Low-energy dipole strength and the critical case of 48Ca

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ABSTRACT
Recent theoretical work has not led to a consensus regarding the nature of the low-energy E1 strength in the 40,44,48Ca isotopes, for which high-resolution (γ,γ′) data exist. Here we revisit this problem using the first-order quasiparticle random-phase approximation (QRPA) and different interactions. First we examine all even Ca isotopes with N = 14–40. All isotopes are predicted to undergo dipole transitions at low energy, of large and comparable isoscalar strength but of varying E1 strength. Provided a moderate and uniform energetic shift is introduced to the results, QRPA with the Gogny D1S interaction is able to account for the (γ,γ′) data, because, up to N = 28, it yields a rather pure isoscalar oscillation. A neutron-skin oscillation is anticipated for N ⩾ 30. This contradicts existing predictions that 44,48Ca develop a neutron-skin mode. Which theoretical result is correct cannot be resolved conclusively using the available data. We propose that alpha-scattering, possibly followed by an electroexcitation experiment, could resolve the situation and thereby help to improve the different models aspiring to describe reliably the low-energy dipole strength of nuclei.

1. Introduction
Recent theoretical work [1–3] has led to a partial understanding, at best, of the nature of the low-energy E1 strength observed in Ca isotopes [1]. Effects beyond first-order random-phase approximation (RPA), or quasiparticle RPA (QRPA), seem necessary to reproduce certain details of the strength distributions, but contradictory reports exist as to whether a collective transition is present at all in 48Ca [2,3]. Here we show that QRPA with a conventional force, Gogny D1S, also reproduces the observed systematics and provides a new physical interpretation for it. We then propose a strategy to resolve which theoretical scenario is correct.

In the following we avoid the loaded term “pygmy dipole” strength, or resonance. We instead refer to electric dipole transitions below the giant dipole resonance (GDR) simply as low-energy strength, or resonance. To refer specifically to an oscillation of a nucleus 40Ca carries too much E1 strength compared to other nuclei of similar mass [5]. Even so, RPA tends to predict too much E1 strength for this state [6]. For a useful comparison between theory and experiment one must therefore consider 40Ca to be part of the systematics of the Ca isotopic chain, not just a core with negligible LED strength. The acronym IS-LED stands for isoscalar low-energy dipole states or strength. To refer specifically to an oscillation of a neutron skin against a rather inert core, we use the term “neutron-skin mode”, or NSM.

Let us first recapitulate the current situation. High-resolution (γ,γ′) experiments have measured the E1 strength at excitation energies below 10 MeV in the three isotopes, 40,44,48Ca [1]: rather little in 40Ca, and roughly 10 times more in 44,48Ca. That there is comparable and even somewhat less strength in 42Ca than in 44Ca is viewed as a puzzle, because it contradicts the simplistic expectation, founded on early calculations [4], that LED strength should increase with neutron/proton asymmetry. The above reasoning disregards the fact that the strongest LED state in the N = Z nucleus 40Ca carries too much E1 strength compared to other nuclei of similar mass [5]. Even so, RPA tends to predict too much E1 strength for this state [6]. For a useful comparison between theory and experiment one must therefore consider 40Ca to be part of the systematics of the Ca isotopic chain, not just a core with negligible LED strength.

We point out that Ca isotopes cannot be considered too light to develop collective LEDs. Precisely such a model is observed in 40Ca (and even in 16O) – see Ref. [6], where a compelling case was made for its coherent nature. If 48Ca does not develop a collective LED mode at all, its mass is not the underlying reason. Whether or not it is too light to develop a NSM [3], is a different question. We stress that a NSM is not the only collective dipole

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vibration that could develop below the GDR and therefore not the only conceivable collective mechanism for generating LED strength. This becomes obvious in the case of $^{40}$Ca [6] and was stressed also in Ref. [7], devoted to the toroidal dipole mode, and implied in Ref. [8].

