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Effect of a two-phonon admixture on the *M*1 resonance in spherical nuclei

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The effect of a two-phonon admixture on the properties of the $M1$ resonance in spherical nuclei is studied for the mass-number range $60 \leq A \leq 140$. The calculations are based on the quasiparticle–phonon model with factorized multipole and spin–multipole forces. The two-phonon admixture is of minor importance in nuclei with 50 and 82 neutrons; in the other nuclei, especially in those with well expressed pairing in both the proton and neutron systems, this admixture is impotant. The radiative strength functions at the neutron binding energy are calculated. The results are compared with experimental data and calculations by other workers.

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1. INTRODUCTION

The interaction with two-phonon states seems to be generally accepted as playing an important role in shaping the structure of the giant resonances in intermediate and heavy spherical nuclei, in particular, in determining the width of these resonances.[1-5] Quantitative calculations for these effects, however, can be carried out only on the basis of relatively simple nuclear models, and in fact such calculations have been carried out for only a few nuclei. Calculations for doubly magic nuclei were carried out in Refs. 3 and 4, and calculations for nuclei with well-expressed pairing were carried out in Refs. 1 and 5 on the basis of a quasiparticle–phonon model. Recent work has focused primarily on the electric resonances. Calculations have been carried out for the $M1$ resonances in ^{140}Ce, ^{126}Te, and Ba isotopes in Ref. 6 in connection with a study of the behavior of the $M1$ radiative strength functions of these nuclei. The effect of a two-phonon admixture on the $M1$) resonance itself has not been studied, but we believe that this effect is pertinent to many nuclei, especially since the simplicity of the $M1$-resonance structure and its relatively low excitation energy permit calculations with a minimum of simplifying assumptions. More attention has been drawn to this question by experimental data reported recently on the $M1$ resonance (more precisely, on the absence of this resonance) at Darmstadt.[7]

The Darmstadt group[7] carried out careful experiments on the scattering of slow electrons by nuclei, and only in ^{58}Ni were they able to detect 60% of the strength of the $M1$ transitions predicted by the shell model; there was essentially no evidence of the $M1$ resonance in either ^{90}Zr or even ^{208}Pb, where the existence of this resonance has been taken to be a solidly established fact. On the other hand, all the calculations in the random-phase approximation[4,8-11] predict an $M1$ resonance. Two factors may be responsible for the fact that the resonant $M1$ transitions are far less prominent than predicted theoretically in the random-phase approximation. First, there is the interaction with more complex configurations, primarily two-phonon configurations; this interaction may spread the strength of the $M1$ transitions over a substantial interval of excitation energies. The second factor, predicted by Knüpfer *et al.*,[7] is a possible decrease in the effective gyromagnetic ratios g_s^{eff} with increasing mass number A; the result would be a preferential suppression of the $M1$ resonance in heavy nuclei. At the excitation energies corresponding to the $M1$ resonance, the continuous spectrum cannot have a significant effect.

2. CHOICE OF PARAMETERS FOR THE MODEL

In the present calculations we find the energy and probability for the excitation of the $M1$ resonance in $^{58,60}Ni$, ^{90}Zr, $^{118,120}Sn$, $^{124,126}Te$, ^{138}Ba, and ^{140}Ce. The calculations are based on the semimicroscopic quasiparticle—phonon model, which is described in detail in Ref. 12. The model wave function for the 1^+ state incorporates single-phonon and two-phonon components:

$$\Psi_\nu(JM)=\Big\{\sum_i R_\nu(Ji)Q^+_{JMi}+\sum_{\substack{\lambda_1\lambda_2\\ i_1 i_2}} P^{\lambda_2 i_2}_{\lambda_1 i_1}(J\nu)\,[Q^+_{\lambda_1\mu_1 i_1}Q^+_{\lambda_2\mu_2 i_2}]_{JM}\Big\}\Psi_0, \quad (1)$$

