Description of the excitation of one- and two-phonon states in spherical nuclei induced by the inelastic scattering of low-energy protons

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In the framework of the quasiparticle-phonon model (QPM) of the nucleus, we develop an approach to the microscopic description of the excitation of one- and two-phonon states in spherical nuclei, induced by the inelastic scattering of low-energy protons. The interaction potential between the incident proton and the target nucleus is treated with the help of the nuclear transition densities, obtained in the QPM, and the effective nucleon-nucleon interaction M3Y. We calculate the cross sections for the inelastic scattering of protons of different energies from ^{58}Ni and ^{90}Zr. The structure of several of the low-lying electric excitations in ^{58}Ni is studied. The manifestations of a complex structure of these states in the cross sections of (p, p') reactions are discussed.

1. INTRODUCTION

Inelastic scattering of protons is an important source of information on the structure of different kinds of excitations in nuclei.[1] This is especially true for electric excitations, considered in the present paper. In electric transitions the reduced transition probabilities $B(E\lambda)$ are strongly correlated with the cross sections of (p, p') reactions.[2] In recent years data on the scattering of low-energy protons (up to several tens of MeV) from several spherical nuclei have been obtained with high experimental resolution (see for example Refs. 3 and 4). In these experiments the angular distributions are measured over a wide range for many of the low-lying excitation levels of the target nucleus. Analysis of the data in the framework of simple phenomenological models with a large number of parameters shows that many of the excited states have complicated structures (two-phonon or mixtures of one- and two- phonon configurations).[5] Therefore the microscopic study of the structure of such states and the description of the corresponding inelastic reaction cross sections are problems of great current interest.

The microscopic approach to the description of such processes is usually based on the construction of an optical potential and the form factors of inelastic transitions induced by the scattering of protons. Use is made of effective nucleon–nucleon forces and nuclear transition densities calculated in the framework of an appropriate microscopic model.[2,6]

Several different microscopic models have been used successfully in description of the low-lying excitations in nuclei. One of them is the quasiparticle-phonon model (QPM),[7–9] which has been used in recent years in description of low-lying states[10,11] and also of giant resonances.[8,9] In the present paper the nuclear wave functions of the QPM are used to treat microscopically the scattering of low-energy protons from spherical nuclei accompanied by excitation of one- and two-phonon states.

2. FUNDAMENTALS

The scattering of protons from nuclei is treated starting from the effective nucleon-nucleon (NN) interaction $V_{NN}(\mathbf{r}_{12})$ and the nuclear transition densities $\rho(\mathbf{r})$ calculated microscopically. The folding model[2] is then normally used. In this model the proton–nucleus interaction potential is obtained by averaging the NN interaction over the distribution of nucleons in the target nucleus:

$$U_F(\mathbf{R}) = \int \rho(\mathbf{r})\, v_{NN}(\mathbf{R}-\mathbf{r})\, d\mathbf{r}. \tag{1}$$

We discuss first the construction of the nuclear transition densities in the QPM. The model Hamiltonian has the general form[7–9]:

$$H = H_{sp} + H_{pair} + H_M + M_{SM}, \tag{2}$$

where $H_{\rm sp}$ describes the independent motion of the protons and neutrons in their mean fields; $H_{\rm pair}$ is the monopole pairing force and only acts between neutrons and between protons; H_M and H_{SM} are the separable multipole and spin-multipole forces which generate the nuclear excitations. In order to calculate the characteristics of the different nuclear excitations, the QPM Hamiltonian is transformed to the quasiparticle–phonon representation,[7–9] and then the phonon operator has the form

$$Q^+_{\lambda\mu i} = \frac{1}{2}\sum_{jj'} \{\psi^{\lambda i}_{jj'}[\alpha_{jm}{}^+\alpha^+_{j'm'}]_{\lambda\mu} - (-1)^{\lambda-\mu}\varphi^{\lambda i}_{jj'}[\alpha_{j'm'}\alpha_{jm}]_{\lambda-\mu}\}. \tag{3}$$

Here α^+_{jm} and α_{jm} are the quasiparticle creation and annihilation operators. The wave function of the one-phonon state is written as

$$Q^+_{\lambda\mu i}\Psi_0, \tag{4}$$

where $\Psi_0 = |0\rangle$ is the ground-state wave function of an even–even nucleus, which is assumed to be the phonon vacu-

