# New experimental access to the two-phonon octupole vibration in ${ }^{208} \mathrm{~Pb}$ * 

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Received 20 August 1996; revised 20 September 1996


#### Abstract

The feasibility of a new experimental method to identify two-phonon octupole vibrations is investigated for the example of ${ }^{208} \mathrm{~Pb}$. The $2^{+}$member of the $3_{1}^{-} \otimes 3_{1}^{-}$multiplet can be excited with the ( $\gamma, \gamma^{\prime}$ ) reaction and a sizable E1 decay to the $3_{1}^{-}$level serves as a signature of the two-phonon character. Numerical estimates within the quasiparticle-phonon model indicate that such an experiment should be possible with state-of-the-art large volume Ge detectors like the EUROBALL cluster module.


PACS: 21.10.Re; 21.60.Jz; 25.20.Dc
Keywords: ${ }^{208} \mathrm{~Pb}\left(\gamma, \gamma^{\prime}\right), E=7 \mathrm{MeV}$ bremsstrahlung; calculated $E_{\gamma}, I_{\gamma}$, Monte Carlo simulation, resonance fluorescence. ${ }^{208} \mathrm{~Pb}$ deduced two-phonon octupole vibrations identification possibility. Quasiparticle-phonon model calculation.

## 1. Introduction

The search for the two-phonon octupole vibration in the doubly magic nucleus ${ }^{208} \mathrm{~Pb}$ is one of the most outstanding nuclear structure problems with a long history. In a spherical nucleus like ${ }^{208} \mathrm{~Pb}$ collective surface vibrations form excitation modes lowest in energy which are well described by a phonon approach. The relevance of their detailed understanding lies in two aspects. On one hand, coupling to low-lying surface

[^0]oscillations is the decisive mechanism for the damping of nuclear excitations [1]. On the other hand, one expects characteristic multiplets of excited states in the low-energy spectra due to the coupling of two (or more) of these phonons. Investigating their properties provides a measure how far the phonon concept implying purely harmonic vibrations holds in real nuclei. The energy splitting of the multiplets furthermore permits valuable insight into the effects of the Pauli principle and the general properties of the phonon-phonon interaction in the coupling of elementary excitation modes.

States corresponding to $\left|2_{1}^{+} \otimes 2_{1}^{+} ; 0^{+}, 2^{+}, 4^{+}\right\rangle$configurations are well established in many spherical nuclei. There is also evidence for the $1^{-}-5^{-}$quintuplet of $2_{1}^{+} \otimes 3_{1}^{-}$ states near closed shells. In particular, the $1^{-}$members are selectively excited in ( $\gamma, \gamma^{\prime}$ ) experiments (see Refs. [2-4] and references therein) and rather complete experimental information on the multiplet exists [5] e.g. for the case of ${ }^{144} \mathrm{Nd}$. On the other hand, experimental evidence for $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 0^{+}, 2^{+}, 4^{+}, 6^{+}\right\rangle$states is rather sparse [6-10]. Two nuclei, ${ }^{146} \mathrm{Gd}$ and ${ }^{208} \mathrm{~Pb}$, are the most interesting candidates, since their first excited state is due to an octupole vibration and the members of this multiplet should be the lowest states of two-phonon nature in the respective nuclei.

The properties of the $3_{1}^{-} \otimes 3_{1}^{-}$multiplet in ${ }^{208} \mathrm{~Pb}$ have been studied in a variety of theoretical approaches [11-18]. Experimental searches have been performed using the ( $\mathrm{n}, \gamma$ ) reaction [19], internal conversion decay of $0^{+}$levels [20], inelastic hadron scattering [21], heavy ion (HI) Coulomb excitation [22,23] and $\gamma$-spectroscopy after deep inelastic reactions [24], but the contradictions of the results are confusing. The interpretation of the experiments described in Refs. [19,20] is rather model dependent. However, it was demonstrated recently by the observation of a characteristic E3-E3 decay sequence following excitation in inelastic neutron scattering that the $E_{x}=5.241 \mathrm{MeV}$, $J^{\pi}=0^{+}$level must contain a large $3_{1}^{-} \otimes 3_{1}^{-}$component which makes it the best characterized two-phonon candidate in ${ }^{208} \mathrm{~Pb}$ so far [25]. Evidence for the population of the $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 6^{+}\right\rangle$state by two-step Coulomb excitation in the reaction ${ }^{208} \mathrm{~Pb}+{ }^{208} \mathrm{~Pb}$ has been claimed in Ref. [22]. However, because of the low cross sections in this experiment the projectile energy had to be chosen slightly above the Coulomb barrier where a substantial fraction of the total cross section is due to single nucleon transfer. It was therefore questioned in Refs. [21,24] that the $\gamma$-transition taken as signature of the two-phonon state in Ref. [22] belongs to ${ }^{208} \mathrm{~Pb}$, and a recent HI Coulomb excitation experiment at Argonne shows that the state seen must rather be assigned to ${ }^{207} \mathrm{~Pb}$ [23].

