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Damping width of double resonances

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Abstract. Damping width of the double giant dipole resonance of ^{136}Xe excited in relativistic heavy ion collisions is calculated by diagonalizing a microscopic Hamiltonian in a basis containing one-, two- and three-phonon states. The coupling between these states is determined making use of the fermion structure of the phonons. The resulting width of the double giant dipole resonance is close to $\sqrt{2}$ times the width of the single giant dipole resonance.

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Two-phonon giant dipole resonances have been observed in relativistic heavy ion collision. Their centroid energy is found close to the twice the energy of the single resonance, while their damping width displays a value lying between $\sqrt{2}$ and 2 times the width of the one-phonon state (cf. e.g. [1]). From rather general arguments [2], the most important couplings leading to real transitions of the double giant resonances and thus to a damping width of these modes are to configurations built out by promoting three nucleon across the Fermi surface. That is, configurations containing three holes in the Fermi sea and three particles above the Fermi surface (3p-3h configurations). In this paper we present, for the first time, results of calculations taking into account such couplings [3], not included in the microscopic studies carried out previously [4–7]. The nucleus considered is ^{136}Xe. It will be concluded that the resulting width of the double giant resonance is approximately equal to $\sqrt{2}$ times the width of the single resonance state.

The Hamiltonian describing the system under discussion includes mean field terms for protons and neutrons, a monopole pairing interaction and multipole-multipole forces. For details we refer to [8]. Diagonalizing the Hamiltonian in the random phase approximation one obtains a phonon basis of multipolarity λ and parity $\pi = (-1)^\lambda$. The associated creation operators shall be denoted Q^+_α, where $\alpha = (\lambda^\pi, n)$ and where the index n labels whether the phonon with these quantum numbers has lowest, next to lowest, etc. energy (n = 1,2,3 ...). The set of basis states $Q^+_\alpha\ |\rangle_{ph}$, where $|\rangle_{ph}$ is the phonon vacuum, includes both collective and non-collective states. The wave function describing the double giant dipole resonance (DGDR) and their coupling to 1p-1h and to 3p-3h doorway states is written as

$$\Psi^\nu(J) = \left\{ \sum_{\alpha_1} S^\nu_{\alpha_1}(J) Q^+_{\alpha_1} + \sum_{\alpha_2\beta_2} \frac{D^\nu_{\alpha_2\beta_2}(J) Q^+_{\alpha_2} Q^+_{\beta_2}}{\sqrt{1+\delta_{\alpha_2,\beta_2}}} \right.$$
$$\left. + \sum_{\alpha_3\beta_3\gamma_3} \frac{T^\nu_{\alpha_3\beta_3\gamma_3}(J) Q^+_{\alpha_3} Q^+_{\beta_3} Q^+_{\gamma_3}}{\sqrt{1+\delta_{\alpha_3,\beta_3}+\delta_{\alpha_3,\gamma_3}+\delta_{\beta_3,\gamma_3}+2\delta_{\alpha_3,\beta_3,\gamma_3}}} \right\} |\rangle_{ph}, \quad (1)$$

where $J^\pi = 0^+, 2^+$ denote the total angular and parity of the excitation. The index ν (= 1,2,3 ...) labels whether a state J is the first, second, etc., state in the total energy spectrum of the system. It is assumed that any combination α, β, γ of phonons appears only once. The second and the third terms in (1) can include phonons of different multipolarities and parities.

The orthogonality relation associated with the above wave functions reads

$$\langle \Psi^\nu(J) | \Psi^{\nu'}(J) \rangle = \delta_{\nu,\nu'}$$
$$= \sum_{\alpha_1} S^\nu_{\alpha_1}(J) S^{\nu'}_{\alpha_1}(J) + \sum_{\alpha_2\beta_2} D^\nu_{\alpha_2\beta_2}(J) D^{\nu'}_{\alpha_2\beta_2}(J)$$
$$+ \sum_{\alpha_2\beta_2,\alpha'_2\beta'_2} D^\nu_{\alpha_2\beta_2}(J) D^{\nu'}_{\alpha'_2\beta'_2}(J)\ 2\tilde{K}_J(\beta_2\alpha_2|\alpha'_2\beta'_2)$$
$$+ \sum_{\alpha_3\beta_3\gamma_3} T^{\nu_2}_{\alpha_3\beta_3\gamma_3}(J) T^{\nu'_2}_{\alpha_3\beta_3\gamma_3}(J)\ . \quad (2)$$

The coefficients $\tilde{K}_J(\beta_2\alpha_2|\alpha'_2\beta'_2)$ arise from the fermion structure of phonons, and have their origin in the Pauli principle [9]. While they are small, they produce shifts in the energy centroid of the double giant resonance. This is the reason why we have kept them. Similar coefficients appear also in connection with the term arising from the “doorway states” containing three phonons in (1). We have neglected them because they again are small and furthermore act only in higher order as compared to the previous term, in defining the properties of the double giant dipole resonance. Finally, the corresponding $\tilde{K}$-coefficient associated with the first term in (2) is proportional to the number of quasiparticles present in the ground state of the system, a quantity which is assumed to be zero within linear response theory.