In general, the first-order RPA and QRPA cannot describe precisely the LED strength observed in Ca isotopes. In self-consistent (Q)RPA calculations, the lowest dipole states tend to lie too high in excitation energy, or the systematics tends to be wrong: E1 strength is predicted to rise almost linearly with neutron number $N$ [4], apparently due to the development of a neutron skin. Moreover, as already mentioned, the LED strength of $^{40}$Ca is often overestimated. The so-called Extended Theory of Finite Fermi Systems (ETFFS), which goes beyond RPA, is actually the only model which has reproduced the properties of LED strength observed in $^{40}$Ca, $^{44}$Ca and $^{48}$Ca quite accurately [2].

Configurations beyond first-order (Q)RPA can affect the LED strength by (1) shifting strength to lower energies, where LED strength is experimentally observed, as first suggested in [9]; (2) introducing fragmentation and possible quenching of strength; (3) introducing additional states of two-phonon character, not described by RPA; (4) redistributing strength so that much LED strength remains above the measurement endpoint of 10 MeV. All the above mechanisms have been touched upon in Refs. [1–3]. A strong statement in Ref. [2] was indeed that in the case of $^{48}$Ca phonon coupling shifts a strong LED state resembling a NSM to energies higher than 10 MeV and thus the experiment missed it. This was proposed as an explanation of the observed systematics. Notably, an early Coulomb-excitation experiment found no evidence of a NSM mode below 12 MeV in $^{48}$Ca [10].

An important concern remains then that RPA-based models predict collective LED vibrations for all Ca isotopes, of NSM type or other, which have not been identified except for $^{40}$Ca. Obviously, RPA, like any model, cannot be expected to account for all possible collective nuclear states. The converse is not true, however: A collective RPA vibration that remains incomprehensibly elusive is not a minor problem.

At this point we note that, if mechanism (1) above is a dominant one, then a reasonable description of LED strength should be possible within (Q)RPA, once a uniform energetic shift is introduced. We will make such an attempt in this work.

We may now give an outline of the present work. First we present the theoretical framework. We then discuss our results with different effective interactions, Gogny and UCOM-based, for the IS and E1 strength distributions of even Ca isotopes with $N = 14–40$ and identify interesting features at the lower end of the spectrum. Next, we make an attempt to describe the experimental systematics within QRPA, by introducing an energetic shift as already mentioned. We find it successful when the Gogny D1S interaction is used. The reason is that, for $N$ up to 28, a predominantly isoscalar state is predicted, as opposed to a NSM. Such a scenario contradicts the ETFFS predictions [2]. We suggest that an electroexcitation experiment on $^{48}$Ca, preceded by alpha scattering, could resolve the situation.

## 2. Theory

We employ the self-consistent Hartree–Fock–Bogolyubov–Quasiparticle-RPA (HFB-QRPA), which reduces to the self-consistent Hartree–Fock–RPA for closed–shell nuclei. The ground-state problem is solved within a single-particle basis spanning 15 harmonic-oscillator shells. The same effective interaction is used to describe the ground state and to construct the QRPA equations, in both the $ph$ and the $pp$ channels. Technical and performance details regarding our implementation can be found in [11]. We consider a two-body Hamiltonian of the form

$$H = T + V_{NN} + V_{Coul} + V_{\rho},$$

where $T$ is the intrinsic kinetic energy, $V_{NN}$ a two-body nuclear interaction, $V_{Coul}$ the Coulomb interaction between protons and

$$V_{\rho} = \gamma_1 (1 + x_2) \delta (\vec{r}) \rho^2 (\vec{R})$$

is a density-dependent contact interaction ($\vec{r}$ the relative and $\vec{R}$ the center-of-mass position vector of the interacting nucleon pair). We employ the phenomenological Gogny D1S [12] parameterization and a unitarily-transformed AV18 realistic potential, supplemented with a phenomenological three-body contact term, which we label UCOM(SRG)$_{3.34N}$. See Refs. [6,13] for information on the latter. For comparison we will also use Second-RPA (SRPA) for $^{40,44,48}$Ca, with the full coupling in the $2p2h$ space, and with the pure two-body UCOM-transformed AV18 potential, UCOM$_{var}$ (UCOM-SRPA). This model is rather well suited for the description of the GDR region [14]. Here we introduce an energy cut-off of 140 MeV in the $2p2h$ space, so that the IS-LED state of $^{40}$Ca appears at approximately 7 MeV, i.e., close to its measured value.

