Influence of the nuclear structure on the suppression factor of the $M1$ resonance in the $(p,p')$ reaction

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In the distorted-wave impulse approximation we calculate cross sections for $(p,p')$ scattering at $E_p = 200$ MeV with excitation of the $M1$ resonance in $^{90-96}$Zr isotopes. Here we take into account the central and tensor components of the free $t_{nn}$ interaction and the exchange knockout of nucleons. The wave function of the $M1$ resonance is calculated in different approximations of the quasiparticle-phonon nuclear model. It is shown that the suppression factor of the excitation probability of the $M1$ resonance in $(p,p')$ scattering $q = \sigma_{exp}/\sigma_{theor}$ depends strongly on the complexity of the model wave function of the resonance. For the wave function containing one- and two-phonon components the mean value of the factor is $q = 0.74$.

1. INTRODUCTION

We study excitation of the magnetic dipole resonance in $^{90-96}$Zr isotopes in proton inelastic scattering at $E_p = 200$ MeV. The first results from the corresponding experiments appeared several years ago and have significantly clarified the situation with the $M1$ resonance, which had "disappeared" at that time in heavy nuclei. The problem was the following: in many experiments on the scattering of slow electrons at large angles performed at the end of seventies no significant strength of $M1$ transitions in nuclei with $A \geq 100$ could be observed in the energy range $E_a$ where according to general belief the $M1$ resonance should have been observed. In the $^{90}$Zr nucleus the existence of several weak $1^+$ levels was more or less unambiguously established in the interval $0.8 < E_a < 10$ MeV with the total transition probability $B(M1) \approx 0.78 \mu_0^2$, and for the $M1$ strength the upper bound was found to be $B(M1) \lesssim 2.6 \mu_0^2$, which is $\sim 20\%$ of the simple shell estimate for the purely neutron configuration $(1g_{9/2},1g_{7/2})$. At the same time it was noted that one of the possible reasons of failure of $(e,e')$ experiments was the influence of an $M2$ resonance, masking $M1$ transitions, situated in the same region of $E_a$ and excited with larger probability than the $M1$ resonance for the values of momentum transfer achieved in electron inelastic scattering.

In proton inelastic scattering at intermediate energies and the angles $\theta = 2-4^\circ$ the momentum transfer to the nucleus is significantly smaller than in $(e,e')$ scattering under the conditions of the experiment of Refs. 2 and 3. This leads to the suppression of the transition intensity with the momentum transfer $L \geq 1$ and to preferential excitation of the states with $L = 0$. In addition, the character of the behavior of different components of the interaction of the incident proton and the nucleus is such that at $E_p = 100-300$ MeV in $(p,p')$ scattering the spin–isospin and isospin modes are excited with larger probability. All these circumstances have led to the fact that just in $(p,p')$ experiments at intermediate energies the $M1$ resonance was again "discovered". In the $^{90-96}$Zr isotopes this resonance is seen most clearly as a peak with the width $\Gamma \approx 1.5$ MeV whose energy decreases slightly with increase of $N$ from 8.9 MeV in $^{90}$Zr to 8.6 MeV in $^{96}$Zr. In heavier nuclei ($^{130,134}$Sn, $^{140,146}$Ce) the $M1$ resonance is not so sharply pronounced against the background of the reaction and it occupies a broader $E_a$ interval, which, apparently, indicates that its fragmentation is stronger. Nevertheless, the existence of the $M1$ resonance in medium and heavy nuclei can be now regarded as firmly established. In $^{90}$Zr this resonance was confirmed in $(p,p')$ experiments with high resolution at $E_p = 319$ MeV. Quite recently $M1$ resonance was discovered in $^{206}$Pb in experiments with polarized $\gamma$ rays.