The diagonalization of the model Hamiltonian within the basis of states defined in (1) leads to secular equation. The one associated with two-phonon type states reads

$$det \left|\left| \; (\omega_{\alpha_2} + \omega_{\beta_2} - E_x)[\delta_{\alpha_2\beta_2,\alpha'_2\beta'_2} + \tilde{K}_J(\beta_2\alpha_2|\alpha'_2\beta'_2)] \right.\right.$$
$$+\Delta_J(\alpha_2\beta_2,\alpha'_2\beta'_2) \; - \; \sum_{\alpha_1} \frac{U^{\alpha_1}_{\alpha_2\beta_2}(J)\; U^{\alpha_1}_{\alpha'_2\beta'_2}(J)}{\omega_{\alpha_1} - E_x}$$
$$\left. - \sum_{\alpha_3\beta_3\gamma_3} \frac{U^{\alpha_2\beta_2}_{\alpha_3\beta_3\gamma_3}(J)\; U^{\alpha'_2\beta'_2}_{\alpha_3\beta_3\gamma_3}(J)}{\omega_{\alpha_3} + \omega_{\beta_3} + \omega_{\gamma_3} - E_x} \right|\right| = 0 \; , \qquad (3)$$

where ω_α is the RPA energy for the i^{th}_α state of multipolarity λ. The quantities $U^{\alpha_1}_{\alpha_2\beta_2}(J)$ and $U^{\alpha_2\beta_2}_{\alpha_3\beta_3\gamma_3}(J)$ are matrix elements of the interaction connecting two-phonon configurations $\{\alpha_2\beta_2\}$ with one- $\{\alpha_1\}$ and three-phonon $\{\alpha_3\beta_3\gamma_3\}$ configurations, respectively, while $\Delta_J(\alpha_2\beta_2,\alpha'_2\beta'_2)$ is the energy shift of the configuration $\{\alpha_2\beta_2\}$ due to its interaction with the configuration $\{\alpha'_2\beta'_2\}$, both belonging to the two-phonon response (cf. [8]). These quantities are calculated making use of the model Hamiltonian and the microscopic fermion structure of phonons.

The solution of (3) provides the eigenvalues E^ν_x associated with the states introduced in (1) and the coefficients $D^\nu_{\alpha_2\beta_2}(J)$, minors of the matrices introduced in (3). The other coefficients appearing in (1) are related to the D-coefficients according to

$$S^\nu_{\alpha_1}(J) = \; - \; \frac{\sum_{\alpha_2\beta_2} D^\nu_{\alpha_2\beta_2}(J)\; U^{\alpha_1}_{\alpha_2\beta_2}(J)}{2 \;\cdot\; (\omega_{\alpha_1} - E^\nu_x)} \; ,$$

$$T^\nu_{\alpha_3\beta_3\gamma_3}(J) = \; - \; \frac{\sum_{\alpha_2\beta_2} D^\nu_{\alpha_2\beta_2}(J)\; U^{\alpha_2\beta_2}_{\alpha_3\beta_3\gamma_3}(J)}{2 \;\cdot\; (\omega_{\alpha_3} + \omega_{\beta_3} + \omega_{\gamma_3} - E^\nu_x)} \; .$$

In keeping with the fact that the Q-value dependence of the Coulomb excitation amplitude is rather weak at relativistic energies [10], the cross section associated with the two-step excitation of the double giant dipole resonance is proportional to

$$|\sum_{\nu_1} \langle \Psi^\nu_{0^+(2^+)} | M(E1) | \Psi^{\nu_1}_{1^-} \rangle \cdot \langle \Psi^{\nu_1}_{1^-} | M(E1) | \Psi_{g.s.} \rangle|^2$$
$$= \left| \; 2 \cdot \sum_{\alpha_2\beta_2} D^\nu_{\alpha_2\beta_2}(J) \cdot \left[\frac{M_{\alpha_2} M_{\beta_2}}{\sqrt{1 + \delta_{\alpha_2,\beta_2}}} \right.\right.$$
$$\left.\left. + \sum_{\alpha'_2\beta'_2} M_{\alpha'_2} M_{\beta'_2} \sqrt{1 + \delta_{\alpha'_2,\beta'_2}} \tilde{K}_J(\beta_2\alpha_2|\alpha'_2\beta'_2) \right] \right|^2 \; , \qquad (4)$$

where $M_\alpha = \langle Q_\alpha || M(E1) || 0^+_{g.s.} \rangle$ is the reduced matrix element of the $E1$-operator which acting on the ground state $|\rangle_{ph}$ excites the one-phonon state with quantum numbers $\alpha = (1^-, n)$.

