$

⇒ -3σ away from unitarity!

Negligible ~2x10^{-5}
(B decays)

$$|V_{ud}| = \cos\theta_C \text{ and } |V_{us}| = \sin\theta_C$$

We will come back to this later

Anomalies in Flavour Physics

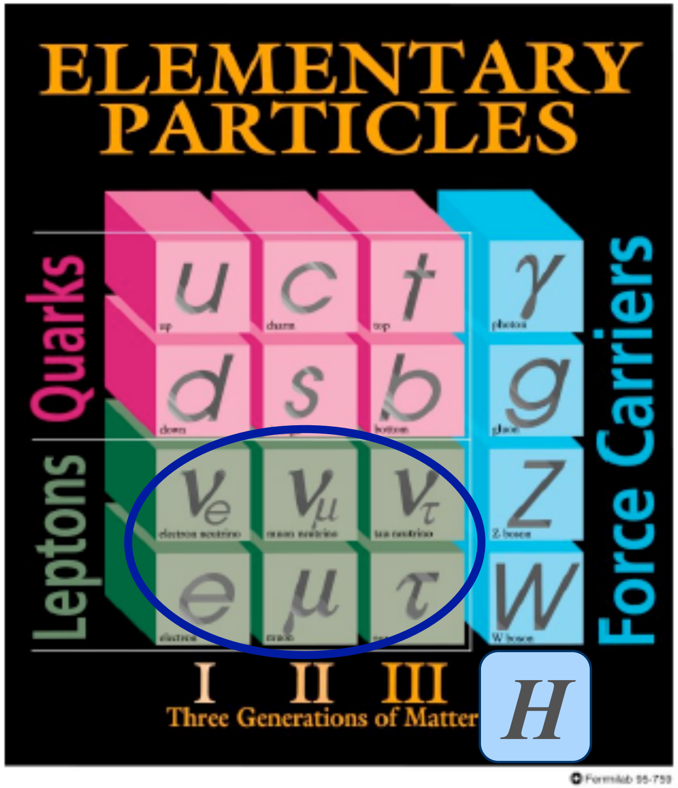
- These anomalies have generated a lot of excitement and theoretical papers to try to explain them using new physics models

- This requires a good understanding of hadronic physics

- New measurements are planned at ATLAS, CMS (dedicated B physics run), LHCb Belle II and NA62

- Better precision within the next decade
  ⇒ match the level of precision theoretically with *hadronic physics*

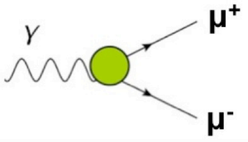
- Can we try to escape considering leptons?
  ⇒ Do not interact through *strong interactions*



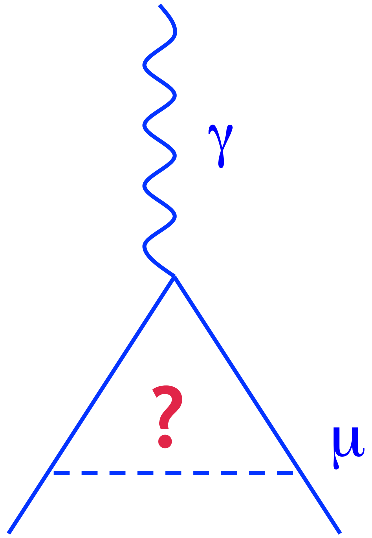
2.3 Anomalous magnetic moment of the muon

$$a_\mu = \frac{(g-2)_\mu}{2}$$

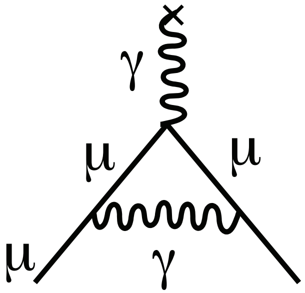
Anomalous magnetic moment

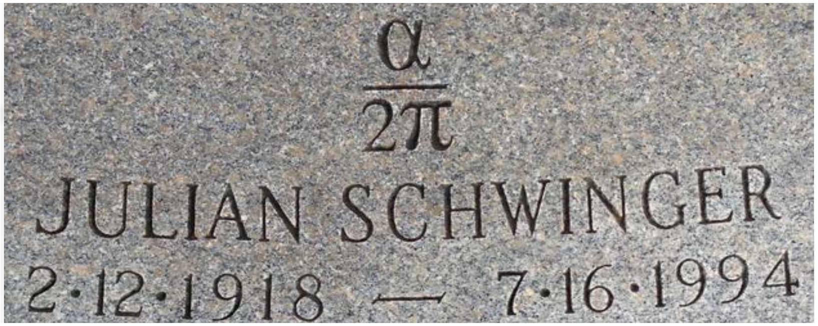


- The gyromagnetic factor of the muon is modified by loop contribution
- Predicted by Dirac to be 2
- Schwinger computed the first order correction



QED





$\frac{\alpha}{2\pi}$

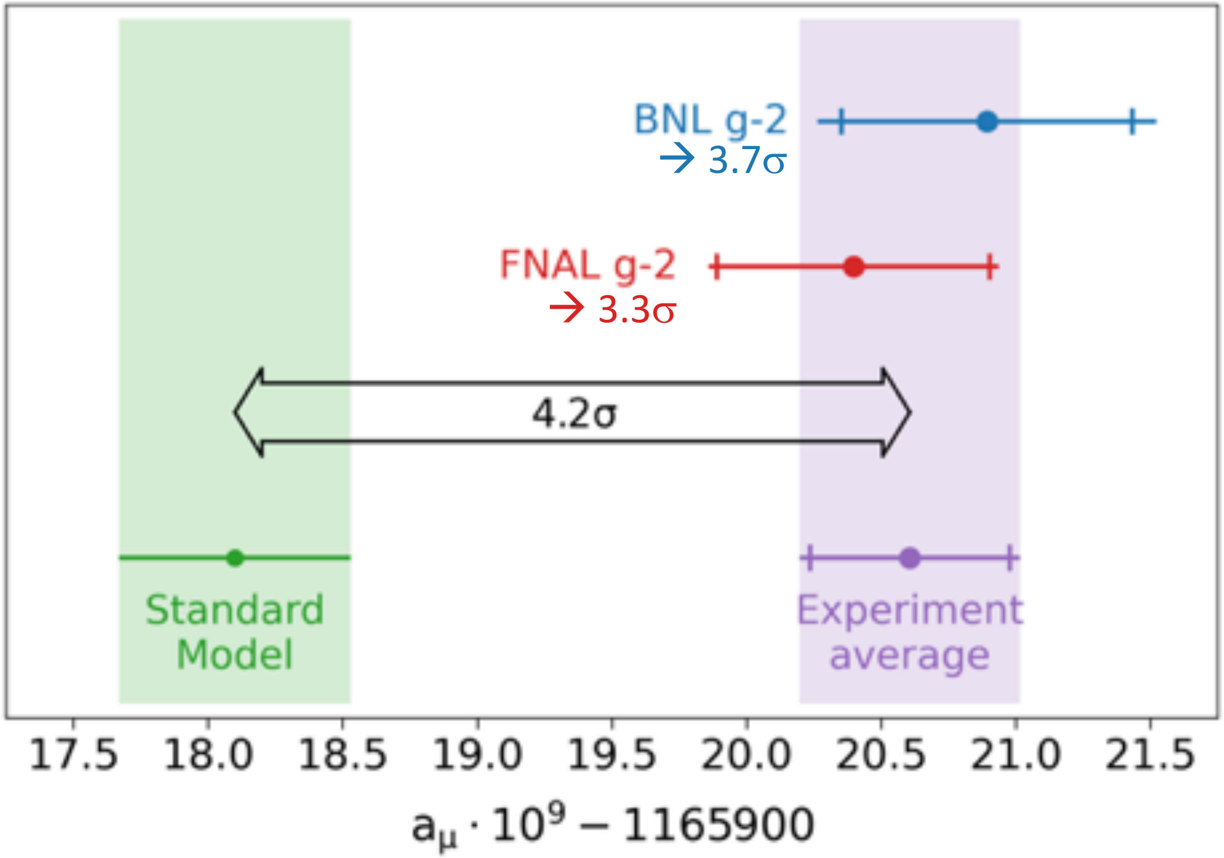
JULIAN SCHWINGER

2·12·1918 — 7·16·1994

2.3 Anomalous magnetic moment of the muon

FNAL g-2
Chris Polly'19

$a_\mu(SM) = 0.00116591810(43) \rightarrow 368$ ppb



- Individual tension with SM
  - BNL: 3.7σ
  - FNAL: 3.3σ

$a_\mu(Exp) - a_\mu(SM) = 0.00000000251(59) \rightarrow 4.2\sigma$

Confronting measurement and prediction

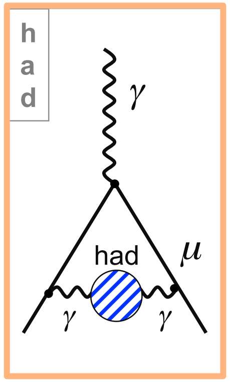
- Theoretical Prediction:

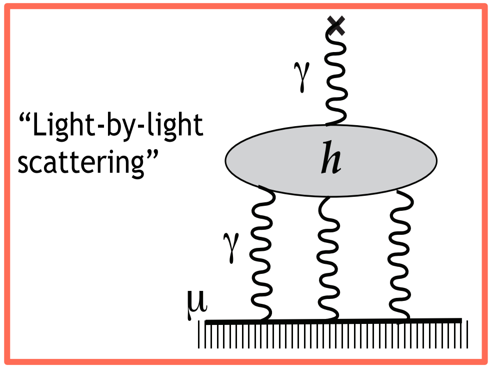
Colangelo et al. Snowmass 2022

Contribution	Result in 10^{-10} units
QED(leptons)	11658471.885 ± 0.004
HVP(leading order)	693.1 ± 4.0
HVP(higher order) (*)	-8.59 ± 0.07
HLBL	9.2 ± 1.8
EW	15.4 ± 0.1
Total	11659181.0 ± 4.3

- Important contribution comes from virtual hadrons in the loop!

- Tackled using :
  - Models
  - Dispersion Relations
  - Lattice QCD

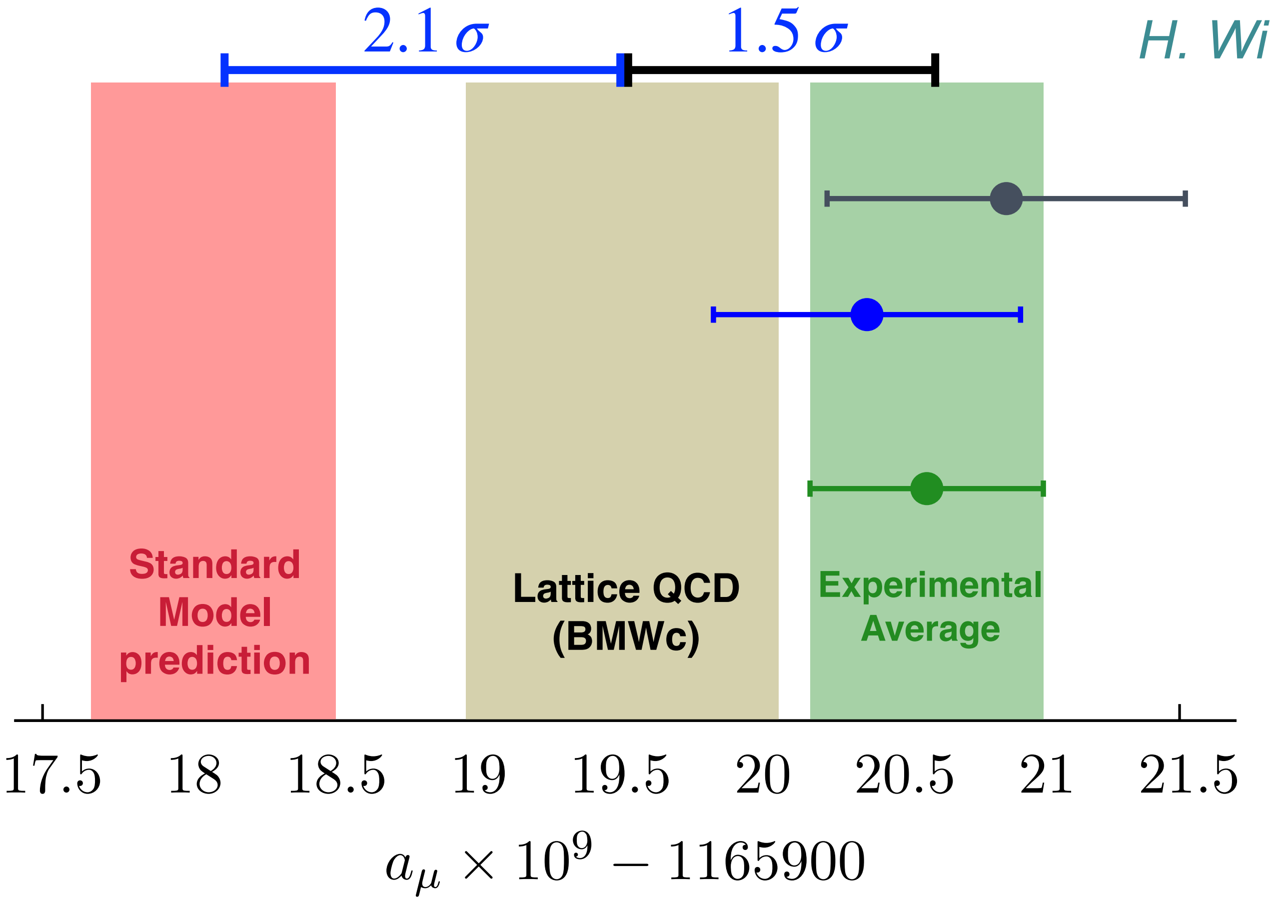




2.3 Anomalous magnetic moment of the muon

- Since 2019: Progress in many fronts: Lattice QCD

H. Wittig@Moriond 2023

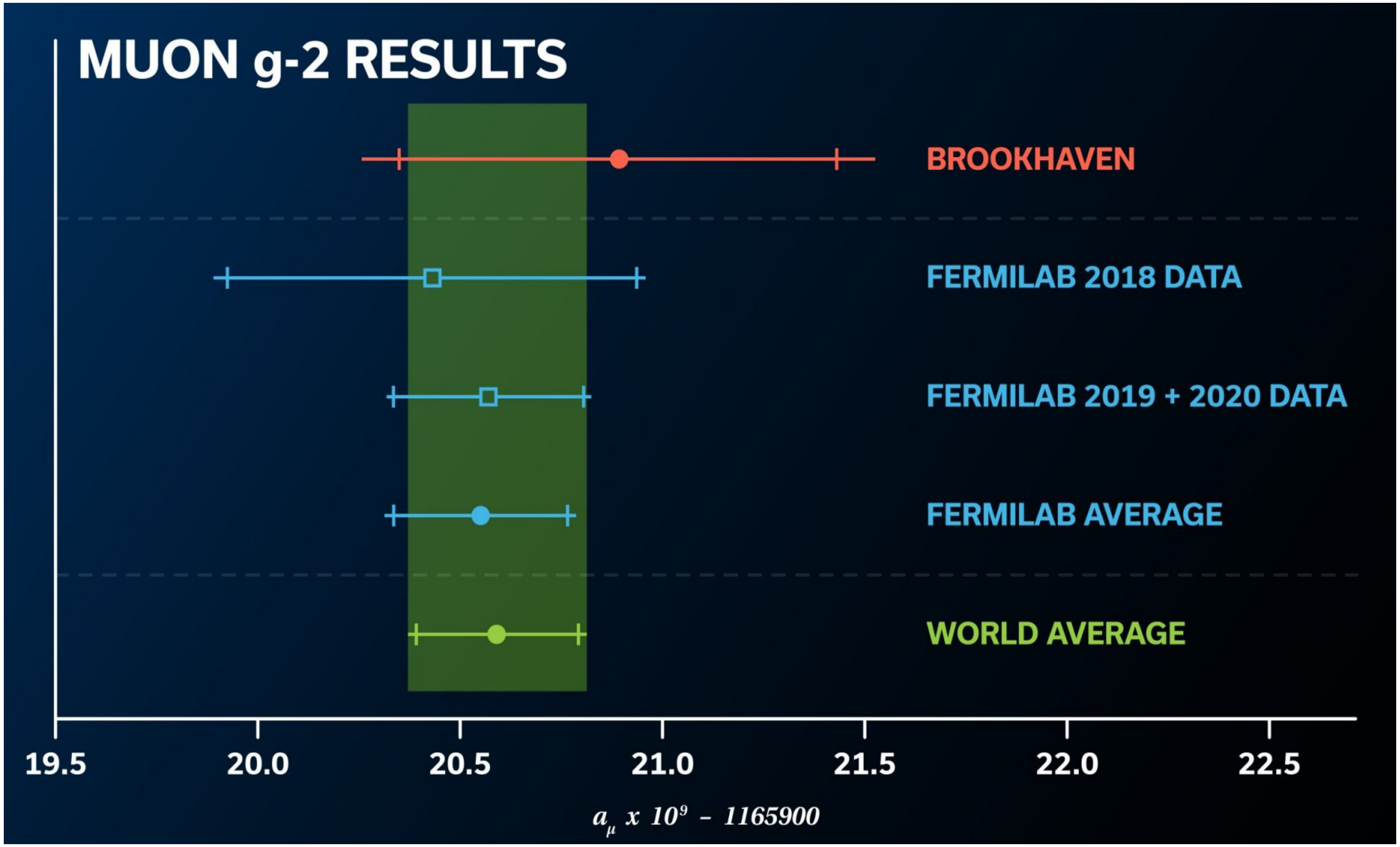


- Also experimentally and analytically in SM predictions

➡ See talks tomorrow morning by *A. Denig*, *B.Kubis* and *D. G. Melo Porras*

2.3 Anomalous magnetic moment of the muon

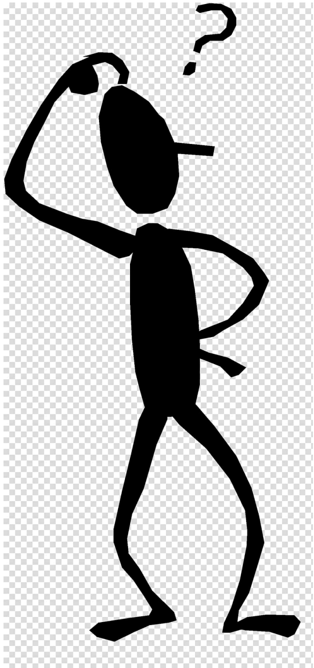
- Since 2019: Progress in many fronts: New result released on August 10 2023 by *Muon g-2 experiment*



https://news.fnal.gov/2023/08/muon-g-2-doubles-down-with-latest-measurement/

2.4 Neutrino Physics

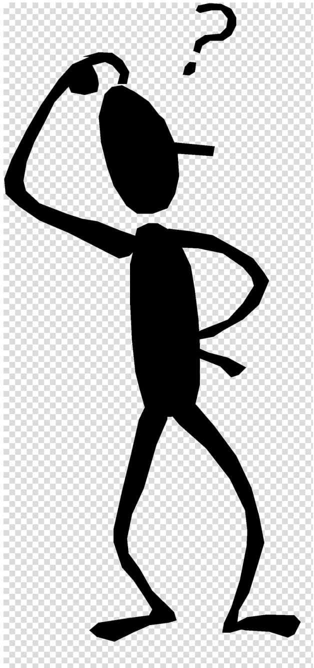
- What about neutrino physics?



⇨ We should be fine!

2.4 Neutrino Physics

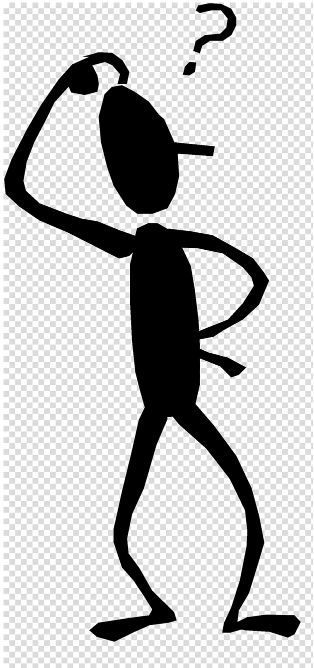
- What about neutrino physics?



⇨ We should be fine! *Really?*

2.4 Neutrino Physics

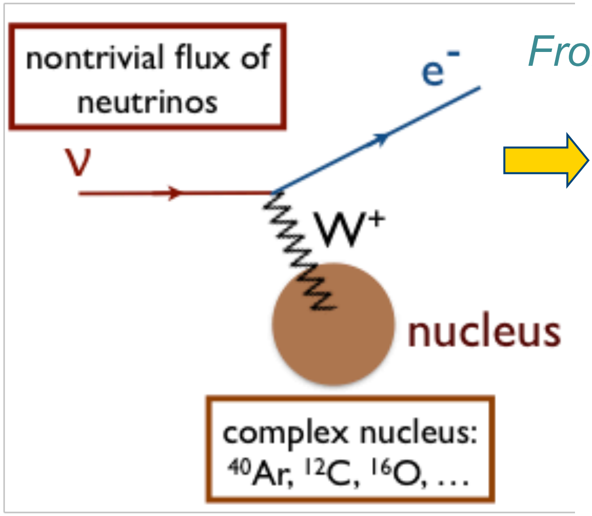
- What about neutrino physics?



⇨ We need to detect the neutrinos!

By detecting the final state leptons and all the product to reconstruct the neutrino energy unknown

Make them interact on Nucleus



From R. Hill

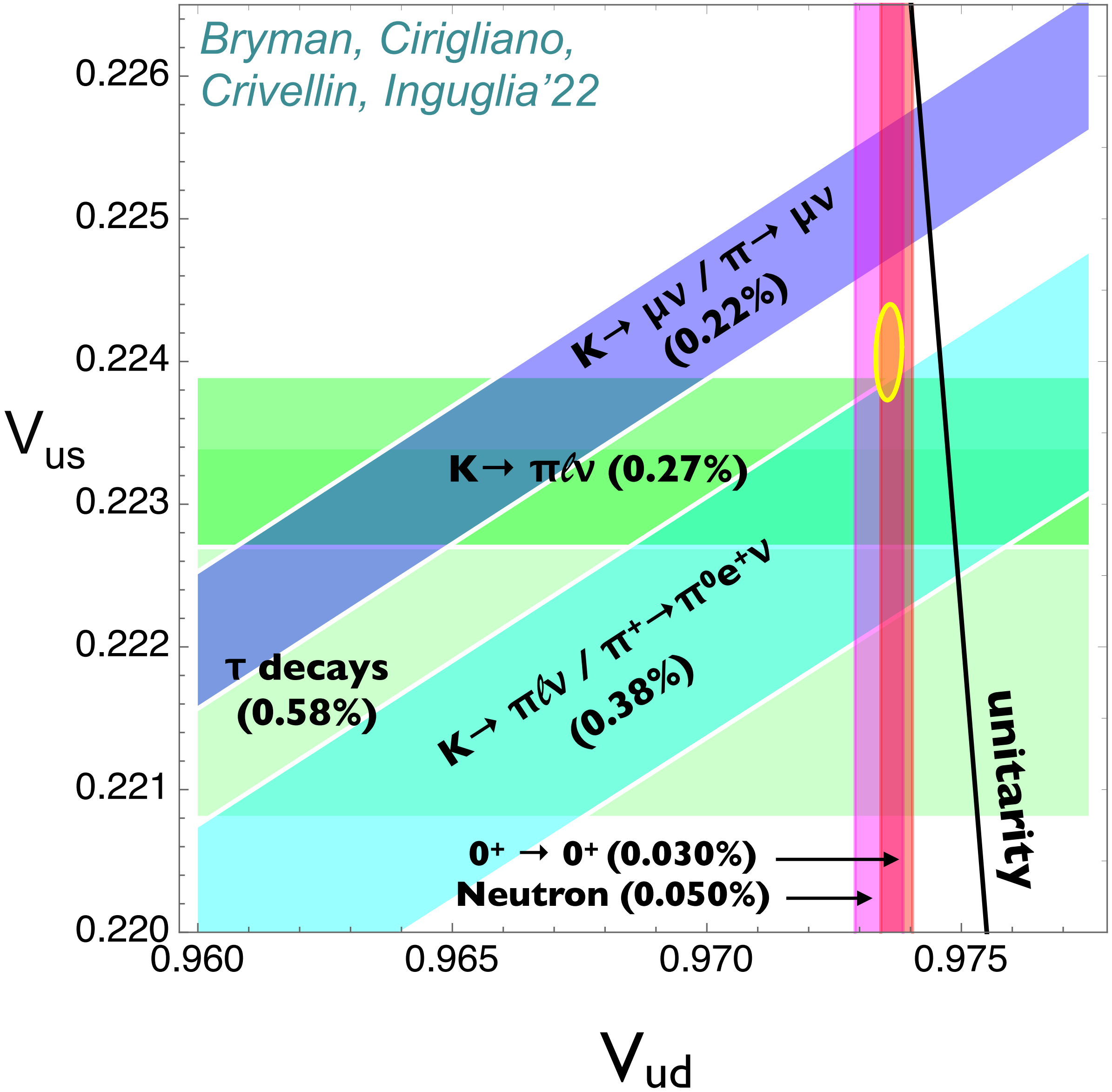
⇨ Compute Cross Section

Hadronic Physics

3. Precision Hadronic Physics : selected examples

3.1 Cabibbo angle anomaly

Moulson &
E.P.@CKM2021



$|V_{ud}| = 0.97373(31)$

$|V_{us}| = 0.2231(6)$

$|V_{us}|/|V_{ud}| = 0.2311(5)$

Fit results, no constraint

$V_{ud} = 0.97365(30)$

$V_{us} = 0.22414(37)$

$\chi^2/\text{ndf} = 6.6/1\ (1.0\%)$

$\Delta_{CKM} = -0.0018(6)$

-2.7σ

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1 + \Delta_{CKM}}$$

Negligible ~2x10^{-5}
(B decays)

Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$

$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \delta_{RC}) \times F_{\text{kin}}$$

Channel-dependent effective CKM element

Hadronic matrix element

Radiative corrections

Key hadronic inputs

- For K_{l3} decays: $K\pi$ form factors

$$\boxed{\left\langle \pi(p_\pi) \right| \bar{s}\gamma_\mu \mathbf{u} \left| K(p_K) \right\rangle = \left[\left(p_K + p_\pi\right)_\mu - \frac{\Delta_{K\pi}}{t}\left(p_K - p_\pi\right)_\mu \right] f_+(t) + \frac{\Delta_{K\pi}}{t}\left(p_K - p_\pi\right)_\mu f_0(s)}$$

$$\hspace{6cm}\underset{\text{vector}}{\uparrow}\hspace{4cm}\underset{\text{scalar}}{\uparrow}$$

with $t = q^2 = (p_K - p_\pi)^2, \ \ \bar{f}_{0,+}(s) = \dfrac{f_{0,+}(s)}{f_+(0)}$