Clearly, this situation is rather unsatisfactory asking for experimental clarification. The purpose of this note is twofold: to provide a microscopic study of the $3_{1}^{-} \otimes 3_{1}^{-}$ multiplet in ${ }^{208} \mathrm{~Pb}$ and its decay properties to the g.s. and the one-phonon $3_{1}^{-}$level, and based on these results to investigate the feasibility of a new experimental method in order eventually to identify the long-sought two-phonon octupole vibration. We propose to look for the excitation of the $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$state in a nuclear resonance fluorescence (NRF) experiment. Although the NRF method is by far most sensitive to dipole excitations the recently demonstrated dramatic improvements of the detection limits [26] with a new generation of large volume HPGe detectors like the EUROBALL Cluster module effectively enable the observation even of weak electric quadrupole transitions. A clear


Fig. 1. Two-phonon octupole vibration multiplet in ${ }^{208} \mathrm{~Pb}$ and levels relevant for the calculations described in the text. The experimental one-phonon energies are taken from the literature [27]. The multiplet energies are from a QPM calculation including of mixing between one- and two-phonon components in the excited state wave functions.
signature for the two-phonon nature of the $2^{+}$state would be a E1 decay to the $3_{1}^{--}$onephonon state, since none of the other $2^{+}$levels in the excitation energy region around the two-phonon octupole multiplet is expected to show a sizable partial decay width to the $3_{1}^{-}$state.

The relevant levels and transitions are schematically summarized in Figure 1. We are interested in the g.s. excitation probability to the $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$level (indicated by the upward arrow) and the partial decay widths to the g.s. and the $3_{1}^{-}$state (denoted by downward arrows). The experimental energies of the $2_{1}^{+}$and $3_{1}^{-}$vibrations in Fig. 1 are taken from the literature [27]. Next, we discuss the model calculations which also provide reliable estimates for the energies of the two-octupole phonon states indicated in Fig. 1.

## 2. Quasiparticle-phonon model calculations

The properties of the $3_{1}^{-} \otimes 3_{1}^{-}$multiplet are calculated within the quasiparticlephonon model (QPM) which successfully describes many different properties of single and multi-phonon low-lying states in spherical nuclei (see e.g. Refs. [28-32]). The model Hamiltonian includes terms corresponding to the mean field for neutrons and protons, monopole pairing and the residual interaction in a separable form. Excited states of even nuclei are described by wave functions which may include one-, twoand higher-phonon components. The properties of phonons, their excitation energies and their internal fermion structure are obtained by diagonalizing the model Hamiltonian in the random phase approximation (RPA); see Ref. [33] for details.

We introduce the phonon operator for the first $3^{-}$state as

$$
\begin{equation*}
Q_{3 \mu}^{+}=\frac{1}{2} \sum_{j_{1} j_{2}}^{N, Z}\left\{\psi_{j_{1} j_{2}}\left[\alpha_{j_{1} m_{1}}^{+} \alpha_{j_{2} m_{2}}^{+}\right]_{3 \mu}+(-1)^{\mu} \phi_{j_{1} j_{2}}\left[\alpha_{j_{1} m_{1}} \alpha_{j_{2} m_{2}}\right]_{3-\mu}\right\} \tag{1}
\end{equation*}
$$