The problem of the strength of the resonance $M1$ transitions is not cleared up completely. Their weakening, of course, is not so strong as it seemed on the basis of the results of $(e,e')$ experiments, but nevertheless it exists. The experimental value of the excitation cross section of the $M1$ resonance in $Zr$ isotopes in $(p,p')$ scattering is approximately three times smaller than the theoretical value. True, the theoretical value $\sigma(\theta)_{theor}$ was calculated for the purely neutron configuration $(1g_{9/2},1g_{7/2})$. Obviously, many factors are responsible for the suppression of the strength of $M1$ transitions, among them also nucleon correlations in nuclei. It makes sense to talk about the existence of other effects and their magnitude only after the investigation of "traditional" causes of this phenomenon and after exhausting their possibilities. From this point of view it is also preferable to calculate directly cross sections for reactions with nuclear wave functions obtained within the framework of the microscopic model and not with such "structure" quantities as reduced probabilities of electromagnetic transitions or nuclear response functions to simple external fields and appropriate symmetry, whose "experimental" values are obtained as a result of a model-dependent processing of experimental cross sections. The present work is one of the attempts of this kind.

2. QUASIPARTICLE–PHONON MODEL AND PROPERTIES OF $1^+$ STATES IN ZR ISOTOPES

In the present paper the structures of $1^+$ excitations in $^{90-96}$Zr isotopes is calculated within the framework of the quasiparticle–phonon model (QPM). The choice of the model is based on two factors. First, in the nuclei considered pairing plays a significant role. Second, we want to investigate the influence of the interaction with complicated configurations on the $M1$ resonance and its excitation probability in the $(p,p')$ reaction. The QPM is one of the few models where these two effects are taken into account. As draw-
backs of the QPM one should regard the use of the schematic separable particle–hole interaction and of the basis of single-particle states limited to bound and relatively narrow quasi-bound states in the single-particle Woods-Saxon potential. The recent calculations of the excitation cross section of the M1 resonance in $^{90}$Zr in $(p,p')$ scattering performed within the framework of the theory of finite Fermi systems (TFFS) are free of these drawbacks. In these calculations, however, pair correlations in the proton system of the $^{90}$Zr nucleus and the interaction with complicated configurations are not taken into account.

The radial form factor of the effective separable interaction in the spin channel was chosen in the form $F(r) = dU/dr$, where $U$ is the central part of the Woods-Saxon potential. We have not taken into account the contribution to the structure made by single-particle 1+ excitations of two-quasiparticle states with the difference of orbital angular moments equal to two. According to the results of Ref. 15 their influence on the M1 resonance is not significant.

The constant $x^{(01)}$ of the spin–isospin interaction was chosen in each nucleus independently of the experimental location of the M1 resonance. The isoscalar spin–spin interaction was chosen to be weak ($x^{(01)}_{s}=0.1x^{(01)}$). This choice was based on the results of Refs. 13 and 16 and also on our calculations of the properties of the recently discovered isoscalar 1+ state ($E_s=5.846$ MeV) in $^{208}$Pb.

In the one-photon approximation (or in the random phase approximation) in the excitation spectrum of all $^{90–96}$Zr isotopes two 1+ states with large values of $B(M1, 0^+_s \rightarrow 1^+)$ are seen (see Table I). The excitation energy of the state with the maximum value of $B(M1)$ is equal to the experimental energy of the M1 resonance (because of the choice of the constant $x^{(01)}$). The energy of the second state is $E_2 \approx 11$ MeV. The main contribution to the structure of the "resonance" 1+ state comes from the neutron two-quasiparticle configuration $(1g_{9/2}1g_{7/2})_n$, and the main contribution to the structure of the 1+ state with the energy $E_2 \approx 11$ MeV from the proton two-quasiparticle configuration $(1g_{9/2}1g_{7/2})_p$. The existence of the second level among the levels considered is entirely related to superfluid correlations in the proton system. The enhancement of the isoscalar spin interaction leads to increase of the value of $B(M1)$ for the "resonance" 1+ level and to its decrease for the 1+ level with $E_2 \approx 11$ MeV.