  - Normalization $f_+(0)$ determined from lattice QCD
  - Shape of the $K\pi$ form factors obtained from a fit to the data using a dispersive parametrization

Bernard, Oertel, E.P., Stern'08,'10

- For K_{l2}/π_{l2} : the decay constant ratio: f_K/f_π

Dispersive parametrization for Kπ form factors

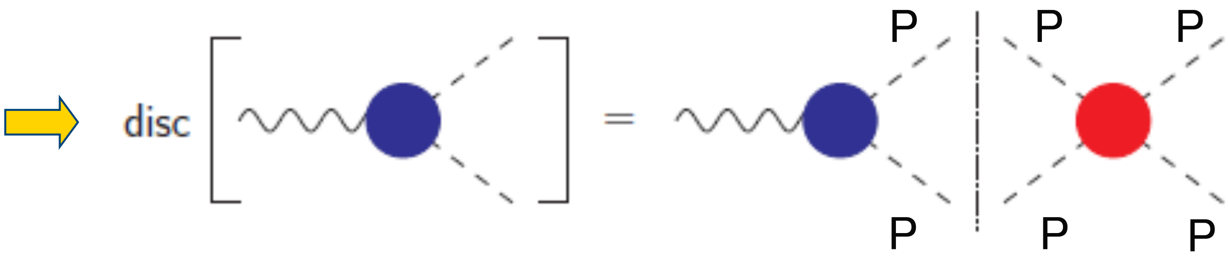
- Advantage of a dispersive approach:
  - Based on analyticity and unitarity ⇒ model independence
  - Summation of rescattering
  - Connect different energy regions

Dispersive parametrization for Kπ form factors

- Unitarity ⇒ the discontinuity of the form factor is known:

$$\boxed{\frac{1}{2i} disc\ F_{PP}(s) = \mathrm{Im}\, F_{PP}(s) = \sum_n F_{PP\to n} \left(\mathrm{T}_{n\to PP}\right)^*}$$

Only one channel n = PP (elastic region)



$$\boxed{\frac{1}{2i} disc\ F_I(s) = \mathrm{Im}\, F_I(s) = F_I(s)\ \sin \delta_I(s) e^{-i\delta_I(s)}}$$

Watson's theorem

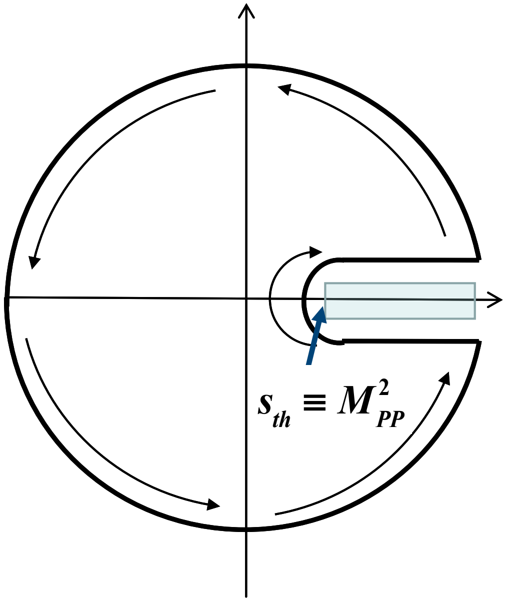
PP scattering phase
known from experiment

Dispersive parametrization for $K\pi$ form factors

- Analiticity : Knowing the discontinuity of F ⇒ write a dispersion relation for it

- Cauchy Theorem and Schwarz reflection principle

$$F(s) = \frac{1}{\pi}\oint \frac{F(s')}{s'-s}\, ds' \Rightarrow \frac{1}{2i\pi}\int_{M^2_{PP}}^{\infty} \frac{disc\left[F(s')\right]}{s'-s-i\varepsilon}\, ds'$$



- If F does not drop off fast enough for $|s| \to \infty$

⇒ subtract the DR

$$\boxed{F(s) = P_{n-1}(s) + \frac{s^n}{\pi}\int_{M^2_{PP}}^{\infty} \frac{ds'}{s'^n}\frac{\mathrm{Im}\left[F(s')\right]}{\left(s'-s-i\varepsilon\right)}}$$

$P_{n-1}(s)$ polynomial

Dispersive parametrization for $K\pi$ form factors

- Solution: Use analyticity to reconstruct the form factor in the entire space

⇨ Omnès representation : $F_I(s) = P_I(s)\ \Omega_I(s)$

polynomial

Omnès function

- Omnès function : $$\boxed{\Omega_I(s) = \exp\left[\frac{s}{\pi}\int_{s_{th}}^{\infty}\frac{ds'}{s'}\frac{\delta_I(s')}{s'-s-i\varepsilon}\right]}$$

- Polynomial: $P_I(s)$ not known but determined from a matching to experiment or to ChPT at low energy

Dispersive parametrization for Kπ form factors

- Scalar Kπ form factor obtained from a twice subtracted Dispersion Relation:

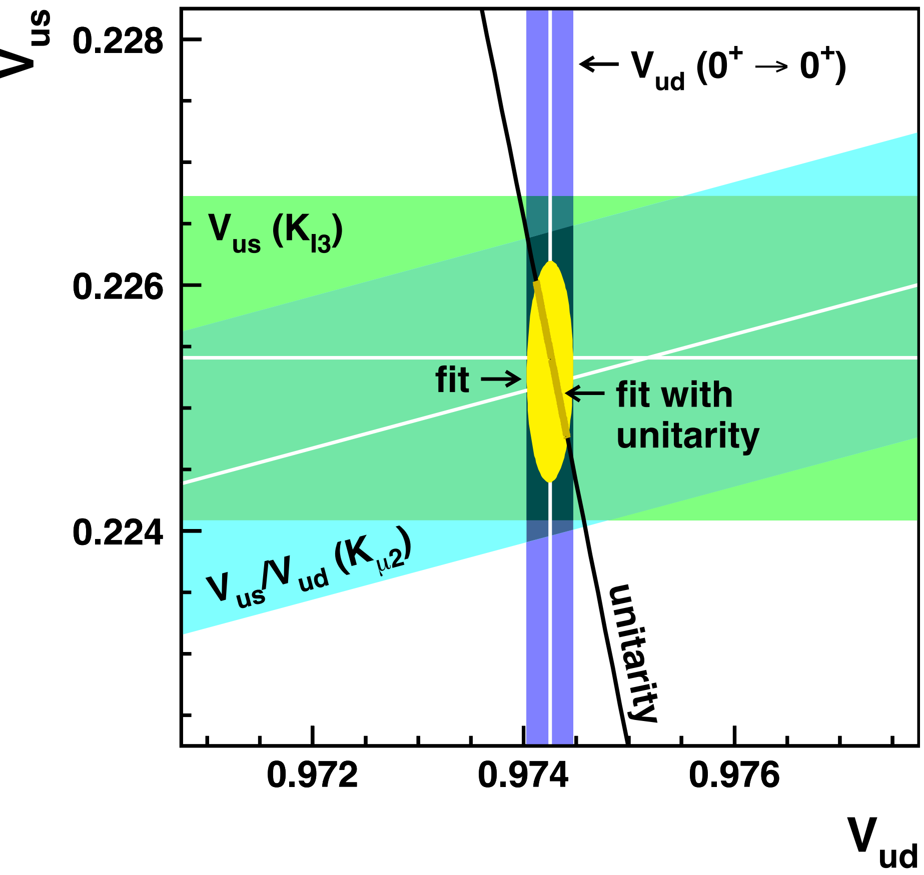
$$\boxed{\overline{f}_0(s) = \exp\left[\frac{s}{\Delta_{K\pi}}\left(\ln C + \frac{(s-\Delta_{K\pi})}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{ds'}{s'}\frac{\phi_0(s')}{(s'-\Delta_{K\pi})(s'-s-i\varepsilon)}\right)\right]}$$

One Subtraction in s=0 and another one in $s = \Delta_{K\pi} = (m_K+m_\pi)^2$ at the Callan-Treiman point where a low energy theorem exists

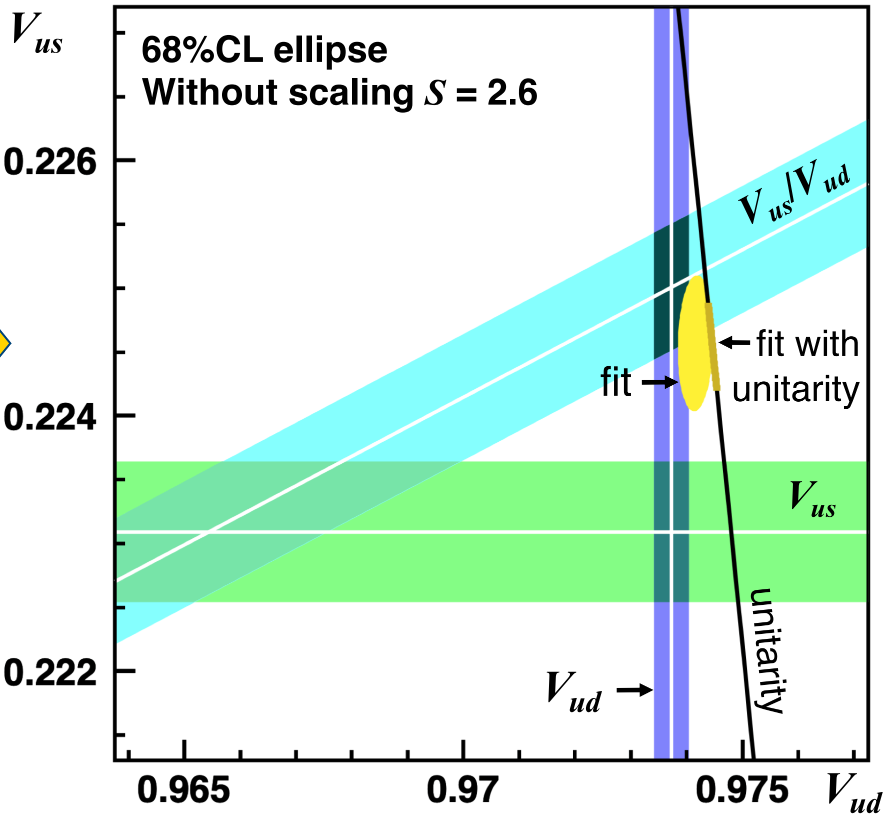
$$\ln C \equiv \overline{f}_0(\Delta_{K\pi})$$

Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*



Moulson & E.P.@CKM2021

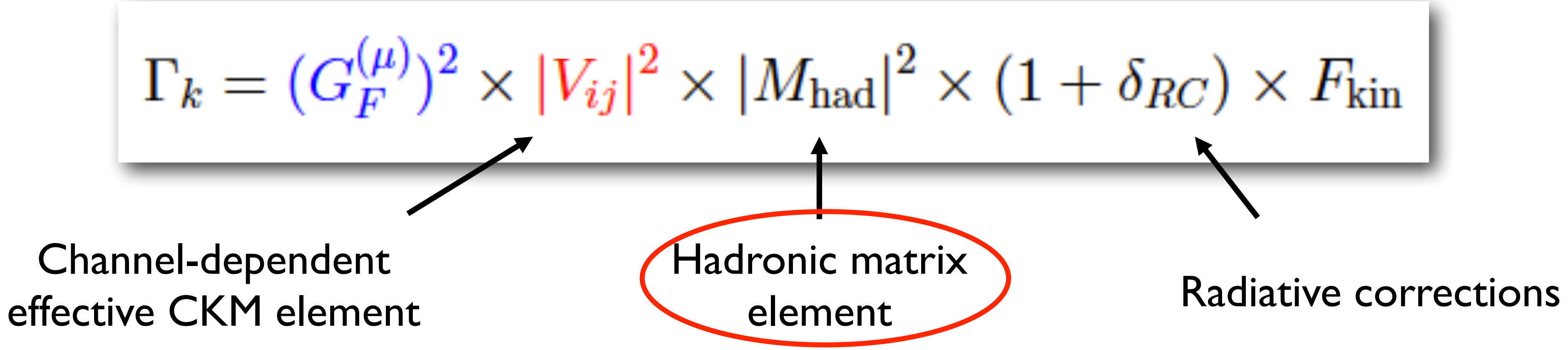


What happened?

Changes on V_{us} and V_{ud} since 2011

- Almost no change on the experimental side since 2011

Flavianet Kaon WG: *Antonelli et al'11*



$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1+\delta_{RC}) \times F_{\text{kin}}$$

Channel-dependent effective CKM element

Hadronic matrix element

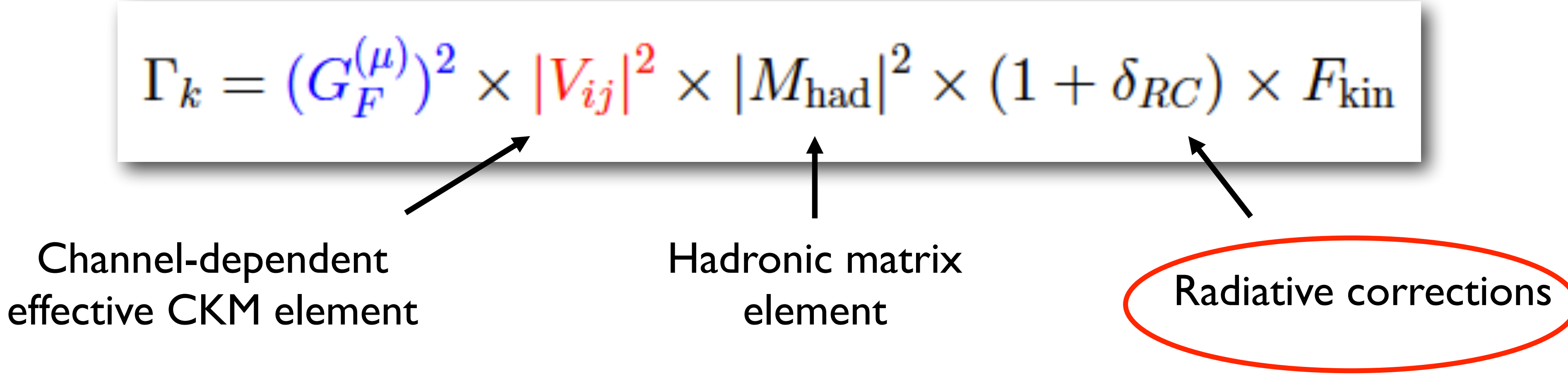
Radiative corrections

- Changes in *theoretical* inputs:
  - Impressive progress on hadronic matrix element computations from *lattice QCD* for V_{us} and V_{us}/V_{ud} extraction from Kaon decays

Changes on V_{us} and V_{ud} since 2011

- Almost no change on the experimental side since 2011

Flavianet Kaon WG: *Antonelli et al'11*



$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{\rm had}|^2 \times (1+\delta_{RC}) \times F_{\rm kin}$$

Channel-dependent effective CKM element

Hadronic matrix element

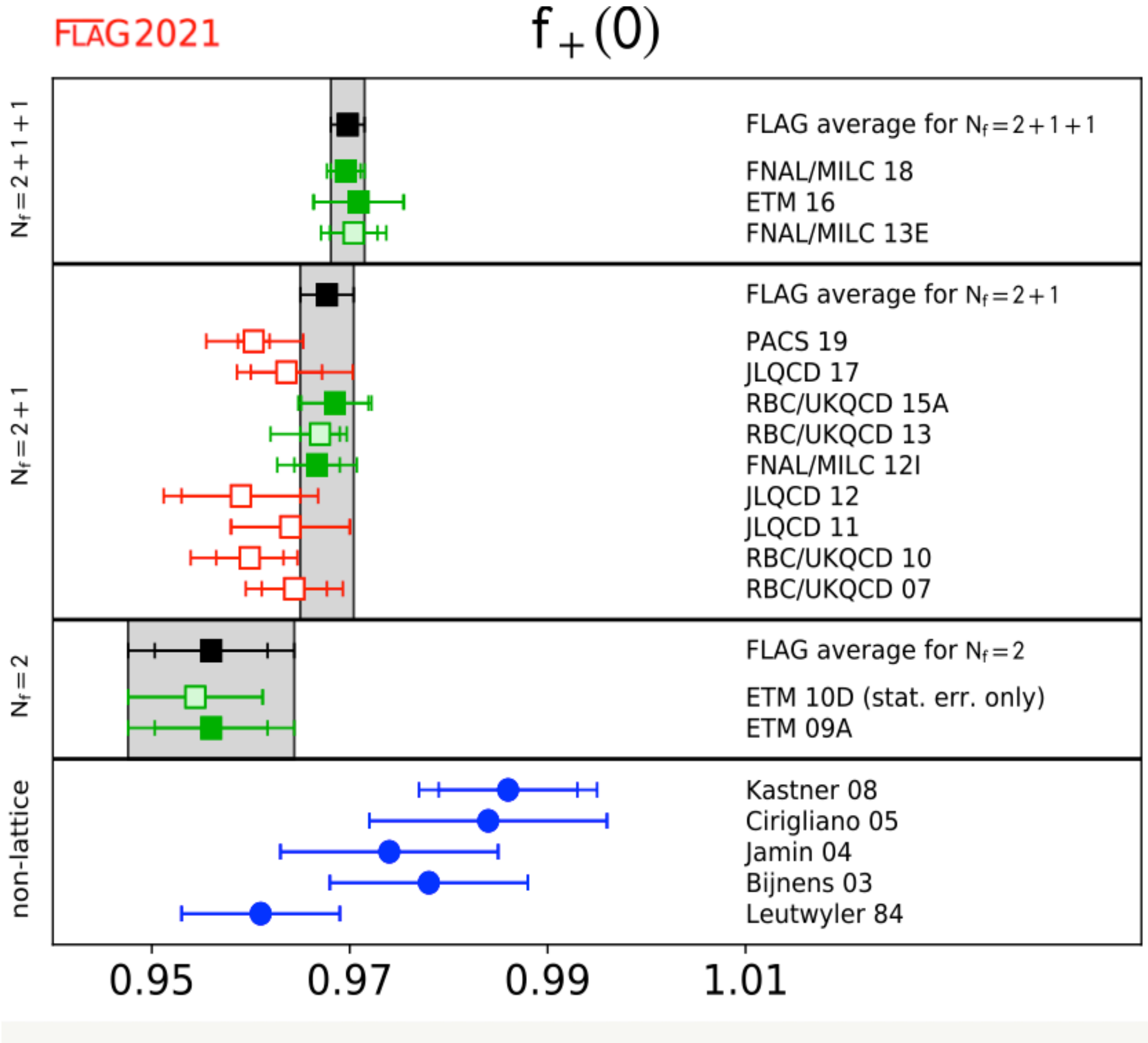
Radiative corrections

- Changes in *theoretical* inputs:
  - Impressive progress on hadronic matrix element computations from lattice QCD for V_{us} and V_{us}/V_{ud} extraction from Kaon decays
  - Radiative corrections from dispersive methods for V_{ud} extraction

    e.g. Seng, Gorchtein, Patel, Ramsey-Musolf'18,'19

$f_+(0)$ from lattice QCD

- Recent progress on Lattice QCD for determining $f_+(0)$



$$f_+(0)^{FLAG21}_{N_f=2+1+1} = 0.9698(17)$$

0.18% uncertainty

to be compared to

$$f_+(0)^{FLAG16}_{N_f=2+1+1} = 0.9704(32)$$

$$f_+(0)^{2010}_{N_f=2+1} = 0.959(50)$$

Uncertainty divided by ~2 w/ 2016 and by 25 w/ 2011!

→ Lattice uncertainties at the same level as exp.

-3.2σ away from unitarity!

2011: $V_{us} = 0.2254(5)_{exp}(11)_{lat}$ → $V_{us} = 0.2231(4)_{exp}(4)_{lat}$

V_{us}/V_{ud} from K_{l2}/π_{l2}

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1-m_\mu^2/m_{\pi^\pm}^2}{1-m_\mu^2/m_{K^\pm}^2}\left(1-\frac{1}{2}\delta_{\text{EM}}-\frac{1}{2}\delta_{SU(2)}\right)$$

- Recent progress on radiative corrections computed on lattice:

Di Carlo et al.'19
Boyle et al.'23

- Main input hadronic input: f_K/f_π

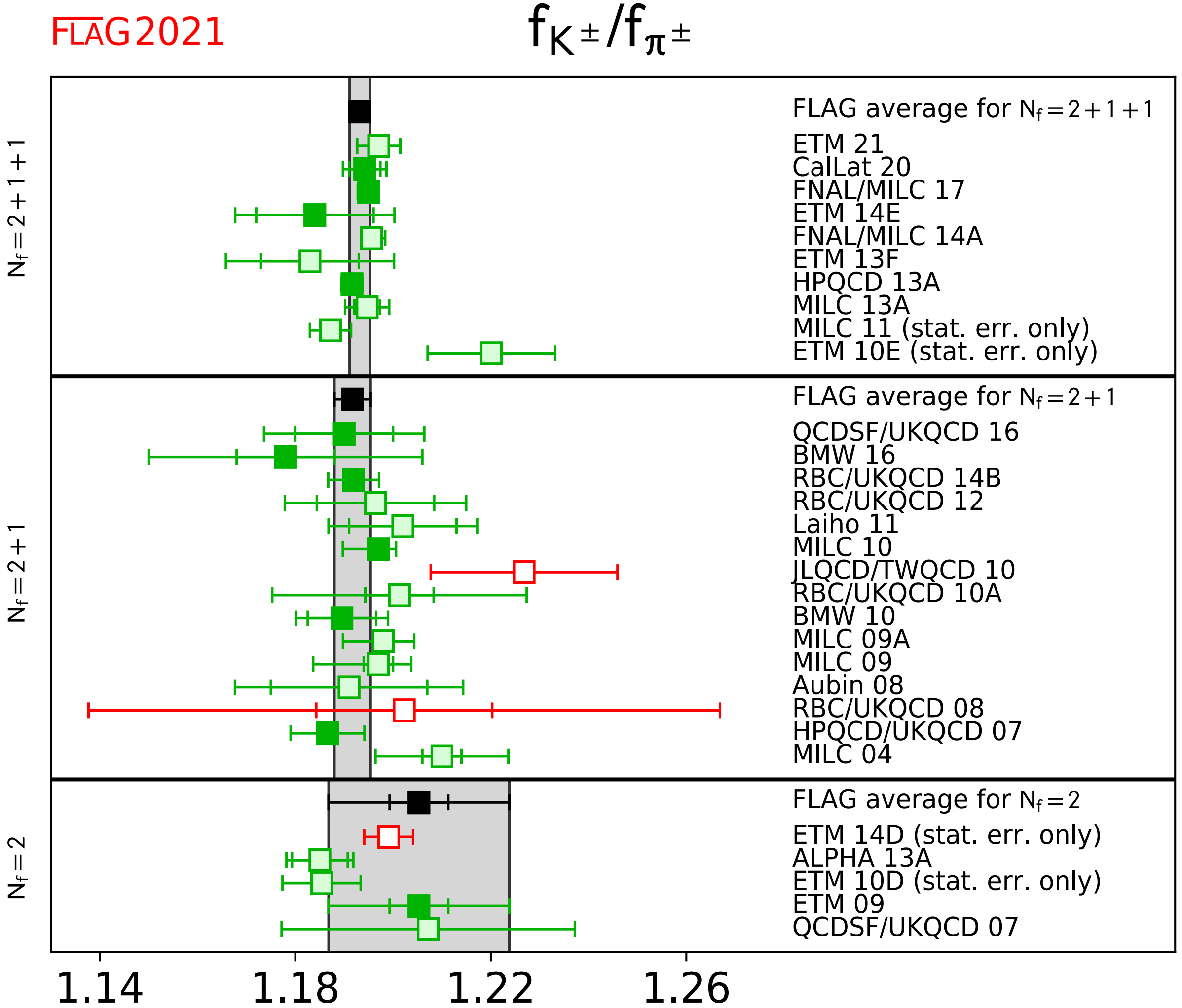
- In 2011: $V_{us}/V_{ud} = 0.2312(4)_{\text{exp}}(12)_{\text{lat}}$

- In 2021: $V_{us}/V_{ud} = 0.\ 2311(3)_{\text{exp}}(4)_{\text{lat}}$ the lattice error is reducing by a factor of 3 compared to 2011! It is now of the same order as the experimental uncertainty.