where $Q^+_{\lambda\mu i}$ is the creation operator for a phonon with angular momentum λ angular-momentum projection μ, and an index i; Ψ_0 is the wave function corresponding to the ground state of the even–even nucleus (the phonon vacuum). To find the energies of the $\Psi_\nu(JM)$ states and the coefficients $R_\nu(Ji)$ and $P^{\lambda_2 i_2}_{\lambda_1 i_1}(J\nu)$, it is necessary to solve a rather complicated nonlinear equation.[13] As before,[1,5,6] however, we will carry out a direct calculation of the strength function of the $M1$ transitions, $b(M1,\eta)$:

$$b(M1,\eta)=\frac{1}{2\pi}\sum_\nu \frac{\Delta}{(\eta-\eta_\nu)^2+\Delta^2/4}B(M1,0^+\to 1^+_\nu). \quad (2)$$

Here

$$B(M1,0^+\to 1^+_\nu)=\Big|\sum_i R_\nu(Ji)\langle\Psi_0|\mathfrak{M}(M1)|1^+_i\rangle\Big|^2.$$

The matrix element of the operator representing the magnetic dipole moment between the ground and the 1^+ i-th single-phonon state is calculated in the random-phase approximation. As has been demonstrated in several places,[1,12] $b(M1,\eta)$ can be calculated directly, without solving complicated equations.

The nuclear Hamiltonian in the quasiparticle–phonon model consists of the average field, chosen in the form of the Woods–Saxon potential; a superfluid pairing interaction; and a long-range force which generates phonon excitations in the nucleus. This long-range force is written in a factorized form, and the calculations are thereby simplified considerably. Phonons with $I,\pi=\lambda,\ (-1)^\lambda$ are generated by multipole forces, while phonons with $I,\pi=\lambda,\ (-1)^{\lambda+1}$ are generated by spin–multipole forces. The parameters of the single-particle potential, chosen on the basis of Ref. 14, are listed in Table I. The single-particle energies, the wave functions, and the matrix elements of the multipole and spin–multipole operators are calculated by the REDMEL program, which implements the numerical method proposed in Ref. 15. for solving the spherical by symmetric Schrödinger equation. As a result, the single-particle spectra used in the present calculations differ from those found in Ref. 16 through a semianalytic solution of the Schrödinger equation.[17] The new spectrum has a slightly higher density of single-particle levels; more importantly, the wave functions of the quasistationary states exhibit the correct asymptotic behavior. The constants of the pairing forces G_N and G_Z are determined by the standard procedure[18] from the experimental pairing energies. The values of $G_{N,Z}$ are also listed in Table I. It is a more difficult matter to correctly choose the constants for the long-range forces: the isoscalar constant $\varkappa_0^{(\lambda)}$ and the isovector constant $\varkappa_1^{(\lambda)}$. For dipole forces, they are determined from the position of the giant dipole resonance in each nucleus and from the condition $\omega_{11}=0$, so that the effect of the ghost state due to violation of translational invariance can be eliminated very accurately.[19] The constants of the quadrupole and octupole interactions are chosen such that the theoretical energies of the 2_1^+ and 3_1^- levels, calculated for the case of a two-phonon admixture in their structure, agree with the experimental values.[16] The multipole constants, for $\lambda>3$, are found by dividing the estimates in Ref. 20 by a factor of 1.5. With these values of the constants, no low-lying collective states with angular momentum greater than 3 appear. The isovector constants $\varkappa_1^{(\lambda)}$ are calculated from the isoscalar constants $\varkappa_0^{(\lambda)}$ through the relation

TABLE I. Parameters of the Woods–Saxon potential and constants of the pairing interaction.