where $j$ denotes a single particle level with quantum numbers $n, l, j$ and $m$ is its projection; $\alpha_{j m}^{+}$is a quasiparticle creation operator. The quantities $\psi_{j_{1} j_{2}}, \phi_{j_{1} j_{2}}$ denote the forward and backward amplitudes, respectively, of the RPA equations. The energy of this state corresponds to the first solution for multipolarity $J^{\pi}=3^{-}$. In the calculation we used the single particle spectrum of Ref. [34]. It was adjusted to properties of low-lying states in odd nuclei of the lead region and later successfully used for different calculations in ${ }^{208} \mathrm{~Pb}$. The parameters of the residual interaction of the Hamiltonian were adjusted to reproduce the collectivity of the lowest $3^{-}$state, i.e. the experimental $B(E 3$, g.s. $\rightarrow 3_{1}^{-}$) value [35] for this level.

In a first step we assume that mixing between one- and two-phonon configurations in the low-energy spectra is negligible. This allows us to define the wave function of the two-phonon $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$state as

$$
\begin{equation*}
\left|\Psi_{\left|3_{1}^{-} \otimes 3_{1}^{-}: 2^{+}\right\rangle_{\mu}}\right\rangle=\frac{1}{\sqrt{2}} \sum_{M_{1} M_{2}}\left\langle 3 M_{1} 3 M_{2} \mid 2 \mu\right\rangle Q_{3^{-} M_{1}}^{+} Q_{3-M_{2}}^{+}\left|\Psi_{\mathrm{g} . \mathrm{s} .}\right\rangle \tag{2}
\end{equation*}
$$

An E1 transition between the states described in Eqs. (1) and (2) as well as a direct decay of the $2^{+}$state to the ground state are forbidden in the pure boson picture. Only through exact commutation relations between phonon and quasiparticle operators, i.e. taking into account the fermion structure of phonons, one gets non-vanishing values. The reduced transition probability for the E1 decay $\left(\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow 3_{1}^{-}\right)$has the form

$$
\begin{align*}
B(\mathrm{E} 1)= & 171.5 \left\lvert\, \sum_{j_{1} j^{\prime} j^{\prime} j^{\prime \prime}}\left\langle j_{2}\|M(\mathrm{E} 1)\| j_{1}\right\rangle u_{j_{1} j_{2}}^{(+)} \sum_{J}(-1)^{J}\left\{\begin{array}{ccc}
j_{2} & j_{1} & 1 \\
j^{\prime \prime} & j^{\prime} & 3 \\
3 & 3 & J
\end{array}\right\}\right. \\
& \times e_{p(n)}\left[\left(\psi_{j_{1} j^{\prime}} \psi_{j_{2} j^{\prime \prime}} \psi_{j^{\prime \prime} j^{\prime}}+\phi_{j_{1} j^{\prime}} \phi_{j_{2} j^{\prime \prime}} \phi_{j^{\prime \prime} j^{\prime}}\right) \delta_{J, 2}\right. \\
& \left.+\left(\psi_{j_{1} j^{\prime}} \phi_{j_{2} j^{\prime \prime}} \phi_{j^{\prime \prime} j^{\prime}}+\phi_{j_{1} j^{\prime}} \psi_{j_{2} j^{\prime \prime}} \psi_{j^{\prime \prime} j^{\prime}}\right)(2 J+1)\left\{\begin{array}{lll}
1 & 3 & 2 \\
3 & 3 & J
\end{array}\right\}\right]\left.\right|^{2} \tag{3}
\end{align*}
$$

and for the E2 decay $\left(\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow\right.$ g.s. $)$

$$
B(\mathrm{E} 2)=19.6\left|\sum_{j_{1} j_{2} j^{\prime}} e_{p(n)}\left(j_{2}\|M(\mathrm{E} 2)\| j_{1}\right) v_{j_{1} ._{2}}^{(-)} \psi_{j_{1 j^{\prime}}} \phi_{j^{\prime} j_{2}}\left\{\begin{array}{ccc}
2 & 3 & 3  \tag{4}\\
j^{\prime} & j_{2} & j_{1}
\end{array}\right\}\right|^{2}
$$

where $\left\langle j_{2}\|M(E \lambda)\| j_{1}\right\rangle$ is the reduced matrix element of the electromagnetic transition; $v_{j_{1}, 2_{2}}^{(-)}=u_{j_{1}} u_{j_{2}}-v_{j_{1}} v_{j_{2}}$ and $u_{j_{1} j_{2}}^{(+)}=u_{j_{1}} v_{j_{2}}+v_{j_{1}} u_{j_{2}}$ are combinations of coefficients of the Bogoliubov transformation from particle $\left(a_{j m}^{+}\right)$to quasiparticle ( $\alpha_{j m}^{+}$) operators: $\alpha_{j m}^{+}=$