-1.8σ away from unitarity

f_K/f_π from lattice QCD

Progress since 2018: ⇨ new results from *ETM'21* and *CalLat'20*



Now Lattice collaborations include SU(2) IB corr.
For N_f=2+1+1, FLAG2021

$$f_{K^+}/f_{\pi^+} = \mathbf{1.1932(21)}$$

0.18% uncertainty

Results have been stable over the years

For average substract IB corr.

$$f_K/f_\pi = \mathbf{1.1967(18)}$$

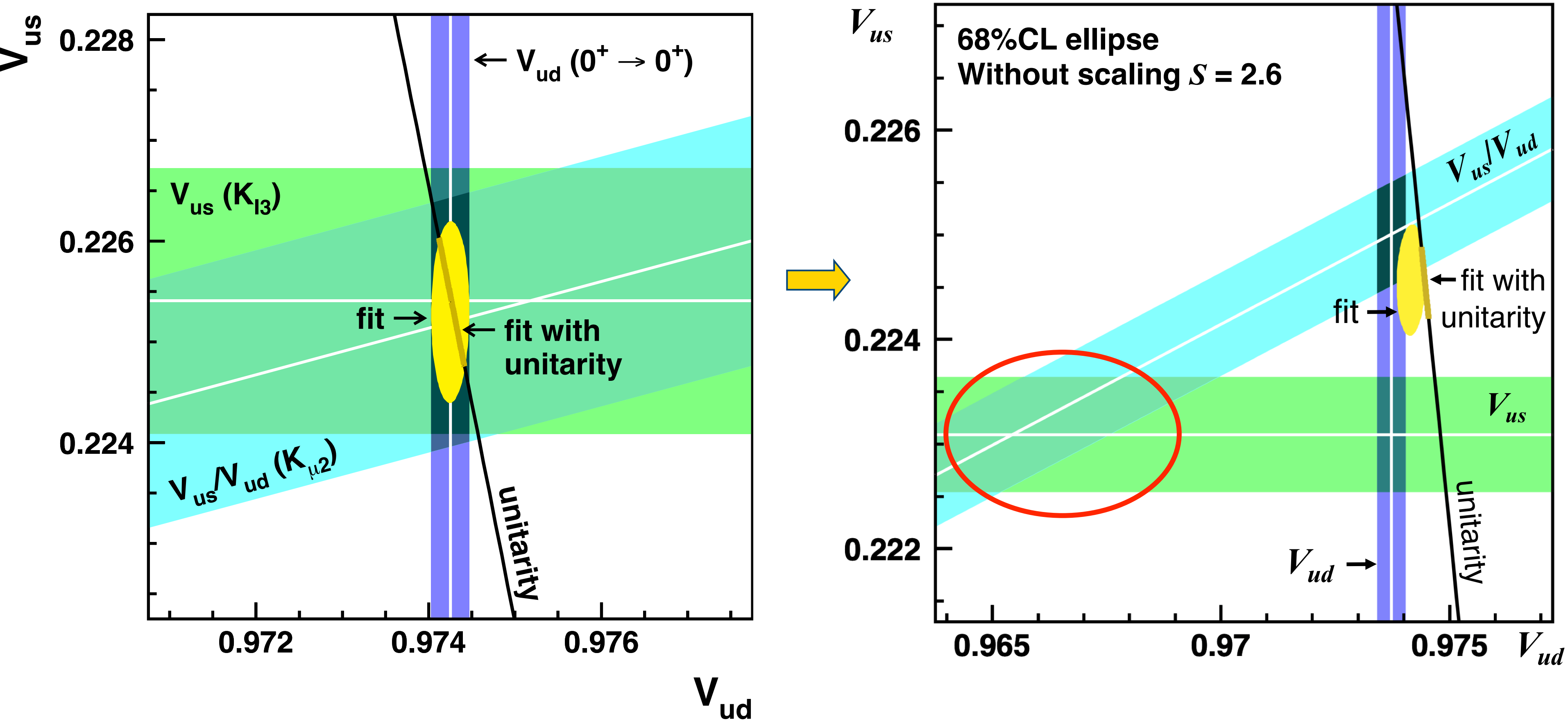
In 2011: $f_K/f_\pi = \mathbf{1.193(6)}$

⇨ $V_{us}/V_{ud} = 0.\ 23108(29)_{exp}(42)_{lat}$

Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*

Moulson & E.P.@CKM2021

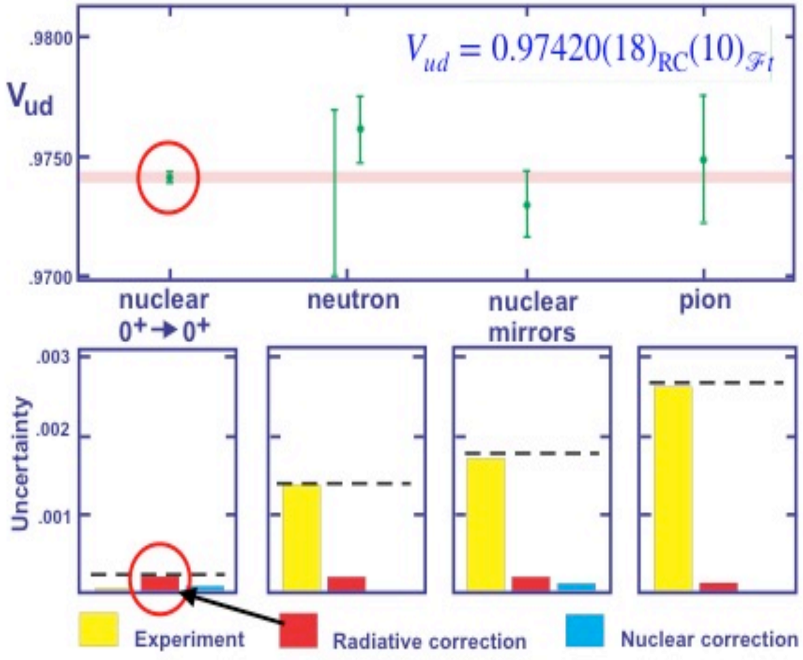


$|V_{ud}|$ from $0^+ \rightarrow 0^+$ superallowed β decays

See Talk by Misha Gorshteyn @CKM2021

PDG 2018:

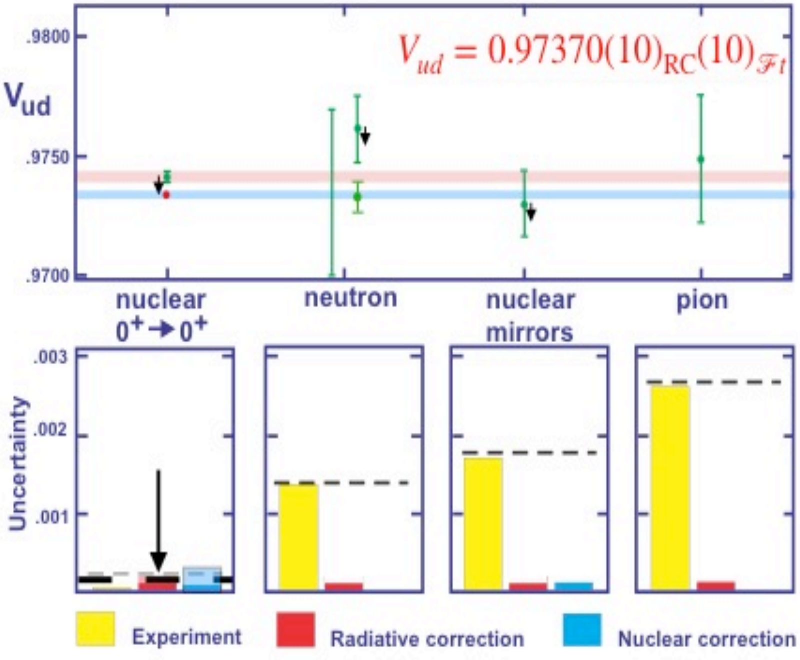
$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9994(4)_{V_{ud}}(2)_{V_{us}}$$



PDG 2020:

Figure adapted from J. Hardy

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9985(3)_{V_{ud}}(4)_{V_{us}}$$

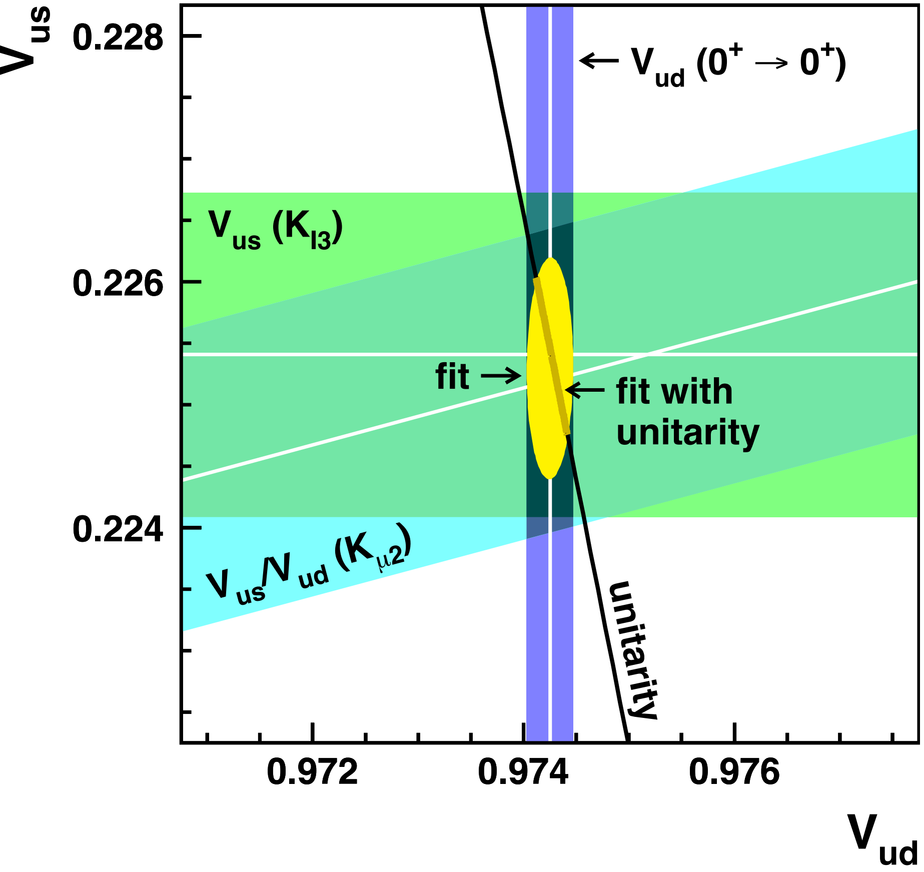


Recent improvement on the theoretical RCs +Nuclear Structure Corrections

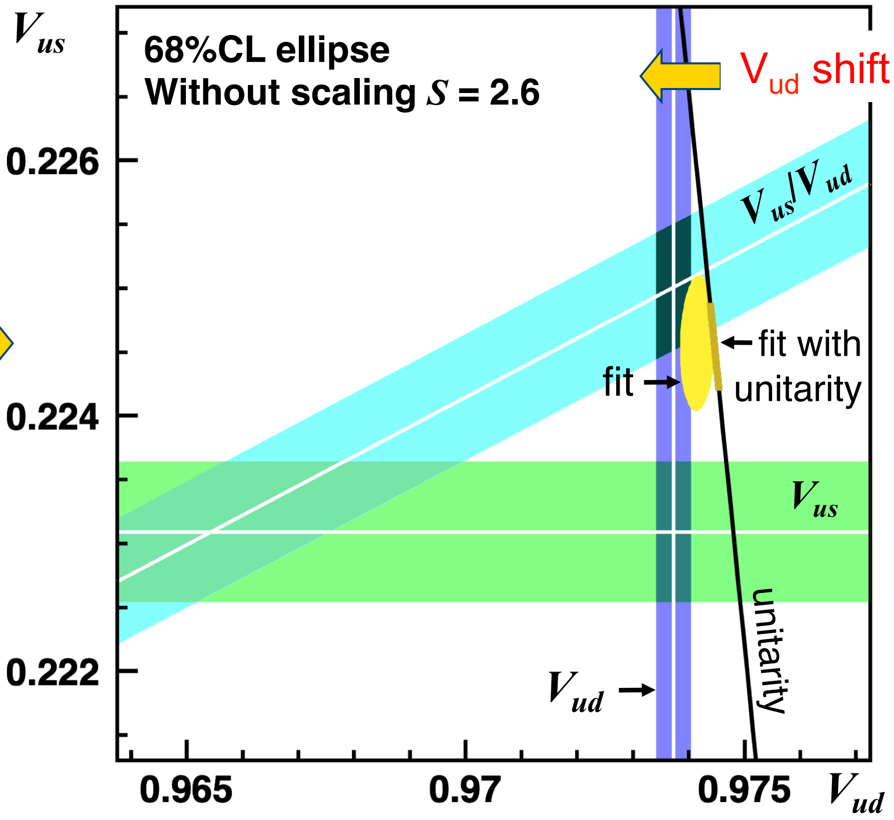
⇨ Use of a data driven dispersive approach *Seng et al.'18'19, Gorshteyn'18*

Changes on V_{us} and V_{ud} since 2011

Flavianet Kaon WG: *Antonelli et al'11*



Moulson & E.P.@CKM2021



Prospects

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$

- V_{us} from Hyperon decays and from Tau physics

- V_{ud} from *neutron decays : very impressive progress recently
  *pion β decay $\pi^+ \to \pi^0 e^+ \nu$: PIONEER experiment

- Lattice Progress on hadronic matrix elements: decay constants, FFs

  ⇨ Full QCD+QED decay rate on the lattice

3.2 Search for a light scalar mixing with the Higgs

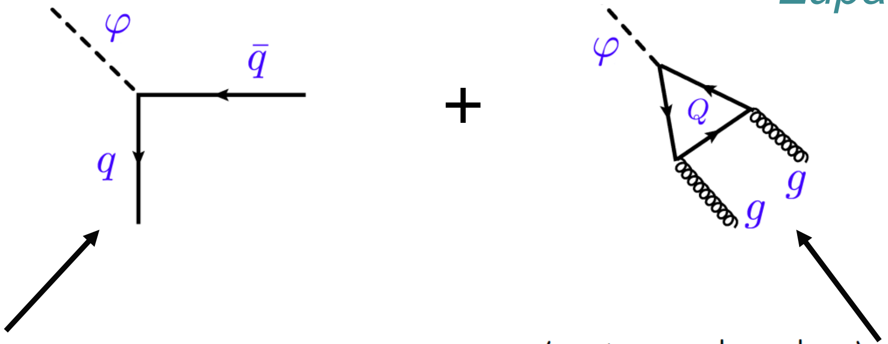
Blackstone, Tarrus Castella, E. P., Zupan in preparation

- Motivation: relaxion, dark matter and inflation models, see e.g. *Goudelis, Lebedev, Park'11,*

$$\boxed{\mathcal{L}_{\text{eff}} = -\sum_q c_q \frac{m_q}{v_W}\bar{q}q\phi - \sum_\ell c_\ell \frac{m_\ell}{v_W}\bar{\ell}\ell\phi + c_g \frac{\alpha_s}{12\pi v_W}\phi G^a_{\mu\nu}G^{a\mu\nu}}$$

Key hadronic inputs

Blackstone, Tarrus Castella, E. P., Zupan in preparation



$$s = \left(p_{\pi^+} + p_{\pi^-}\right)^2$$

$$\langle \pi^+\pi^- \,| m_u \bar{u}u + m_d \bar{d}d |\, 0 \rangle \equiv \Gamma_\pi(s) \qquad \langle \pi^+\pi^- \,| \theta^\mu_\mu |\, 0 \rangle \equiv \theta_\pi(s)$$

$$\langle \pi^+\pi^- \,| m_s \bar{s}s |\, 0 \rangle \equiv \Delta_\pi(s)$$

$$\boxed{\theta^\mu_\mu = -9\frac{\alpha_s}{8\pi}G^a_{\mu\nu}G^{\mu\nu}_a + \sum_{q=u,d,s} m_q \bar{q}q}$$

$$\Gamma_{PP} \propto \frac{s_\theta^2\,\beta_P}{m_\phi}\left|\frac{2}{9}\theta_P + \frac{7}{9}(\Gamma_P + \Delta_P)\right|^2$$

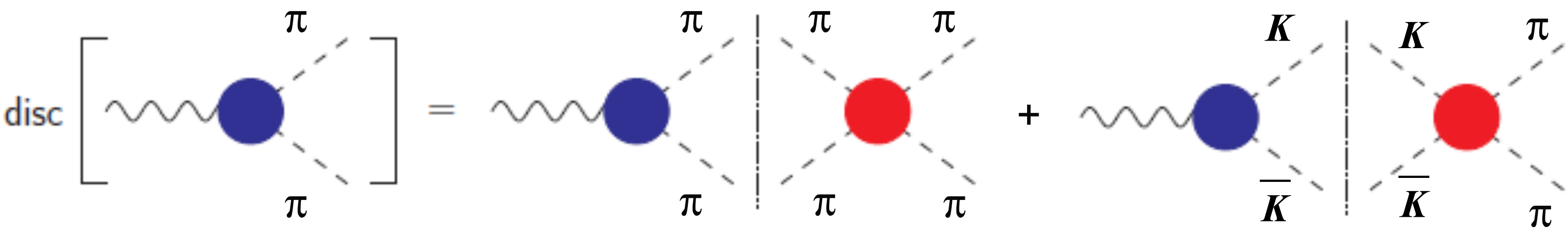
with $c_q = c_\ell = c_g = s_\theta$

Determination of the form factors

- No experimental data on the FFs → *Coupled channel analysis*
  up to √s~1.4 GeV
  Inputs: I=0, S-wave ππ and KK data

Donoghue, Gasser, Leutwyler'90
Moussallam'99
Daub, Dreiner, Hanhart, Kubis, Meissner'12
Celis, Cirigliano, E.P.'14
Winkler'19

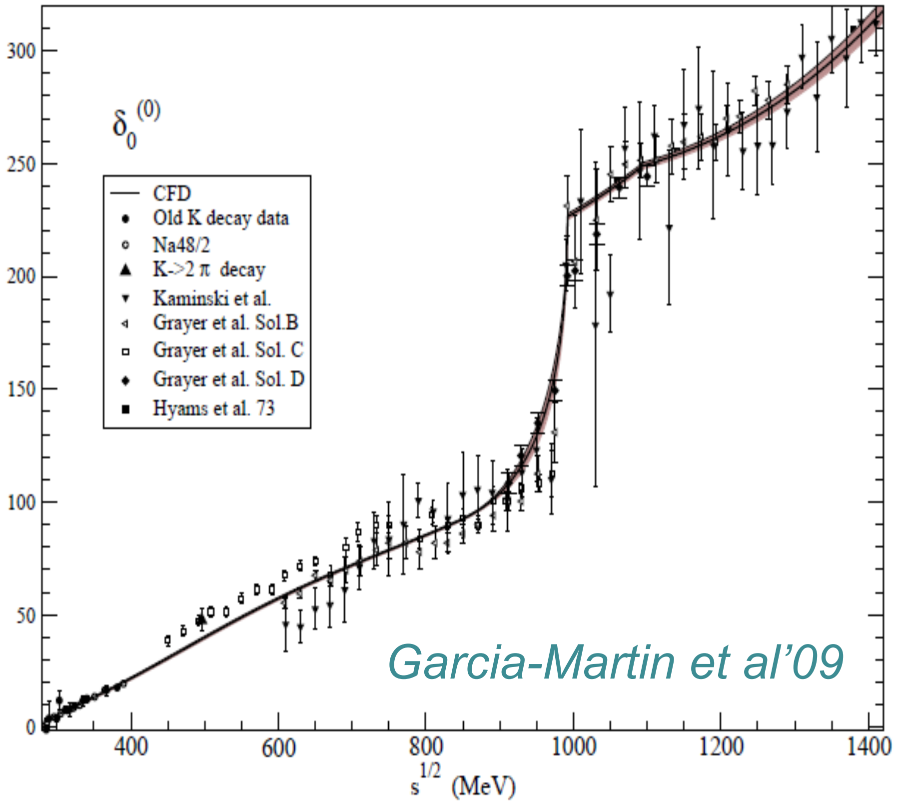
- Unitarity:



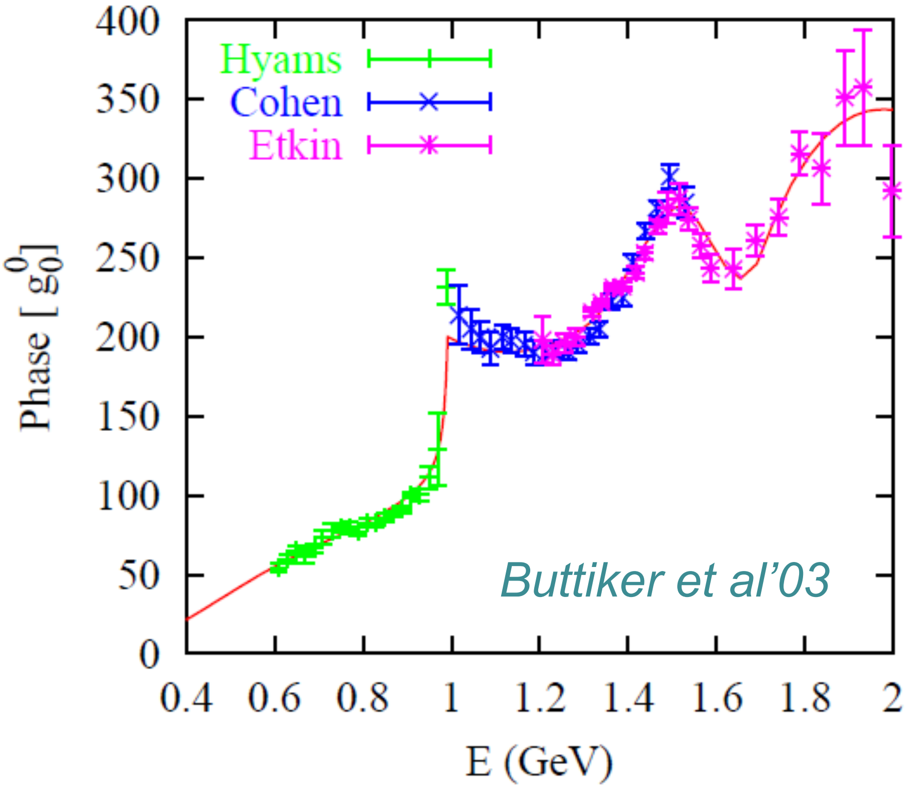
→ $$\boxed{\mathrm{Im}F_n(s) = \sum_{m=1}^{2} T^*_{nm}(s)\sigma_m(s)F_m(s)}$$ $n = \pi\pi, K\bar{K}$

Determination of the form factors

- Inputs : $\pi\pi \to \pi\pi, KK$



Garcia-Martin et al'09



Buttiker et al'03

- A large number of theoretical analyses *Descotes-Genon et al'01, Kaminsky et al'01, Buttiker et al'03, Garcia-Martin et al'09, Colangelo et al.'11* and all agree
- 3 inputs: $\delta_\pi(s)$, $\delta_K(s)$, η from *B. Moussallam* ⇨ *reconstruct T matrix*

Determination of the form factors

- General solution:

$$\boxed{\begin{pmatrix} F_\pi(s) \\ \frac{2}{\sqrt{3}} F_K(s) \end{pmatrix} = \begin{pmatrix} C_1(s) & D_1(s) \\ C_2(s) & D_2(s) \end{pmatrix} \begin{pmatrix} P_F(s) \\ Q_F(s) \end{pmatrix}}$$

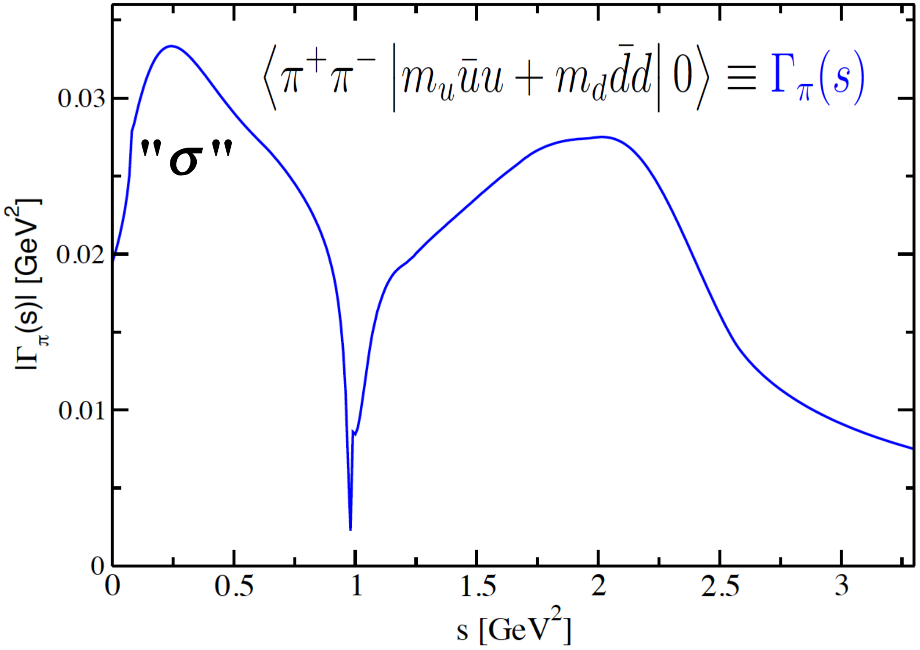
Canonical solution

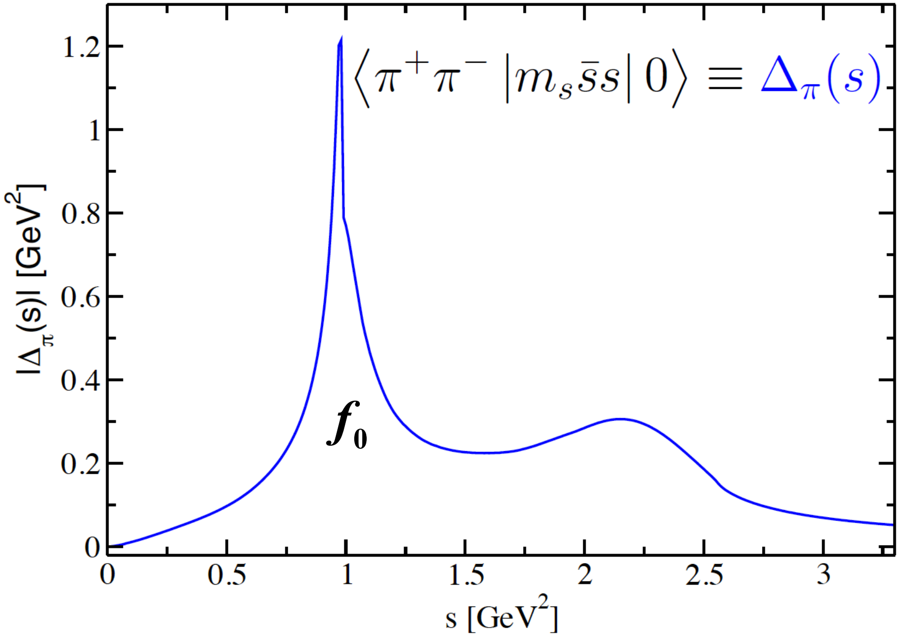
Polynomial determined from a matching to ChPT + lattice