A	NZ	r_0, fm	V_0, MeV	$\varkappa$, fm^2	α, fm^{-1}	$G_{N,Z}$, MeV
59	$N=31$	1.31	46.2	0.413	1.613	0.280
	$Z=27$	1.24	53.7	0.308	1.587	0.302
91	$N=53$	1.29	44.7	0.413	1.613	0.168
	$Z=39$	1.24	56.9	0.338	1.587	0.194
121	$N=71$	1.28	43.2	0.413	1.613	0.122
	$Z=51$	1.24	59.9	0.346	1.587	0.136
127	$N=73$	1.28	43.4	0.413	1.613	0.124
	$Z=53$	1.24	59.7	0.350	1.587	0.130
141	$N=83$	1.27	46.0	0.413	1.613	0.116
	$Z=59$	1.24	57.7	0.349	1.587	0.122

$$\varkappa_1=-\varkappa_0^{(\lambda)}\cdot 0.2(2\lambda+3). \quad (3)$$

Equation (3) is the renormalized version of Eq. (6.127) of Ref. 20. This equation yields a value for the quadrupole isovector constant $\varkappa_1^{(2)}$ such that the position of the $E2$ resonance with $T=1$ agrees with the experimental position. This value is also approximately equal to the estimates of the hydrodynamic model. The spin–spin interaction constants were studied in Ref. 8, where it was found that the values

$$\varkappa_0=0,\quad \varkappa_1=-4\pi\cdot 28/A\ \text{MeV} \quad (4)$$

lead to a prediction of the position of the $M1$ resonance in the random-phase approximation which is in basically satisfactory agreement with the few experimental results available. These values for the constants also agree with the data reported by the Copenhagen group.[21] For some nuclei, calculations have been carried out for values of $\varkappa_0$ and $\varkappa_1$ other than those in (4). For the spin–multipole forces with $\lambda>1$ the following values have been adopted:

$$\varkappa_0^{(\lambda)}=0,\quad \varkappa_1^{(\lambda)}=-28\cdot 4\pi/A\langle r^{\lambda-1}\rangle^2\ \text{MeV}\cdot\text{fm}^{-2\lambda+2}.$$

The effective gyromagnetic ratios were also taken from Ref. 8: $g_s^{eff}=0.8g_s^{free}$; $g_l^n=0.0, g^p=1.0$. We assign the parameter Δ the value 0.1 MeV; in most cases this value makes it possible to distinguish the states with large values of $B(M1)$ and does not lead to a further broadening of the $M1$ resonance.

Since the single-phonon excitations can be both collective and two-quasiparticle in nature, there are components in the two-phonon part of the wave function in (1) which violate the Pauli principle. To suppress this effect, we impose a further requirement on the two-phonon part of the wave function in (1): One of the phonons, $Q^+_{\lambda_1\mu_1i_1}$ or $Q^+_{\lambda_2\mu_2i_2}$, must be collective. This approach is of course not completely systematic, but to systematically take the Pauli principle into account in terms of boson operators would seriously complicate the problem.

