E2 decay strength of the M1 scissors mode of $^{156}$Gd and its first excited rotational state

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The E2/M1 multipole mixing ratio $\delta_{1,2}$ of the $1^+_1 \rightarrow 2^+_1$ γ-ray decay in $^{156}$Gd and hence the isovector E2 transition rate of the scissors mode of a well-deformed rotational nucleus has been measured for the first time. It has been obtained from the angular distribution of an artificial quasimonochromatic linearly polarized γ-ray beam of energy 3.07(6) MeV scattered inelastically off an isotopically highly enriched $^{156}$Gd target. The data yield first direct support for the deformation dependence of effective proton and neutron quadrupole boson charges in the framework of algebraic nuclear models. First evidence for a low-lying $J^p = 2^+$ member of the rotational band of states on top of the $1^+$ band head is obtained, too, indicating a significant signature splitting in the $K = 1$ scissors mode rotational band.

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Introduction.—Orbital out-of-phase oscillations of a coupled two-component many-body quantum system are generally called scissors modes (SMs). A SM has been discovered in deformed atomic nuclei [1]. It has later been identified in Bose-Einstein condensed gases [2,3] and is expected to occur in Fermi gases [4], in metallic clusters [5–7], and in deformed quantum dots [8]. SMs are interesting quantum modes because their properties are sensitive to the restoring forces between the many-body subsystems. They inevitably break spherical symmetry and hence lead to a sequence of quantum states of the many-body system that form a rotational band.

The isovector low-lying $J^p = 1^+$ SM of deformed even-even nuclei is the most prominent example for a SM. Its occurrence has been conjectured in 1978 by Lo Iudice and Palumbo [9] in the framework of the semiclassical two-rotor model of coupled quadrupole-deformed proton and neutron subsystems. In the framework of the algebraic interacting boson model (IBM-2) [10] it was explicitly considered as a valence-shell mode by Iachello [11], who identified it as one example of an entire class [12] of nuclear valence-shell excitations with nontrivial symmetry properties with respect to the two coupled subsystems. Within the IBM-2 the proton-neutron symmetry of a wave function is quantified by the $F$-spin quantum number $F = F_{\text{max}} - 1$, where $F_{\text{max}}$ is given by half of the number of proton and neutron bosons $N = N_p + N_n$. We address states with $F = F_{\text{max}} - 1$ in this context as “isovector valence-shell excitations.”

While the nuclear SM occurs due to the quadrupole deformation of the proton and neutron subsystems, its signature is the electromagnetic coupling to the ground-state band via strong magnetic dipole ($M1$) transitions caused by the predominant isovector character of its decay transitions to low-energy nuclear states with proton-neutron symmetry. Indeed, the SM has been discovered [1] in the quadrupole-deformed nucleus $^{156}$Gd in inelastic electron-scattering experiments at Darmstadt. It manifested itself as an accumulation of $M1$ excitation strength concentrated in a few $J^p = 1^+$ states at excitation energies around 3 MeV. The SM of deformed nuclei has been studied extensively in inelastic electron-scattering ($e,e'$), photon scattering ($\gamma,\gamma'$), and neutron-scattering ($n,n'$) experiments [13,14 and references therein]. The observed correlation of the total $M1$ strength of the SM to the size of the nuclear quadrupole-deformation parameter [15–18] has proven the quadrupole-collective nature of the nuclear SM. Despite its quadrupole-collective origin, the electric quadrupole-decay ($E2$) properties of the SM are, however, still unknown. Consequently, the predicted [19] deformation dependence of effective quadrupole charges in the IBM-2 has remained an open question. The SM is expected to form a rotational band of states with spin and parity quantum numbers.
$J^p = 1^+, 2^+, 3^+$, etc. Evidence for an $E2$ excitation at an excitation energy about 21(5) keV above the dominant $1^+$ state of the SM of $^{156}$Gd has been reported from ($e$, $e'$) experiments [20]. However, the data did not allow for establishing the corresponding $2^+$ state as a member of the rotational band built on top of the $1^+$ band head of the SM.