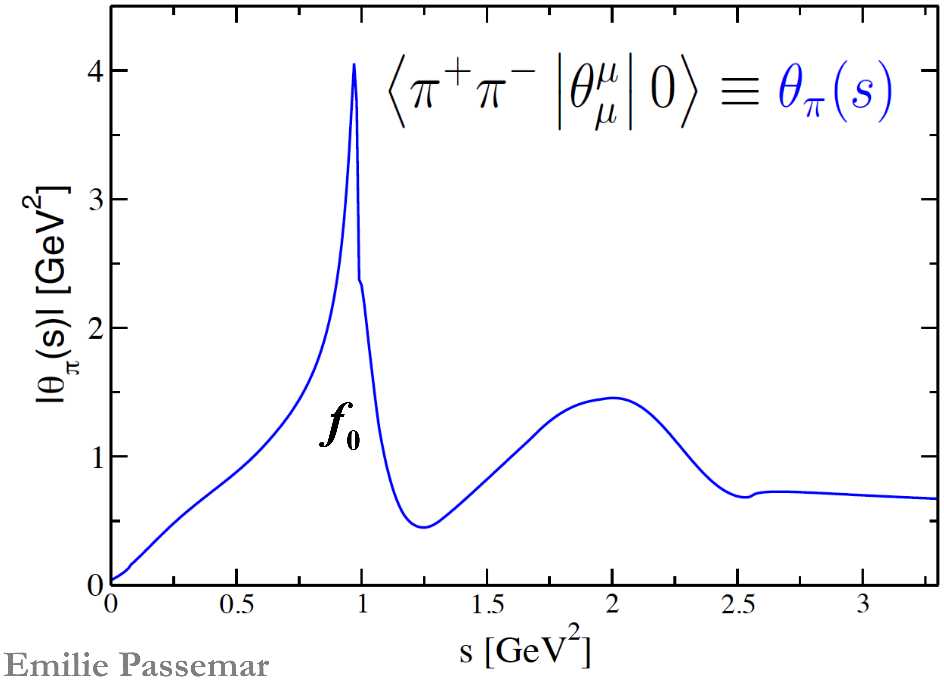
- Canonical solution found by solving the dispersive integral equations iteratively starting with Omnès functions

$$\boldsymbol{X(s) = C(s), D(s)}$$

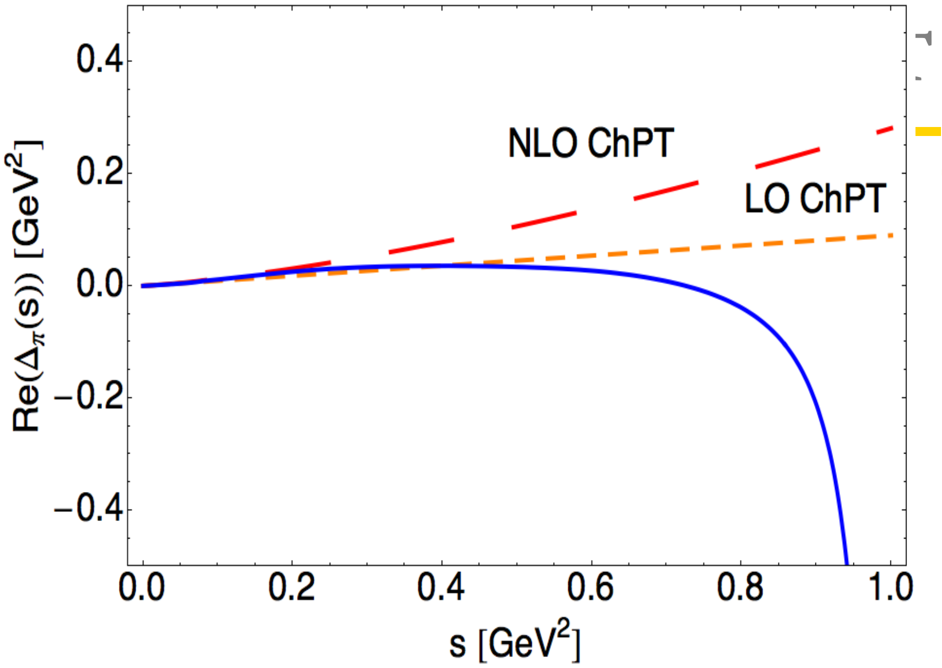
$$\boxed{\mathrm{Im} X_n^{(N+1)}(s) = \sum_{m=1}^{2} \mathrm{Re}\left\{T^*_{nm}\sigma_m(s) X_m^{(N)}\right\}} \longrightarrow \boxed{\mathrm{Re} X_n^{(N+1)}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s} \mathrm{Im} X_n^{(N+1)}}$$

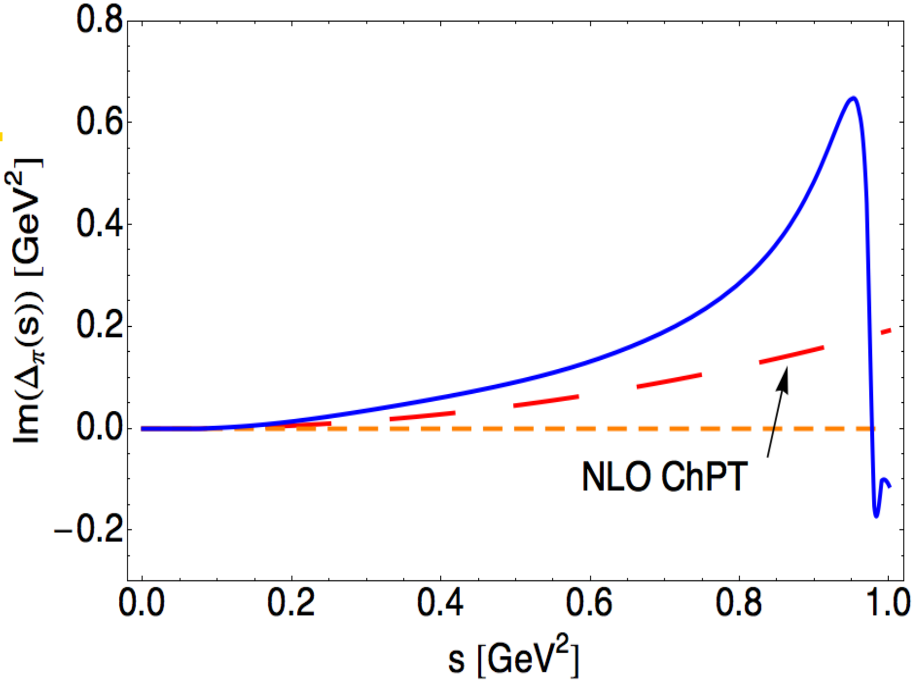


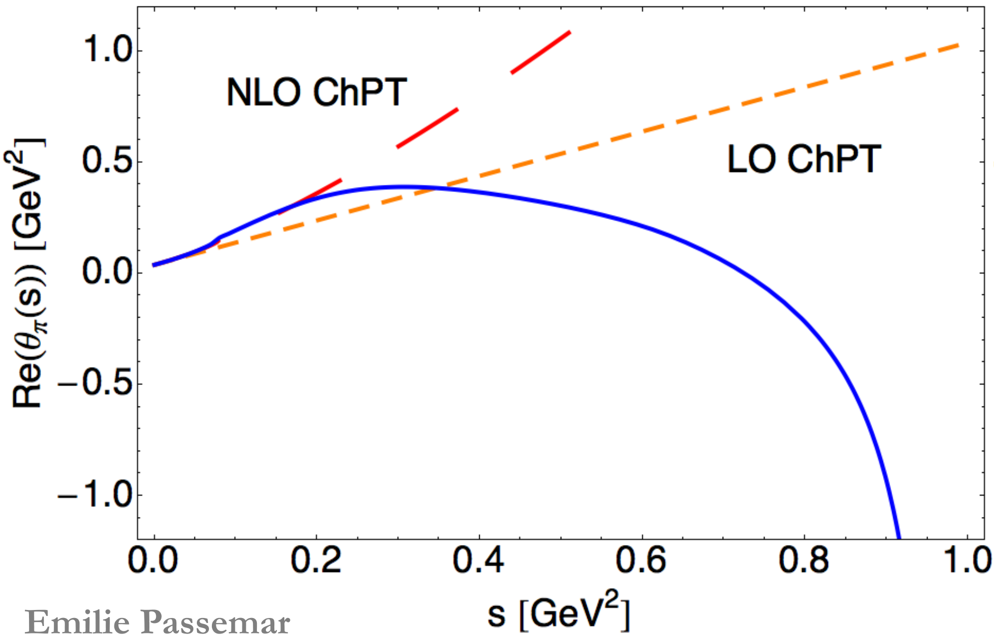


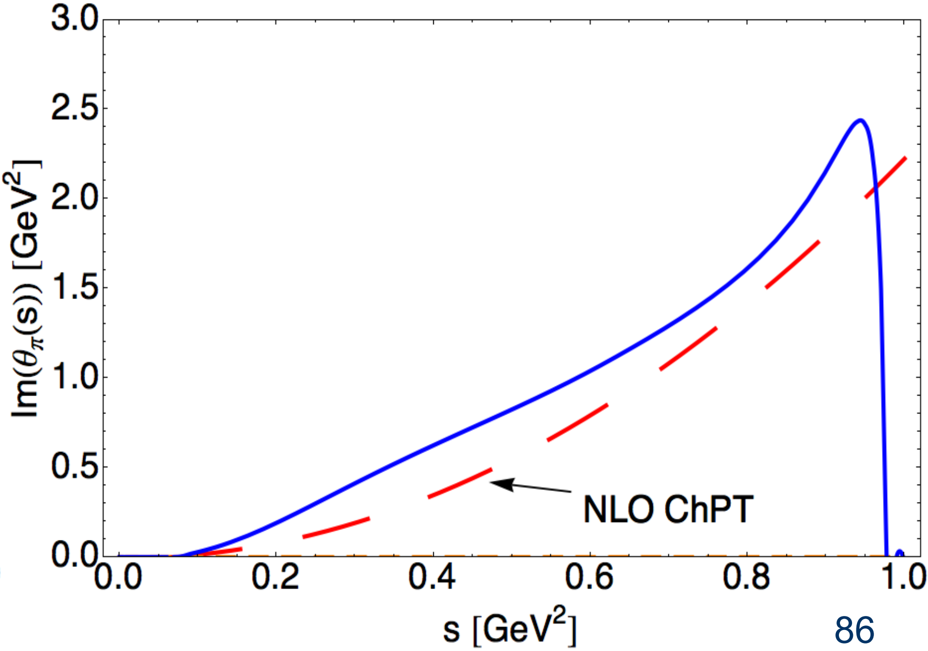


Dispersion relations:
Model-independent method,
based on first principles
that extrapolates ChPT
based on data

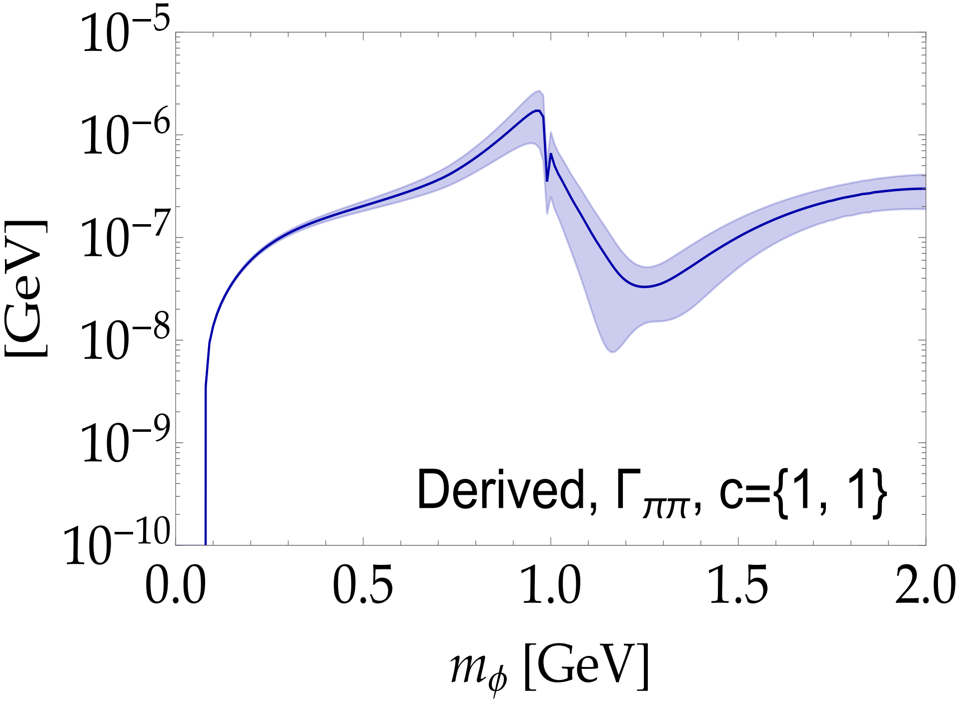

NLO ChPT
LO ChPT
0.4
0.2
0.0
−0.2
−0.4
$\mathrm{Re}(\Delta_\pi(s))$ [GeV^2]
0.0
0.2
0.4
0.6
0.8
1.0
s [GeV^2]


0.8
0.6
0.4
0.2
0.0
−0.2
$\mathrm{Im}(\Delta_\pi(s))$ [GeV^2]
NLO ChPT
0.0
0.2
0.4
0.6
0.8
1.0
s [GeV^2]

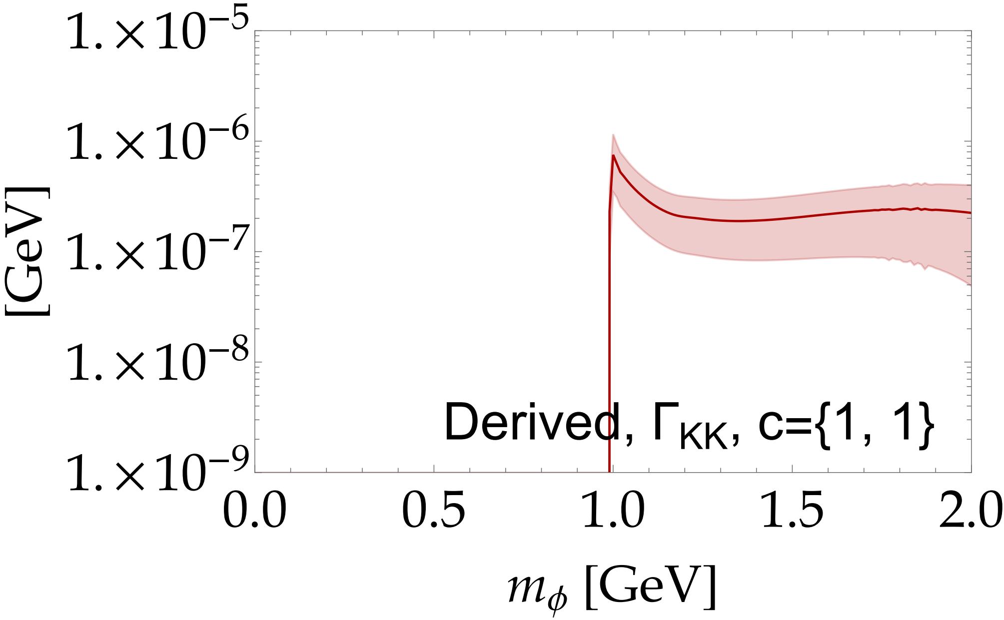

NLO ChPT
LO ChPT
1.0
0.5
0.0
−0.5
−1.0
$\mathrm{Re}(\theta_\pi(s))$ [GeV^2]
0.0
0.2
0.4
0.6
0.8
1.0
s [GeV^2]


3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\mathrm{Im}(\theta_\pi(s))$ [GeV^2]
NLO ChPT
0.0
0.2
0.4
0.6
0.8
1.0
s [GeV^2]

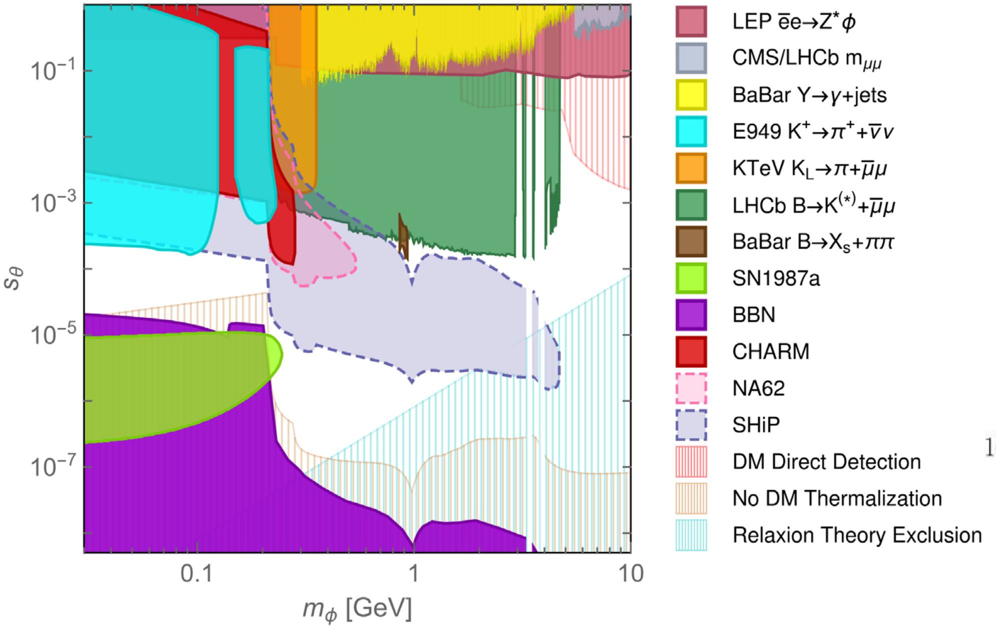
Results



$$\Gamma_{PP} \propto \frac{s_\theta^2\, \beta_P}{m_\phi} \left| \frac{2}{9}\theta_P + \frac{7}{9}(\Gamma_P + \Delta_P) \right|^2$$



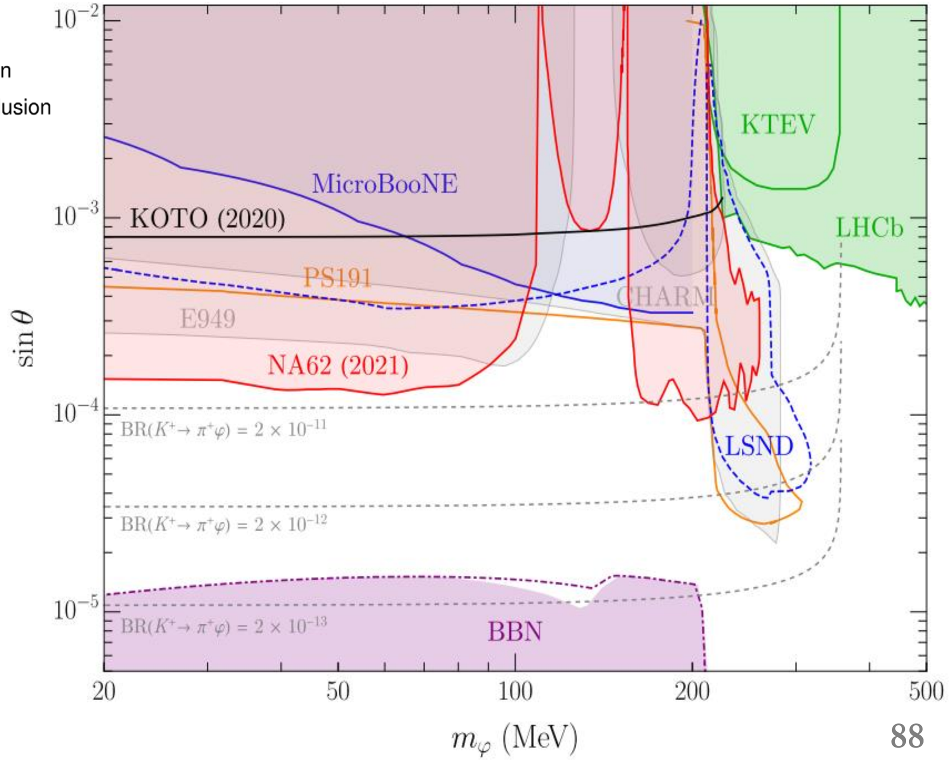
Constraints on a light scalar mixing with the Higgs



Adapted from Wrinkler'19

Blackstone, Tarrus Castella, E. P., Zupan in preparation

Goudzovski et al. (Snowmass)'22



4. Conclusion and Outlook

4.1 Conclusion

- Hadronic physics is crucial to understand fundamental laws of physics and new physics phenomena

- K, D and B mesons measurements more accurate → require inputs from hadronic physics

- To reach this quest, studying interactions of quarks, leptons and even neutrinos with high precision requires a precise knowledge of hadronic physics: directly for quark interactions or indirectly for leptons and neutrinos

- Hadronic physics relies on non-perturbative techniques to treat QCD at low energies: → synergies between lattice QCD and analytical methods: ChPT, dispersion relations, etc.

- We have enter a precision era in all domains of particle physics requiring an unprecedent effort in taming the hadronic uncertainties

4.2 Outlook

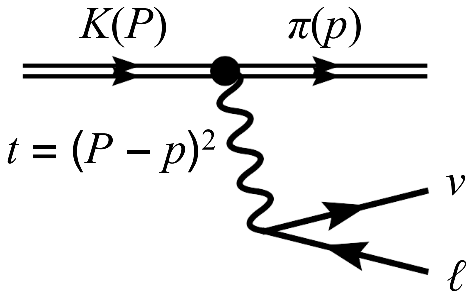
- To answer these new demands, we can use precision hadronic physics *combining* ChPT, dispersion relations with lattice results

- I showed two examples:
  - Cabibbo Anomaly
  - Constraints on a light scalar mixing with the Higgs

- Still some challenges which need to be addressed:
  - Bridge the gap between dispersive analyses and Perturbative QCD
  - Radiative corrections: Electromagnetic and Isospin Breaking

5. Back up

2.2 V_{us} from K_{l3} ($K \to \pi l \nu_l$)



- Master formula for $K \to \pi l \nu_l$: $K = \{K^+, K^0\}$, $l = \{e, \mu\}$

$$\boxed{\Gamma\left(K \to \pi l \nu\left[\gamma\right]\right) = Br(K_{l3})/\tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl} \left(1 + 2\Delta_{\mathrm{EM}}^{Kl} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)}$$

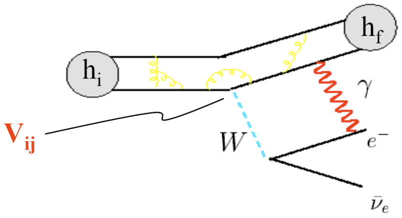
Average and work by Flavianet Kaon WG *Antonelli et al'11* and then by *M. Moulson*, see e.g. *Moulson.@CKM2021*

Theoretically

- Update on long-distance EM corrections for K_{e3} *Seng et al.'21*
- Improvement on Isospin breaking evaluation due to more precise dominant input: quark mass ratio from $\eta \to 3\pi$ *Colangelo et al.'18*
- Progress from lattice QCD on the $K \to \pi$ FF

$$\boxed{\left\langle \pi^-(p) \right| \bar{\mathrm{s}}\gamma_\mu \mathrm{u} \left| \mathrm{K}^0(\mathrm{P}) \right\rangle = f_+^{K^0\pi^-}(0)\left[\left(P+p\right)_\mu \bar{f}_+^{K^0\pi^-}(t) + \left(P-p\right)_\mu \bar{f}_-^{K^0\pi^-}(t)\right]}$$

4.3 V_{ud} from $0^+ \longrightarrow 0^+$



$$\frac{1}{t} = \frac{G_\mu^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} f(Q)\,(1+RC) \longrightarrow f\,t\,(1+RC) = \frac{2984.48(5)\ s}{|V_{ud}|^2}$$

$$(1+RC) = (1-\delta_C)\,(1+\delta_R)\,(1+\Delta_R)$$

$$\langle f|\tau_+|i\rangle = \sqrt{2}\,(1-\delta_C/2)$$

Coulomb distortion
of wave-functions

$$\delta_C \sim 0.5\%$$

Towner-Hardy
Ormand-Brown

Nucleus-dependent
rad. corr.
(Z, E^{max} ,nuclear structure)

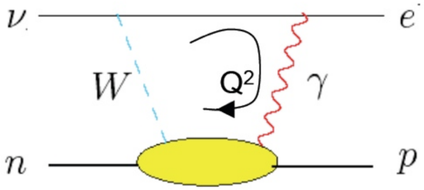
$$\delta_R \sim 1.5\%$$

Sirlin-Zucchini '86
Jaus-Rasche '87

Nucleus-independent
short distance rad. corr.

$$\Delta_R \sim 2.4\%$$

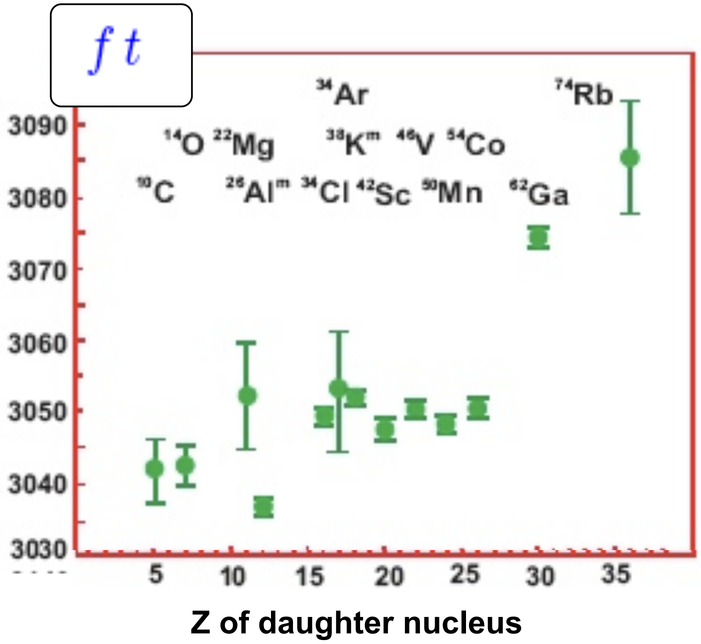
Marciano-Sirlin '06

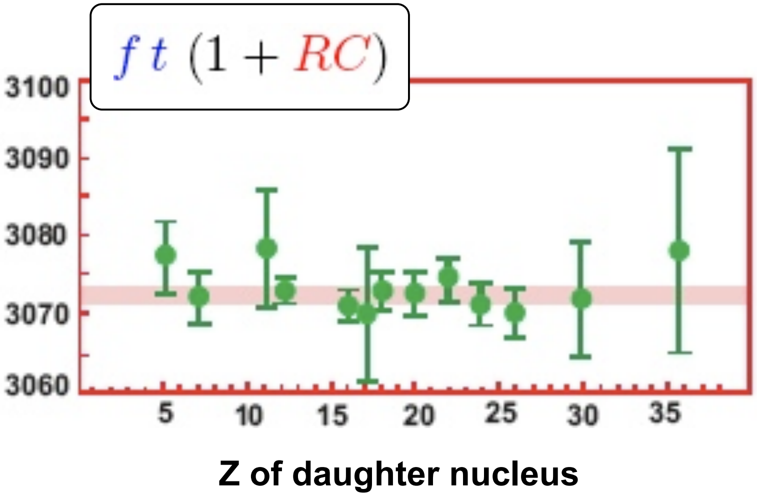


From V. Cirigliano

4.3 V_{ud} from $0^+ \rightarrow 0^+$

Hardy@Amherst'19

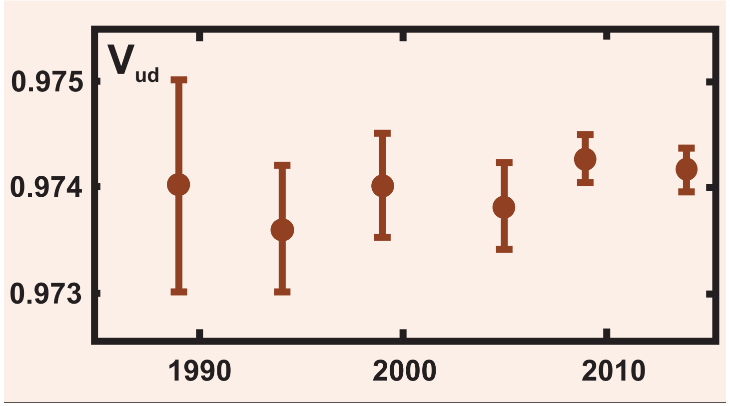




$$\boxed{|V_{ud}| = 0.97418(21)}$$

Improvements over years :

