Quark Symmetries

only the quark sector is relevant. For two flavors :

$$\mathcal{L}_{QCD} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{m_u + m_d}{2}\bar{\psi}\psi - \frac{m_u - m_d}{2}\bar{\psi}\tau_3\psi$$

$$\psi = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} \qquad \bar{\psi} = (\bar{\psi}_u, \bar{\psi}_d)$$

vector symmetry : $\psi \to e^{i\alpha_k \frac{\tau_k}{2}}\psi$

$$\mathcal{V}_k^\mu = \bar{\psi}\gamma^\mu\frac{\tau_k}{2}\psi; \quad Q_k^V = \int d^3r\psi^\dagger\frac{\tau_k}{2}\psi$$

axial symmetry : $\psi \to e^{i\beta_k \frac{\tau_k}{2}\gamma_5}\psi$

$$\mathcal{A}_k^\mu = \bar{\psi}\gamma^\mu\gamma_5\frac{\tau_k}{2}\psi; \quad Q_k^A = \int d^3r\psi^\dagger\gamma_5\frac{\tau_k}{2}\psi$$

since $m_u, m_d \ll M_h \sim 1$ GeV

$$[Q_k^V, H_{QCD}] = 0; \quad [Q_k^A, H_{QCD}] = 0$$

conservation of vector- and axial currents !