It is the purpose of this Letter to present first data on the $E2$ decay transition strength of the SM. This has been achieved by measuring a finite value for the $E2/M1$ multipole-mixing ratio of a $\gamma$-ray transition between the SM and the ground-state rotational band of the nucleus $^{156}$Gd. It represents the first measurement of an $F$-vector $E2$ transition in axially deformed nuclei. The data yield a finite difference of effective boson quadrupole-charges for proton and neutron bosons in the IBM-2 of a deformed nucleus fitted locally to sensitive $F$-scalar and $F$-vector transition rates in a single rotational nucleus. Moreover, the size of the measured $F$-vector $E2$ decay matrix element enables us to estimate the $E2$ excitation strength of the $2^+_c$ state of the SM rotational band from an Alaga-rule constraint. The data indicate the existence of this state consistent with the estimated $E2$ rotational band from an Alaga-rule constraint. The energy profile of the $\gamma$-ray transition between the $2^+_c$ state and to excited final states was determined. The measured decay intensity ratio to the $1^+_f$ state is consistent within two standard deviations of the SM.

Experiment and results.—Nuclear resonance fluorescence (NRF) experiments [13,21] with linearly polarized quasimonenergetic $\gamma$-ray beams [22] have been performed at the High-Intensity $\gamma$-Ray Source (HiyS) [23] at Duke University, in Durham, North Carolina. The photon beams were scattered off a Gd$_2$O$_3$ target which contained 10 g of Gadolinium with an enrichment of 93.79(3)% in $^{156}$Gd. The target was mounted in the $\gamma^3$ setup [24], which included four high-purity Germanium (HPGe) detectors at a polar angle of $\vartheta = 135^\circ$ with respect to the incoming beam and at azimuthal angles $\varphi$ of 0°, 90°, 180°, and 270° with respect to the horizontal polarization plane. The linear polarization of the incident photons causes an anisotropic azimuthal distribution of the scattered photons, which is sensitive to the $E2/M1$ mixing ratio of the $1^+_c \rightarrow 2^+_c$ transition [25,26]. Its angular distribution function is given by

$$W(\vartheta, \varphi; \delta) = 1 + \frac{3}{40} \left( 1 + 6\sqrt{5}\delta + 5\delta^2 \right) \times \cos^2 \vartheta + (1 - \cos^2 \vartheta) \cos 2\varphi - 1/3$$

with the $E2/M1$ multipole-mixing ratio

$$\delta_{1-2} = \frac{\sqrt{3} E_2 \langle 2^+_c | \hat{T}(E2)| 1^+_c \rangle}{10 \hbar^2 \langle 2^+_c | \hat{T}(M1)| 1^+_c \rangle}$$

in the phase convention of Krane et al. [27]. The quantities $\hat{T}(E2)$ and $\hat{T}(M1)$ denote the electric quadrupole and magnetic dipole transition operators.

Figure 1 shows the ($\vec{\gamma}$, $\gamma'$) spectra of $^{156}$Gd measured in the polarization plane (a) and perpendicular to it (b). The energy profile of the beam with a width of about 3.5% of the centroid energy $3.070$ MeV is indicated by a dashed Gaussian in the lower plot. Prominently observed—besides other fragments of the SM—are the $\gamma$-ray transitions from the dominant $1^+_c$ state at $3.070$ MeV to the ground state and to the $2^+_1$ state with $\gamma$-ray energy of $2.981$ MeV. Four additional $1^+_c,i$ states of the SM are observed at $3.158$, $3.050$, $3.010$, and $2.974$ MeV ($i = 2$–$5$). For the most strongly populated $1^+_c,1$ state the experimental decay intensity ratio $I_{1c,1} = \Gamma_{1c}/\Gamma_0$ to the $2^+_1$ state to other low-lying final states were determined. Here, $\Gamma_0$ and $\Gamma_1$ denote the partial decay widths to the ground state and to excited final states. The measured decay intensity ratio to the $2^+_1$ state is consistent within two standard deviations with the literature [28], but is more precise. Decays to the $0^+_{2,3}$ and $2^+_2$ states were observed with relative intensities below 1%. The measured relative intensities are given in Table I.

The $I_{rel}$ value to the ground state can be compared to expectations from the Alaga-rule [30]

$$R(\Delta K) = \frac{B(\pi\lambda; J_{K_i} \rightarrow J_{K_f})}{B(\pi\lambda; J_{K_i} \rightarrow J_{K_i})} = \left( \frac{C_{JK_i,\Delta K}^{JK_f}}{C_{JK_f,\Delta K}^{JK_i}} \right)^2$$

for relative transition strengths between arbitrary states of two rotational bands of an axially deformed rotor differing by $\Delta K = K_f - K_i$ in their intrinsic angular-momentum.