- Survey of 150 measurements of 13 different $0^+ \rightarrow 0^+$ β decays
- 27 new *ft* measurements including Penning-trap measurements for QEC
- Improved EW radiative corrections
  Marciano & Sirlin'06
- New *SU*(2)-breaking corrections
  Towner & Hardy'08



4.4 New Radiative Corrections for $0^+ \rightarrow 0^+$

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

- Conventional calculation:

$$\Delta_R^V = 0.02361(38)$$

Marciano & Sirlin'06

- Dispersion Relations:

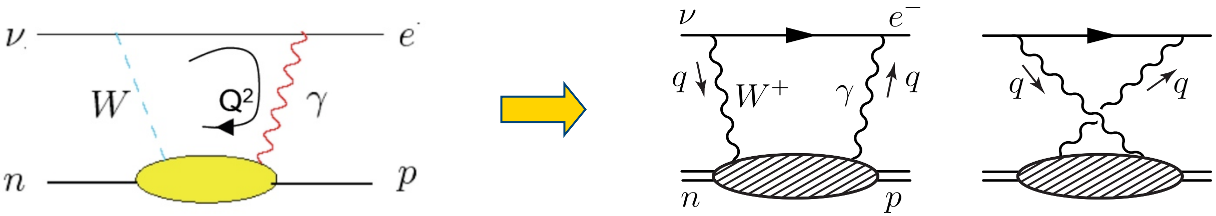
$$\Delta_R^V = 0.02467(22)$$

Seng, Gorchtein, Patel & Ramsey-Musolf'18

$$\boxed{|V_{ud}| = 0.97418(10)_{Ft}(18)_{\Delta_R^V}} \Rightarrow \boxed{|V_{ud}| = 0.97370(10)_{Ft}(10)_{\Delta_R^V}}$$

~1.8σ smaller

4.4 New Radiative Corrections for $0^+ \rightarrow 0^+$



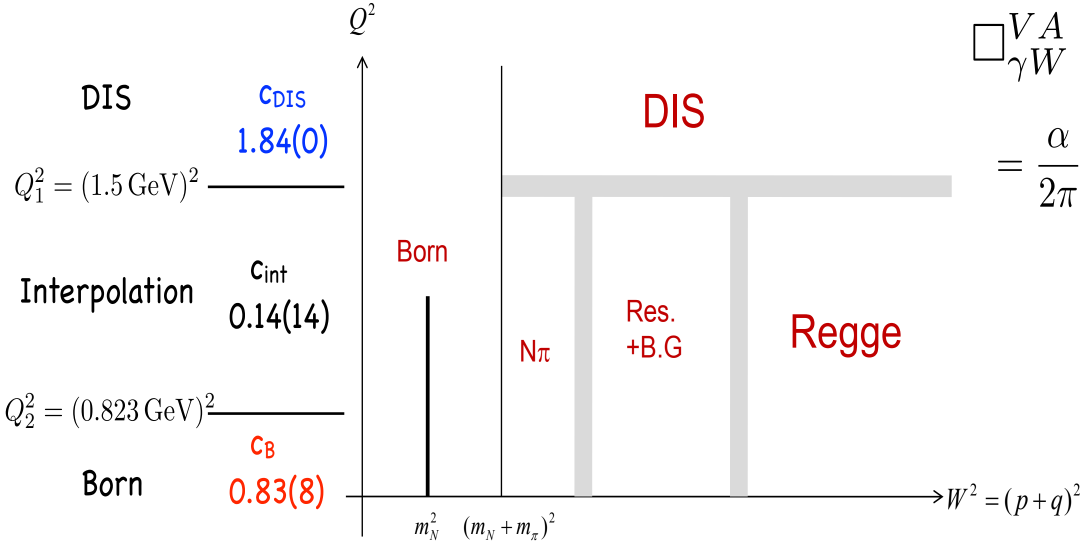
Gorchtein@CIPANP'18
Amherst'19

Marciano & Sirlin'06

$$\Box^{VA}_{\gamma W} = \frac{\alpha}{2\pi}[c_B + c_{int} + c_{DIS}] = \frac{\alpha}{2\pi}[0.83(8) + 0.14(14) + 1.84(0)]$$

$$\Box^{MS}_{\gamma W} = \frac{\alpha}{2\pi}2.79(17) = 3.24(20) \times 10^{-3}$$

New evaluation: *Seng, Gorchtein, Patel & Ramsey-Musolf'18*



$$\Box^{VA}_{\gamma W} = \frac{\alpha}{2\pi}[c_B + c_{piN} + c_{\mathrm{Res}} + c_{\mathrm{Regge}} + c_{DIS}]$$

$$= \frac{\alpha}{2\pi}[0.91(5) + 0.044(5) + 0.01(1) + 0.238(14) + 1.84(0)]$$

$$\boxed{\Box^{\mathrm{New}}_{\gamma W} = \frac{\alpha}{2\pi}3.03(5) = 3.51(6) \times 10^{-3}}$$

2.8 $|V_{ud}|$ from Neutrons

See Talk by Chen Yu Liu this afternoon

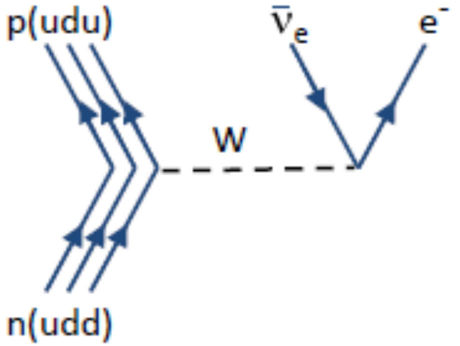
- Master Formula:

$$\boxed{|V_{ud}|^2 = \frac{5024.7s}{\tau_n\left(1+3\lambda^2\right)\left(1+\Delta_R\right)}}$$

Lifetime

$$\lambda = g_A / g_V$$

$$n \rightarrow p + e^- + \overline{\nu}_e$$



- Needs δλ/λ ≈ 3 × 10^{-4} and $\delta\tau_n \approx 0.3$ s to compete with $0^+ \rightarrow 0^+$ transitions.
- Theoretically, the radiative corrections are under control (same as for $0^+ \rightarrow 0^+$)
- Recent progress :
  - New Perkeo III result: *PERKEO III* result improves world-average of beta asymmetry by factor 5! Half of it is due to the reduction of the scale factor

$$\boxed{A = -0.11958(21),\ S = 1.2 \quad \lambda_A = -1.2757(5)}$$

  - Tension with *aSPECT* result:

$$\boxed{\lambda_{\text{avg}} = -1.2754(13),\ S = 2.7}$$

2.8 $|V_{ud}|$ from Neutrons

See Talk by Chen Yu Liu this afternoon

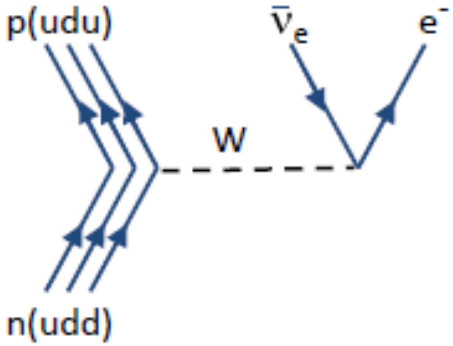
- Master Formula:

$$\boxed{|V_{ud}|^2 = \frac{5024.7s}{\tau_n\left(1+3\lambda^2\right)\left(1+\Delta_R\right)}}$$

Lifetime

$$\lambda = g_A / g_V$$

$$n \rightarrow p + e^- + \overline{\nu}_e$$



- Needs $\delta\lambda/\lambda \approx 3 \times 10^{-4}$ and $\delta\tau_n \approx 0.3$ s to compete with $0^+ \rightarrow 0^+$ transitions.
- Theoretically, the radiative corrections are under control (same as for $0^+ \rightarrow 0^+$)
- Recent progress :
  - New Perkeo III result: *PERKEO III* result improves world-average of beta asymmetry by factor 5! Half of it is due to the reduction of the scale factor

⇨ $$\boxed{A = -0.11958(21),\ S = 1.2 \quad \lambda_A = -1.2757(5)}$$

  - New result for Lifetime from *UCNτ* $\tau_n = 877.75 \pm 0.28^{+0.22}_{-0.16}$ s

⇨ improvement by a factor of 2.25 compared to previous result

2.9 $|V_{ud}|$ from pion β decay: $\pi^+ \to \pi^0 e^+ \nu$

- Theoretically cleanest method to extract V_{ud} : corrections computed in SU(2) ChPT

Sirlin'78, Cirigliano et al.'03, Passera et al'11

- Present result: *PIBETA* Experiment (2004) → **Uncertainty: 0.64%**

$$B(\pi^+ \to \pi^0 e^+ \nu) = (1.036 \pm 0.004_{\rm stat} \pm 0.004_{\rm syst} \pm 0.003_{\pi e2})\ \mathrm{x}10^{-8}\left(\pm 0.6\%\right)$$

➡ $\boxed{|V_{ud}| = \mathbf{0.9739(28)_{exp}(1)_{th}}}$ to be compared to $|V_{ud}| = \mathbf{0.97373(31)}$

- Reduction of the theory error thanks to a new lattice calculation for RC *Feng et al'20*

- Next generation experiment PIONEER Phase II and III measurement at 0.02% level ➡ will be competitive with current $0^+ \to 0^+$ extraction

- Would be completely independent check! No nuclear correction and different RCs compared to neutron decay

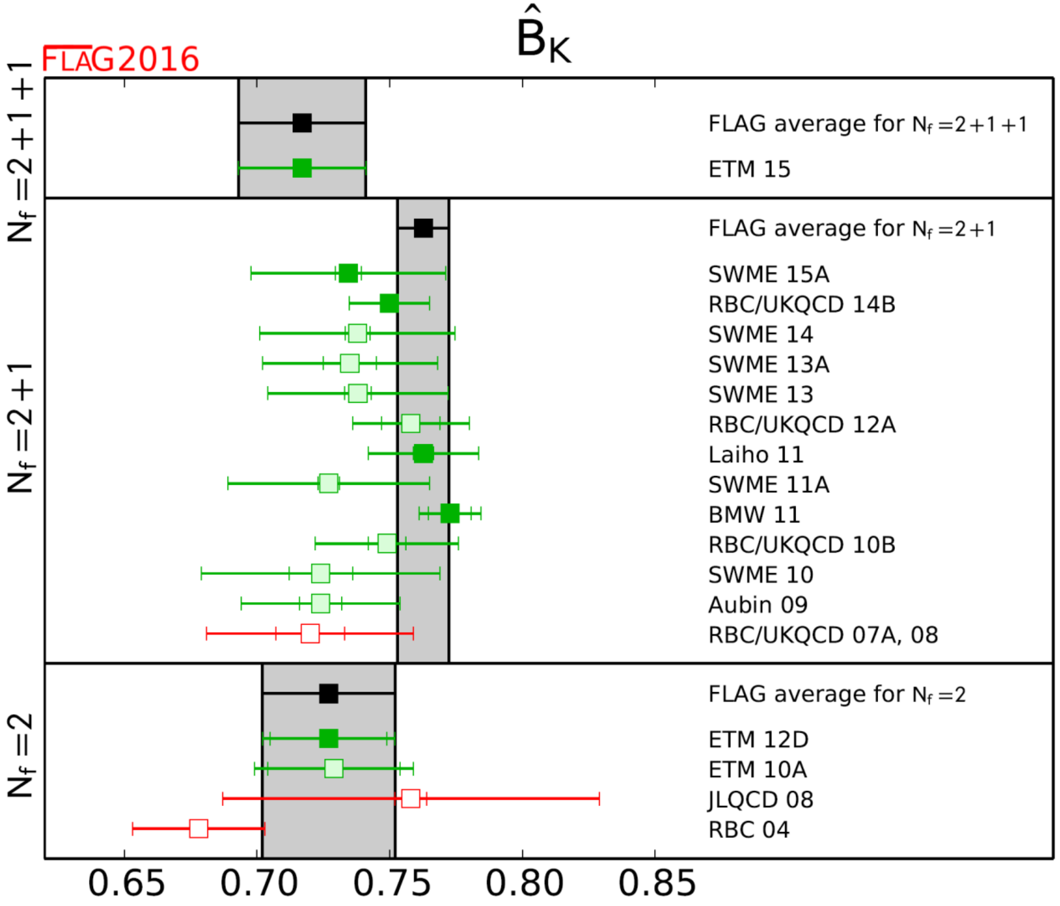
Czarnecki, Marciano, Sirlin'20

- **Opportunity to extract V_{us}/V_{ud}** from $\dfrac{B(K \to \pi l \nu)}{B(\pi^+ \to \pi^0 e^+ \nu)}$ EW Rad. Corr. cancel

Improve precision on $B(\pi^+ \to \pi^0 e^+ \nu)$ by x3 ➡ $V_{us}/V_{ud} < \pm 0.2\%$

Lattice results for BK

$B_K^{\overline{\text{MS}}}(2\,\text{GeV}) = 0.557 \pm 0.007$, $\hat{B}_K = 0.763 \pm 0.010$ $\left(N_f = 2+1\right)$



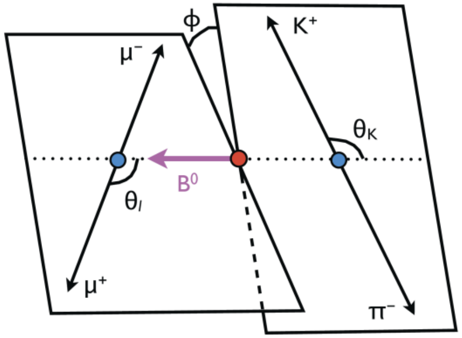
Flavianet Lattice Averaging Group

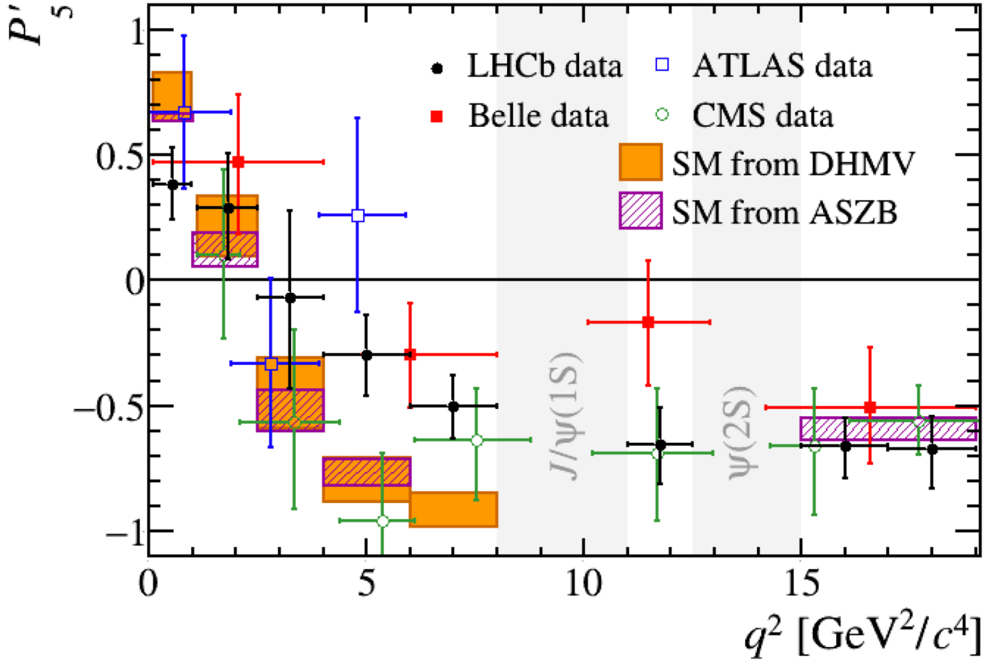
$B \rightarrow K^*\mu^+\mu^- \rightarrow K\pi\mu^+\mu^-$

$$\frac{1}{\mathrm{d}\Gamma/dq^2}\frac{\mathrm{d}^4\Gamma}{\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_K\,\mathrm{d}\phi\,\mathrm{d}q^2} = \frac{9}{32\pi}\Big[\frac{3}{4}(1-F_\mathrm{L})\sin^2\theta_K + F_\mathrm{L}\cos^2\theta_K + \frac{1}{4}(1-F_\mathrm{L})\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_\mathrm{L}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+ S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+ S_6\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

$$q^2 = s_{\mu^+\mu^-}$$

$$P'_{i=4,5,6,8} = \frac{S_{j=4,5,7,8}}{\sqrt{F_L(1-F_L)}}$$





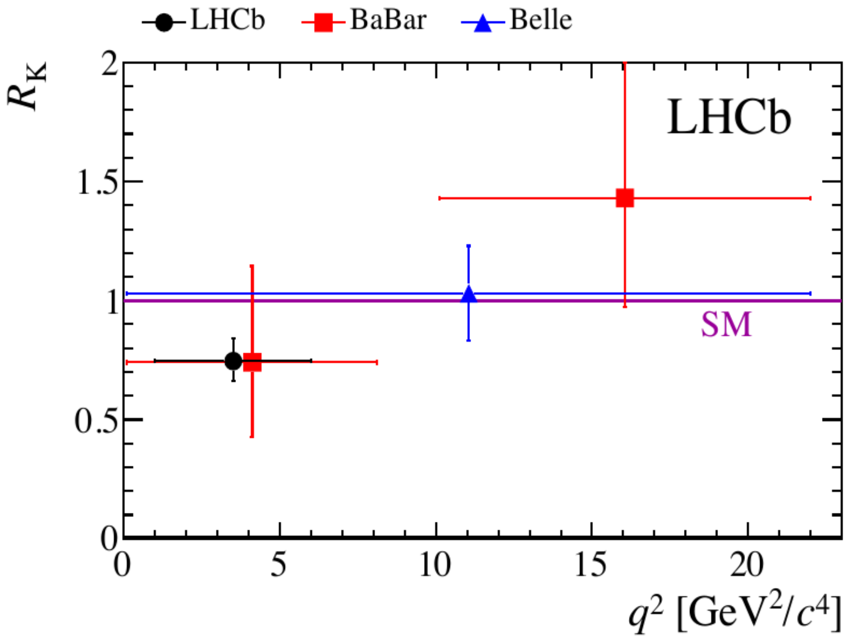
- Build an observable the less sensitive possible to hadronic uncertainties ⇨ P5’

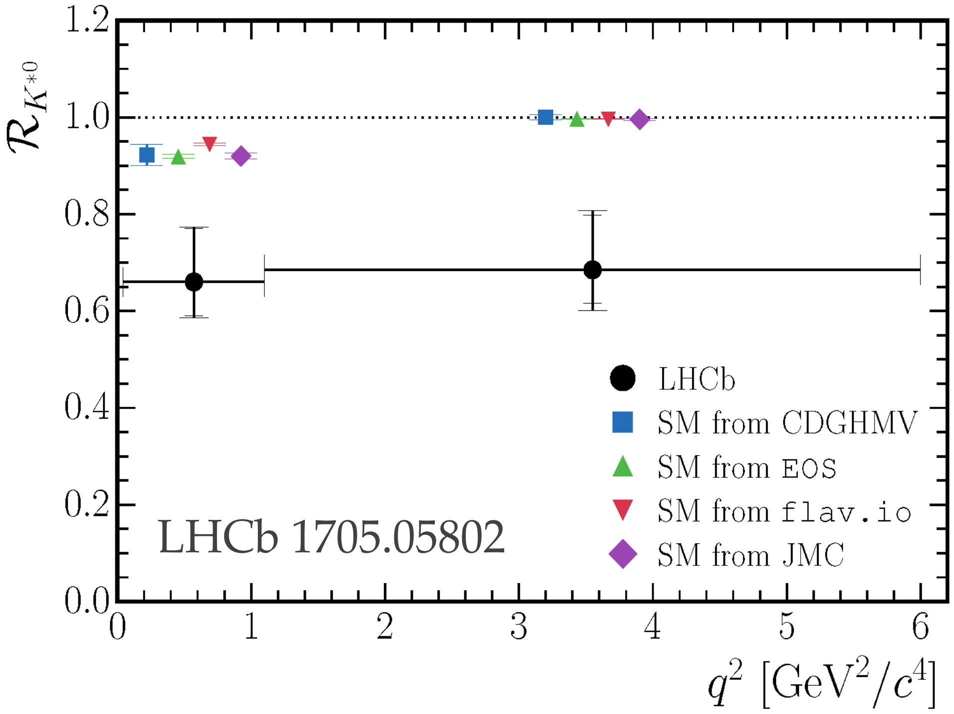
DHMV: Descotes-Genon et al.’15
ASZB: . Altmannshofer et al.

- But new physics contributions involve *hadronic physics*!

R_K, R_{K*}

LHCb 1406.6482



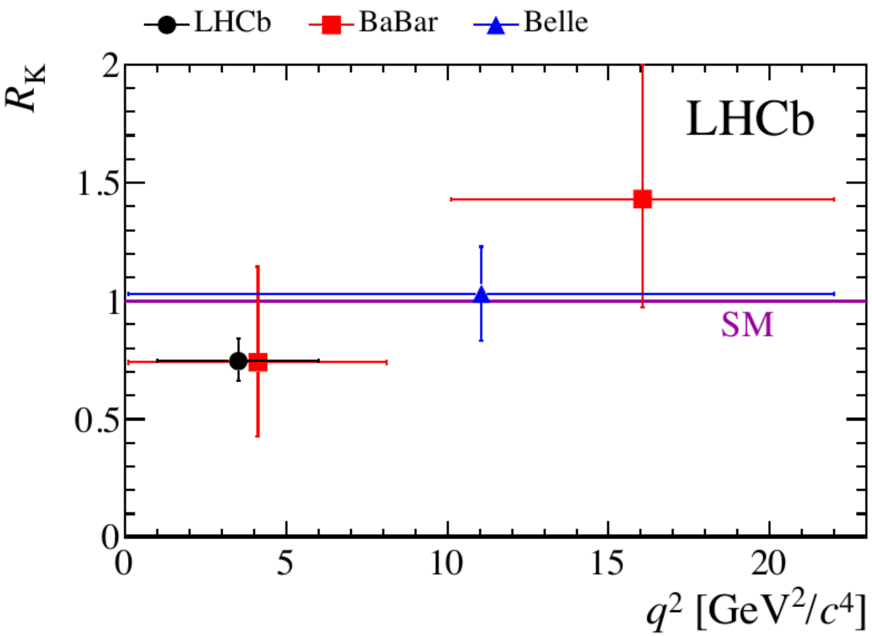

LHCb 1705.05802

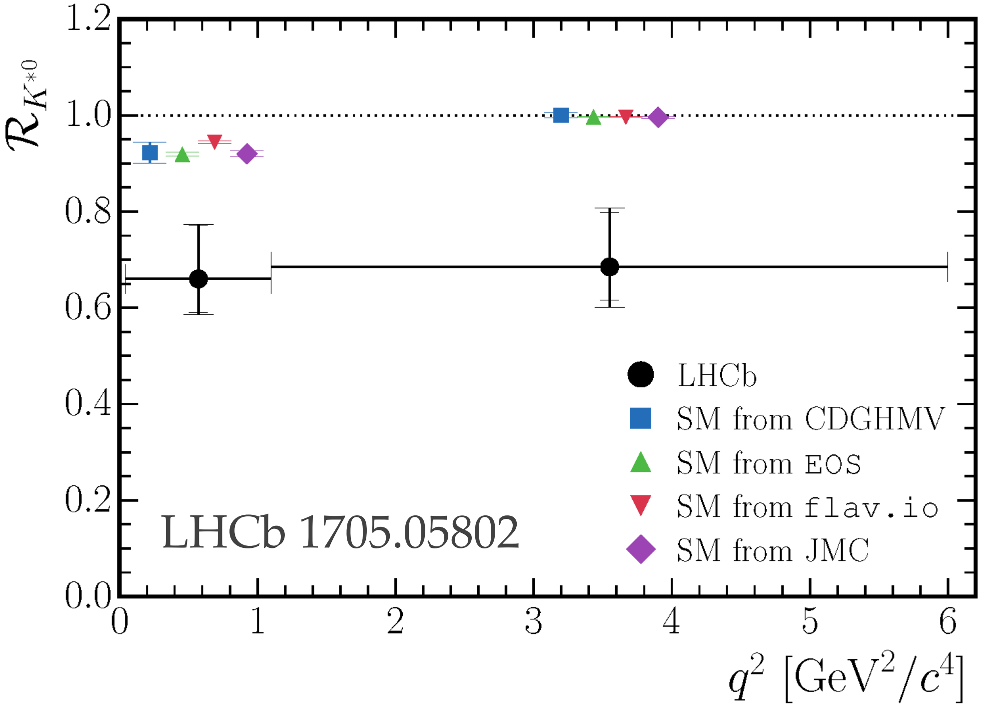
$$R_{K^{(*)}} = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)}\mu^+\mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)}e^+e^-)}$$

- Hadronic uncertainties cancel in the ratio

R_K, R_{K*}

LHCb 1406.6482





$$R_{K^{(*)}} = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)} \mu^+ \mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)} e^+ e^-)}$$

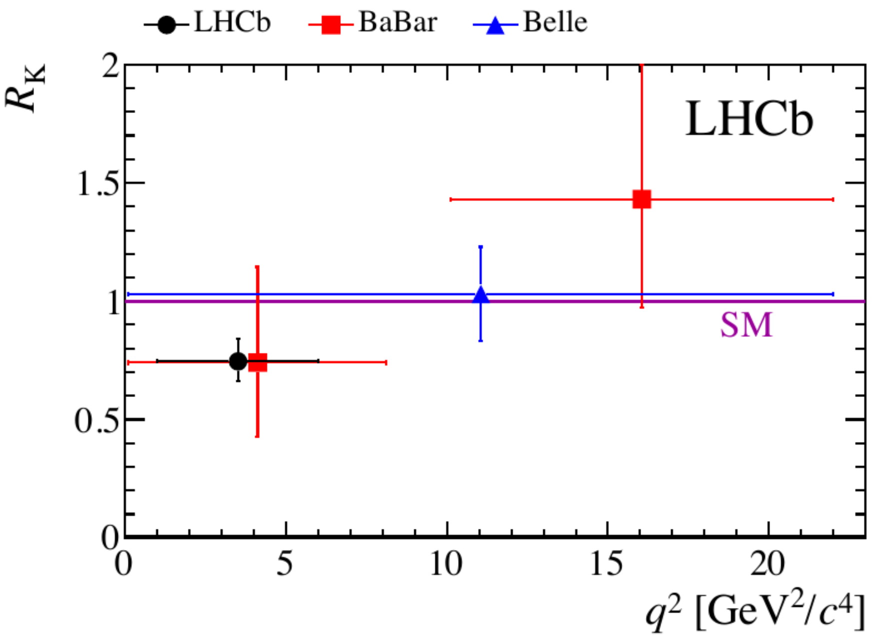
- Hadronic uncertainties cancel in the ratio
- Update from LHCb and Belle

