- QCD sum rules

T. Hatsuda et al. PRC (1991)

F. Klingl et al. NPA (1997)

S. Leupold et al. PRC (1998)

vacuum dispersion relation:

$$\begin{aligned}\Pi^\circ(q^2) &= \frac{i}{3}\int d^4x\, e^{iqx}\langle 0|\mathcal{T}(J^\mu(x), J_\mu(0))|0\rangle \\ &= \Pi^\circ(0) + q^2\int_0^\infty \frac{ds}{s}\,\frac{(-1/\pi)\mathrm{Im}\Pi^\circ(s)}{q^2 - s + i\eta}\end{aligned}$$

rhs related to the spectral density:

$$\rho^\circ(s) = -(1/\pi s)\mathrm{Im}\Pi^\circ(s)$$

large space-like momenta $Q^2 = -q^2 \to \infty$ probe short-distance part of the correlator:

OPE: $(d_\rho = 3/2 \quad d_\omega = 1/6 \quad d_\phi = 1/3)$

$$\begin{aligned}\frac{\Pi^\circ(q^2 = -Q^2)}{Q^2} &= \int_0^\infty \frac{ds}{s}\,\frac{(-1/\pi)\mathrm{Im}\Pi^\circ(s)}{s + Q^2} \\ &= \frac{d_V}{12\pi^2}\left[-c_0 \ln\frac{Q^2}{\mu} + \frac{c_1}{Q^2} + \frac{c_2}{Q^4} + \frac{c_3}{Q^6} + \cdots\right]\end{aligned}$$