As was noted above, the advantage of the QPM is the fact that within its framework a formalism is constructed which makes it possible to take into account the influence of the interaction with two-photon configurations on highly excited states. The interaction of one- and two-photon states leads to the fact that the strength of one-photon magnetic dipole states is distributed among a large number of levels and the M1 resonance acquires the so-called spreading width $\Gamma$. The quantity $\Gamma$ depends on the strength of the interaction of one- and two-photon states, which in the QPM is calculated microscopically. We shall not dwell in detail on the formalism of the QPM, because it is described in detail in the review of Ref. 12, and the fragmentation of the M1 resonance within the framework of this model was investigated in Ref. 17.

In what follows we concentrate our attention only on the one-photon 1+ state with $E_2 \approx 9$ MeV, because we identify it with the M1 resonance. Other 1+ states are outside the region occupied by the M1 resonance in the experimental spectrum ($7 < E < 10$ MeV) and are weakly excited in $(p,p')$ scattering. The strongest among them, the 1+ state with $E_2 \approx 11$ MeV, is in addition strongly fragmented because of the interaction with two-photon states.

### TABLE I. Energies ($E_2$), reduced probabilities of M1 transitions [$B(M1)\mu^2$], and principal components of wave functions of the single-particle 1+ states with maximum values of $B(M1)$ in $^{90–96}$Zr isotopes.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$E_2$, MeV</th>
<th>$B(M1)\mu^2$</th>
<th>Structure of wave functions, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{90}$Zr</td>
<td>8.9</td>
<td>4.35</td>
<td>$(</td>
</tr>
<tr>
<td>$^{92}$Zr</td>
<td>11.0</td>
<td>3.64</td>
<td>$(</td>
</tr>
<tr>
<td>$^{94}$Zr</td>
<td>8.8</td>
<td>4.85</td>
<td>$(</td>
</tr>
<tr>
<td>$^{96}$Zr</td>
<td>8.8</td>
<td>3.27</td>
<td>$(</td>
</tr>
<tr>
<td>$^{98}$Zr</td>
<td>6.7</td>
<td>6.20</td>
<td>$(</td>
</tr>
<tr>
<td>$^{100}$Zr</td>
<td>8.6</td>
<td>5.44</td>
<td>$(</td>
</tr>
</tbody>
</table>

Note. Values of $B(M1)$ are calculated with the effective gyromagnetic factors $g^{eff}_p = 0.8g^{eff}_p$. 

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MeV has been considered within the framework of this approximation in Refs. 5 and 14. In the present work in order to calculate the cross section $\sigma_{\text{theor}}$ as accurately as possible we take into account in addition to the central component also the tensor component of the $t_{\text{NN}}$ interaction. We take into account exactly effects of the exchange knockout, because we calculate directly the nonlocal form factor using the QPM transition densities. The calculations of the cross sections are performed with the help of the computer program which is described in detail in Ref. 19. In order to determine scattering phase shifts in the field of the target nucleus we use the optical potential, which depends on energy and mass number and whose parameters are adjusted on the basis of data on the cross section for proton elastic scattering and on the analyzing power for a large group of nuclei.

Transition densities of $1^+$ states are calculated within the framework of the following three approximations: 1) in the independent-quasiparticle approximation when the $M_1$ resonance is described as a purely two-quasiparticle state $(\ell_{g_{9/2}}^2, \ell_{g_{7/2}}^1)$; 2) in the Tamm-Dancoff approximation (TDA); 3) in the random phase approximation (RPA). This makes it possible to trace how the complication of the wave function of the $M_1$ resonance affects the cross section for excitation of the $M_1$ resonance in $(p,p')$ scattering. The transition densities $\rho_{\pi,\sigma}^N(r)$ of the resonance $1^+$ state of the $^{90}$Zr nucleus evaluated in different approximations are shown in Fig. 1. The transition densities for the values of the orbital angular momentum $\Delta L = 0$ and 2 are shown separately. We note that the magnitude of $\rho_{011}^N$ is significantly larger than the magnitude of $\rho_{211}^N$. The complication of the wave function of the $1^+$ state leads to a small shift of the maximum of $\rho_{\pi,\sigma}^N(r)$ into the nucleus and to its decrease with the simultaneous broadening of the transition density. On the whole, the changes are quite insignificant; however, as we shall see below, they lead to significant changes in the magnitude of cross sections for $(p,p')$ scattering. It is interesting that changes of the quantity $B(M_1, 0^+\rightarrow 1^+)$ for the $M_1$ resonance in the transition from the TDA to the RPA are small ($< 10\%$).