Making use of the elements discussed above we calculated the distribution of the quantity (4) over the states (1) in ^{136}Xe. We considered only $J^\pi = 0^+$ and 2^+ components of the two-phonon giant dipole resonance. It was shown recently [11] that its $J^\pi = 1^+$ component cannot be excited in the second order perturbation theory and is sufficiently quenched in coupled-channels calculation. The fifteen configurations $\{1^-n, 1^-n'\} = \{\alpha_2, \beta_2\}$ displaying the largest B(E1)×B(E1) values were used in the calculation. They are built up out of the five most collective RPA roots associated with the one-phonon giant dipole resonance carrying the largest B(E1) values and exhausting 77% of energy weighted sum rule (EWSR). Two-phonon states of collective character and with quantum numbers different from 1^- lie, as a rule, at energies few MeV away from the double giant dipole states and were not included in the calculations. The three-phonon states $\{\alpha_3\beta_3\gamma_3\}$ were built out of phonons with angular momentum and parity 1^-, 2^+, 3^- and 4^+. Only those configurations where either α_3, β_3 or γ_3 were equal to α_2 or β_2 were chosen. This is because other configurations lead to matrix elements $U^{\alpha_2\beta_2}_{\alpha_3\beta_3\gamma_3}(J)$ of the interaction, which are orders of magnitude smaller than those associated with the above mentioned three-phonon configurations, and which contain in the present calculation 5742 states up to an excitation energy 38 MeV. The single particle continuum has been approximated in the present calculation by quasibound states. As demonstrated by our previous studies [6], this approximation provides rather good description of the single GDR properties in ^{136}Xe. This means that our (2p-2h)$_{[1^- \times 1^-]}$ spectrum is also rather complete for the description of the DGDR properties although it is located at higher energies.

If one assumes a pure boson picture to describe the phonons, without taking into account their fermion structure, the three-phonon configurations omitted in the present calculation do not couple to two-phonon states under consideration. Furthermore, although the density of 3p-3h configurations is quite high in the energy region corresponding to the DGDR, a selection of the important doorway configurations in terms of the efficiency with which configurations couple to the DGDR, can be done rather easily. The above considerations testify to the advantage of employing a microscopic phonon picture in describing the nuclear excitation spectrum, instead of a particle-hole representation. One can more readily identify the regularities typical of the collective picture of the vibrational spectrum, and still deal with the fermion structure of these excitations. As far as the one-phonon term appearing in (1) is concerned, essentially all phonons with angular momentum and parity 0^+ and 2^+ were taken into account within the energy interval 20–40 MeV.

A rather general feature displayed by the results of the present calculation is that all two-phonon configurations of the type $\{1^-n, 1^-n'\}$ building the DGDR in the “harmonic” picture are fragmented over a few MeV due to the coupling to 3p-3h “doorway states”. The maximum amplitude with which each two-phonon configuration enters in the wave function (1) does not exceed a few percent. Two-phonon configurations made out of two different 1^- phonons are fragmented stronger then two-phonon configurations made out of two identical 1^- phonons. This in keeping with the fact that, as a rule, states of the type $\{1^-n, 1^-n'\}$ with $n \neq n'$ are less harmonic than states with $n = n'$ and consequently are coupled to a larger number of three-phonon configurations.

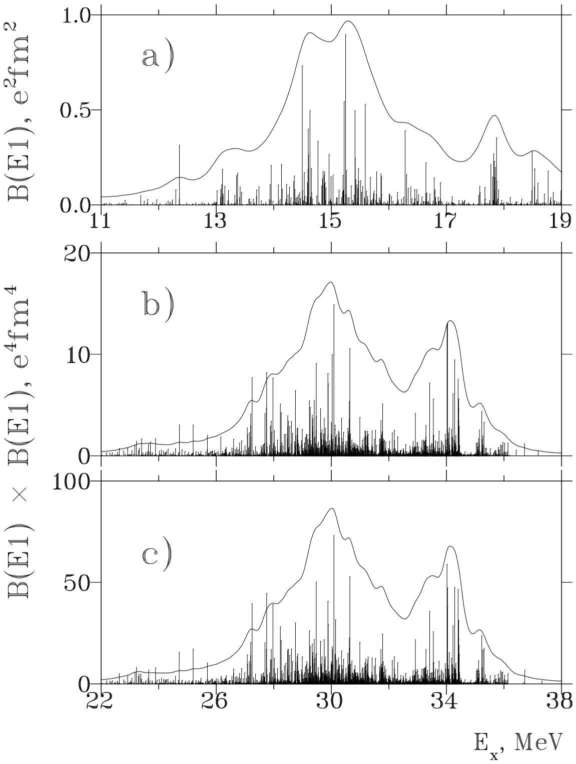

Fig. 1. a B(E1) values for the GDR and **b,c** B(E1)×B(E1) values (4) for the DGDR associated with Coulomb excitation in ^{136}Xe in relativistic heavy ion collision. **b** and **c** correspond to $J = 0^+$ and $J = 2^+$ components of the DGDR, respectively. A smooth curve is a result of averaging over all states with a smearing parameter Γ = 0.5 MeV. See text for details