Table 1
Excitation and decay properties of $2^{+}$member of two-phonon $3_{1}^{-} \otimes 3_{1}^{-}$multiplet in ${ }^{208} \mathrm{~Pb}$. The results given in column I neglect the mixing of one- and two-phonon configurations in the wave functions of excited states, while it is included in the full calculation presented in column II. In brackets we present the results of test calculations varying the strength of the residual interaction. See text for details

|  | $\mathrm{I}(\mathrm{Ia}, \mathrm{Ib})$ | $\mathrm{II}(\mathrm{IIa}, \mathrm{IIb})$ |
| :--- | :---: | :---: |
| $B\left(\mathrm{E} 2, \mathrm{~g} . \mathrm{s} . \rightarrow\left[3_{1}^{-} \otimes 3_{1}^{-}\right]_{2^{+}}\right), \mathrm{e}^{2} \mathrm{fm}^{4}$ | $10.4(5.1,17.7)$ | $40.2(19.0,54.6)$ |
| $\Gamma_{0}\left(\mathrm{E} 2,\left[3_{1}^{-} \otimes 3_{1}^{-}\right]_{2^{+}} \rightarrow \mathrm{g} . \mathrm{s}.\right), \mathrm{meV}$ | $6.58(3.21,11.2)$ | $25.4(12.0,34.5)$ |
| $\Gamma_{3_{1}^{-}}\left(\mathrm{E} 1,\left[3_{1}^{-} \otimes 3_{1}^{-}\right]_{2^{+}} \rightarrow 3_{1}^{-}\right), \mathrm{meV}$ | $14.0(12.6,17.0)$ | $13.7(12.9,14.3)$ |
| $\Gamma_{3_{1}^{-}} / \Gamma_{0}$ | $2.13(3.93,1.52)$ | $0.54(1.08,0.41)$ |

$u_{j} a_{j m}^{+}+(-1)^{j-m} v_{j} a_{j-m}$; for ${ }^{208} \mathrm{~Pb}$ these coefficients are either 0 or 1 . The numerical factors appearing in Eqs. (3) and (4) are products of $(2 \lambda+1)$ factors for the phonon operators taken into account. We use effective charges $e_{p(n)}=1(0)$ and $N / A(-Z / A)$ for E2 and E1 transitions, respectively.

## 3. Results and discussion

The partial decay widths of these transitions can be calculated from

$$
\begin{align*}
& \Gamma(\mathrm{E} 1)=1.048\left(E_{i}-E_{f}\right)^{3} B\left(\mathrm{E} 1, J_{i} \rightarrow J_{f}\right) \mathrm{eV}  \tag{5}\\
& \Gamma(\mathrm{E} 2)=8.076 \times 10^{-7}\left(E_{i}-E_{f}\right)^{5} B\left(\mathrm{E} 2, J_{i} \rightarrow J_{f}\right) \mathrm{eV} \tag{6}
\end{align*}
$$

where the $B(E \lambda)$ values are in units of $\mathrm{e}^{2} \mathrm{fm}^{2 \lambda}$ and energies in MeV .
The results of the calculation are displayed in column I of Table 1. The resulting $B(E 2)$ excitation probability of the $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$state from the ground state is about 300 times weaker than the known $B(E 2)$ value [36] for the transition to the first $2^{+}$ state. The calculation also indicates a large branching ratio for the decay of this state into the first $3^{-}$level. For the latter one has also a competing collective E3 transition with a large matrix element $\left\langle 3_{1}^{-}\|E 3\| 3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$corresponding to the one-phonon annihilation. However, because of the large multipolarity its contribution to the decay width is negligible.