4. RESULTS OF THE CALCULATIONS OF CROSS SECTIONS FOR $(p,p')$ SCATTERING

We now analyze the calculated cross sections for $(p,p')$ scattering. First of all, we shall consider some methodical aspects of the calculations. In Fig. 2 we show cross sections for the excitation of the $M_1$ resonance in $^{90}$Zr calculated with inclusion of different components of the $t_{\text{NN}}$ interaction with the RPA transition densities. It is seen clearly that after the first minimum (i.e., at scattering angles $\theta > 10^\circ$) the behavior of the cross section is influenced strongly by both the tensor component of the $t_{\text{NN}}$ matrix and the transitions with $\Delta L = 2$. For angles $\theta < 10^\circ$ their contribution increases the cross section for $(p,p')$ scattering by 5–10%. In Fig. 2 we show also the cross section for excitation of the $1^+$ state with $E_x \approx 11$ MeV. Since protons with energy $E_p = 200$ MeV interact slightly more strongly with the neutrons of the target nucleus than with the protons, this state is excited much more weakly than the $M_1$ resonance.

We note that the ratio of the quantities $B(M_1)$ for these two states is $\approx 1.2$ (see Table II), while the ratio of the excitation cross sections in $(p,p')$ scattering for those two states at $\theta = 3–4^\circ$ is equal to two. The differences of the shapes of the cross sections calculated with transition densities corresponding to the RPA and TDA and to the purely two-quasiparticle configuration $(\ell_{g_{9/2}}^2, \ell_{g_{7/2}}^1)$, for small scattering angles are small, and the theoretical and experimental behaviors of the cross section as function of $\theta$ for $\theta < 8^\circ$ are close (see Fig. 3 below). The absolute value of the cross sections changes much more strongly. In the calculations with the purely particle-hole configuration $(\ell_{g_{9/2}}^2, \ell_{g_{7/2}}^1)$ in $^{90}$Zr the ratio of the cross sections $q = (\sigma_{\pi,\sigma}/\sigma_{\pi,\sigma})_{\text{theo}} / (\sigma_{\pi,\sigma}/\sigma_{\pi,\sigma})_{\text{expt}}$ is close to two.

FIG. 1. Transition densities of the $M_1$ resonance in $^{90}$Zr calculated in different approximations for the wave function of the $M_1$ resonance: (a)—$\rho_{101}^N(r)$, and (b)—$\rho_{211}^N(r)$. The solid lines show transition densities for the RPA wave function, the dashed lines for the TDA, and the dash-dot for the pure configuration $(\ell_{g_{9/2}}^2, \ell_{g_{7/2}}^1)$.

FIG. 2. Cross section for excitation of the single-phonon $1^+$ states in $^{90}$Zr in $(p,p')$ scattering as a function of the scattering angle $\theta$. The solid curve is the cross section for excitation of the $M_1$ resonance in the case when in the $t_{\text{NN}}$ matrix the central and tensor components and exchange knockout are taken into account exactly; the dashed curve is the cross section for excitation of the $M_1$ resonance in which transitions with $\Delta L = 2$ are not taken into account; the dash-dot curve is the cross section for excitation of the $1^+$ state with $E_x = 11.0$ MeV in the $t_{\text{NN}}$ matrix the central and tensor components and exchange knockout are taken into account exactly. $E_p = 200$ MeV.
TABLE II. The suppression factor of the excitation probability of the $M1$ resonance in $(p,p')$ scattering $q = \sigma_{\exp}/\sigma_{\text{theor}}$ calculated in different approximations for the wave function of the $M1$ resonance.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$\langle \psi_{1}, \psi_{2} \rangle_{V}$</th>
<th>TDA</th>
<th>RPA</th>
<th>$q^{+} + q^{-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{92}$Zr</td>
<td>0.32</td>
<td>0.48</td>
<td>0.64</td>
<td>0.79</td>
</tr>
<tr>
<td>$^{94}$Zr</td>
<td>0.25</td>
<td>0.38</td>
<td>0.47</td>
<td>0.56</td>
</tr>
<tr>
<td>$^{96}$Zr</td>
<td>0.34</td>
<td>0.48</td>
<td>0.62</td>
<td>0.75</td>
</tr>
<tr>
<td>$^{98}$Zr</td>
<td>0.54</td>
<td>0.69</td>
<td>0.60</td>
<td>0.88</td>
</tr>
</tbody>
</table>

