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LETTER TO THE EDITOR

Magnetic dipole and quadrupole states of ^{58}Ni

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Abstract. The cross sections for the inelastic scattering of electrons with excitation of the 1^+ and 2^- states of ^{58}Ni are calculated using the DWBA. Their structure is calculated within the quasiparticle–phonon nuclear model taking the interaction of one- and two-phonon states into account. The interaction is shown to be important for the explanation of the experimental data of Lindgren *et al.*

Several years ago Lindgren *et al* (1976) performed experiments on inelastic scattering of electrons with energy $E_0 = 40$–75 MeV on the isotopes ^{58}Ni and ^{60}Ni. The scattering at large angles was studied and, in ^{58}Ni, the 1^+ and 2^- states were observed in the excitation energy interval $5.5 < E_x < 11$ MeV. The theoretical calculations of Ponomarev *et al* (1979) within the semi-microscopic quasiparticle–phonon nuclear model are in good agreement with the results of Lindgren *et al* (1976). The properties of 2^- states in ^{58}Ni were shown to be greatly influenced by the interaction of one-phonon states with two-phonon states. However, Ponomarev *et al* studied the distribution of the reduced M1 and M2 transition probabilities, whereas the (e, e′) scattering cross sections were not calculated. It is evident that the calculation of the excitation cross sections of the states and the form factors is more informative and allows one to compare the theoretical calculations with the experimental data more accurately. These are the calculations presented in this Letter.

The structure of 1^+ and 2^- states has been calculated in the framework of the semi-microscopic quasiparticle–phonon nuclear model (QPM) (Soloviev 1976, 1978, 1979). It is assumed in this model that the nucleons move in the average field (which is traditionally approximated by the Saxon–Woods potential) and interact through the two-nucleon effective forces. The effective forces are chosen as a sum of monopole nn- and pp-pairing forces in the particle–particle channel and of separable multipole and spin-multipole forces with a simple radial dependence r^λ in the particle–hole channel. The residual forces of different multipolarities generate phonon excitations with different momenta and parities in doubly even nuclei. In this model the one-phonon M1 and M2 states are generated using the spin and spin-dipole forces, respectively.

The wavefunction of the one-phonon excitation $Q^+_{LMi}\Psi_0$ is a superposition of a certain number of two-quasiparticle components and is written through the creation and annihilation operators of quasiparticles (α^+_{jm}, α_{jm}) as follows:

$$Q^+_{LMi}\Psi_0 = \tfrac{1}{2}\sum_{j_1 j_2}(\psi^{Li}_{j_1 j_2}[\alpha^+_{j_1 m_1}\alpha^+_{j_2 m_2}]_{LM} - (-1)^{L-M}\varphi^{Li}_{j_1 j_2}[\alpha_{j_2 m_2}\alpha_{j_1 m_1}]_{L-M})\Psi_0. \quad (1)$$

Ψ_0 is the ground-state wavefunction of a doubly even nucleus. Depending on the relation

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of the amplitudes $\psi^{Li}_{j_1 j_2}$ determining the contribution of the different two-quasiparticle components to the phonon wavefunction, the phonon describes either the collective nuclear excitation (with several amplitudes $\psi^{Li}_{j_1 j_2}$ of the same order of magnitude) or the non-collective, purely two-quasiparticle nuclear excitation (when the contribution of one component is very large).

The approximation in which the phonon excitations of a doubly even nucleus are thought to be non-interacting is insufficient in many cases for an adequate description of the experimental data. The QPM allows one to take into account the interaction between the phonons H_{qph} (Soloviev 1976, 1978, 1979). We should like to note that this interaction is derived consistently within the model, has no special parameters introduced and is calculated on the basis of the microscopic phonon structure. In the first approximation the matrix elements H_{qph} between the states, which differ by one phonon, for instance

$$U^{\lambda_2 i_2}_{\lambda_1 i_1}(Li) = \langle \Psi_0 Q_{LMi} \| H_{\text{qph}} \| [Q^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2}]_{LM} \Psi_0 \rangle,$$

turn out to be different from zero. In this Letter the calculations have been made within this approximation. The excited-state wavefunction of a doubly even nucleus $\Psi_\nu(LM)$ has the form

$$\Psi_\nu(LM) = \left(\sum_i R_\nu(Li) Q^+_{LMi} + \sum_{\lambda_1 \lambda_2 i_1 i_2} P^{\lambda_2 i_2}_{\lambda_1 i_1}(L\nu) [Q^+_{\lambda_1 \mu_1 i_1} Q^+_{\lambda_2 \mu_2 i_2}]_{LM} \right) \Psi_0. \tag{2}$$