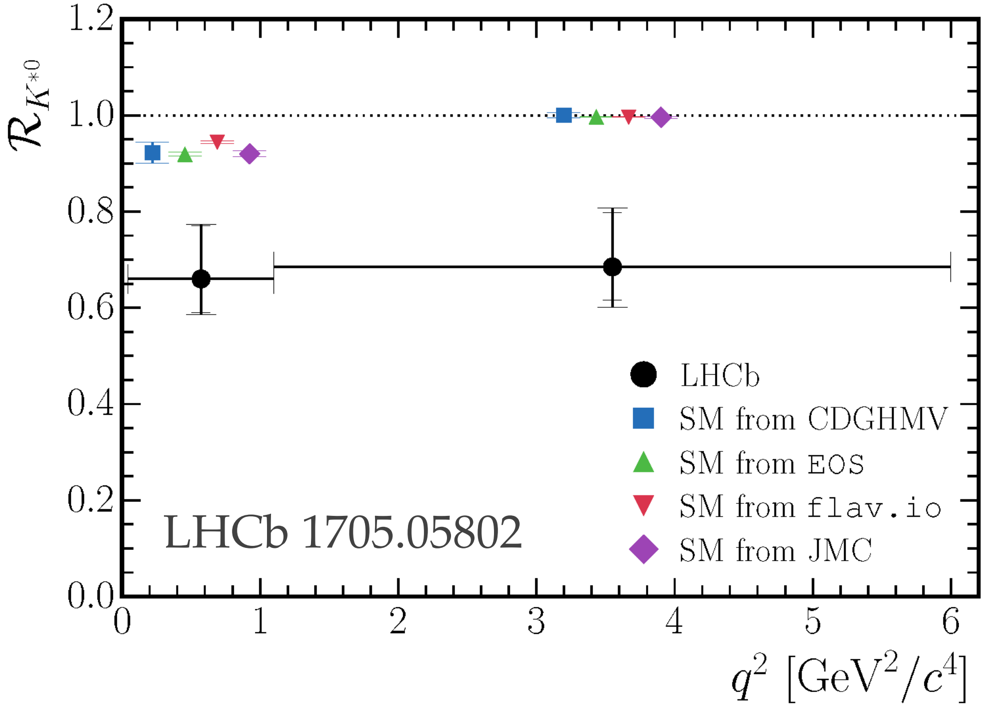
❖ Original LHC*b* result (2.6σ):

$$R_K = 0.745^{+0.090}_{-0.074}\,(\text{stat}) \pm 0.036\,(\text{syst})$$

R_K, R_{K*}

LHCb 1406.6482





LHCb 1705.05802

$$R_{K^{(*)}} = \frac{\Gamma(\bar{B} \to \bar{K}^{(*)}\mu^+\mu^-)}{\Gamma(\bar{B} \to \bar{K}^{(*)}e^+e^-)}$$

- Hadronic uncertainties cancel in the ratio
- Update from LHCb and Belle

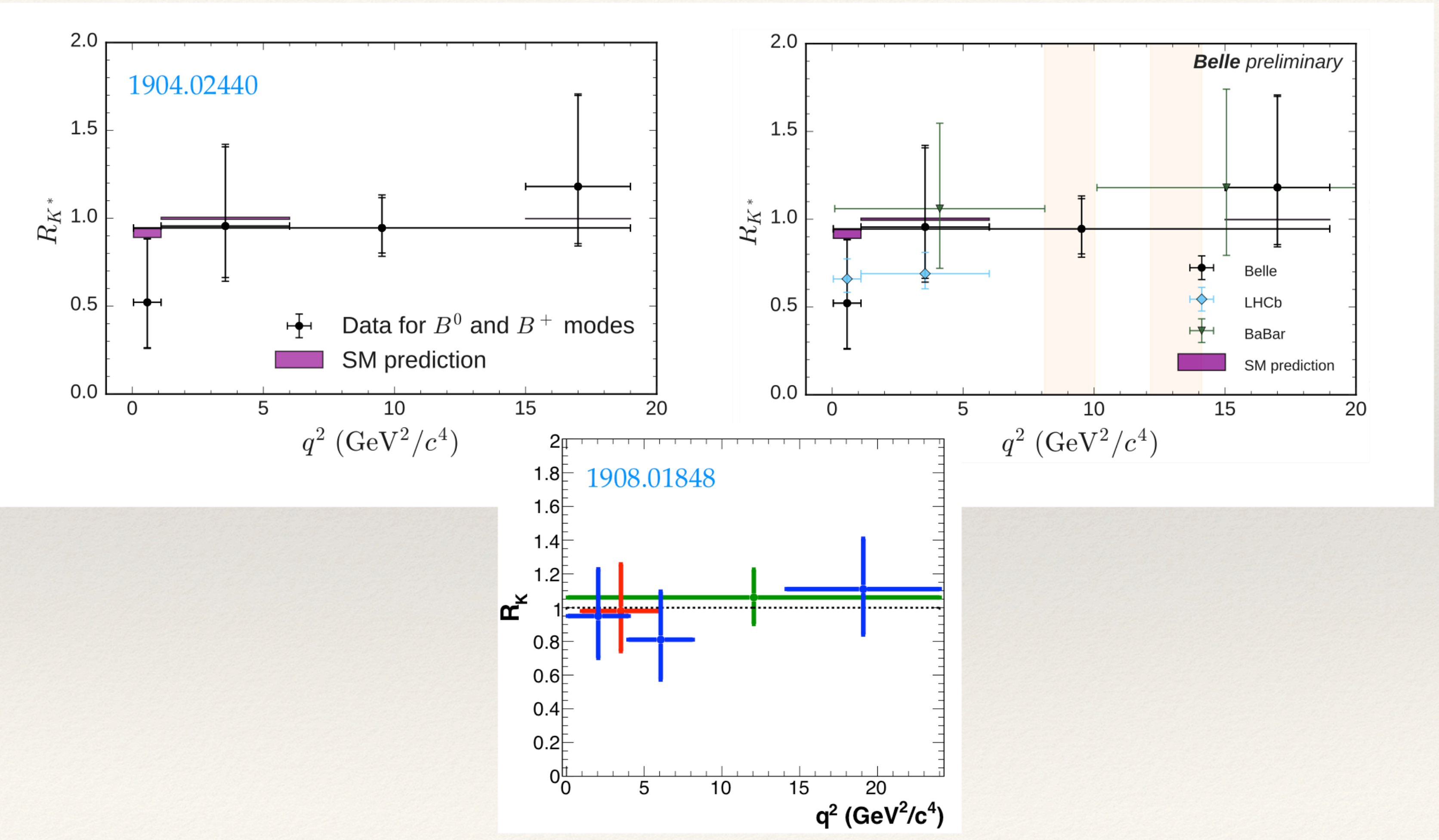
❖ Original LHC*b* result (2.6σ):

$$R_K = 0.745^{+0.090}_{-0.074}\,(\text{stat}) \pm 0.036\,(\text{syst})$$

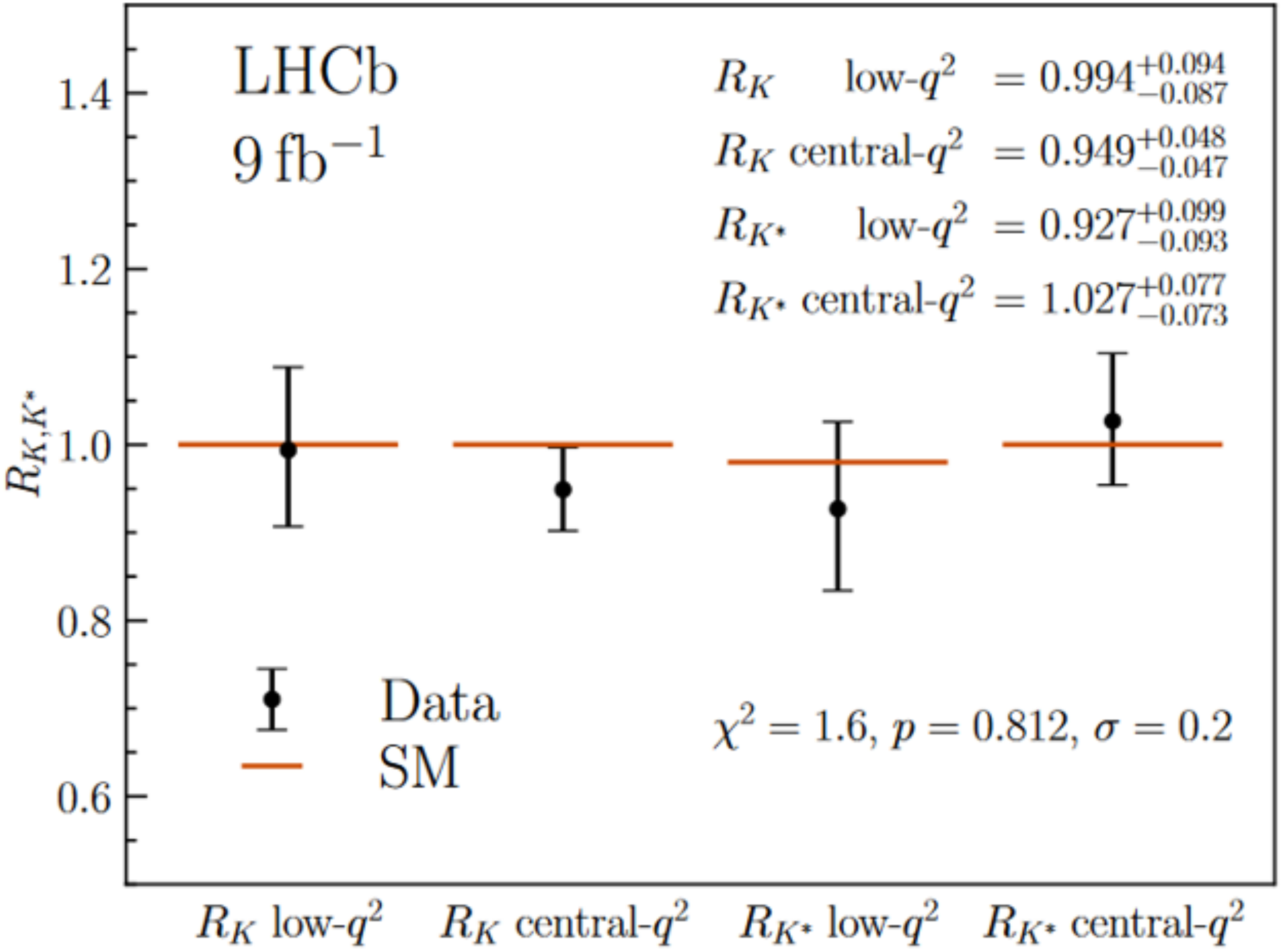
❖ New result including data until 2016 (2.5σ):

$$R_K = 0.846\,^{+\,0.060}_{-\,0.054}\,^{+\,0.016}_{-\,0.014}$$

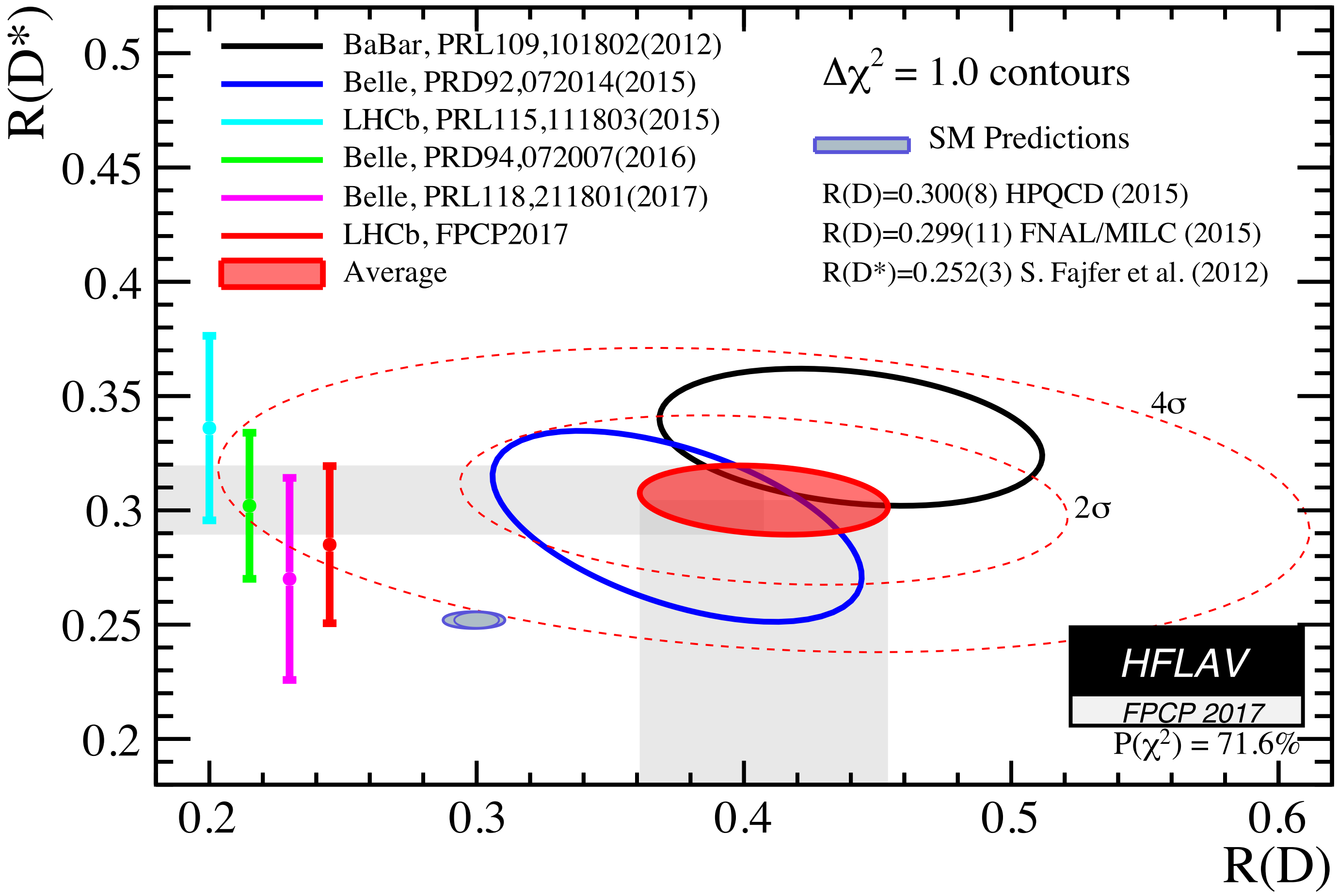
R_K, R_{K^*}: Belle results



R_K, R_{K*}: LHCb 2022 update, Christmas Present

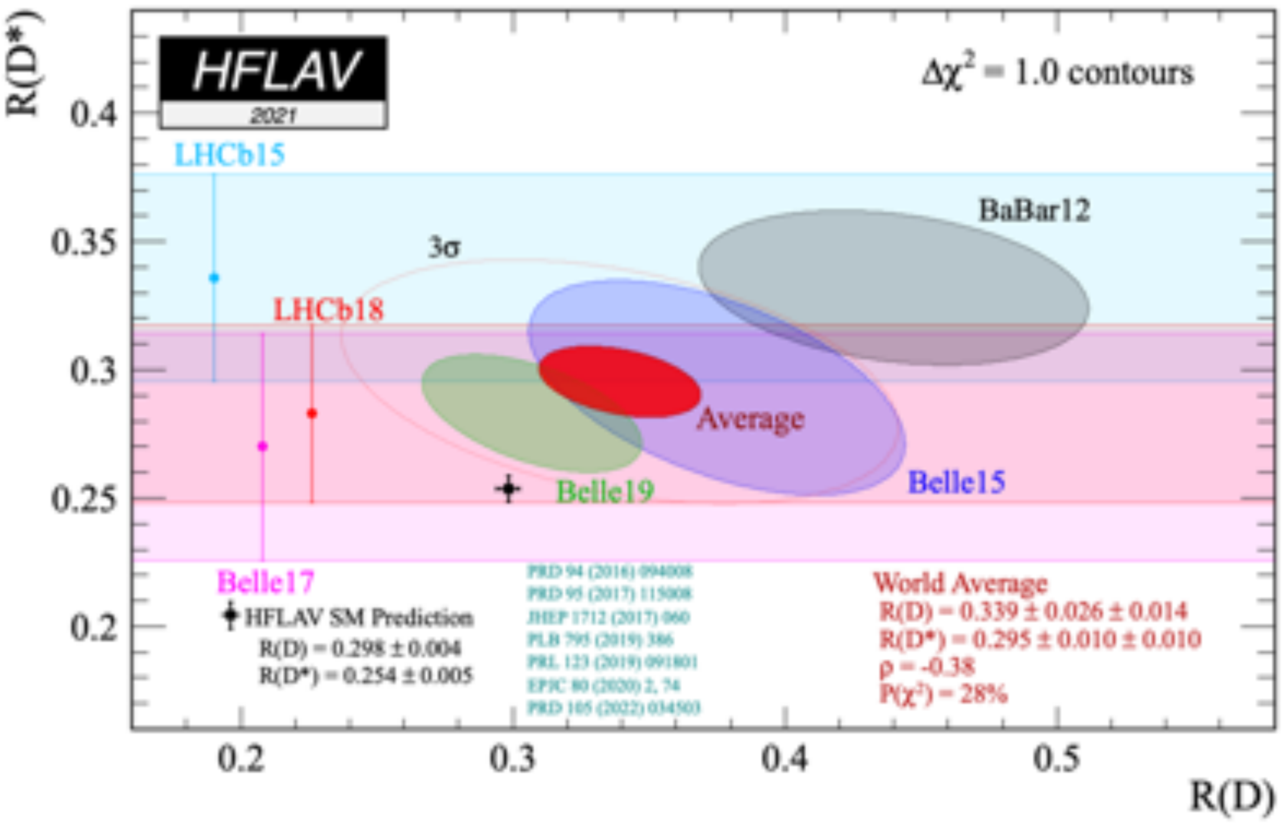


R_D, R_{D*}

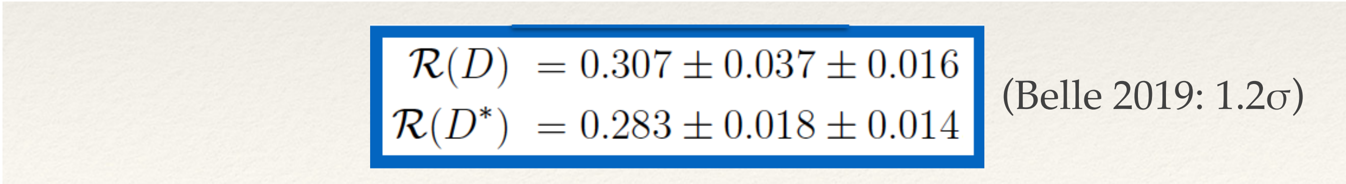


$$R_{D^{(*)}} = \frac{\Gamma(\bar{B} \to D^{(*)}\tau\bar{\nu})}{\Gamma(\bar{B} \to D^{(*)}\ell\bar{\nu})} \, ; \quad \ell = e, \mu$$

R_D, R_{D*} : update from Belle in 2019



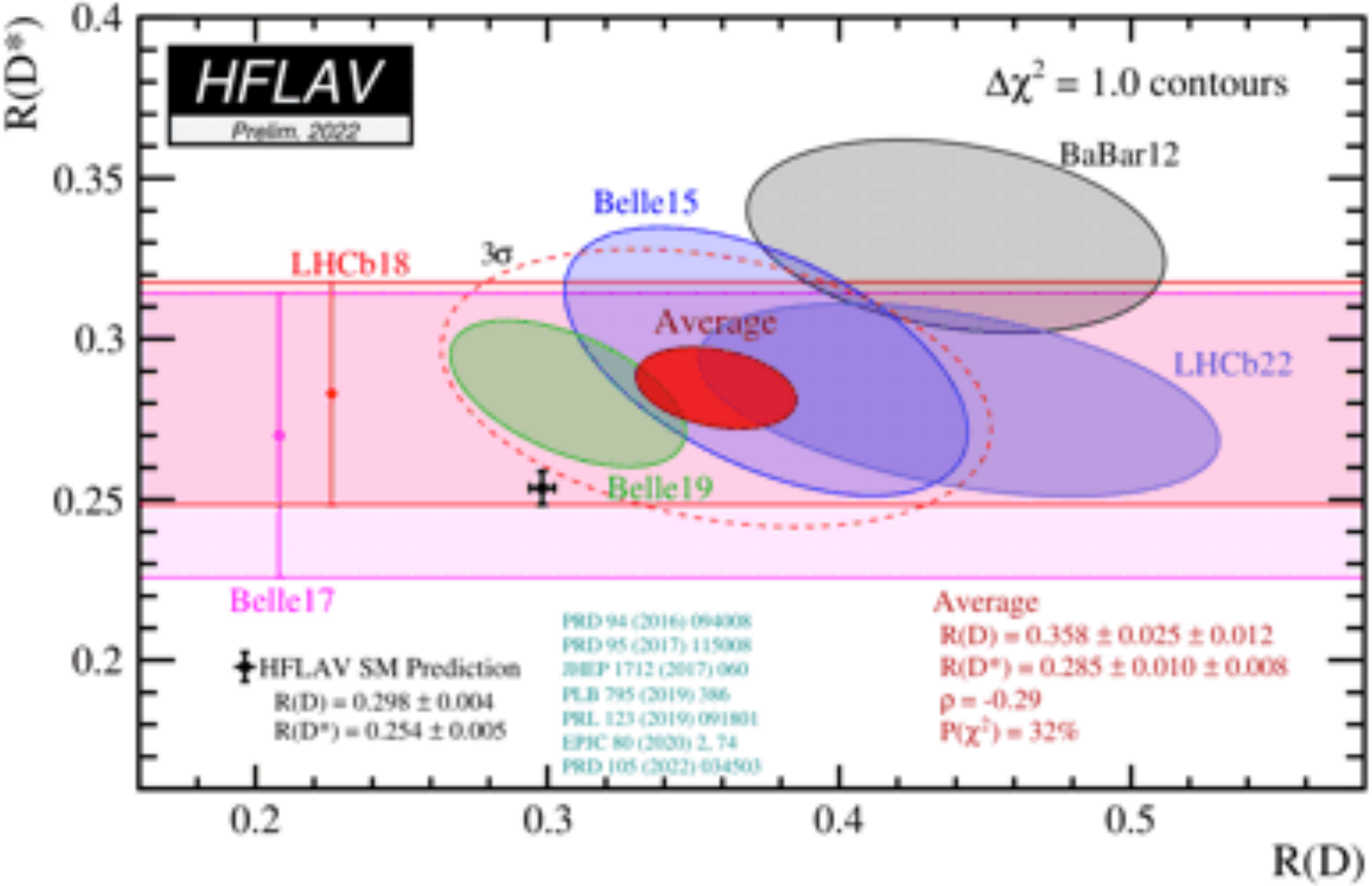
Significance reduced from 4.1 to 3.1σ 🙁



$$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$$
$$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$$

(Belle 2019: 1.2σ)

R_D, R_{D*} : recent update from LHCb in 2022



3.1 Experimental Prospects for V_{us}

On Kaon side

Cirigliano et al'22

- *NA62* could measure several BRs: $K_{\mu3}/K_{\mu2}$, $K \to 3\pi$, $K_{\mu2}/K \to \pi\pi$
- Note that the high precision measurement of BR($K_{\mu2}$) (0.3%) comes only from a single experiment: KLOE. It would be good to have another measurement at the same level of accuracy

- *LHCb* : could measure BR($K_S \to \pi\mu\nu$) at the < 1% level?
  $K_S \to \pi\mu\nu$ measured by KLOE-II but not competitive
  τ_S known to 0.04% (vs 0.41% for τ_L, 0.12% for $\tau_\pm$)

- V_{us} from Tau decays at *Belle II*:

Belle II with 50 ab^{-1} and ~4.6 x 10^{10} τ pairs will improve V_{us} extraction from τ decays
Inclusive measurement is an opportunity to have a complete independent extraction of V_{us} ⇒ not easy as you have to measure many channels

⇒ $$\boxed{|V_{us}| = 0.2184 \pm 0.0018_{\exp} \pm 0.0011_{th}}$$

To be competitive theory error will have to be improved as well

HFLAV'21

V_{us} from Hyperon decays

V_{us} can be measured from Hyperon decays:

- **$\Lambda \to p e \nu_e$** Possible measurement at *BESIII, Super τ-Charm factory?*

- Possibilities at *LHCb*?