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um state. The energies $\omega_{\lambda i}$ of the one-phonon state (4) and the amplitudes $\psi_{jj}^{\lambda i}$, and $\varphi_{jj}^{\lambda i}$, are calculated by solving the secular equation in the RPA.[8]

The approximation resulting from the assumption that the phonon excitations are independent is often too crude. In this case the wave function of the excited states in the even–even nucleus is written in the more complicated form[9]:

$$\Psi_\nu(JM)=\left\{\sum_i R_i(J\nu)\,Q^+_{JMi}+\sum_{\lambda_1 i_1 \lambda_2 i_2} P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\,[Q^+_{\lambda_1\mu_1 i_1} Q^+_{\lambda_2\mu_2 i_2}]_{JM}\right\}|0\rangle. \tag{5}$$

We note that in general it is possible to add to (5) three–phonon and more complicated terms. The wave function (5) is normalized in the following way:

$$\sum_i [R_i(J\nu)]^2+2\sum_{\lambda_1 i_1 \lambda_2 i_2}[P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)]^2 \times\left\{1+\frac{1}{2}\mathcal{K}^J(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)\right\}=1. \tag{6}$$

The functions $\mathcal{K}^J(\lambda_2 i_2,\lambda_1 i_1|\lambda_1 i_1,\lambda_2 i_2)$ appear because of the Pauli principle.[9] The fundamental equations of the QPM for even–even nuclei and with account of the Pauli principle have been obtained in Ref. 12. The calculations here use the quasiboson approximation and the Pauli principle is not taken into account. (The results of Ref. 10 show that for the collective excitations in even–even spherical nuclei the Pauli principle affects the results insignificantly.) In this case the system of fundamental equations[9,12] has the form

$$\begin{aligned}&(\omega_{Ji}-\eta_{J\nu})R_i(J\nu)+\sum_{\lambda_1 i_1\lambda_2 i_2}U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)\,P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)=0,\\ &2(\omega_{\lambda_1 i_1}+\omega_{\lambda_2 i_2}-\eta_{J\nu})P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)+\sum_{i'}U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')R_{i'}(J\nu)=0,\end{aligned} \tag{7}$$

where the $U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)$ are the interaction matrix elements of the one- and two-phonon states. From the condition that there exist a nontrivial solution of the system of equations (7), we obtain the secular equation for the energies $\eta_{J\nu}$ of the states (5):

$$\mathcal{F}(\eta_{J\nu})=\det\left|(\omega_{Ji}-\eta_{J\nu})\delta_{ii'}-\frac{1}{2}\times\sum_{\lambda_1 i_1\lambda_2 i_2}\frac{U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji)\,U^{\lambda_1 i_1}_{\lambda_2 i_2}(Ji')}{\omega_{\lambda_1 i_1}\pm\omega_{\lambda_2 i_2}-\eta_{J\nu}}\right|=0. \tag{8}$$

Therefore substituting the solutions of (8) into the system of equations (7), one can obtain the structure coefficients R and P of the wave function (5) for the one and two-phonon parts.

We next write the transition density of the nucleus $\rho(\mathbf{r})=\langle f|\Sigma_k\delta(\mathbf{r}-\mathbf{r}_k)|i\rangle$ in the form of a multipole expansion

$$\rho(\mathbf{r})=\sum_{\lambda\mu}C_\lambda\langle J_i M_i\lambda\mu|J_f M_f\rangle\rho_\lambda(r)\,(-i)^\lambda Y_{\lambda\mu}{}^*(\theta,\varphi). \tag{9}$$

We have

$$C_\lambda\rho_\lambda(r)=\langle J_f\|\sum_k r_k^{-2}\delta(r-r_k)\,i^\lambda Y_\lambda(\theta_k,\varphi_k)\|J_i\rangle, \tag{10}$$

and the normalization coefficients C_λ are given by[2]: $C_0=\sqrt{4\pi}$, $C_\lambda=1$ (for $\lambda\neq 0$). We obtain the following result for the nuclear ground-state density ($J_f=J_i=\lambda=0$) after some reductions:

$$\rho_0(r)=\frac{1}{4\pi}\sum_j(2j+1)\,|R_j(r)|^2 v_j^2, \tag{11}$$

where $R_j(r)$ is the radial part of the wave function of the single-particle state $j\equiv(n,l,j)$, and v_j is the coefficient of the Bogolyubov transformation, which arises because of pairing.[7,9]