3. THE $M1$ RESONANCE IN SPHERICAL NUCLEI WITH $60 < A < 130$

A. The $M1$ resonance in ^{90}Zr

In terms of the general effect of the two-phonon admixture on the $M1$ resonance, the nuclei treated in these calculations can be divided into two groups: first, the nuclei with a magic number of neutrons, in which the effect of the interaction of the quasiparticles with phonons on the $M1$ resonance is small; second, all the other nuclei (Ni, Sn, Te), in which this interaction significantly affects the strength distribution of the $M1$ transitions in the resonance region. Included in the first group is ^{90}Zr, whose $M1$ resonance has been studied by many workers.[4,8,10] we will accordingly begin the discussion of the present results with this nucleus. Figure 1 shows the $M1$ resonances in ^{90}Zr calculated in the random-phase approximation in the present calculations (a-c) and by other workers[4,10] (d-f). The present calculations are shown for three values of the constant $\varkappa_0$: a) $\varkappa_0=0$; b) $\varkappa_0=0.5\varkappa_1$; c) $\varkappa_0=0.9\varkappa_1$. As $|\varkappa_0|$ increases, the states with the largest values of $B(M1)$ move closer together, primarily because of an increase in the excitation energy of the lowest of these states. The results shown in Figs. 1d–1f were calculated from the theory of finite Fermi systems. Figure 1d is taken from Ref. 4, where the pairing interaction in the proton system is ignored, so that the $M1$ resonance is represented by a single neutron particle—hole state, the $1g_{7/2}$-$1g_{9/2}$ state. The two other figures (1e and 1f) show the results of the calculations of Ref. 10 for two values of the "dimension-eliminating" factor $(dn/d\varepsilon_F)^{-1}$, which appears in the expression for the effective particle-hole potential. We have presented these results in order to demonstrate that different semimicroscopic models in the random-phase approximation lead to approximately equal results for the $M1$ resonance, if the strength parameters are chosen appropriately. The differences in the integrated probability for the excitation of the $M1$ resonance according to different workers are due primarily to different values of g_s^{eff}. The interaction with two-phonon states has only a slight effect on the $M1$ resonance in ^{90}Zr. As can be seen from Fig. 2a, which shows the function $b(M1,\eta)$ in the nucleus ^{90}Zr for the case $\varkappa_0=0.9\varkappa_1$, the position of the $M1$ resonance remains the same; the only change is a slight decrease in its center of gravity. In the calculations incorporating the interaction of the $1p$-$1h$ and $2p$-$2h$ configurations[4] (Fig. 2b), the $2p$-$2h$ states have a far greater effect on the $M1$ resonance. The single 1^+ state which arises in the random-phase approximation (Fig. 1d) decays into three states which span an interval $\Delta E\simeq 2.5$ MeV. In addition, a weakly defined state with $E\simeq 15.5$ MeV appears. We would like to point out, however, that the "final" forms of the $M1$ resonance are approximately the same in Ref. 4 and the present calculations. At any rate, neither the present results nor those of Ref. 4 permit the "disappearance" of a large part of the strength of the $M1$ transitions in ^{90}Zr—the disappearance observed (?) in Ref. 7—to be explained on the basis of an interaction with two-phonon configurations. The two-phonon admixtures have an equally slight effect on the structure of the $M1$ resonance in other nuclei which have the magic number of neutrons.

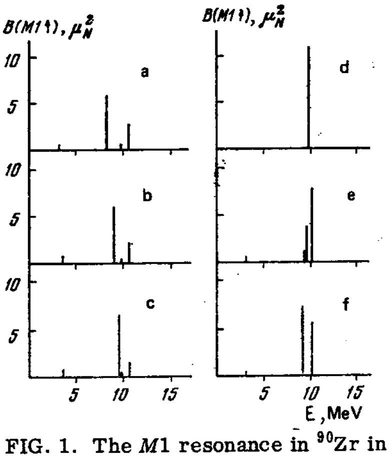

FIG. 1. The $M1$ resonance in ^{90}Zr in the random-phase approximation. a–c) Present calculations, with the effective $\sigma\sigma$ strengths $\varkappa_1=-28\cdot 4\pi/A$ MeV and $\varkappa_0=0$ (a), $\varkappa_0=0.5\varkappa_1$ (b), $\varkappa_0=0.9\varkappa_1$ (c). d–f) Calculations from the theory of finite Fermi systems in Refs. 4 (d) and 9 (e and f).

B. The $M1$ resonance in ^{58}Ni

Extremely reliable experimental data on the $M1$ resonance have been obtained[22] for ^{58}Ni. Experiments on inelastic electron scattering by this nucleus have revealed two groups of states, with excitation energies of 6.0–8.5 MeV ($\sum B(M1\uparrow)\simeq 2.7\mu_N^2$) and 9.8–11 MeV ($\sum B(M1\uparrow)\simeq 5.7\mu_N^2$), which can be tentatively identified as levels with $I^\pi=1^+$, although the group with the lower excitation energies apparently contains both 1^+ and 2^- levels. Calculations for the $M1$ resonance in ^{58}Ni with isovector $\sigma\sigma$ forces in the random-phase approximation (Fig. 3a; see also Ref. 8) predict a single 1^+ level with $\omega=10$ MeV and $B(M1\uparrow)=10\mu_N^2$ (with $g_s^{eff}=0.8g_s^{free}$). The incorporation of isoscalar $\sigma\sigma$ forces leads to a splitting of this state, with a separation of 200 keV between

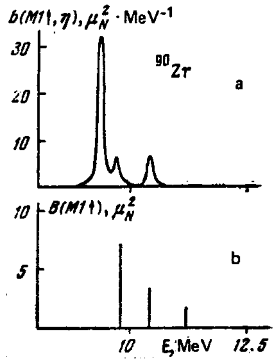

FIG. 2. The $M1$ resonance in ^{90}Zr, with the interaction in two-phonon states. a) Present calculations, $\varkappa_0=0.9\varkappa_1$; b) calculations of Ref. 4.

