Weinberg Sum Rules

In the vector sector chiral symmetry is encoded in Weinberg sum rules

vacuum:

$$\int_0^\infty ds\,(\rho_V(s)-\rho_A(s)) = F_\pi^2 \qquad \text{polarizability}$$

$$\int_0^\infty ds s\,(\rho_V(s)-\rho_A(s)) = 0 \qquad \text{EWSR}$$

pole approximation: (chiral limit)

$$\rho_V(s) = \frac{m_\rho^4}{g_\rho^2}\frac{1}{s}\delta(s-m_\rho^2)$$

$$\rho_A(s) = \frac{m_{a_1}^4}{g_{a_1}^2}\frac{1}{s}\delta(s-m_{a_1}^2)$$

then

$$m_\rho^2 = a g_\rho^2 F_\pi^2; \quad a = \left(1-\frac{m_\rho^2}{m_{a_1}^2}\right)^{-1}$$

for $m_{a_1} = \sqrt{2}m_\rho$ (a=2 KFSR relation)