Dealing with such a weak transition one has to consider possible small admixtures of other configurations which can be excited by strong transitions allowed in the boson space. The most important one in the region of the octupole two-phonon multiplet should be the $2_{1}^{+}$level, since the energy difference between these two states is estimated to be only a little more than 1 MeV , see Fig. 1. To take into account mixing in the $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$state the model Hamiltonian was diagonalized for a set of $2^{+}$wave functions

$$
\begin{equation*}
\left|\Psi_{2^{+}}\right\rangle=\left\{\sum_{i} S_{i} Q_{2^{+} i}^{+}+\sum_{\lambda_{1} i_{1} \lambda_{2} i_{2}} \frac{D_{\lambda_{1} i_{1}}^{\lambda_{1} i_{2}}\left[Q_{\lambda_{1} i_{1}}^{+} Q_{\lambda_{2} i_{2}}^{+}\right]_{2^{+}}}{\sqrt{1+\delta_{\lambda_{1}, \lambda_{2}} \delta_{i_{1}, i_{2}}}}\right\}\left|\Psi_{\mathrm{g} . \mathrm{s} .}\right\rangle, \tag{7}
\end{equation*}
$$

where the index $i$ labels RPA root numbers for a given multipolarity $\lambda^{\pi}$. In this calculation all one- and two-phonon configurations built of $\lambda^{\pi}=2^{+}, 3^{-}, 4^{+}$phonons up to 13 MeV (i.e. including the complete isoscalar giant quadrupole resonance) were taken into account. The diagonalization with the wave functions of Eq. (7) yields eigen values and coefficients $S_{i}, D_{\lambda_{1} i_{1}}^{\lambda_{2} i_{2}}$ expressing the contributions of each one- and two-phonon configuration in the wave function, respectively.

Similar calculations were performed for all members of the two-octupole multiplet and the resulting energies are displayed in Fig. 1. The relevant transition strengths are summarized in column II of Table 1. One finds a negligibly small contribution of the isoscalar quadrupole resonance in the low-energy region, but a few percent admixture of the $2_{1}^{+}$configuration to the $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$state increases the excitation probability roughly by a factor of four. The decay width to the $3_{1}^{-}$state is practically unchanged since the magnitude of the E1 transition matrix element $\left\langle 3_{1}^{-}\|M(\mathrm{E} 1)\| 2_{1}^{+}\right\rangle$is comparable to the matrix element $\left\langle 3_{1}^{-}\|M(\mathrm{E} 1)\| 3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$.

To illustrate the effect of an admixture of the $2_{1}^{+}$component in the wave function of the $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$consider the following simplified two-state model. Let us write the wave function of the $3_{1}^{-}$and $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$states as

$$
\begin{align*}
\left|\Psi_{\left|3_{1}^{-}\right\rangle}\right\rangle & =3_{1}^{-}\left|\Psi_{\text {g.s. }}\right\rangle,  \tag{8}\\
\left|\Psi_{\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle}\right\rangle & =\left\{\sqrt{\delta} 2_{1}^{+}+\sqrt{1-\delta}\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle\right\}\left|\Psi_{\text {g.s. }}\right\rangle, \tag{9}
\end{align*}
$$

where $\delta \ll 1$ (in our calculation it is $\approx 0.01$ ). We denote by the capital letter $M$ the matrix element of a collective transition corresponding to one-phonon exchange and use the small letter $m$ for weak transitions which are possible only due to rearranging of the fermions in the phonons. Then the $\gamma$-decay of the $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$state into the $3_{1}^{-}$state is determined by

$$
\begin{align*}
B\left(\mathrm{E} 1 ;\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow 3_{1}^{-}\right) & \left.\sim\left|\sqrt{\delta} m_{2_{1}^{+} \rightarrow 3_{1}^{-}}^{\mathrm{El}}+\sqrt{1-\delta} m_{\mid 3_{1}^{-}}^{\mathrm{El}} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow 3_{1}^{--}\right|^{2} \\
& \approx \mid m_{\left.\left|3_{1}^{-} \otimes 3_{1}^{-}\right|_{2^{+} \rightarrow 3_{1}^{-}}^{\mathrm{El}}\right|^{2}} \tag{10}
\end{align*}
$$