The ISD response is determined by the transition matrix elements of the operator

$$\hat{O}_{\text{ISD}} = \sum_{i=1}^{A} \delta (r^2 - \frac{3}{5} (r^2_{i})) Y_{1m}(\Omega_1)$$

and the electromagnetic response by

$$\hat{O}_{\text{E1}} = \frac{Ze}{A} \sum_{n=1}^{N} r_n Y_{1m}(\Omega_2) - \frac{Ne}{A} \sum_{p=1}^{Z} r_p Y_{1m}(\Omega_3)$$

in an obvious notation, where the subscripts $p$ and $n$ refer to protons and neutrons, respectively. We calculate the excitation strength, $B(E1)\uparrow$. The above operators include corrections to explicitly restore translational invariance. Within our self-consistent (Q)RPA calculations, we obtain practically the same values of strength if we use the uncorrected forms of these operators, except of course for the spurious state, which appears at practically zero energy, namely below 15 keV in all considered cases. In SRPA spurious admixtures are unavoidable.

Electroexcitation cross sections are calculated by using the proton transition density, $\delta \rho_p(r)$. For the longitudinal form factors in plane-wave Born approximation (PWBA) we use the convention

$$F_1(q^2) = \frac{\sqrt{12\pi}}{Z} \int_0^\infty \delta \rho_p(r) j_1(qr) r^2 \, dr.$$
We take the opportunity to point out again [6] that E1 strength alone is not a good indicator of collectivity, in particular the lack thereof: a collective IS dipole state could in principle carry less E1 strength than a 1p1h transition. It may be telling that LED strength is often overestimated, even when it is interpreted as due to non-collective transitions [16].

The nature of a state is further illuminated by looking at the radial behavior of its proton and neutron transition densities.

3. Results

3.1. General considerations

In Fig. 1 we show the ISD and E1 strength distributions of the even Ca isotopes calculated using the Gogny D1S and the UCOM(SRG)$_{S,2N}$ interactions. For $^{40,48,50}$Ca, UCOM-SRPA results are shown as well. The patterns and trends with respect to neutron number become clear in this mode of presentation, where the amount of strength at each value of excitation energy is represented by the area of a circle. A different scale is used for the IS (discs) and E1 (open circles) strength. IS and E1 strength functions of selected isotopes, calculated within QRPA, are shown also in Fig. 2.

The theoretical peaks of the GDR are visible as the larger open circles in Fig. 1. The GDR peaks of the $N = 20–28$ isotopes are found experimentally at an excitation energy of, roughly, 18–21 MeV. All three models reproduce this fact rather well. In all cases we find strong ISD states at the lower end of the spectrum (see also Fig. 2). The same observation is made in a previous QRPA study using a different SRG-based interaction [11] and is in agreement with existing results using Skyrme functionals [17,18]. These results are not surprising, because IS-LED states below threshold exhausting 4–14% of the IS EWSR have been observed in various nuclei since years [19]. For $^{40}$Ca, the lowest-lying RPA state has been identified recently as the isospin-forbidden E1 transition [6], experimentally observed at about 7 MeV [20,21]. Notably, the IS strength of the lowest strongly IS state of all isotopes and for both interactions is of the same order of magnitude, but the same does not hold for the E1 strength of those states.