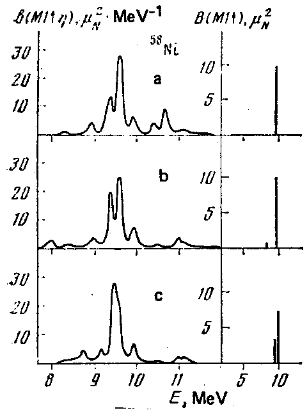

FIG. 3. The $M1$ resonance in ^{58}Ni. Right) Calculations in the random-phase approximation; left) the strength function $b(M1, \eta)$ incorporating the interaction with two-phonon states. $\varkappa_1=4\pi\cdot28/A$ MeV, $\varkappa_0=0$ (a), $\varkappa_0=0.5\varkappa_1$ (b), or $\varkappa_0=0.9\varkappa_1$ (c).

the two components (Fig. 3c); the resultant value of $B(M1)$ is the same. The interaction with two-phonon states significantly affects the $M1$ resonance in ^{58}Ni, leading to a spread of the strength of the one single-phonon state with a large value of $B(M1)$ over an interval of $\simeq$2MeV. The maximum of the function $b(M1, \eta)$ turns out to be 400 keV lower than for the 1$^+$ single-phonon state itself (Fig. 3a). The position of the maximum of $b(M1, \eta)$ is essentially independent of $|\varkappa_0|$, as can be seen from Figs. 3b and 3c, but the distribution of the strength of the $M1$ transitions in the resonance region changes considerably. For the values of the constants in (4), the maximum of $b(M1, \eta)$ is 600 keV below the experimentally observed group of 1$^+$ states with strong $M1$ transitions,[22] and the integrated excitation probability $B(M1\uparrow)$ over an interval $\Delta E=1$ MeV symmetric with respect to the $b(M1, \eta)$ maximum is $7.8\,\mu_N^2$. The position of the $b(M1, \eta)$ maximum can be brought into coincidence with the experimental energy by renormalizing the constant $\varkappa_1$. As can be seen from Figs. 4a–4c, the shape of the $M1$ resonance changes upon an increase in $|\varkappa_1|$ ($\varkappa_0=0$). The best agreement with the experimental excitation energy is reached at the value $\varkappa_1=-37\cdot4\pi/A$ MeV. As can be seen from

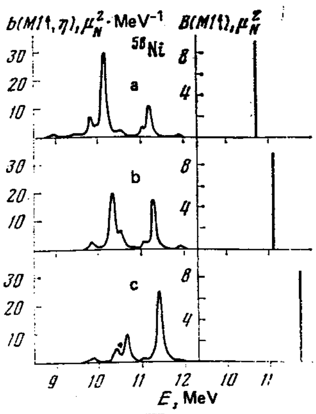

FIG. 4. The $M1$ resonance in ^{58}Ni. Right) Calculations in the random-phase approximation; left) the strength function $b(M1, \eta)$ incorporating the interaction with two-phonon states. $\varkappa_0=0$, $\varkappa_1=-37\cdot4\pi/A$ MeV (a), $\varkappa_1=-42\cdot4\pi/A$ MeV (b), or $\varkappa_1=-50\cdot4\pi/A$ MeV (c).