When $\lambda\neq 0$ one can transform the single-particle operator in (10) to the quasiparticle-phonon representation. We then have

$$\sum_k r_k^{-2}\delta(r-r_k)\,i^\lambda Y_{\lambda\mu}(\theta_k,\varphi_k)\to\sum_{j_1 j_2}\langle j_1\|T_\lambda\|j_2\rangle \times\left\{\frac{1}{2}u^{(+)}_{j_1 j_2}\sum_i(\psi^{\lambda i}_{j_1 j_2}+\varphi^{\lambda i}_{j_1 j_2})\,[Q^+_{\lambda\mu i}+(-1)^{\lambda-\mu}Q_{\lambda-\mu i}] +v^{(-)}_{j_1 j_2}B(j_1 j_2;\lambda\mu)\right\}, \tag{12}$$

where

$$\langle j_1\|T_\lambda\|j_2\rangle=(-1)^{j_2+\lambda-1/2}i^{l_2+\lambda-l_1}\frac{\hat{j}_1\hat{j}_2}{4\sqrt{\pi}}\begin{pmatrix} j_1 & j_2 & \lambda\\ 1/2 & -1/2 & 0\end{pmatrix} \times R_{j_1}{}^*(r)R_{j_2}(r)\,[1+(-1)^{l_2-l_1+\lambda}], \tag{12a}$$

$$B(j_1 j_2;\lambda\mu)=\sum_{m_1 m_2}(-1)^{j_2+m_2}\langle j_1 m_1 j_2 m_2|\lambda\mu\rangle\alpha^+_{j_1 m_1}\alpha_{j_2-m_2};$$

and $u^{(+)}_{j_1 j_2}$ and $v^{(-)}_{j_1 j_2}$ are combinations of the Bogolyubov transformation coefficients u_j and v_j (see Refs. 7–9). Substituting (12) into (10), we obtain an expression for the nuclear transition density:

a) for the transition from the ground state into a one-phonon excited state (4) we have

$$\rho_\lambda(r)=\sum_{j_1 j_2}\langle j_1\|T_\lambda\|j_2\rangle\frac{1}{2}u^{(+)}_{j_1 j_2}\,(\psi^{\lambda i}_{j_1 j_2}+\varphi^{\lambda i}_{j_1 j_2}); \tag{13}$$

b) for the transition from the ground state into the state (5) (mixture of one- and two-phonon configurations) we have

$$\rho_J(r)=\sum_{j_1 j_2}\langle j_1\|T_J\|j_2\rangle\left[\frac{1}{2}u^{(+)}_{j_1 j_2}\sum_i R_i(J\nu)\,(\psi^{Ji}_{j_1 j_2}+\varphi^{Ji}_{j_1 j_2}) -\sum_{\lambda_1 i_1\lambda_2 i_2 j}P^{\lambda_1 i_1}_{\lambda_2 i_2}(J\nu)\,v^{(-)}_{j_1 j_2}\hat{\lambda}_1\hat{\lambda}_2\begin{Bmatrix}\lambda_1 & \lambda_2 & J\\ j_1 & j_2 & j\end{Bmatrix}(\psi^{\lambda_1 i_1}_{j_2 j}\varphi^{\lambda_2 i_2}_{j j_1}+\varphi^{\lambda_1 i_1}_{j_2 j}\psi^{\lambda_2 i_2}_{j j_1})\right]; \tag{14}$$

c) for the transition from the one-phonon state (4) into the state (5) we have

$$\rho_{\lambda'}(r)=\sum_{j_1 j_2}\langle j_1\|T_{\lambda'}\|j_2\rangle\Big[u_{j_1 j_2}^{(+)}\sum_{i'}P_{\lambda i}^{\lambda' i'}(Jv)\,(\psi_{j_1 j_2}^{\lambda' i'}+\varphi_{j_1 j_2}^{\lambda' i'})$$
$$-\sum_{i' j}v_{j_1 j_2}^{(-)}R_{i'}(Jv)\hat{\lambda}\hat{\lambda}'\begin{Bmatrix}J & \lambda & \lambda'\\ j_1 & j_2 & j\end{Bmatrix}(\psi_{j_1 j}^{J i'}\psi_{j j_2}^{\lambda i}+\varphi_{j_1 j}^{J i'}\varphi_{j j_2}^{\lambda i})\Big]. \tag{15}$$