FIG. 1. Gamma-ray spectra from the $^{156}$Gd($\vec{\gamma}, \gamma'$) reaction taken at the HiyS facility [23]. Detectors were placed at a polar angle of $\vartheta = 135^\circ$ and azimuthally in the horizontal polarization plane (a) of the incident $\gamma$-ray beam and perpendicular to it (b). The $1^+_c \rightarrow 0^+_1$ $M1$ transition with $3.070$ MeV transition energy and the mixed $E2/M1$ transition to the $2^+_1$ state at $2.981$ MeV dominate the spectra. Other fragments of the SM are indicated by arrows in (a). The energy profile of the $\vec{\gamma}$-ray beam is indicated by the dashed Gaussian curve in (b). The data points were obtained from the relative luminosity determined from the known cross sections [28] for the corresponding $0^+_1 \rightarrow 1^+ \rightarrow 0^+_1$ photon scattering cascades.
TABLE I. Comparison of measured relative intensities $I_{\text{rel}}$ of transitions from the strongest fragment of the SM to values from Ref. [28]. In addition to the $1^+_\text{sc} \rightarrow 0^+_1, 2^+_1$ transitions, recently discovered [29] decay paths of the $1^+_\text{sc}$ state at 3.070 MeV to other low-lying states are considered. The intensities are normalized to 100 for the ground-state transitions. Furthermore, the determined multipole-mixing ratio for the $1^+_\text{sc} \rightarrow 2^+_1$ transition is given.

<table>
<thead>
<tr>
<th>Transition</th>
<th>$E_\gamma$ (keV)</th>
<th>$\delta_{1-2}$</th>
<th>$I_{\text{rel}}$ (%)</th>
<th>$I_{\text{rel}}$ (%) [28]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1^+_\text{sc} \rightarrow 0^+_1$</td>
<td>3070</td>
<td>100.0(3)</td>
<td>100(27)</td>
<td></td>
</tr>
<tr>
<td>$1^+_\text{sc} \rightarrow 2^+_1$</td>
<td>2981</td>
<td>-0.07(1)</td>
<td>59(6)</td>
<td></td>
</tr>
<tr>
<td>$1^+_\text{sc} \rightarrow 0^+_2$</td>
<td>2020</td>
<td>0.35(8)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1^+_\text{sc} \rightarrow 2^+_2$</td>
<td>1941</td>
<td>0.48(10)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1^+_\text{sc} \rightarrow 0^+_3$</td>
<td>1902</td>
<td>0.36(9)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$^a$Assuming pure M1 character.
$^b$Assuming pure E2 character.

projection quantum number. The reduced relative decay-intensity ratio $R_{\text{exp}} = |I_{\text{rel},1^+_\text{sc} \rightarrow 0^+_1}/(E_\gamma,1^+_\text{sc} \rightarrow 2^+_1)|^2/|I_{\text{rel},1^+_\text{sc} \rightarrow 0^+_1}/(E_\gamma,1^+_\text{sc} \rightarrow 0^+_1)|^2 = 0.52(1)$ is consistent within two standard deviations with the value $R(\Delta K = 1) = 0.5$ from the Alaga rule for pure dipole transitions from the SM with intrinsic projection $K = 1$ to the $K = 0$ ground-state band. This already indicates that the $1^+_\text{sc} \rightarrow 2^+_1$ transition is dominated by the M1 component and a possible E2/M1 multipole mixing ratio must be close to zero, i.e., $\delta_{1-2} \ll 1$.