3.2. Analysis of the lowest-energy strongly IS states

Let us examine the three sets of results separately, with emphasis on the lowest part of the spectrum. We start with QRPA and the Gogny D1S interaction. We first point out that the IS-LED strength is split in the neutron-deficient isotopes and even more fragmented for $N \geq 30$. We find that the IS strength of the lowest LED states doubles between $^{48}$Ca and $^{50}$Ca and remains large for the heavier isotopes, while the E1 strength increases fiftyfold from $^{48}$Ca to $^{50}$Ca and remains in that order of magnitude in the heavier isotopes. In particular, for the lowest-lying peaks in the $N = 18, 20, 22, 24, 26, 28, 30$ isotopes we find, respectively, 590 (2 peaks below 10 MeV), 832, 860, 868, 855, 845, 1731 (two lowest peaks) $[e^2\text{fm}^2]$ of IS strength and 26.7 (two peaks), 4.5, 1.9, 1.6, 3.0, 6.0, 3.15 (two peaks) $[\times 10^{-3}e^2\text{fm}^2]$ E1 strength. At $N = 30$ and beyond the energy of the lowest peak decreases.

The E1 strength (but not the IS strength) reaches a minimum at $N = 24$ and not at $N = 20$. A similar counter-intuitive result appears in a recent study of shell effects in the isotopic behavior of LED strength [22]. The transition densities of these states reveal that they resemble a neutron-skin oscillation only for $N \geq 30$, not the lighter $N > Z$ isotopes. The NSM is of mixed isospin character and appears strong in both the IS and E1 channels [cf. also $^{52}$Ca in Fig. 2]. By contrast, the stronger IS-LED states of the lighter $N > Z$ isotopes show a locally isoscalar character instead, resembling that of $^{40}$Ca, and therefore carry little E1 strength. Nevertheless, these strongly IS states are found quite collective. In the case of $^{48}$Ca, for example, eight out of the 18 possible 1h0p configurations (four of them proton configurations) contribute at least 1/18 each to the strength of the IS-LED state (see [15]) and many ph configurations contribute coherently to its IS strength. Remarkably, the hole states in $^{48}$Ca are found very similar for protons and neutrons, a situation that is not realized in $^{40}$Ca.
Finally, we remark that states dominated by proton transitions are found in the neutron-deficient isotopes. The results for the UCOM(SRG)$_{\delta,3N}$ interaction are more in line with earlier predictions \[4\]: a minimum in E1 strength is reached for 40Ca, and a monotonic behavior is observed on either side of N = 20. At both N = 22 and N = 30 the position of the lowest peak shows a kink, pointing to structural changes of this state. Here a neutron-skin mode, characterized by a translational-type $\delta \rho_p(r)$ and an extended $\delta \rho_n(r)$ with a node, is predicted already for the lightest N > Z isotopes. However, these states carry too much E1 strength to be compatible with the data, namely 0.021 and 0.4 e$^2$ fm$^2$ for 40Ca and 48Ca, respectively. In the case of 48Ca, this state is of NSM type and dominated by $(\nu f^7/2)^{-1}$ transitions. Again the NSM scenario fails to describe the data.

Within RPA with the UCOM(SRG)$_{\delta,3N}$ interaction as well as within the UCOM-SRPA model, we find that other IS states of 48Ca, at somewhat higher energies but still below the GDR, are of locally isoscalar character with a node in the transition densities of both nucleon species.

3.4. Comparison with measured E1 strength

We now return to our main topic, which is the LED strength of 40,44,48Ca, and ask whether the scenario predicted by the use of the Gogny D1S interaction is compatible with the $(\gamma,\gamma')$ data. A revealing comparison is made in Fig. 3. Here we show the summed E1 strength observed below 10 MeV, the existing prediction within ETFFS, and our new sets of results: The E1 strength of the lowest dipole state in each isotope within (Q)RPA using the Gogny D1S interaction; the summed strength below 13.25 MeV, 13.75 MeV, and 14.25 MeV within the same model. The lowest states are too weak with respect to experiment. A similar observation was made in Ref. [18], where the Skyrme SkM* interaction was used, and in Ref. [11]. The energy cut-off strongly influences the quality of state alone carries too much E1 strength to be compatible with the data, namely 0.021 and 0.4 e$^2$ fm$^2$ for 40Ca and 48Ca, respectively.
the comparison. In fact, minor uncertainties in the data and the theoretical results can likewise affect the systematics. Therefore an attempt to reproduce the data very precisely in this region appears rather meaningless. Nevertheless, the measurements are rather well reproduced by QRPA for the higher cutoffs, indicated in Fig. 1 by the shaded area. This generic observation remains true if we change certain ingredients of our calculation such as the harmonic-oscillator parameter. That it takes an energy shift by about 3–4 MeV for the QRPA to describe the data, is in excellent agreement with the early calculations of Ref. [9]. Higher-order configurations are expected to shift the strength to lower energies and could introduce the fragmentation necessary to describe the E1 values more precisely [2,3,9,26–28], although they are not expected to influence dramatically the total strength below the GDR.