With the help of the microscopically calculated nuclear transition densities (11), (13) through (15), and the *NN*interaction $v(\mathbf{r})$, we can construct the optical potential and the form factor of the inelastic transitions induced by the scattering of protons. Using the multipole expansion (9) we obtain the following expression for the folding potential (1) for the transition of the nucelus into the excited state with moment λ (Ref. 2):

$$U_F(\mathbf{R})=C_\lambda U_\lambda(R)\,Y_{\lambda\mu}^{*}(\theta_R,\ \varphi_R), \tag{16}$$

where

$$U_\lambda(R)=\frac{1}{2\pi^2}\int_0^\infty dk\,k^2 j_\lambda(kR)\,\tilde{v}(k)\,\tilde{\rho}_\lambda(k). \tag{17}$$

Here

$$\tilde{v}(k)=4\pi\int_0^\infty dr\,r^2 j_0(kr)\,v(r),\qquad \tilde{\rho}_\lambda(k)=4\pi\int_0^\infty dr\,r^2 j_\lambda(kr)\,\rho_\lambda(r). \tag{18}$$

Then $U_\lambda(r)$ is included in the calculation of the angular distribution of (p,p') as the real form factor of the corresponding inelastic transition (elastic scattering corresponds to $\lambda=0$ in these formulas). Since for low-energy protons (in the corresponding approximations[13]) the effective *NN* interaction between the incident nucleon and a bound nucleon in the nucleus is real, in our calculations the imaginary form factor is introduced phenomenologically from the collective model.[14] We chose the so-called M3Y effective *NN* interaction[13]

$$v_0(r)=7999\exp(-4r)/4r-2134.25\exp(-2.5r)/2.5r. \tag{19}$$

In calculations with the M3Y interaction,[15,16] the exchange term corresponding to the antisymmetrization between the incident nucleon and the nucleons in the target nucleus is usually chosen as a pseudopotential of zero range[17]:

$$v_{EX}(\mathbf{r}_{12})=-276(1-0.005E)\,\delta(\mathbf{r}_{12}), \tag{20}$$

where E is the energy of the incident nucleon. In the present paper, all calculations of the reaction cross sections were done with the microscopic densities (11), (13) through (15), and the effective M3Y interaction (19) and (20). We note that with these *NN* forces and the densities (11) and (13), calculations in the *QPM* have been successfully carried out for the elastic and inelastic scattering cross sections of heavy ions with the excitation of one-phonon states of the type (4) (Ref. 18).

3. RESULTS AND DISCUSSION

With the help of the form factors calculated from (1), (16), and (17), and using the nuclear transition densities given by (11) and (13) through (15), we calculated the cross sections for the elastic and inelastic scattering of protons of various energies from ^{58}Ni and ^{90}Zr. We note that the imaginary part of the optical potential is introduced in our calculations phenomenologically from the usual optical model,[19] and the imaginary transition form factors are determined by deformation[14] of the imaginary part of the optical potential.

The calculation of the cross sections in the distorted-wave approximation (DWA) and in the coupled-channel method (CCM) were carried out with the help of the programs DWUCK (Ref. 20) and ECIS (Ref. 21), respectively. The Coulomb transition form factor was calculated microscopically using the same formulas (16) and (17), but in place of the nuclear transition densities and the M3Y interaction in these equations we used the charge transition density and the Coulomb interaction between two protons. In the nuclear densities (11) and (13) through (15) the one-phonon energies ω and the coefficients ψ and φ were calculated using the program RPAS (Ref. 22), and the energies η_ν and the coefficients R and P in the wave function (5) were calculated using the program GIRES (Ref. 23).

It is known that for inelastic scattering accompanied by the excitation of natural-parity states in even–even nuclei, one can assume a central interaction of the type (19) (Refs. 2 and 16). Fig. 1 shows the results for two versions of the calculation in the distorted-wave approximation for the cross section of ^{90}Zr(p,p) at 61.2 MeV with the excitation of the first 2 + state in ^{90}Zr. The solid curve corresponds to the inclusion of the isotopic dependence of the M3Y interaction, where the form factor $U_{0^+\to 2^+}$ is calculated with the help of the effective (p,p) and (p,n) forces and the proton and neutron parts of the transition densities (13), respectively. The dashed curved corresponds to the case when $U_{0^+\to 2^+}$ is calculated with the isotopically independent *NN* interaction (19) and the total nuclear transition density (13). Fig. 1 shows that these two versions of the calculation give nearly