Fig. 4, an increase in $|\varkappa_1|$ leads to not only an increase in the energy of the main peak of the $M1$ resonance but also to a decrease in the height of this peak. As a result, for the value $\varkappa_1=-37\cdot4\pi/A$ MeV over an interval $\Delta E=1$ MeV centered at the $b(M1, \eta)$ maximum the resultant value of $B(M1)$ is $5.9\,\mu_N^2$, in good agreement with the experimental value. For no values of $\varkappa_0$ and $\varkappa_1$, however, are we able to find a 1$^+$ state in the range 6–8 MeV with a value of $B(M1)$ near the experimental value. The one 1$^+$ single-phonon state in this region has the value $B(M1)\simeq0.5\,\mu_N^2$, and the interaction with two-phonon states does not affect this value. An increase in $B(M1)$ for the states in this region occurs only in the heavier Ni isotopes, because of an enhancement of the $M1$ transition with the two-quasiparticle 1$^+$ neutron state, $2p_{3/2}-2p_{1/2}$, in these nuclei.[1] This result is evidence in favor of the assumption that the lower group of states in ^{58}Ni consists primarily of states with $I^\pi=2^-$. For more reliable conclusions, the spectrum of 2$^-$ states in this range of excitation energies should be studied.

C. The $M1$ resonance in ^{118}Sn and ^{124}Te

Figures 3a–3c show the variation of the shape of the $M1$ resonance in ^{58}Ni with the ratio of the constants $\varkappa_0$ and $\varkappa_1$. The situation is even more interesting in those cases in which the $M1$ resonance in the single-phonon approximation is formed by several states with slightly different values of $B(M1\uparrow)$. As examples, we show in Figs. 5a–5c and Fig. 6 the $M1$ resonance in ^{118}Sn and ^{124}Te. In ^{118}Sn (Fig. 5), the $M1$ resonance is shown for three values of $\varkappa_0$: a) $\varkappa_0=0$; b) $\varkappa_0=0.5\varkappa_1$; c) $\varkappa_0=0.9\varkappa_1$. As $|\varkappa_0|$ increases, the two strongest single-phonon 1$^+$ states move closer together, and there is a simultaneous decrease in $B(M1)$ for the state with the higher excitation energy (the right side of Figs. 5a–5c). Because of the interaction with the two-phonon states, the rate at which the two $b(M1, \eta)$ peaks are moving toward each other decreases sharply, and the integrated values of $B(M1)$, which are concentrated at these peaks, become more uniform. There is a "pumping" of the strength of the $M1$ transitions in the region ~1.5 MeV, which is occupied by the $M1$ resonance. In ^{124}Te (Fig. 6), the $M1$ resonance is represented in the random-phase approximation by three 1$^+$ states with approximately equal values of $B(M1\uparrow)$. The primary effect of

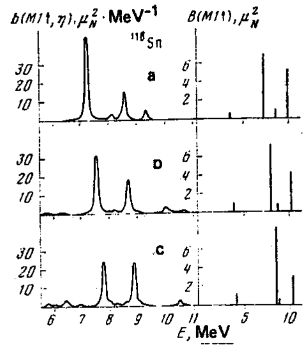

FIG. 5. The $M1$ resonance in ^{118}Sn. Right) Calculations in the random-phase approximation; left) the strength function $b(M1, \eta)$ incorporating the interaction with two-phonon states. $\varkappa_1=-4\pi\cdot28/A$ MeV, $\varkappa_0=0$ (a), $\varkappa_0=0.5\varkappa_1$ (b), or $\varkappa_0=0.9\varkappa_1$ (c).

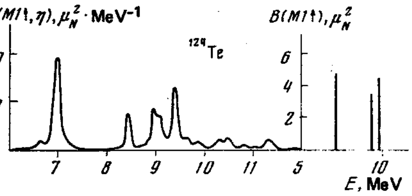

FIG. 6. The $M1$ resonance in ^{124}Te. Right) Calculation in the random-phase approximation; left) the strength function $b(M1, \eta)$ incorporating the interaction with two-phonon states. $\varkappa_0 = 0$, $\varkappa_1 = -4\pi \cdot 28/A$ MeV.

the two-phonon admixture is on states with relatively high excitation energy, which become severely fragmented. The corresponding strength of the $M1$ transitions is distributed over an interval of more than 2 MeV. The state at 7 MeV exhibits essentially no change in either position or value of $B(M1)$, apparently because of the far higher density of 1^+ two-phonon states at 10 MeV.