In order to obtain information on the size of the E2 decay matrix element from the SM to the ground-state band, we have analyzed the E2 contribution to the $1^+_\text{sc} \rightarrow 2^+_1$ transitions. The ratio $N_{\parallel}/N_{\perp}$ is sensitive to the multipole mixing ratio $\delta_{1-2}$. Here, $N_{\parallel}$ and $N_{\perp}$ are the $\gamma$-ray intensities registered in and perpendicular to the polarization plane of the incident $\gamma$-ray beam, respectively. The observed ratio is compared to the respective ratio of angular distributions $W(135^\circ, 0^\circ; \delta_{1-2})/W(135^\circ, 90^\circ; \delta_{1-2})$ from Eq. (1) for the $0^+_1 \rightarrow 1^+_\text{sc} \rightarrow 2^+_1$ sequence, integrated over the opening angles of the individual detectors. The result for the $1^+_\text{sc} \rightarrow 2^+_1$ decay transition at 2.981 MeV (cf. Fig. 1) is shown in Fig. 2. It features two solutions; one close to zero, the other corresponding to dominant E2 character. The first solution, $\delta_{1-2} = -0.07(1)$, is the only one consistent with both, the data for the azimuthal angular distribution and with the Alaga rule. Evidence for possible $K$ mixing has been shown to be small (cf. Ref. [31] and Fig. 9 in Ref. [14]). In the case of two-state mixing, mixing ratios $\delta_{1-2} = -2.69(19)$ for the multipole mixing ratio can be shown to be incompatible with the fact that the $1^+_1$ state at 3.070 MeV is the strongest $M1$ excitation of $^{156}$Gd [1,28]. Hence, application of the Alaga rule is well justified for this predominantly axially deformed nucleus $^{156}$Gd, in particular, for transitions whose strengths dominate in the given energy interval. The $1^+_\text{sc,2,3,4,5} \rightarrow 2^+_1$ transitions lack the intensity to extract finite multipole-mixing ratios.

**Discussion.**—We concentrate the subsequent discussion on the dominant fragment $1^+_\text{sc}$ of the SM at 3.070 MeV, which carries about 1/3 of the entire $M1$ excitation strength [28], and on a possible rotational band built on top of it. In the following, the uncertainty of a quantity deduced from the multipole mixing ratio is given as the square root of the sum of the systematic and the random uncertainty for simplicity.

From the squared multipole mixing ratio $\delta_{1-2}^2$ and the known partial decay width $\Gamma_{1^+_\text{sc} \rightarrow 2^+_1} = 70.8 \pm 15.1$ meV obtained from the value for $\Gamma_{1^+_\text{sc} \rightarrow 2^+_1}^2 = 80.5 \pm 14.8$ meV from Pitz et al. [28] and corrected for previously unobserved or redetermined branching ratios, we obtain an $E2$ decay strength from the main fragment of the SM of $^{156}$Gd of

$$B(E2; 1^+_\text{sc} \rightarrow 2^+_1) = 1.9(13) e^2 f m^4 = 0.037(26) \text{ W.u.}$$

This value represents the first measurement of the $E2$ decay strength of a $1^+$ state of the SM in a well-deformed nucleus and, correspondingly, the first experimental information on the strength of an $F$-vector $E2$ transition in an axially deformed nucleus.

In the IBM-2 an $F$-vector $E2$ transition strength is proportional to the square of the difference of the effective boson quadrupole charges for proton and neutron bosons $(e_p - e_n)^2$. Using the $F$-spin limit [32] of the SU(3) dynamical symmetry of the IBM-2 and considering that the SM fragment at 3.070 MeV carries about 1/3 of the
entire SM $M1$ (and $E2$) strength, we obtain for the first time local values $e_x = 0.131(4)eb$ and $e_y = 0.106(6)eb$ for the effective boson quadrupole charges directly from the $F$-vector $E2$ decay of the SM. These two values are more precise but agree within uncertainties with the charges determined in Ref. [20] under the assumption that the $2^+$ state at 3.089 MeV was the rotational excitation of the SM. Furthermore, they are closer to each other by two standard deviations as compared to those obtained from previous approaches [32]. These fitted effective charges to $B(E2; 2_1^+ \rightarrow 0_1^+)$ values from a chain of isotopes assuming that their structure does not differ, and that the model qualitatively reproduces the evolution of the data as a function of boson number. Instead, in the present work they are locally determined from an $F$-vector $E2$ transition discussed above and from the transition strength of 189(3) W.u. [33] for the $F$-scalar $E2$ transition from the $2_1^+$ state to the ground state of the very same nucleus, $^{156}$Gd. The corresponding reduction of $(e_x^2 - e_y^2)$ by about one order of magnitude with respect to previous estimates [32], and a correspondingly small $E2$ excitation strength from the ground state, explains why the $2^+_2$ state has remained largely undetected in the past. The new data agree with early predictions [19] for the deformation dependence of the effective $E2$ boson charges in the IBM-2.