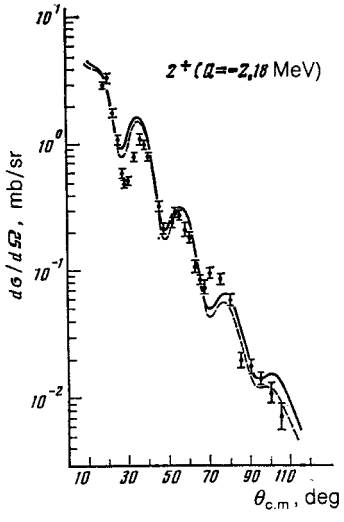

FIG. 1. Experimental data of Ref. 32 (points) and calculated results (solid and dashed curves) for the inelastic cross section of ^{90}Zr(p,p') at 61.2 MeV with excitation for the 2_1^+ state.

identical angular distributions. We note that the structure of the first 2^+ state in ^{90}Zr was calculated in the RPA. The calculations give the following results: the excitation energy $\omega_{2_1^+} = 2.66$ MeV, the reduced transition probability $B(E2\uparrow) = 708.92\ e^2 \cdot F^4$ (the corresponding experimental data for ^{90}Zr are $\omega_{2_1^+} = 2.19$ MeV and $B(E2\uparrow) = 600\text{-}720\ e^2 \cdot F^4$ (Ref. 24).

All in all, the calculations show that the experimental cross section for the excitation of low-lying collective states with a dominant one-phonon component in the wave function (5) is described accurately in the distorted-wave approximation with the transition densities (13) calculated in the RPA (Ref. 8). For example, consider the lowest 2^+ and 3^- states in ^{58}Ni, which have an obvious one-phonon structure. Calculations in the RPA with the multipole constants $\varkappa_0^{(\lambda)}$ and $\varkappa_1^{(\lambda)}$ of Ref. 8, which reproduce the transition probabilities $B(E2\uparrow) = 833.15\ e^2 \cdot F^4$ and $B(E3\uparrow) = 20.34 \cdot 10^3\ e^2 \cdot F^6$, give the excitation energies $\omega_{2_1^+} = 1.754$ MeV and $\omega_{3_1^-} = 4.8$ MeV (the corresponding experimental data of Refs. 24 and 25 are: $\omega_{2_1^+} = 1.45$ MeV, $\omega_{3_1^-} = 4.48$ MeV, and $B(E2\uparrow) = 657\text{-}900\ e^2 \cdot F^4$, $B(E3\uparrow) = (18.6\text{-}27) \cdot 10^3\ e^2 \cdot F^6$).

Calculations with the wave function (5), that is, with inclusion of the interaction between the one- and two-phonon configurations, gives, after solving (7) and (8), $\eta_{2_1^+} = 1.697$ MeV, $B(E2\uparrow) = 807.82\ e^2 \cdot F^4$ and $R_1(2_1^+) = 0.9846$. Hence it follows that the one-phonon component in the first 2^+ state in ^{58}Ni is $[R_1(2_1^+)]^2 = 97\%$. Similar results are obtained for the state 3_1^-.

Figure 2 shows the results of the calculation and the experimental data for the excitation cross sections of the states 2_1^+ and 3_1^- in ^{58}Ni for incident-proton energies 18.6 (Ref. 26) and 40 MeV (Ref. 5). The transitition densities

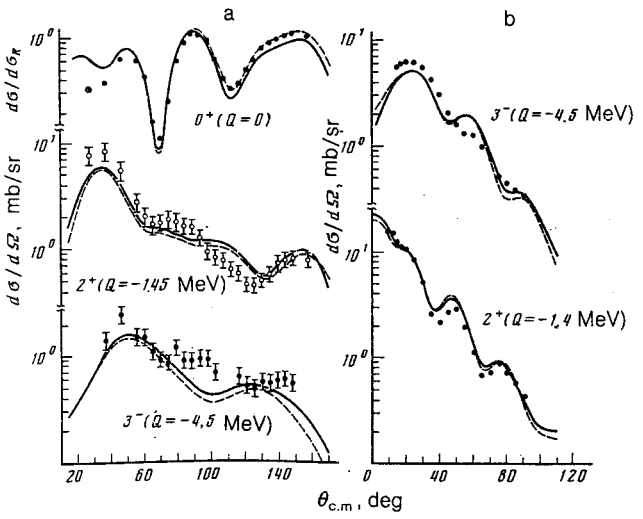

FIG. 2. Experimental data of Refs. 26 and 5 (points) and calculated resutls (solid and dashed curves) for the inelastic cross sections of ^{58}Ni(p,p') [(a) for 18.6 MeV; (b) for 40 MeV] with excitation of the states 2_1^+ and 3_1^-.

used in constructing the form factors $U_{0^+\to 2^+}$ and $U_{0^+\to 3^-}$ were calculated in the RPA [see (13)]. The solid curves correspond to the coupled-channel method with two channels ($0^+_{g.s.} \to 2_1^+, 3_1^-$), and the dashed curves to the distorted-wave approximation.