4. RADIATIVE $M1$ STRENGTH FUNCTIONS

Yet another source of information on the distributions of the strength of the $M1$ transitions in nuclear spectra is the radiative strength function $k(M1)$. At energies near the neutron binding energy B_n, the value of $k(M1)$ is found from data on (n, γ) and (γ, n) reactions. At other energies, this strength function is determined by studying, for example, the elastic scattering of γ rays by nuclei, even though the values of $k(M1)$ found in the (γ, γ') reaction are higher than those found from the (n, γ) or (γ, n) data, because of the mechanism for inelastic scattering of γ rays by nuclei; this mechanism singles out the 1^+ states with the highest values of $B(M1)$ in a narrow energy interval. A systematic classification of the experimental data on $k(M1)$ in various nuclei[23] yields an average value $\langle k(M1) \rangle \approx (15-20) \cdot 10^{-9}$ MeV^{-3}, but this value is much higher than the single-particle estimates. The sharp increase in $\langle k(M1) \rangle$ observed experimentally[24,25] in nuclei with $A \simeq 140$ can be attributed tentatively to the proximity of the $M1$ resonance to B_n in these nuclei. This assumption has been confirmed by theoretical calculations based on the quasiparticle–phonon model.[6] We emphasize that a systematic microscopic calculation of $k(M1)$ in spherical nuclei would be possible only by taking into account the fragmentation of the single-phonon states over the two-phonon states because of the very low density of 1^+ single-phonon levels and the large differences in the values of $B(M1)$ among these levels.

Knowing the strength function for the $M1$ transitions, $b(M1, \eta)$, we can calculate $\langle k(M1) \rangle$ extremely easily, using the following equations:

$$\langle k(M1) \rangle = \sum_{\nu \in \Delta E} \Gamma_{\gamma_0}(M1, \eta_\nu)/E_\gamma^3 \Delta E,$$

$$\sum_{\nu \in \Delta E} \Gamma_{\gamma_0}(M1, \eta_\nu) = 3.76 \cdot 10^{-3} \int_{E-\Delta E/2}^{E+\Delta E/2} \eta^3 b(M1, \eta)\, d\eta \quad \text{(eV)}.$$

Here $\Gamma_{\gamma 0}$ is in electron volts, and E_γ and ΔE are in MeV. The values calculated for $\langle k(M1) \rangle$ for most of the nuclei studied in these calculations are listed in Table II. In

TABLE II. The $M1$ radiative strength functions.

Nucleus	E_γ, MeV	$\langle k(M1) \rangle \cdot 10^9$ MeV^{-3}			
		Experimental	Theoretical		
			$\varkappa_0 = 0$	$\varkappa_0 = 0.5\,\varkappa_1$	$\varkappa_0 = 0.9\,\varkappa_1$
^{118}Sn	9.33	–	4.5	2.6	9.5
^{120}Sn	9.11	–	8.9	9.5	17.8
^{124}Te	9.41	–	18.2	14.9	11.3
^{126}Te	7.915	39	3.0	18.0	24.0
	9.09	–	30	22	15
^{138}Ba	8.54	90±35	44	37	5.6
^{140}Ce	9.1	37	7.7	35	31