since $\left|m_{2_{1}^{+} \rightarrow 3_{1}^{-}}^{\mathrm{El}}\right| \approx\left|m_{\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow 3_{1}^{-}}^{\mathrm{E}}\right|$. On the other hand, for the $\gamma$-decay into the ground state

$$
\begin{equation*}
B\left(\mathrm{E} 2 ;\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow \mathrm{g} . \mathrm{s} .\right) \sim\left|\sqrt{\delta} M_{2_{1}^{2} \rightarrow \mathrm{g.s.}}^{\mathrm{E} 2}+\sqrt{1-\delta} m_{\left.\mid 3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right) \rightarrow \mathrm{g.s} .}^{\mathrm{E} 2}\right|^{2} \tag{11}
\end{equation*}
$$

one has $\left|M_{2_{1}^{+} \rightarrow \mathrm{g} . \mathrm{s} .}^{\mathrm{E} 2}\right|^{2} \gg\left|m_{\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow \mathrm{g} . \mathrm{s} .}^{\mathrm{E}}\right|^{2}$ and both terms give a contribution of the same order of magnitude. Thus, a small admixture of the $2_{1}^{+}$component affects the $\gamma$-decay properties of the two-phonon state into the ground state (see Table 1), but not into the one-phonon level.

The deduced transition probability for E1 decay $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow 3_{1}^{-}$is equal to $0.73 \times 10^{-3} \mathrm{e}^{2} \mathrm{fm}^{2}$. This is in good correspondence ${ }^{2}$ to experimental values $B\left(\mathrm{E} 1, \mid 3_{1}^{-} \otimes\right.$

[^1]$\left.\left.3_{1}^{-} ; 2^{+}\right\rangle \rightarrow 3_{1}^{-}\right)=0.95 \times 10^{-3} \mathrm{e}^{2} \mathrm{fm}^{2}$ and $1.20 \times 10^{-3} \mathrm{e}^{2} \mathrm{fm}^{2}$ for possible candidates reported in ${ }^{96} \mathrm{Zr}$ [8] and ${ }^{144} \mathrm{Sm}$ [10], respectively.

The QPM predictions presented here are mainly sensitive to the collectivity of the $3_{1}^{-}$state which determines the strength parameter of the residual interaction. To estimate how robust these predictions are we performed test calculations in which it was varied artificially by $\pm 30 \%$ with respect to the value determined by the experimental $B\left(\mathrm{E} 3\right.$, g.s. $\left.\rightarrow 3_{1}^{-}\right)$. The results are included in columns Ia, IIa and Ib, IIb of Table 1, respectively. The $\gamma$-width of the $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle$state for the decay into the ground state approximately changes by a factor of two for such a variation of the strength parameter while the decay width into the $3_{1}^{-}$state is stable within $20 \%$.

## 4. Proposed experimental test

The numbers presented in Table 1 can be converted to cross sections in a ( $\gamma, \gamma^{\prime}$ ) experiment. The experimental goal is to identify the transitions and to determine spins of the intermediate and final state from the angular correlations $(0 \rightarrow 2 \rightarrow 0$ and $0 \rightarrow 2 \rightarrow 3$ ) by measuring at two angles ( $90^{\circ}$ and $130^{\circ}$ ). Based on the recent experimental experience [26] the g.s. transition of the two-phonon $2^{+}$level should be easily measurable with a EUROBALL Cluster detector even for the least favourable case (column Ia). The observation of the $2^{+} \rightarrow 3^{-}$transition with roughly half the $\gamma$-energy is more challenging because of the steeply rising background in NRF experiments towards lower $\gamma$-energies.