We see the at the results in the distorted-wave approximation for the cross section are qualitatively the same as in the coupled-channel method. The inclusion of coupling between the channels in these cases leads essentially to changes in the deformation parameters $\beta_{2,3}$ used to construct the imaginary inelastic form factor. The best agreement with experiment is obtained for $\beta_2 = 0.189$ and $\beta_3 = 0.163$; these results correspond closely to those of the phenomenological analysis (see Ref. 5 for example, where the values $\beta_2 = 0.186$ and $\beta_3 = 0.177$ were obtained).

The situation is signifcantly more complicated for the excitation cross section of states with more complicated structure, such as a mixture of one and two-phonon configurations. Here to illustrate the approach, we concentrate on the first excited state 4^+ in ^{58}Ni. Although it has long been known that the 4_1^+ state in ^{58}Ni probably belongs to a two-phonon triplet, constructed from the first two 2^+ phonons, different analyses of the excitation cross section of this state are in rather poor agreement with each other.

For example, in Ref. 27 analysis of the date of ^{58}Ni(α,α') at 34.4 MeV showed that the data for 4_1^+ are described accurately in the distorted-wave approximation, assuming that the state is purely one-phonon, although the possibility of a two-phonon nature of this state was also pointed out. Analysis of the data on ^{58}Ni(p,p') at 40 MeV (Ref. 5) and ^{58}Ni$(^6$Li,^{6}Li$')$ at 71 MeV (Ref. 28) in the framework of the collective vibration model[14] showed that the state 4_1^+ in ^{58}Ni is a mixture of one- and two-phonon states. It was shown in Ref. 28 that the contributions of the one- and two-phonon components in this state are 47% and 53% (in these calculations the one- and two-phonon contributions were introduced phenomenologically by fitting the calculated cross sections to experiment). Finally microscopic[4] and semi-microscopic[29] analyses of the data from ^{58}Ni(p,p') at 27.1 MeV (Ref. 4) show that the one-phonon component is dominant in the state 4_1^+.

The results of our microscopic calculations in the RPA and with the wave functions (5) for different values of the hexadecapole constants $\varkappa_0^{(4)}$ and $\varkappa_1^{(4)}$ are shown in Table I for the first 4^+ state in ^{58}Ni. We see that the inclusion of anharmonocity leads to a strong mixing of the one and two phonon configurations in the structure of the state 4_1^-, and the strongest mixing occurs for the two-phonon state consisting of the first two quadrupole phonons. Of the different sets of $\varkappa^{(4)}$ constants, a good description of the experimental data is given by the results shown in sections 1b and 1c of Table I (the data from inelastic electron scattering were taken from Refs. 24 and 25: $\omega_{4_1^+} = 2.45$ MeV, and $B(E4; 0^+_{g.s.} \to 4_1^+) = (0.684\text{—}1.311) \cdot 10^5\ e^2 \cdot F^8$).

We note that all of the theoretical values of the reduced transition probabilities were calculated with the help of the

TABLE I. Results of microscopic calculations in the QPM for the structure of the first 4^+ state in ^{58}Ni for different sets of the constants $\varkappa_0^{(4)}$, $\varkappa_1^{(4)}$ (1,2,3).