$d\Omega_{\text{theor}}$ is equal to 0.32. This value of $q$ is close to the value obtained in Ref. 7 in the same approximation for the wave function of the $M1$ resonance. We note that in Ref. 7 the calculations were performed with the program DWBA.70. In the transition to the wave function of the $M1$ resonance calculated in the TDA the ratio $q$ increases to 0.48, and in the RPA $q = 0.64$. A similar situation is observed also in other nuclei (see Table II).

From Fig. 3 we can judge the quality of agreement between our calculations and the experimental angular distributions. We show experimental cross sections for all four zirconium isotopes and theoretical cross sections normalized relative to experiment; here the calculations are performed with the RPA wave functions. Normalization coefficients are given in Table II. It is important that the values of $q$ in all isotopes are close and vary in the same manner as a function of the approximation chosen for the wave function of the $M1$ resonance. Only in the isotope $^{92}$Zr is the value of $q$ smaller by 25–30% than in other isotopes; here this discrepancy also does not depend on the form of the wave function of the $M1$ resonance. The point is that the experimental cross section for excitation of the $M1$ resonance in $^{92}$Zr is much smaller than in other Zr isotopes. Theoretical calculations do not reproduce this difference. We do not understand why the cross section for excitation of the $M1$ resonance in $^{92}$Zr is smaller. It is possible that part of the $M1$ strength is lost in the empirical procedure of separation of the background of the reaction. The systematic excess of the experimental cross section $\sigma(\theta)_{\exp}$ over $\sigma(\theta)_{\text{theor}}$ for $\theta > 8^\circ$ is related, apparently, to the increase of the contribution to the cross section made by states with the angular momenta $L > 1$.

Thus, the calculations in the TDA predict for the cross section for excitation of the $M1$ resonance values which are larger than experimental values for a factor 1.5–2.0, but they in their turn are two times smaller than the values obtained in the calculations with a pure configuration. In this way, the nucleon correlations play an important role in suppression of the strength of $M1$ transitions. Here not only admixtures in the wave function of the $M1$ resonance are important, but also the correlations in the nuclear ground state are important. This is related to the increase of $q$ in the transition from the TDA wave functions to the RPA wave functions. The values of $q$ obtained in this stage are very close to those obtained by the authors of Ref. 14 within the framework of the TFFS. We note, however, that the reasons of suppression of $M1$ strength are different. In Refs. 13 and 14, where the pairing in the proton system of the $^{90}$Zr nucleus was not taken into account, the main contribution to the wave function of the $M1$ resonance comes from neutron components; here, as the authors of Ref. 13 note, an important role is played by the transitions into the continuous spectrum. We, because of the restrictions of the single-particle basis, take into account only part of these transitions, and the weakening of the strength of the $M1$ resonance is related mainly to admixtures of proton components [first of all, the component $\langle \gamma_{9/2}, \gamma_{7/2} \rangle_{+}$]. Just because of this, in our calculations the quantity $q$ is sensitive to the ratio of the constants $k_{0}^{(01)}$ and $k_{1}^{(01)}$ and it decreases with increase of the ratio $k_{0}^{(01)}/k_{1}^{(01)}$.