A Monte Carlo simulation of the background expected in a ${ }^{208} \mathrm{~Pb}\left(\gamma, \gamma^{\prime}\right)$ experiment at a bremsstrahlung end point energy of 7 MeV was performed with the code GEANT [39]. Two EUROBALL Cluster detectors were assumed under $90^{\circ}$ and $130^{\circ}$ in a geometry very similar to the one described in Ref. [26]. The contributions of resonant nuclear excitations to the background were neglected. The incoming photon flux was normalized to correspond to one week of beam time under the experimental conditions described in Ref. [26]. The background spectrum resulting after consideration of the Cluster efficiency properties with its statistical fluctuations is displayed in Fig. 2 for a photon energy interval $E_{\gamma}=2.52-2.58 \mathrm{MeV}$. It can be well described by a simple exponential function (dashed line). The effect of a $\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow 3_{1}^{-}$transition with a photon energy of $5.16-2.62=2.54 \mathrm{MeV}$ (see Fig. 1) is demonstrated by the superposition of a Gaussian (dotted line). Its width is taken from the experimentally determined resolution of the Cluster module and the height is normalized to the partial decay width predicted by the QPM (column II of Table 1). The transition can be clearly identified above the background. At $90^{\circ}$, despite the somewhat poorer peak-to-background ratio this still holds true on a $3 \sigma$ level.

[^2]

Fig. 2. Monte Carlo simulation of a spectrum of the ${ }^{208} \mathrm{~Pb}\left(\gamma, \gamma^{\prime}\right)$ reaction for a bremsstrahlung end point energy of 7 MeV and a scattering angle of $130^{\circ}$ using a EUROBALL Cluster detector. The statistics of the "data points" presented in form of a histogram correspond to one week of beam time under the experimental conditions of Ref. [26]. The background is well described by a simple exponential function (dashed line) for the shown energy interval. The signal of the predicted $B\left(\mathrm{El},\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow 3_{1}^{-}\right)$transition at 2.546 MeV (dotted line) is simulated by a Gaussian with a width determined by the experimental resolution of the Cluster module and the height normalized to the decay width predicted by the QPM.

## 5. Conclusions

To summarize, we have discussed the feasibility of a new experimental approach to identify the long-sought two-phonon octupole vibration in ${ }^{208} \mathrm{~Pb}$. By measuring simultaneously the E2 decay to the g.s. and the E1 decay to the $3_{1}^{-}$level in the $\left(\gamma, \gamma^{\prime}\right)$ reaction, the $2^{+}$member of the $3_{1}^{-} \otimes 3_{1}^{-}$multiplet can be unambiguously identified. The description of the relevant transition probabilities has been derived in the QPM including mixing of one- and two-phonon configurations in the wave functions of excited states. The numerical results indicate that such an experiment should be possible with the new generation of large volume HPGe detectors like the EUROBALL Cluster module. At present, the only machine worldwide providing a sufficient bremsstrahlung flux in the energy range required is the Superconducting DArmstadt electron LINear ACcelerator S-DALINAC in Darmstadt. From the experimental information on the $0^{+}$and $2^{+}$states (eventually combined with new results on the $6^{+}$state from recent efforts to repeat the HI Coulomb excitation experiment of Ref. [22] with much improved sensitivity [40]) one may finally elucidate the degree of harmonicity of octupole vibrations in a doubly magic nucleus.

## Acknowledgements

We are indebted to J. Gerl and H.J. Wollersheim for illuminating discussions. V.Yu.P. thanks the International Science Foundation and the Russian Government (grant N6N300) and the RFFI (grant 95-02-05701) for financial support.

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[^0]:    * Work supported by the German Federal Minister for Education, Research and Technology (BMBF) under contract number 06DA 665 I, by the Heisenberg-Landau program and through a Max-Planck-Forschungspreis of Germany.
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[^1]:    ${ }^{2}$ Theoretically, this transition was studied for ${ }^{144} \mathrm{Sm}$ within nuclear field theory [37] in reasonable agreement with the data, and within the QPM [38] which disagrees by several orders of magnitude. Therefore, we

[^2]:    repeated the calculation for ${ }^{144} \mathrm{Sm}$ using the present QPM approach. The result is $B\left(\mathrm{E} 1,\left|3_{1}^{-} \otimes 3_{1}^{-} ; 2^{+}\right\rangle \rightarrow\right.$ $3_{1}^{-}$) $=2.9 \times 10^{-3} \mathrm{e}^{2} \mathrm{fm}^{2}$ in excellent agreement with Ref. [37] taking into account their smaller effective charges. We have no explanation for the extremely small value reported in Ref. [38].