		Structure of the state 4_1^+	$\eta_{4_1^+}$, MeV	$B(E4\uparrow)$, $e^2\cdot F^8$
1	a	$100\%\,[4_1^+]$	3.99	$0.986\cdot10^5$
	b	$41.53\%\,[4_1^+]+58.47\%\,[2_1^+\otimes2_1^+]_{4^+}$	2.33	$1.24\cdot10^5$
	c	$45.57\%\,[4_1^+]+49.08\%\,[2_1^+\otimes2_1^+]_{4^+}$ $+3.04\%\,[2_1^+\otimes2_2^+]_{4^+}+2.31\%\,[2_1^+\otimes4_1^+]_{4^+}$	2.16	$1.326\cdot10^5$
2	a	$100\%\,[4_1^+]$	3.81	$1.833\cdot10^5$
	b	$45.29\%\,[4_1^+]+54.71\%\,[2_1^+\otimes2_1^+]_{4^+}$	2.06	$1.909\cdot10^5$
	c	$48.71\%\,[4_1^+]+45.41\%\,[2_1^+\otimes2_1^+]_{4^+}$ $+3,5\%\,[2_1^+\otimes2_2^+]_{4^+}+2.37\%\,[2_1^+\otimes4_1^+]_{4^+}$	1.86	$2.05\cdot10^5$
3	a	$100\%\,[4_1^+]$	3.61	$2.841\cdot10^5$
	b	$48.54\%\,[4_1^+]+51.46\%\,[2_1^+\otimes2_1^+]_{4^+}$	1.77	$2.622\cdot10^5$
	c	$51.95\%\,[4_1^+]+40.3\%\,[2_1^+\otimes2_1^+]_{4^+}$ $+3.81\%\,[2_1^+\otimes2_2^+]_{4^+}+3.93\%\,[2_1^+\otimes4_1^+]_{4^+}$	1.47	$2.825\cdot10^5$

Note: (a) calculation in the RPA; (b) calculation with the wave function (b) but with the first 2^+ phonons only; (c) calculation with the total wave function (5).

transition densities (13) through (15) according to the equation[2]

$$B(E\lambda; J_i\to J_f)=\frac{2J_f+1}{2J_i+1}\left|\int_0^\infty dr\, r^{\lambda+2}\rho_\lambda^{(p)}(r)\right|^2(e^2\cdot F^{2\lambda}), \qquad (21)$$

where $\rho_\lambda{}^{(p)}(r)$ is the proton part of $\rho_\lambda(r)$.

Next the inelastic form factors $U_{0^+\to2_1^+}$, $U_{0^+\to4_1^+}$, and $U_{2_1^+\to4_1^+}$, constructed with the help of the densities (13) through (15) and the M3Y interaction (19) and (20), were included in the calculation of the three-channel approximation of the coupled-channel method ($0^+_{g.s.}\to2_1^+\to4_1^+$). The results are shown in Fig. 3 for different sets of the constants $\varkappa_0^{(4)}$, $\varkappa_1^{(4)}$, along with the experimental data of Ref. 5 for the excitation cross section of the 4_1^+ state in ^{58}Ni for 40-MeV incident protons. The deformation parameter β_4, extracted from the experimental data of Refs. 24 and 25 for $B(E4\uparrow)_{\text{exp}}$ according to the relation for the Coulomb excitation[2] fluctuates about the value 0.1. The results for the imaginary transition form factor with different versions of the form factors $U_{0^+\to4_1^+}$ and $U_{2_1^+\to4_1^+}$ and with the fixed value $\beta_4=0.1$ are shown in the lower portion of Fig. 3. As can be seen from Fig. 3 and Table I, there is a correlation between the theoretical values of $B(E4\uparrow)$ and the cross sections.

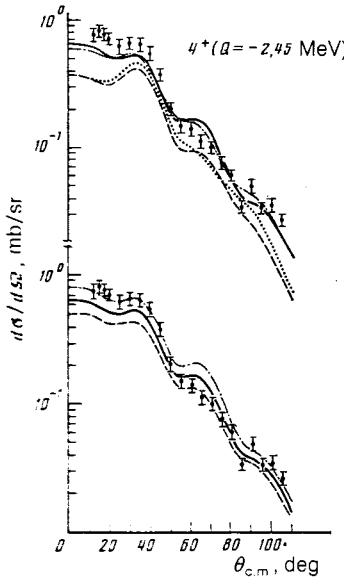

FIG. 3. Experimental data of Ref. 5 and calculated results (different versions; see Sec. 3) of the present work for the inelastic cross section of ^{58}Ni(p,p') at 40 MeV with excitation of the state 4_1^+.