these calculations we use the value of $\varkappa_1$ calculated from Eq. (4) for three values of $\varkappa_0$: $\varkappa_0 = 0$, $\varkappa_0 = 0.5\varkappa_1$, $\varkappa_0 = 0.9\varkappa_1$. The averaging interval is $\Delta E = 1$ MeV (see Ref. 6 for a discussion of how the results depend on ΔE). For those nuclei for which no experimental data are available (and this means most of the nuclei), we give the values of $\langle k(M1) \rangle$ at B_n. It should be noted that the present results differ from those of Ref. 6 in the cases of ^{140}Ce and ^{138}Ba. The reason for the difference is the slight difference in the single-particle schemes and the associated superfluid parameters. The $M1$ resonance in ^{140}Ce is slightly fragmented and is also very close to B_n, so that a slight change in its position with respect to B_n leads to a large change in $\langle k(M1) \rangle$. With $\varkappa_0 = 0$, for example, the center of gravity of the $M1$ resonance in this nucleus is at 8.4 MeV (in comparison with the experimental value of 8.7 MeV; Ref. 26), so that the value of $\langle k(M1) \rangle$ is lower than the experimental value. An increase in $|\varkappa_0|$ leads to an increase in the energy of the resonance and to an increase in $\langle k(M1) \rangle$. The same effects are observed in ^{140}Ce as $|\varkappa_1|$ is increased. The experimental and theoretical values of the resonance energy agree for the value $\varkappa_1 = -34 \cdot 4\pi/A$ MeV; here $\langle k(M1) \rangle \simeq 25 \cdot 10^{-9}$ MeV^{-3}. It should be noted that the value of $B(M1\uparrow)$ for the $M1$ resonance in ^{140}Ce found in the present calculations is significantly lower than the experimental value (although the experimental error is ~50%; Ref. 26), so that the value of $\langle k(M1) \rangle$ is also too low. The apparent reason for the large experimental value of $\langle k(M1) \rangle$ at the energy of 7.915 MeV in ^{126}Te is that this value is obtained in a (γ, γ') reaction.[27] An increase in $\varkappa_1$ in this nucleus cannot significantly change the theoretical value of $\langle k(M1) \rangle$, which is governed primarily by the distribution of the strength of $M1$ transitions in a rather broad vicinity ($\simeq 2$ MeV) of the resonance. Despite the approximately equal values of B_n and the energies of the $M1$ resonance in the Sn and Te isotopes, the radiative strength function is lower in the Sn isotopes. This effect results from a weaker fragmentation of the $M1$ resonance in the tin isotopes. Since the $M1$ resonance for ^{58,60}Ni and ^{90}Zr is 2–3 MeV below B_n, the values of $\langle k(M1) \rangle$ at B_n are small, and we have not listed them in the table.

5. CONCLUSION

The results of these calculations show that in many nuclei a two-phonon admixture has an important effect on the strength distribution of the $M1$ transitions near the magnetic dipole resonance, although this effect is not so strong that we could say that the resonance "disappears" or is completely smoothed over. Despite its simplicity, the separable $\sigma\sigma$ interaction leads to a completely satisfactory description of the experimental data, if the constants of the interaction are chosen appropriately. The drawback of these calculations, as in all calculations based on models with effective forces, is some uncertainty in the choice of parameters (the strength constants, the effective charges, etc.). For the constant for the isovector $\sigma\sigma$ forces, for example, we should apparently use the value $\varkappa_1 = 35 \cdot 4\pi/A$ MeV, rather than the value proposed in Ref. 8 [see Eq. (4)] on the basis of calculations in the random-phase approximation. The choice of $\varkappa_0$ is essentially undetermined. The values which we use for the effective gyromagnetic ratios lead to a satisfactory description of $B(M1\uparrow)$ in ^{58}Ni, but the values are too low for $B(M1\uparrow)$ in ^{140}Ce (this result is characteristic of other calculation also[9,10]). The very large experimental value of $B(M1\uparrow)$ for the $M1$ resonance in ^{140}Ce, which can be reproduced in the calculations only by assuming $g_s^{\text{eff}} \simeq g_s^{\text{free}}$, contradicts the suggestion[7] that the effective g_s ratios decrease with increasing A. A calculation of the radiative strength functions with the same parameters as for the $M1$ resonances leads to a satisfactory agreement with experiment. Again we emphasize that for these quantities the effect of the two-phonon states is of fundamental importance.

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1) The state $(2p_{1/2}, 2p_{3/2})^N_{1^+}$, which is at ~6.5 MeV in the Ni isotopes, becomes a particle–hole state beginning at ^{60}Ni, and this change leads to the enhancement of $B(M1)$.

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