Thermodynamics on the Lattice

aim: 'exact evaluation' of the QCD EoS ($\mu = 0$)

free energy density:

$$\Omega = -\frac{T}{V}\ln \mathcal{Z}$$

energy density and pressure:

$$\epsilon = \frac{T^2}{V}\frac{\partial \ln \mathcal{Z}}{\partial T}; \quad p = T\frac{\partial \ln \mathcal{Z}}{\partial V}$$

(in the thermodynamic limit $p = -\Omega$)

entropy density and 'interaction measure':

$$\begin{aligned} s &\equiv \frac{\epsilon + p}{T} = \frac{\partial p}{\partial T}\,, \\ \Delta &\equiv \frac{\epsilon - 3p}{T^4} = T\frac{\partial}{\partial T}(p/T^4) \end{aligned}$$

condensates:

$$\langle\langle \bar{\psi}\psi \rangle\rangle = \frac{\partial \Omega}{\partial m_q} \qquad \langle\langle G^2 \rangle\rangle = -\frac{8}{9}\left[T^4\Delta + m_q\frac{\partial \Omega}{\partial m_q}\right]$$