The next step in the way of a more complete inclusion of nucleon correlations is the transition to the wave functions which take into account the influence of the interaction with two-photon states on the $M1$ resonance. In this case the cross section for $(p,p')$ scattering can be calculated with the method of strength functions.\(^{23}\) These calculations are quite similar to the calculations of cross sections for $(e,e')$ scattering which are described in detail in Ref. 22. As usual, we have used the Lorentz type weighting function with the half-width $\Delta = 0.1$ MeV. This value of $\Delta$ is much smaller than the observed width $\Gamma \approx 1.5$ MeV, and it should not introduce noticeable errors into the total probability of excitation of the $M1$ resonance.\(^{25}\)

As an example, we show in Fig. 4 the strength function $b(d\sigma/d\Omega_{xx})$ which describes the distribution of the excitation probability of $1^{+}$ states in $(p,p')$ scattering at $E_{x} = 200$ MeV and $\theta = 3^\circ$. Also in Fig. 4 we present the results of similar calculations in the RPA. The location of the center of gravity of the $M1$ resonance has not changed. As a result of the interaction with two-photon states the $M1$ resonance splits. Its strength is divided among a large number of states: the substructures at $E_{x} = 9.5$ MeV and the

FIG. 3. Experimental\(^1\) and theoretical cross sections for excitation of the $M1$ resonance in $^{90,92,94}$Zr isotopes.
group of the weak states forming the low-energy tail of the resonance. However, the resonance width $\Gamma$ is significantly smaller than the experimental value. Nevertheless, the interaction with the two-phonon states pushes out part of the $M_1$ strength from the resonance region, and 81% of the strength of the state with $E_x = 8.9$ MeV remains in this region. A similar situation is observed in other Zr isotopes.

Of course, the total $M_1$ strength in the large interval $\Delta E_x$ is conserved; it is simply redistributed: part of the strength is pushed out of the resonance region and is concentrated in the large number of weak states which are lost in the background of the reaction. The values of the factor $q$ obtained by taking into account the interaction of one- or two-phonon states in different Zr isotopes are collected in the last column of Table II. If we take into account the remarks made above about the isotope, $^{92}$Zr we can say that in experiments on proton inelastic scattering at intermediate energies in Zr isotopes 70–80% of the theoretically expected strength of the $M_1$ resonance is observed.

It is possible that our calculations underestimate the interaction with complex configurations. First, we do not take into account correlations in the nuclear ground state arising from the photon interaction (it, as before, is regarded as the phonon vacuum). The difference in the values of $q$ calculated with the TDA and RPA wave functions shows that their effect can be noticeable. Second, in the calculations performed by other authors the fragmentation of the $M_1$ resonance in $^{90}$Zr is stronger than in our calculations. Thus, the authors of Ref. 26 point to the existence of a long high-energy tail in the distribution of $M_1$ strength. This part of the distribution is lost irretrievably in the background of the reaction.

5. CONCLUSION

The present calculations above all prove irrefutably that the role of nucleon correlations in the observed suppression of the strength of the $M_1$ resonance is very important, if not fundamental. In any case "ordinary" structure effects diminish the excitation probability of the $M_1$ resonance in $(p,p')$ scattering by a factor 2–2.2 and reduce the difference between the theoretical and experimental probabilities to 30–40%. It is worthwhile to note that the values of the factor $q$ obtained are in agreement with the values of the effective gyromagnetic factors ($g^\text{eff} = 0.8 g^\text{(free)}$), for which theoretical values calculated in the QPM$^{22}$ of the excitation probability in $(e,e')$ scattering of $M_1$ and $M_2$ resonances in a number of spherical nuclei$^{24}$ and also magnetic moments of odd spherical nuclei$^{27}$ calculated in the QPM are in agreement with experiment.

The values of $q$ given in the last column of Table II are, apparently, the largest among those already published (see, for example, Refs. 13, 14, 28, and 29); they are closest to the results of the calculations performed within the framework of the TFFS$^{15,14,29}$ (in the RPA our results agree essentially with the results of Refs. 13 and 14). It cannot be excluded that the schematic form of our effective interaction is partially responsible for the large value of $q$. But in any case the values of $q$ obtained within the framework of the QPM for various processes and the quantities related to $M_1$ and $M_2$ resonances are mutually consistent.

Translated by O. Dumbrajs

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23L. A. Malov, JINR, R4-81-228, Dubna, 1981.