We conclude that the scenario whereby a strong isoscalar mode with little E1 strength is present in $^{48}$Ca, and not a NSM, is very well compatible with the measurements.

4. Resolving the situation in $^{48}$Ca

We have seen that a strongly isoscalar state developing in $^{48}$Ca is as compatible with the existing photon-scattering data as a mixed-isospin NSM appearing above 10 MeV. We now ask how the correct scenario can be resolved. Measurements of the full dipole spectrum of $^{48}$Ca could be useful, because a NSM would be visible as a strong E1 transition close to particle threshold. Such information can be obtained from relativistic Coulomb excitation in proton scattering under extreme forward angles as recently demonstrated for the case of $^{208}$Pb [29]. Data for $^{48}$Ca have been taken [30] and are presently analyzed. But if no such strong state is observed, the problem remains unresolved. We now note that both types of mode, namely pure IS or NSM, would carry much ISD strength below threshold [19,31–33], but has not been used on Ca isotopes other than $^{40}$Ca [21]. Such an experiment on $^{48}$Ca would be most useful in resolving the present situation. Inelastic α-scattering above threshold has been performed on $^{40}$Ca [34]. A resonance beyond 16 MeV was found, but no strong state was clearly observed in the energy region of the NSM mode as predicted by ETFFS. The onset of another concentration of IS strength below 10 MeV could be inferred from the data, but not conclusively.

If a strong isoscalar state is found below 10 MeV, it would likely not be of NSM type. An electroexcitation experiment could then reveal its nature. Inelastic electron scattering has been performed on $^{48}$Ca by different groups, but the analyses have mostly focused on $^1\!\!^1$ states and other multipolarities. In Fig. 4 we show the transition densities of the IS-LED state as predicted within RPA by the Gogny D1S interaction and of the NSM mode as predicted, in this case, by UCOM(SRG)$_{3,3N}$. We also show the corresponding longitudinal form factors within DWBA. The NSM has a large form factor already at low momentum transfer, while a completely different result is obtained for the other IS-led state. A minimum at low momenta would make this state analogous to the IS-LED state of $^{40}$Ca [6]. Significant differences between the transverse form factors of the two different modes are also expected. A more dedicated study shall be the subject of upcoming work.

5. Conclusion

Our QRPA results suggest that all even Ca isotopes with $N = 14–40$ develop strong IS-LED states. IS and E1 strength is present in the whole energy region up to the GDR. The nature of the states varies with the neutron number and the energy. Results with the Gogny D1S interaction suggest that the lowest, strongly IS dipole transition becomes a neutron-skin mode only for $N \geq 30$. This result is compatible with the systematics of E1 strength observed in $^{40,44,48}$Ca, but contradicts an earlier interpretation of the data [2]. We propose that α-scattering followed by an electroexcitation experiment on $^{48}$Ca could resolve the situation conclusively.

Our study demonstrates that, in order to understand the nature of the whole low-energy dipole spectrum, one must examine not just the E1 strength, but at the same time the response to isoscalar operators. Ideally these should include both toroidal and compressional ones [25]. These remarks are relevant for all isotopic chains and for problems including the nature of pygmy dipole strength in very neutron-rich nuclei, the so-called isospin splitting of pygmy dipole strength and the origin of E1 and isoscalar strength throughout the region below the GDR.

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References