In Fig. 1b-c, the B(E1)×B(E1) quantity (4) associated with Coulomb excitation of the almost degenerate $J^\pi = 0^+$ and $J^\pi = 2^+$ components of double giant dipole resonance are shown. For comparison, the B(E1) quantity associated with the Coulomb excitation of the one-phonon giant dipole resonance is also shown in Fig. 1a. The reason why the two angular momentum components of the DGDR are almost degenerate can be traced back to the fact that the density of one-phonon configurations to which the DGDR couple and which is different for $J^\pi = 0^+$ and $J^\pi = 2^+$ type states is much lower than the density of states associated with 3p-3h "doorway states", density of states which is the same in the present calculation for the two different angular momentum and parity. Effects associated with the J-dependence of the $\tilde{K}_J$ and Δ_J coefficients are not able to remove the mentioned degeneracy, because of the small size of these coefficients. These coefficients can also affect the excitation probability with which the $J^\pi = 0^+$ and $J^\pi = 2^+$ states are excited (cf. (4)). The effect however is rather small, leading to a decrease of the order of 2-3% in both cases. The J-degeneracy would be probably somehow broken if one goes beyond a one-boson exchange picture in the present approach of interaction between different nuclear modes. The next order term of interaction would couple the DGDR states to many other 3p-3h configurations, not included in the present studies, some of these 3p-3h configurations would be different for different J^π values. Unfortunately, such calculation is not possible the present moment.

Table 1. Position, width and the ratio values R (5) for of $J = 0^+$ and $J = 2^+$ components of the DGDR in respect to the ones of the single GDR in ^{136}Xe. The third row corresponds to pure harmonic picture.

J	$\langle E_{DGDR}\rangle - 2\cdot\langle E_{GDR}\rangle$, keV	$\Gamma_{DGDR}/\Gamma_{GDR}$	R
0^+	–120	1.44	1.94
2^+	–90	1.45	1.96
	0	$\sqrt{2}$	2

The calculated excitation functions displayed in Fig. 1b-c yield the following values for the centroid and width of the DGDR in ^{136}Xe: $< E_{0^+} >$ = 30.68 MeV and Γ_{0^+} = 6.82 MeV for the 0^+ component of the DGDR and $< E_{2^+} >$ = 30.71 MeV and Γ_{2^+} = 6.84 MeV for the 2^+ component. These values have to be compared to $< E_{1^-} >$ = 15.40 MeV and Γ_{1^-} = 4.72 MeV for the single GDR in this nucleus from our calculation. The correspondence between these values is presented in Table 1 in comparison with the prediction of the harmonic model. Also shown is the ratio

$$R = \frac{\sum_\nu \sum_{\nu_1} \langle \Psi^\nu_{0^+(2^+)} | M(E1) | \Psi^{\nu_1}_{1^-}\rangle \cdot \langle \Psi^{\nu_1}_{1^-} | M(E1) | \Psi_{g.s.}\rangle|^2}{|\sum_{\nu_1}\langle \Psi^{\nu_1}_{1^-} | M(E1) | \Psi_{g.s.}\rangle|^4}, \quad (5)$$

between the two-step excitation probability of the DGDR normalized to the summed excitation probability of the one-phonon GDR. The numerical results lie quite close to the predictions of the harmonical model (see also a discussion of this problems in [12]). While the on-the-energy-shell transitions are easier to identify and calculate properly, off-the-energy shell corrections are considerably more elusive. In fact, it may be argued that the calculated shift of the energy centroid of the DGDR with respect to that expected in the harmonic picture is somewhat underestimated, because of the limitations used in selecting two-phonon basis states used in the calculation. On the other hand, including the two-phonon states made up of 2^+ and 3^- phonons as done in the case of ^{40}Ca [4], even if only within the pure boson picture, leads to the value of $\Delta E = 2\langle E_{GDR}\rangle - \langle E_{DGDR}\rangle$ of the order of 200 keV. In keeping with expected $A^{-2/3}$ scaling of ΔE (cf. [13]), this result is consistent with that shown in Table 1.

We conclude that the magnitude as well as the first and second moments of the Coulomb excitation cross section associated with the double giant dipole resonance calculated taking into account couplings between one-, two- and three-phonon states, are well accounted for within the harmonic picture.

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