Talk by Dettori@FPCP20

Channel	$\mathcal{R}$	ϵ_L	ϵ_D	$\sigma_L(\mathrm{MeV}/c^2)$	$\sigma_D(\mathrm{MeV}/c^2)$
$K^0_S \to \mu^+\mu^-$	1	1.0 (1.0)	1.8 (1.8)	~ 3.0	~ 8.0
$K^0_S \to \pi^+\pi^-$	1	1.1 (0.30)	1.9 (0.91)	~ 2.5	~ 7.0
$K^0_S \to \pi^0\mu^+\mu^-$	1	0.93 (0.93)	1.5 (1.5)	~ 35	~ 45
$K^0_S \to \gamma\mu^+\mu^-$	1	0.85 (0.85)	1.4 (1.4)	~ 60	~ 60
$K^0_S \to \mu^+\mu^-\mu^+\mu^-$	1	0.37 (0.37)	1.1 (1.1)	~ 1.0	~ 6.0
$K^0_L \to \mu^+\mu^-$	~ 1	2.7 (2.7) $\times 10^{-3}$	0.014 (0.014)	~ 3.0	~ 7.0
$K^+ \to \pi^+\pi^+\pi^-$	~ 2	9.0 (0.75) $\times 10^{-3}$	41 (8.6) $\times 10^{-3}$	~ 1.0	~ 4.0
$K^+ \to \pi^+\mu^+\mu^-$	~ 2	6.3 (2.3) $\times 10^{-3}$	0.030 (0.014)	~ 1.5	~ 4.5
$\Sigma^+ \to p\mu^+\mu^-$	~ 0.13	0.28 (0.28)	0.64 (0.64)	~ 1.0	~ 3.0
$\Lambda \to p\pi^-$	~ 0.45	0.41 (0.075)	1.3 (0.39)	~ 1.5	~ 5.0
$\Lambda \to p\mu^-\bar{\nu_\mu}$	~ 0.45	0.32 (0.31)	0.88 (0.86)	–	–
$\Xi^- \to \Lambda\mu^-\bar{\nu_\mu}$	~ 0.04	39 (5.7) $\times 10^{-3}$	0.27 (0.09)	–	–
$\Xi^- \to \Sigma^0\mu^-\bar{\nu_\mu}$	~ 0.03	24 (4.9) $\times 10^{-3}$	0.21 (0.068)	–	–
$\Xi^- \to p\pi^-\pi^-$	~ 0.03	0.41(0.05)	0.94 (0.20)	~ 3.0	~ 9.0
$\Xi^0 \to p\pi^-$	~ 0.03	1.0 (0.48)	2.0 (1.3)	~ 5.0	~ 10
$\Omega^- \to \Lambda\pi^-$	~ 0.001	95 (6.7) $\times 10^{-3}$	0.32 (0.10)	~ 7.0	~ 20

R $=$ ratio of production

$\epsilon =$ ratio of efficiencies

- To be able to extract V_{us} one needs to compute form factors precisely

  Lattice effort from *RBC/UKQCD*

3.2 Theoretical Prospects for V_{us}

- Lattice Progress on hadronic matrix elements: decay constants, FFs

- Full QCD**+**QED decay rate on the lattice,for **Leptonic decays of kaons and pions** ⇒ Inclusion of EM and IB corrections :
  - Perturbative treatment of QED on lattice established
  - Formalism for K_{l2} worked out

- Application of the method for **semileptonic Kaon** (K_{l3}) **and Baryon decays**

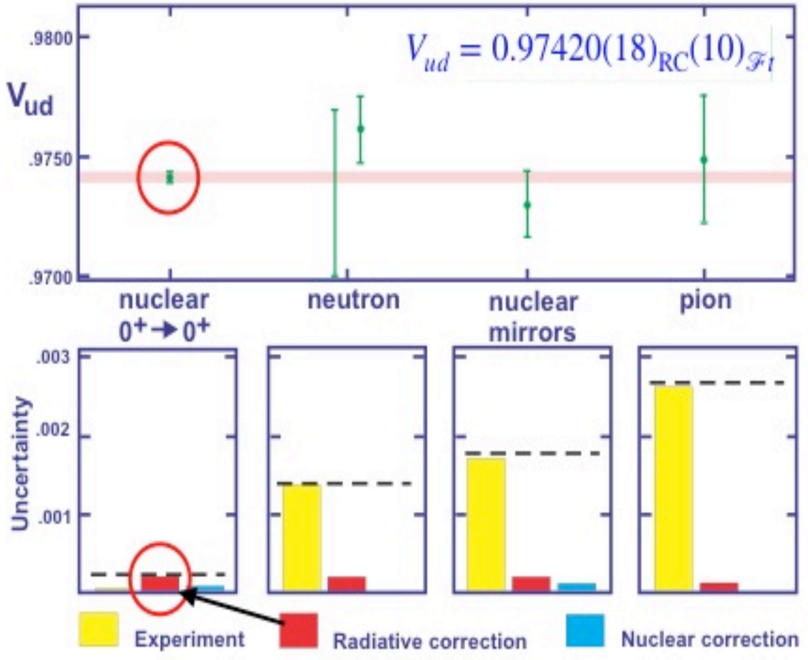
⇒ **Aim: Per mille level within 10 years**

3.3 Prospects for $|V_{ud}|$

See Talk by Misha Gorshteyn @CKM2021

PDG 2018:

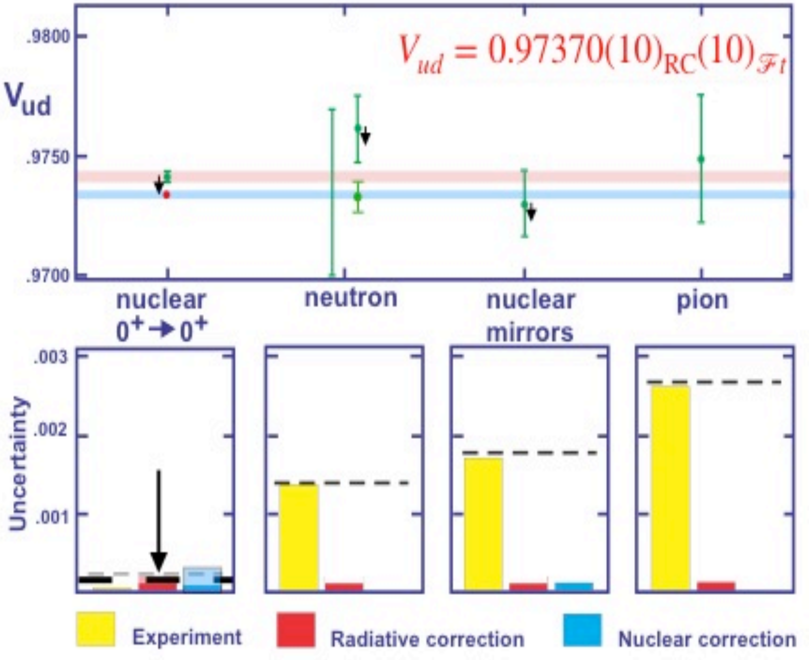
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(4)_{V_{ud}}(2)_{V_{us}}$$



PDG 2020:

Figure adapted from J. Hardy

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(3)_{V_{ud}}(4)_{V_{us}}$$



- From neutron decays : very impressive progress recently
- From pion β decay $\pi^+ \rightarrow \pi^0 e^+ \nu$: PIONEER experiment

3.3 Prospects for $|V_{ud}|$

See Talk by Misha Gorshteyn @CKM2021

PDG 2018:

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=0.9994(4)_{V_{ud}}(2)_{V_{us}}$$

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Figure adapted from J. Hardy

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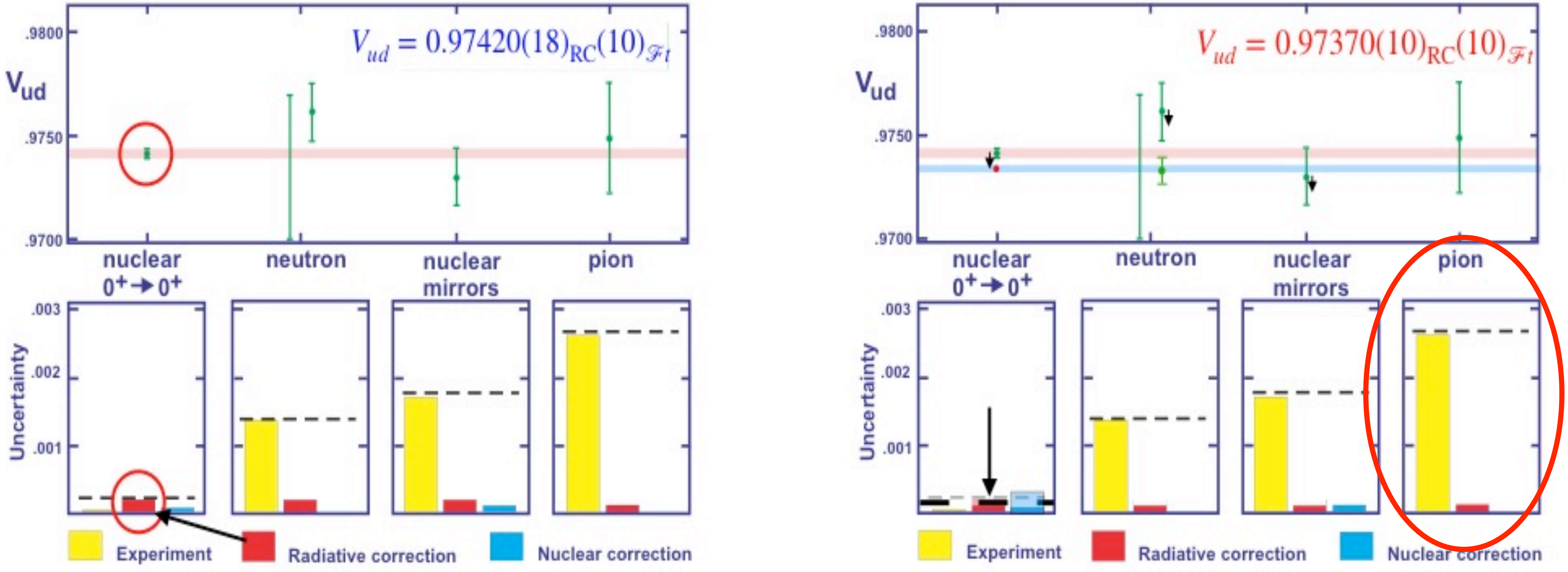


- From neutron decays
- From pion β decay $\pi^+ \to \pi^0 e^+ \nu$: PIONEER experiment
  ⇨ (Phase-I) approved at PSI, physics starting in ~2029

$|V_{ud}|$ from pion β decay: $\pi^+ \to \pi^0 e^+ \nu$

- Theoretically cleanest method to extract V_{ud} : corrections computed in SU(2) ChPT

Sirlin'78, Cirigliano et al.'03, Passera et al'11

- Present result: *PIBETA* Experiment (2004) → **Uncertainty: 0.64%**

$$B(\pi^+ \to \pi^0 e^+ \nu) = (1.036 \pm 0.004_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.003_{\pi e2})\ x10^{-8} (\pm 0.6\%)$$

⇨ $\boxed{|V_{ud}| = \mathbf{0.9739(28)_{exp}(1)_{th}}}$ to be compared to $|V_{ud}| = \mathbf{0.97373(31)}$

- Reduction of the theory error thanks to a new lattice calculation for RC *Feng et al'20*

- Next generation experiment PIONEER Phase II and III measurement at 0.02% level ⇨ will be competitive with current $0^+ \to 0^+$ extraction

- Would be completely independent check! No nuclear correction and different RCs compared to neutron decay

Czarnecki, Marciano, Sirlin'20

- **Opportunity to extract V_{us}/V_{ud}** from $\dfrac{B(K \to \pi l \nu)}{B(\pi^+ \to \pi^0 e^+ \nu)}$ EW Rad. Corr. cancel

Improve precision on $B(\pi^+ \to \pi^0 e^+ \nu)$ by x3 ⇨ $V_{us}/V_{ud} < \pm 0.2\%$

Pion decays and LFU tests

- Lepton Flavor Universality test in $R^{theory}_{e/\mu} = \dfrac{\Gamma\left(\pi \to e\nu(\gamma)\right)}{\Gamma\left(\pi \to \mu\nu(\gamma)\right)}$
  - ➢ Early insight into the **V−A** structure of weak interactions
  - ➢ Exceptional precision of the SM prediction using ChPT

$$R_{e/\mu}(\mathrm{SM}) = 1.23524(015) \times 10^{-4}$$

Cirigliano & Rosell'07

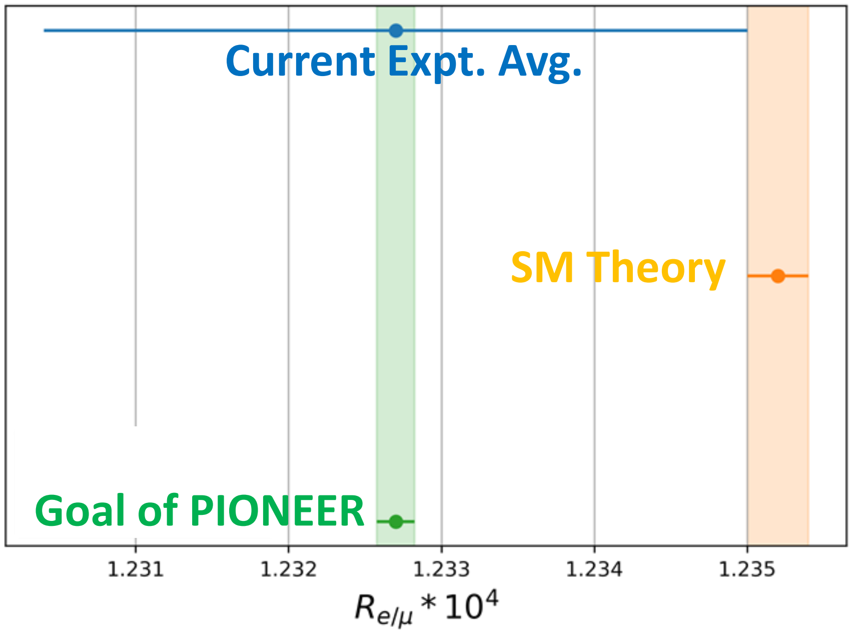
  - ➢ World average (mainly *PIENU* at TRIUMF):

$$R_{e/\mu}(\mathrm{Exp}) = 1.23270(230) \times 10^{-4}$$

15 times worse than theory! ➡

$$\frac{g_e}{g_\mu} = 0.9990 \pm 0.0009 \quad (\pm 0.09\%)$$

(dominated by PIENU expt.)



Goal of PIONEER: reduce unc. by a factor of 10 ! ➡ by far most precise test of LFU

2.6 Why a new dispersive analysis?

- Several new ingredients:
  - New inputs available: extraction ππ phase shifts has improved

    Ananthanarayan et al'01, Colangelo et al'01
    Descotes-Genon et al'01
    Kaminsky et al'01, Garcia-Martin et al'09

  - New experimental programs, precise Dalitz plot measurements

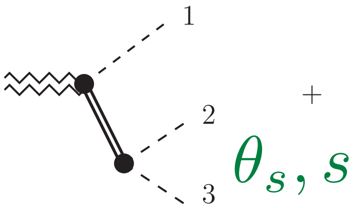
    TAPS/CBall-MAMI (Mainz), WASA-Celsius (Uppsala), WASA-Cosy (Juelich)
    CBall-Brookhaven, CLAS, GlueX (JLab), KLOE I-II (Frascati)
    BES III (Beijing)

  - Many improvements needed in view of very precise data: inclusion of
    - Electromagnetic effects ($\mathcal{O}(e^2m)$) *Ditsche, Kubis, Meissner'09*
    - Isospin breaking effects

2.7 Method

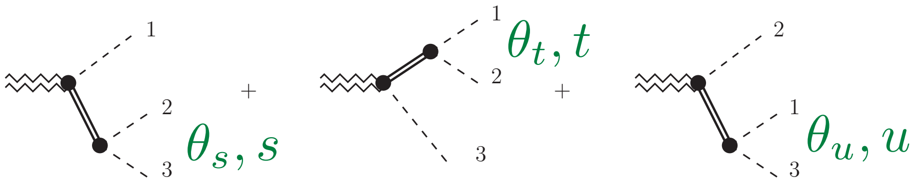
- S-channel partial wave decomposition

$$A_\lambda(s,t) = \sum_J^{\infty} (2J+1) d^J_{\lambda,0}(\theta_s) A_J(s)$$



- One truncates the partial wave expansion : ➡ Isobar approximation

$$\begin{aligned} A_\lambda(s,t) = & \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s) \\ & + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t) \\ & + \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u) \end{aligned}$$

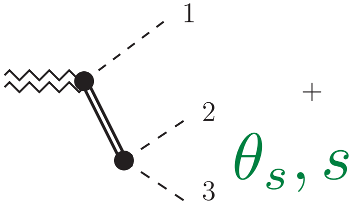


3 BWs (ρ^+, ρ^-, ρ^0) + background term

➡ Improve to include final states interactions

2.7 Method

- S-channel partial wave decomposition



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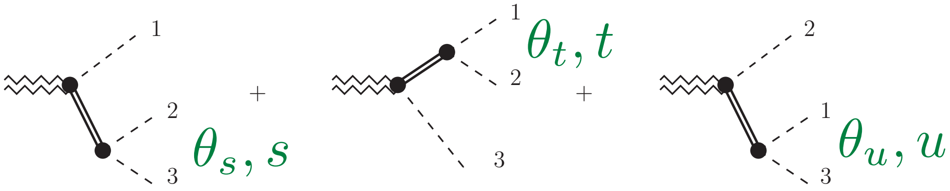
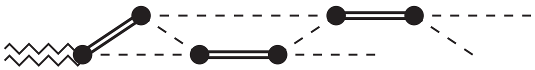
- One truncates the partial wave expansion : ⇨ Isobar approximation

$$A_\lambda(s,t) = \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_s) f_J(s)$$
$$+ \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_t) f_J(t)$$
$$+ \sum_J^{J_{\max}} (2J+1) d^J_{\lambda,0}(\theta_u) f_J(u)$$





- Use a Khuri-Treiman approach or dispersive approach

⇨ Restore 3 body unitarity and take into account the final state interactions in a systematic way

2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$\boxed{M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)}$$

Fuchs, Sazdjian & Stern'93

Anisovich & Leutwyler'96

- ➢ M_I isospin *I* rescattering in two particles
- ➢ Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
- ➢ Main two body rescattering corrections inside M_I

2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

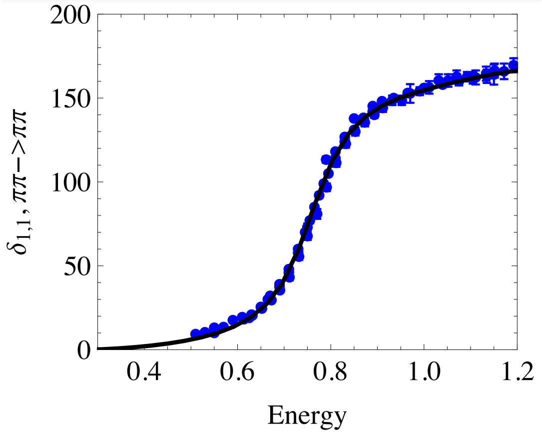
$$\boxed{M(s,t,u) = M_0^0(s) + (s-u)M_1^1(t) + (s-t)M_1^1(u) + M_0^2(t) + M_0^2(u) - \frac{2}{3}M_0^2(s)}$$

- Unitarity relation:

$$\boxed{disc\left[M_\ell^I(s)\right] = \rho(s)\overset{\pi\pi\to\pi\pi}{t_\ell^*(s)}\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)}$$

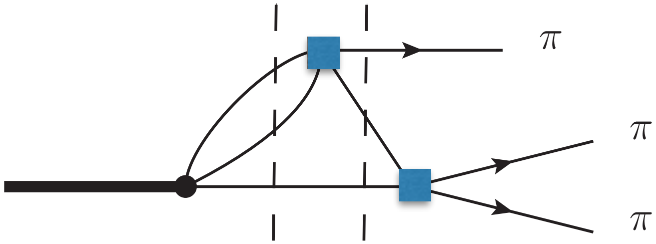
input

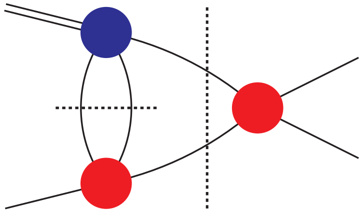
Roy analysis
Colangelo et al.'01



right-hand cut

left-hand cut





2.8 Representation of the amplitude

- Decomposition of the amplitude as a function of isospin states

$$\boxed{M(s,t,u) = M_0(s) + \left(s-u\right)M_1(t) + \left(s-t\right)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)}$$

- Unitarity relation:

$$\boxed{disc\left[M_\ell^I(s)\right] = \rho(s)t_\ell^*(s)\left(M_\ell^I(s) + \hat{M}_\ell^I(s)\right)}$$

- Relation of dispersion to reconstruct the amplitude everywhere:

$$\boxed{M_I(s) = \Omega_I(s)\left(P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\sin\delta_I(s')\,\hat{M}_I(s')}{\left|\Omega_I(s')\right|\left(s'-s-i\varepsilon\right)}\right)}$$

$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}ds'\frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

Omnès function

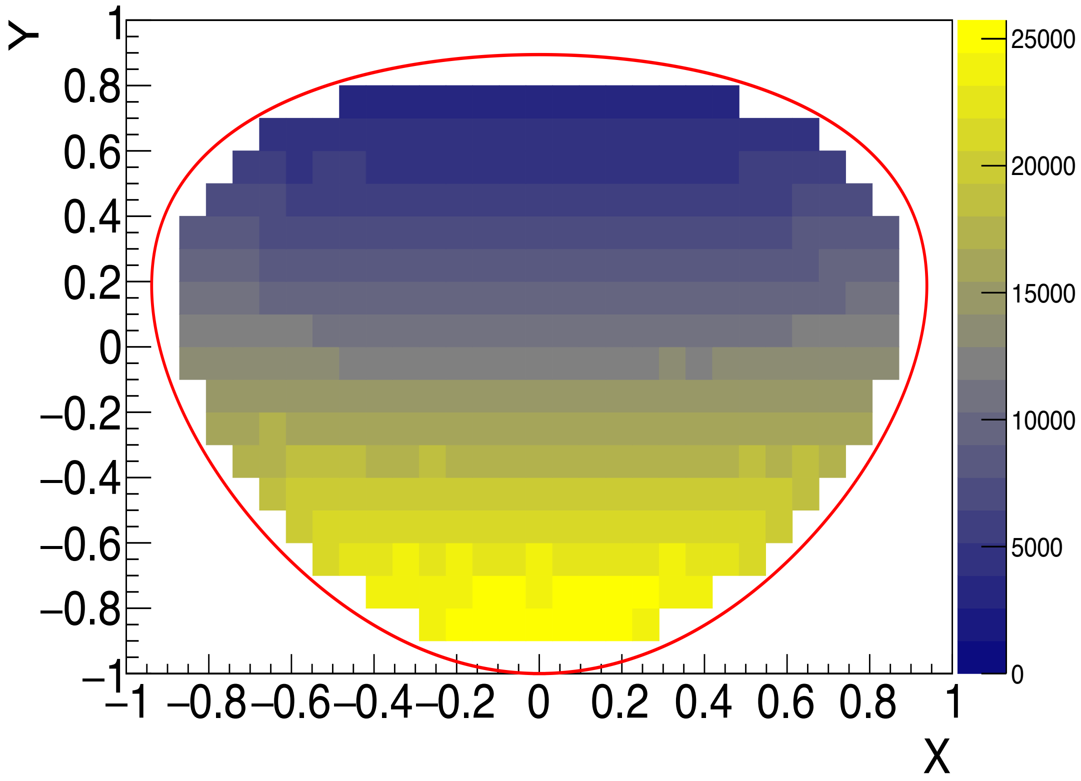
Gasser & Rusetsky'18

- $P_I(s)$ determined from a fit to NLO ChPT + experimental Dalitz plot

2.9 η → 3π Dalitz plot

- In the charged channel: experimental data from *WASA, KLOE, BESIII*

KLOE'16



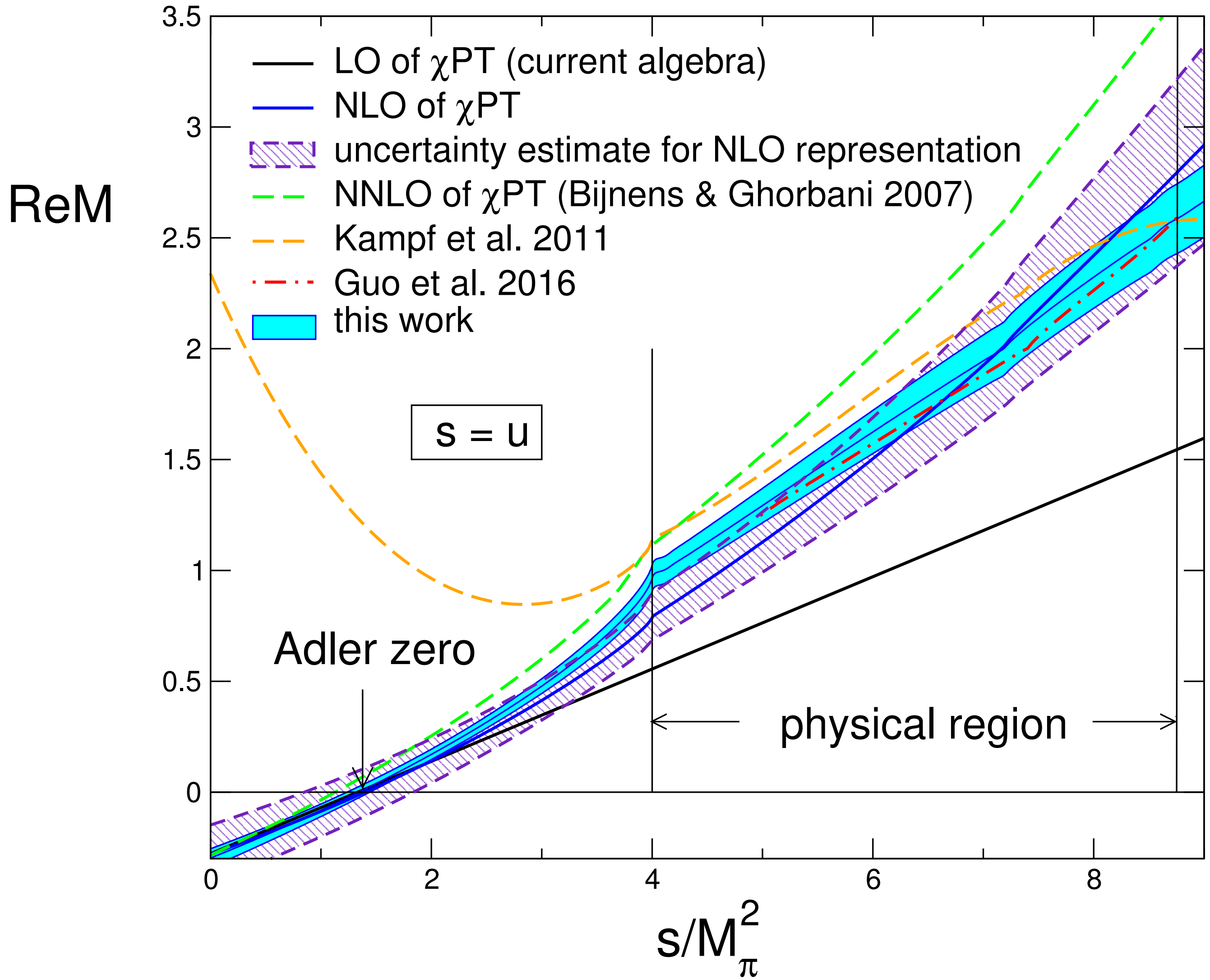
$$X = \sqrt{3}\,\frac{T_+ - T_-}{Q_c} = \frac{\sqrt{3}}{2M_\eta Q_c}\left(u - t\right)$$

$$Y = \frac{3T_0}{Q_c} - 1 = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