Close agreement with experiment is obtained with the set of constants 2*b* of Table I (solid curve, $\varkappa^2=5.36$). Here

$$\chi^2=\frac{1}{N_\sigma}\sum_{i=1}^{N_\sigma}\left[\frac{\sigma_{\text{Theor}}(\theta_i)-\sigma_{\text{exp}}(\theta_i)}{\Delta\sigma_{\text{exp}}(\theta_i)}\right]^2, \qquad (22)$$

where σ_{theor} and σ_{exp} are the theoretical and experimental cross sections, N_σ is the number of measured cross sections, and $\Delta\sigma_{\text{exp}}$ is the experimental error in σ_{exp}. Calculations with the constants 1b (dashed curve, $\varkappa^2=9.12$) and 3*b* (dash-dotted curve, $\varkappa^2=10.93$) somewhat underestimate and overestimate the experimental data, respectively. However the diffraction pattern is identical for all three of the calculated cross sections. This is explained by the identical radial dependence for the densities, which is determined by the reduced single-particle matrix elements (12a). For the same values of the matrix elements $\langle j_1\|\mathbf{T}_\lambda\|j_2\rangle$ the collectivity, or transition strength, can increase or decrease as the result of a corresponding increase or decrease of the phonon amplitudes ψ and φ. This also explains the observed correlation between the values of $B(E\lambda)$ and the corresponding excitation cross sections.

Next (see the upper portion of Fig. 3) the value β_4 was varied about a value 0.1 according tothe fitting procedure of the program ECIS. Among the versions of the calculation considered here, the best agreement with experiment for the transition densities (14) and (15) was obtained with ver-

sions 1b (dash-dotted curve, $\varkappa^2 = 5.16, \beta_4 = 0.118$) and 2b (solid curve, $\varkappa^2 = 5.34, \beta_4 = 0.099$).

The values for β_4 obtained here, and used in the construction of the imaginary transition form factor, are in agreement with the results of different analyses of the excitation cross section of the state 4_1^+ in ^{58}Ni; these values vary from 0.054 to 0.125 (Refs. 4 and 27 through 31). We note that analysis of the same data[5] with the collective vibration model[14] to second order reproduces the angular dependence and the absolute value of the cross section ($\varkappa^2 = 7$) less accurately than our calculations, particularly for small angles.

Calculations using the total wave function (5) for the structure of the state 4_1^+ lead to an insignificant change in the calculated cross section. For example, calculation with version 2c gives about the same agreement with experiment ($\varkappa^2 = 5.6, \beta_4 = 0.095$) as version 2b (these two curves are not distinguished on the graph). This is also observed for calculation with versions 1b and 1c and shows the dominant contribution of 2_1^+ phonons in the formation of the two-phonon impurity of the state 4_1^+. Finally we note that calculations in the RPA for the state 4_1^+ give results (see the upper portion of Fig. 3) which agree poorly with experiment. For example, version 1a corresponds to the dotted curve ($\varkappa^2 = 15.91, \beta_4 = 0.118$), and version 2a corresponds to the dashed curve ($\varkappa^2 = 19.4, \beta_4 = 0.099$).

Hence both the structure calculations of the QPM and the microscopic analysis of the excitation cross sections of the state 4_1^+ in ^{58}Ni show that this state contains a strong two-phonon impurity. The best agreement with the data on ^{58}Ni(p,p') at 40 MeV is given by the constants $(\varkappa_0^{(4)}, \varkappa_1^{(4)})$, which reproduce the energy and transition probability: $\eta 4_1^+ = (2.33—2.06)$ MeV, $B(E4;\ 0_{g.s.}^+ \to 4_1^+) = (1.24\text{-}1.91)\cdot 10^5\, e^2\cdot F^8$. (These values also agree with the present experimental data[24,25] for the state 4_1^+ in ^{58}Ni).

4. CONCLUSION

In the framework of the QPM we have developed an approach to the microscopic analysis of the scattering of low-energy protons by spherical nuclei. Using the example of ^{58}Ni, it is shown that both the one-phonon states and also the excitation of states of more complicated structure are described accurately in the QPM. The calculated nuclear transition densities for these states are used successfully in the microscopic analysis of the excitation cross sections. From all of the analysis ^{58}Ni one concludes that, unlike the 2_1^+ and 3_1^- states, the first 4_1^+ state is a mixture of one- and two-phonon configurations; the one- and two-phonon components are 42–49% and 58–51%, respectively.

There is great interest in the further application of this approach to the theoretical study of one- and two-phonon excitations in various nuclei induced by the scattering of low-energy protons, and in the study of the effect of the Pauli principle on the structure of the excited states and of the corresponding excitation cross sections. A more exact treatment of the antisymmetrization between the incident proton and the bound nucleons and between the bound nucleons themselves would be required in the study of these effects.

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