The applicability, predictive power, and internal consistency of a modified BCS (MBCS) model suggested by Dang and Arima have been analyzed in detail in [1]. That analysis concluded that the \( T \)-range of the MBCS applicability can be determined as being far below the critical temperature \( T_c \), i.e., \( T \ll T_c \). Unfortunately, the source of our conclusions has been misrepresented in [2], which referred to MBCS predictions at \( T \gg T_c \).

Since above \( T_c \), particles and holes contribute to an MBCS gap with opposite signs, the model results are rather sensitive to details of a single particle spectrum (s.p.s.) (e.g., discussion in Sec. IV A 1 of [3]). As so, it is indeed possible to find conditions under which the MBCS simulates reasonable thermal behavior of a pairing gap. This can be achieved, e.g., by introducing some particular \( T \)-dependence of the s.p.s. (item (i) in [2]) or by adding an extra level to a picket fence model (PFM) (item (ii) in [2]). But such results are very unstable, and accordingly, the model has no predictive power.

Dang and Arima explain poor MBCS results for the PFM (\( N = \Omega = 10 \)) discussed in [1] by referring to strong asymmetry in the line shape of the quasiparticle-number fluctuations \( \delta N_j \) above \( T \sim 1.75 \) MeV (symmetry of \( \delta N_j \) is announced as a criterion of the MBCS applicability).

The space limitation is blamed for that in [2]. Remember, particle-hole symmetry is an essential feature of the PFM with \( N = \Omega \). Thus, strong asymmetry is reported from the MBCS calculation in an ideally symmetric system.

It has been found that a less symmetrical example \( N = 10, \Omega = 11 \) satisfies better the MBCS criterion [2]. Indeed, the model mimics the behavior of a macroscopic theory in this case [see Fig. 1(b)]. But this example is the only one in which the MBCS does not breakdown, in a long row of physically very close examples with more limited or less limited s.p.s. In all other examples, we witness either negative heat capacity \( C_v \) [Fig. 1(a)] or negative gap \( \Delta \) [Fig. 1(c)] at rather moderate \( T \) (see also [4]).

Unfortunately, the conclusion in [2] that “within extended configuration spaces . . . the MBCS is a good approximation up to high \( T \) even for a system with \( N = 10 \) particles,” is based on a single example, while in all other \( N = 10 \) examples the MBCS yields unphysical predictions.

The most serious problem of the MBCS is its thermodynamic inconsistency. It is not sufficient to declare two quantities, \( \langle H \rangle = \text{Tr}(HD) \) and \( \varepsilon \) representing the system energy, as being analytically equal by definition (as is done in footnote [8] of [2]) to prove the model consistency. It is easy to find that the expression for \( \varepsilon_{\text{MBCS}} \) [in the form of Eq. (83) in [3]] can be obtained in the same way as all other MBCS equations have been derived: straightforwardly replacing the Bogoliubov \( \{u_j, v_j\} \) coefficients in \( \varepsilon_{\text{BCS}}(T = 0) \) expression by \( \{u_j, v_j\} \) coefficients. Numeric results in Fig. 9 of [1] show that \( \langle H \rangle_{\text{MBCS}} \) and \( \varepsilon_{\text{MBCS}} \) have nothing in common, while \( \langle H \rangle_{\text{BCS}} \approx \varepsilon_{\text{BCS}} \), as it should be for thermodynamically consistent theory.

Another example of the MBCS thermodynamic inconsistency is shown below. We calculate the system entropy \( S \) as

\[
S_1 = \int_0^T \frac{1}{t} \frac{d\varepsilon}{dt} dt,
\]

and

\[
S_2 = -\sum_j (2j + 1)[n_j \ln n_j + (1 - n_j) \ln(1 - n_j)],
\]

where \( n_j \) are thermal quasiparticle occupation numbers. In Fig. 2, we compare \( S_1 \) and \( S_2 \) quantities, which refer to thermodynamic and statistical mechanical definitions of entropy, respectively. The calculations have been performed for the neutron system of \(^{120}\text{Sn} \) with a realistic s.p.s.

It is not possible to visually distinguish \( S_1 \) and \( S_2 \) in the FT-BCS calculation (solid curve in Fig. 2) represents both quantities) as it should be for thermodynamically consistent theory. The MBCS \( S_1 \) and \( S_2 \) quantities are shown by dashed and dot-dashed lines, respectively. They are different by orders of magnitude in the MBCS prediction.
We stress that the low $T$ part is presented in Fig. 2. Dramatic disagreement between $S_1$ (MBCS) and $S_2$ (MBCS) representing the system entropy remains at higher $T$ as well, but we do not find it necessary to extend the plot: the model does not describe correctly a heated system even at $T \sim 200$ keV.

We show in the insert of Fig. 2 another MBCS prediction: entropy $S_1$ decreases as temperature increases. This result is very stable against variation of the pairing strength $G$ within a wide range and contradicts the second law of thermodynamics.

Finally, as we stated before, the conclusion in [1]—that the $T$-range of the MBCS applicability can be determined as being far below the critical temperature $T_c$—is based on the analysis of the model predictions from $T \ll T_c$ and not on $T \gg T_c$ results as presented in [2].

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[4] Dang and Arima put in doubt the validity of the PFM calculations in [1] claiming that “the limitation of the configuration space with $\Omega = 10$ causes a decrease of the heat capacity $C$ at $T_m > 1.2$ MeV... Therefore, the region of $T > 1.2$ MeV, generally speaking, is thermodynamically unphysical.” [2]. It is well-known that such a behavior of the heat capacity is a characteristic feature of finite systems of bound fermions and “does not concern the validity of statistical mechanics. [5]”