- New data expected from *CLAS* and *GlueX* with very different systematics

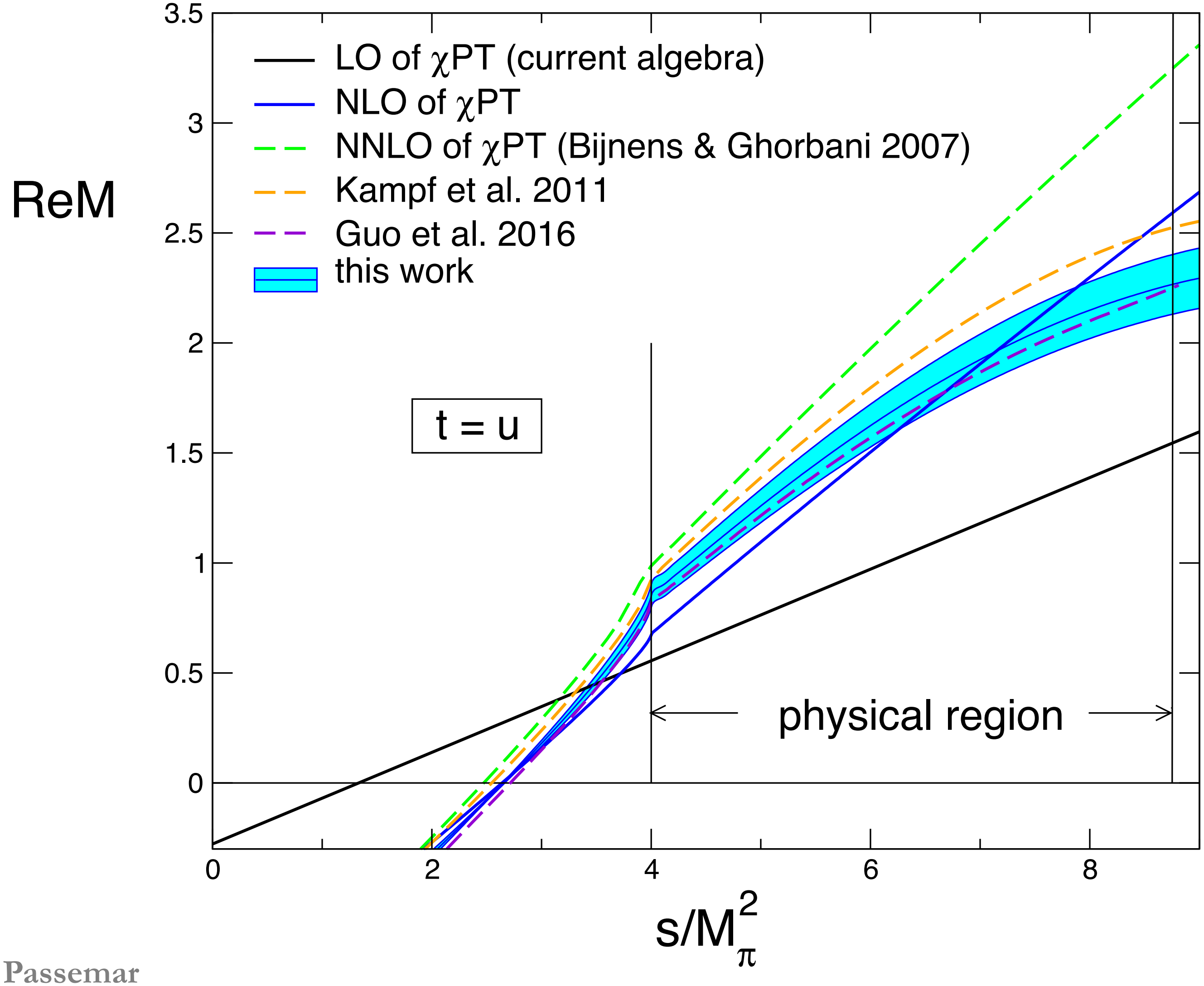
2.10 Results: Amplitude for $\eta \to \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line s = u :



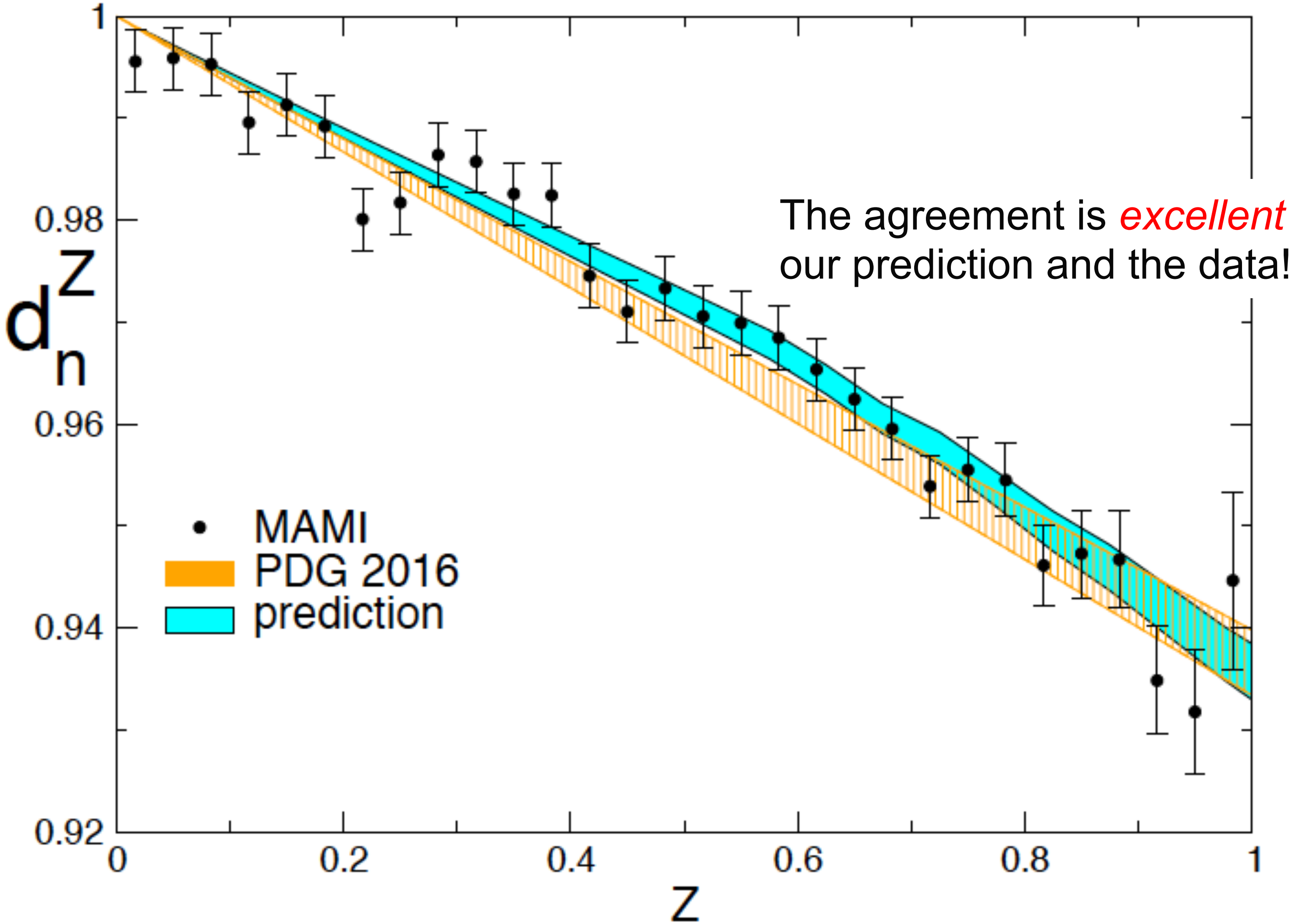
2.10 Results: Amplitude for $\eta \rightarrow \pi^+ \pi^- \pi^0$ decays

- The amplitude along the line t = u :



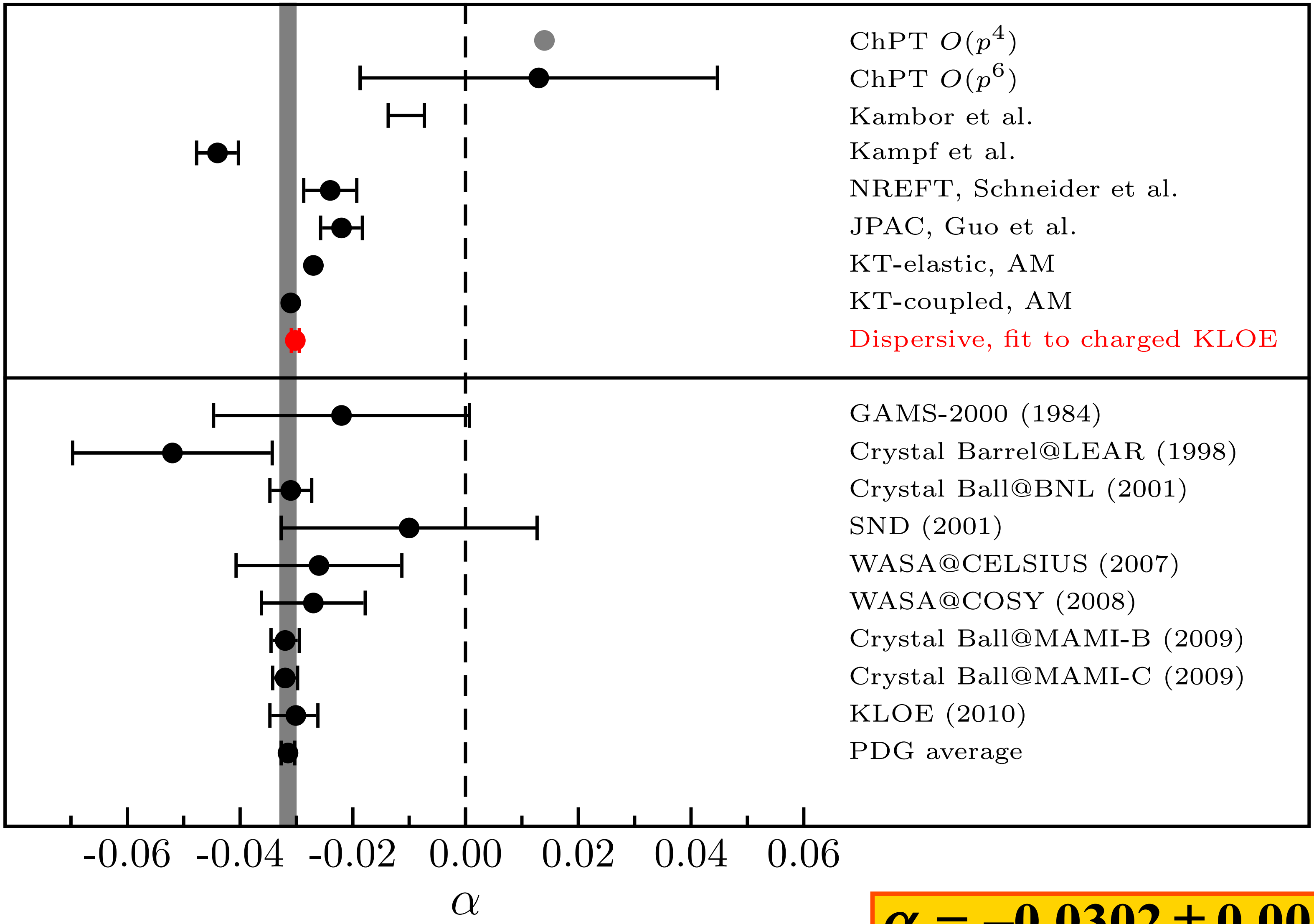
2.11 Z distribution for $\eta \to \pi^0 \pi^0 \pi^0$ decays

- The amplitude squared in the neutral channel is



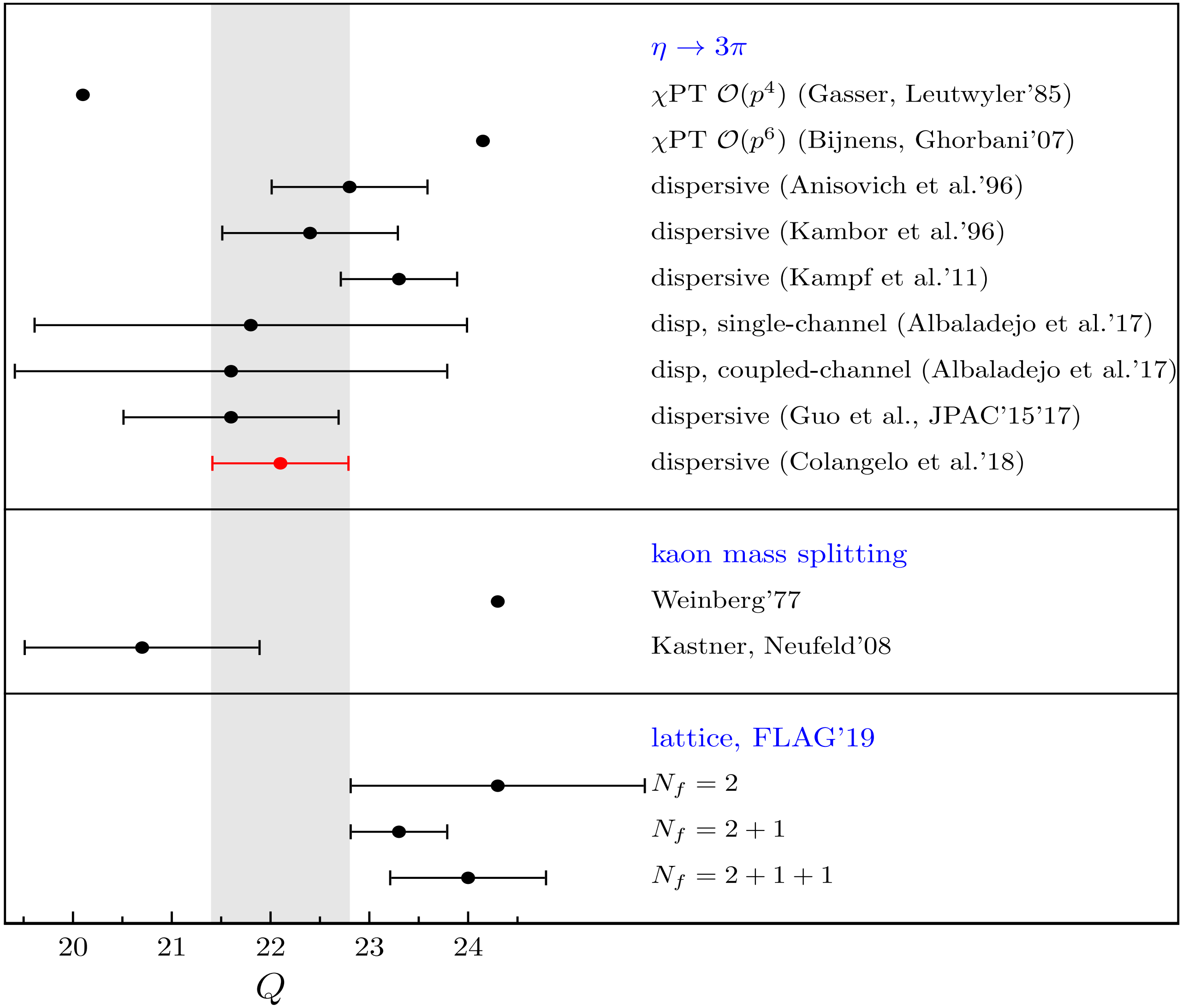
The agreement is *excellent* between our prediction and the data!

2.12 Comparison of results for α



$\alpha = -0.0302 \pm 0.0011$

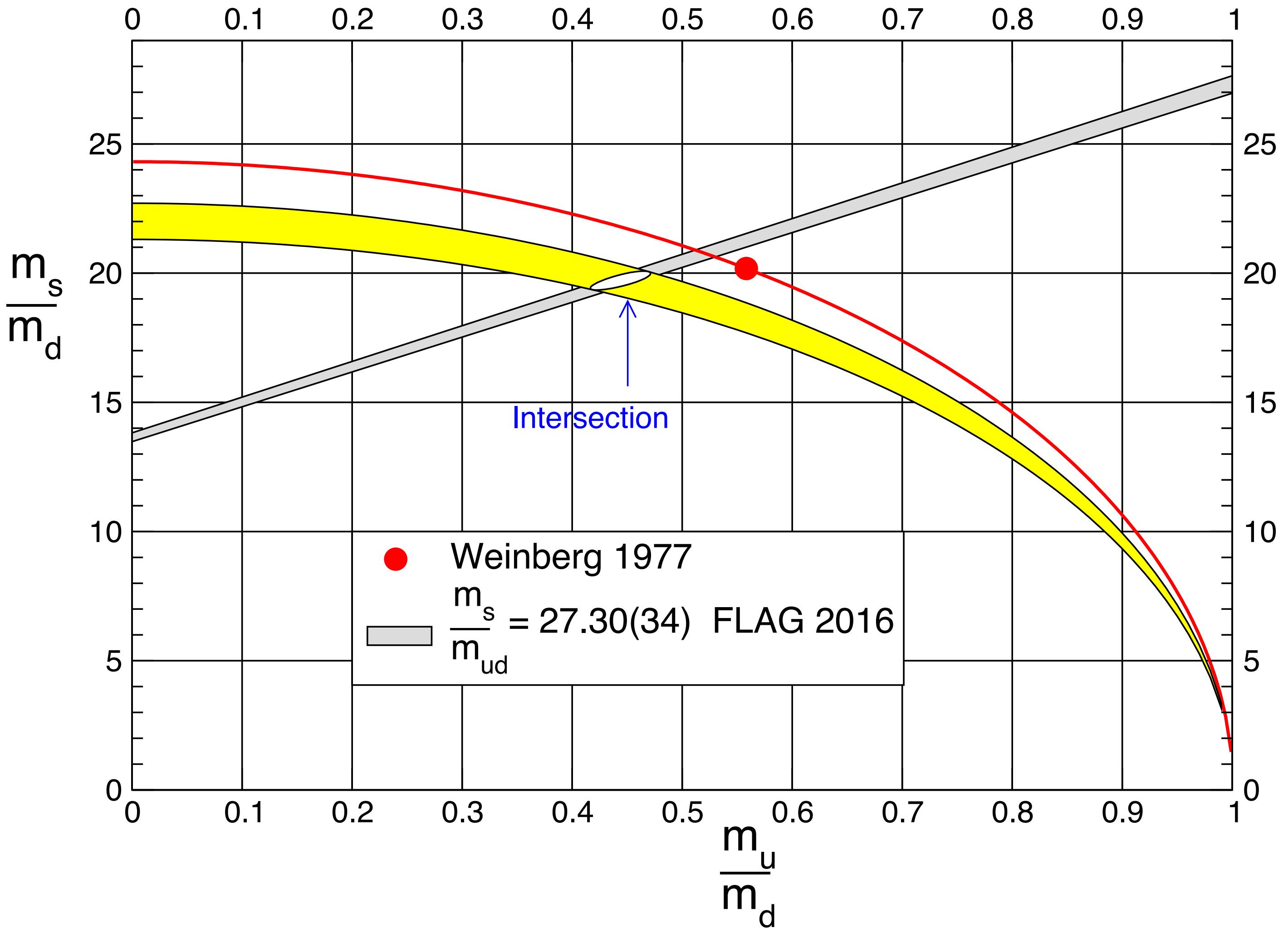
2.13 Quark mass ratio



$\boxed{Q = 22.1 \pm 0.7}$

- No systematics taken into account ⇨ collaboration with experimentalists

2.14 Light quark masses

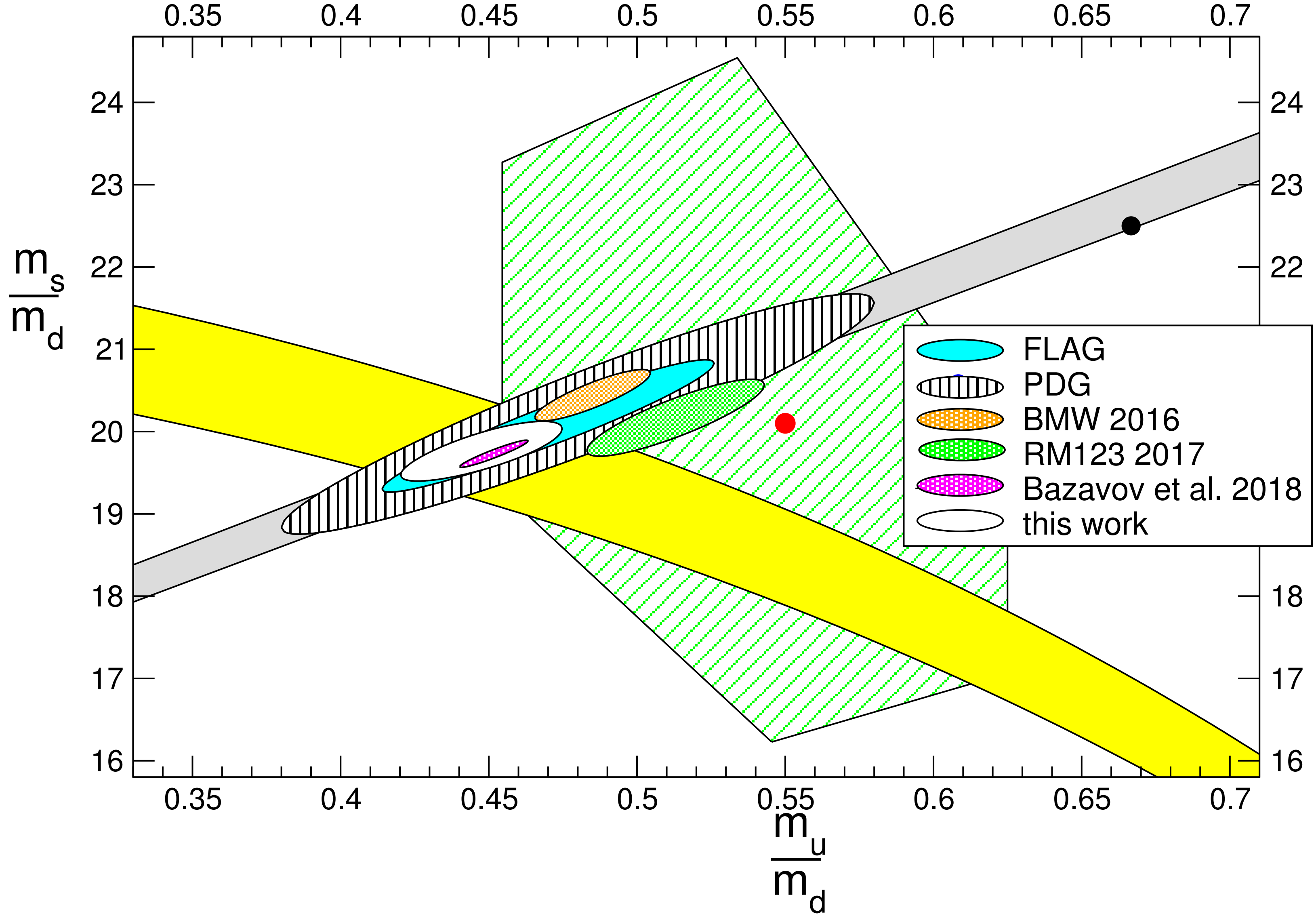


$$Q = 22.1 \pm 0.7$$

$$\frac{m_u}{m_d} = 0.44 \pm 0.03$$

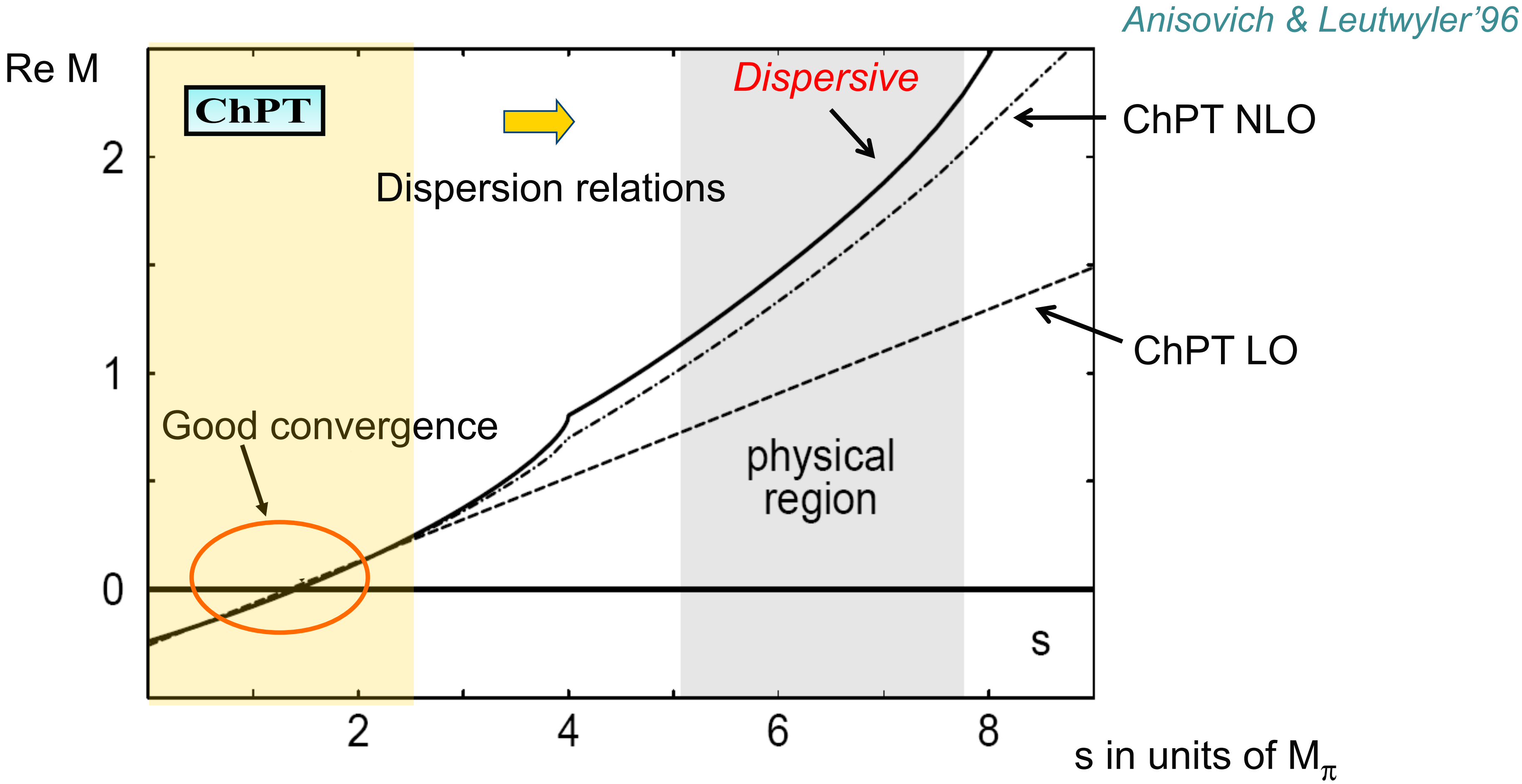
- Smaller values for Q ⇨ smaller values for m_s/m_d and m_u/m_d than LO ChPT

2.14 Light quark masses



Dispersive approach

- Dispersion Relations: extrapolate ChPT at higher energies



- Important corrections in the physical region taken care of by the *dispersive treatment!*