The Alaga rule can be used for estimating the $M1$ and $E2$ transition rates from the expected states of the rotational bands, built on top of the fragments of the SM, to the ground-state band from the measured $B(M1; 1_{sc,1}^+ K = 1 \rightarrow 0_{sc,1}^+, K = 0)$ and $B(E2; 1_{sc,1}^+ K = 1 \rightarrow 2_{sc,1}^+, K = 0)$ values. Applying Eq. (3) one obtains the estimates given in Table II for the $E2$ and $M1$ strengths from the first two $2_{sc,1}^+, 3_{sc,1}^+$ members of the rotational band expected to be built on top of the $1_{sc,1}^+$ state at 3.070 MeV.

From the estimated transition strengths one must expect that the $2_{sc,1}^+$ state on top of the $1_{sc,1}^+$ state would decay to 99% to the $2_{sc,1}^+$ state with an $E2/M1$ multipole mixing ratio of $\delta^2 \approx 0$. This expectation, together with the estimated $E2$ excitation strength of that $2_{sc}^+$ state and the experimental luminosity curve (cf. Fig. 1), is consistent with the following experimental observation: A suitable peak at 3.000(1) MeV, observed in all detectors and interpreted as a signal for the $2_{sc}^+ \rightarrow 2_{sc}^+$ decay transition of a $2_{sc}^+$ state at 3.089(1) MeV excitation energy, is shown in Fig. 3. The experimental luminosity at 3.089 MeV together with the assumption that the Alaga estimates for the $2_{sc}^+$ state from Table II are correct, would produce an excitation yield of about 500 counts in a peak at the transition energy of 3.000(1) MeV with a unique azimuthal intensity asymmetry of 0.176 for a $0^+ \rightarrow 2_{sc}^+ M1^+ 2_1^+$ sequence. The measured asymmetry of the peak at 3.000(1) MeV amounts to

$$\frac{N_\| - N_{\perp}}{N_\| + N_{\perp}} = 0.20(11),$$

excluding all reasonable alternatives to the spin sequence indicated above.

Hence, our data provide evidence for a $2^+$ state located about 19(1) keV above the $1_{sc,1}^+$ state, with an $E2$ excitation strength that matches the $E2$ decay strength of the dominant $1^+ \rightarrow 0^+$ SM level. This observation further coincides within uncertainties with the $2^+$ state found in previous inelastic electron scattering experiments [20] at 21(5) keV above the $1_{sc,1}^+$ state as the strongest $E2$ excitation in that entire energy range. If indeed the $2^+$ state at 3.089(1) MeV excitation energy is the first rotational excitation of the $1_{sc,1}^+$ state at 3.070 MeV, two cases have to be considered. The scissors mode either has a rotational moment of inertia which exceeds the rigid-body value by more than 50%, or the $K = 1$ SM rotational band would exhibit a significant signature splitting [34] with a decoupling parameter (defined for $K = 1$ according to the definition from Ref. [34] for $K = 1/2$) of 0.34, assuming the moment of

![FIG. 3. Candidate for the $2_{sc}^+ \rightarrow 2_{sc}^+$ transition at 3.000(1) MeV observed in all four HPGe detectors. The peak contains about 640(90) counts. Its intensity and azimuthal asymmetry suggest its interpretation as the decay transition to the $2_{sc,1}^+$ state at an excitation energy of 3.089 MeV.](image-url)
inertia of the scissors-mode band is similar to the one of the ground-state band. This alternative can only be verified by a future observation of the $3^+_2$ state. Its identification, e.g., in $(e, e')$ experiments, is therefore of great importance, also with respect to entanglement in the two-rotor model [35] or to energy shifts due to multistate mixing and, hence, the formation of rotational bands, altogether.

Summary.—For the first time, we have measured the multipole mixing ratio of the decay transition from the scissors mode to the $2^+_1$ state in a deformed nucleus. We have determined the $B(E2; ^2_0^+ \rightarrow 2^+_1)$ value of $^{150}$Gd and estimated the $\gamma$-decay behavior of the scissors-mode band from an Alaga rule constraint. The data provide direct evidence for a decrease of $F$-vector $E2$ boson charges within the IBM-2 as a function of ground-state deformation, and for the $2^+_2$ rotational state at 3.089(1) MeV excitation energy. This excitation energy poses a puzzle in light of the rotational characteristics of the SM.

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