The equation for the energy $\eta_{L\nu}$ of the state $\Psi_\nu(LM)$ and the expression for the coefficient $R_\nu(Li)$ have been obtained by Vdovin *et al* (1974) and Soloviev *et al* (1977) and have the following forms:

$$\mathscr{F}(\eta) \equiv \det \left| (\omega_{Li} - \eta)\delta_{ii'} - \tfrac{1}{2} \sum_{\lambda_1 \lambda_2 i_1 i_2} \frac{U^{\lambda_2 i_2}_{\lambda_1 i_1}(Li) U^{\lambda_2 i_2}_{\lambda_1 i_1}(Li')}{\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - \eta} \right| = 0 \tag{3}$$

$$R^2_\nu(Li) = -\left(\frac{\partial}{\partial \eta} \frac{\mathscr{F}(\eta)}{M_{ii}} \right)^{-1} \Bigg|_{\eta = \eta_{L\nu}} \tag{4}$$

where M_{ii} is the sub-determinant from equation (3).

In calculating the (e, e′) scattering cross sections with excitation of the 1^+ or 2^- states, we shall use the method of strength functions (Soloviev *et al* 1977, Malov and Soloviev 1976). Let us denote by $(\mathrm{d}\sigma/\mathrm{d}\nu)_{Li}$ the electron inelastic cross section in which the state $Q^+_{LMi}\Psi_0$ is excited. Then, we determine the strength function $b_1(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta)$ as

$$b_1(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta) = \frac{\Delta}{2\pi} \sum_i \frac{1}{(\omega_{Li} - \eta)^2 + \frac{1}{4}\Delta^2} \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \right)_{Li}. \tag{5}$$

Analogously we introduce the strength function $b_2(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta)$ to describe the dependence on the excitation energy η of the average (e, e′) cross section, with excitation of the states described by the wavefunction (2):

$$b_2(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta) = \frac{\Delta}{2\pi} \sum_\nu \frac{1}{(\eta_{L\nu} - \eta)^2 + \frac{1}{4}\Delta^2} \left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \right)_{L\nu}. \tag{6}$$

It is not very difficult to calculate the value of $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{Li}$ even for all one-phonon states $Q^+_{LMi}\Psi_0$ with an excitation energy up to 30 MeV. This task is more complicated for the values of $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{L\nu}$ (i.e. the excitation cross sections of the states (2)) due to the complexity of the state structure and the very large number of these states. This is the advantage of the method of strength functions, because for the function $b_2(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta)$ one

can write down an analytical expression which does not depend explicitly on $\eta_{L\nu}$ and $R_\nu(Li)$.

For a clearer representation we derive the expression for the strength function $b_2(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta)$ when $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{L\nu}$ is calculated in the PWBA (note that in the following numerical calculations we used the DWBA). In the PWBA

$$(\mathrm{d}\sigma/\mathrm{d}\Omega)_{Li} = N_i(p_\mathrm{i}, p_\mathrm{f})|F_{Li}(q^2)|^2 \tag{7}$$

where $N_i(p_\mathrm{i}, p_\mathrm{f})$ is the kinematic coefficient depending on the incident p_i and outgoing p_f momenta of the electrons, $F_{Li}(q^2)$ is the form factor of the state $Q^+_{LMi}\Psi_0$ and $\boldsymbol{q}=\boldsymbol{p}_\mathrm{i}-\boldsymbol{p}_\mathrm{f}$. The current transition densities $\rho_{LL}(r)$ of one-phonon states are linear functions of the amplitudes $\psi^{Li}_{j_1j_2}$ and $\varphi^{Li}_{j_1j_2}$ (Vdovin *et al* 1980), and $F_{Li}(q^2)$ is a linear function of $\rho_{LL}(r)$. Hence, the form factor of the state (2) is expressed through the form factors of the one-phonon components contributing to its structure:

$$F_{L\nu}(q^2) = \sum_i R_\nu(Li)F_{Li}(q^2).$$

Then the expression for the strength function $b_2(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta)$ is

$$b_2(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta) = \frac{\Delta}{2\pi}\sum_\nu \frac{1}{(\eta_{L\nu}-\eta)^2+\frac{1}{4}\Delta^2}\left|\sum_i R(Li)F_{Li}(q^2)N_i^{1/2}(p_\mathrm{i}, p_\mathrm{f})\right|^2.$$

Using expression (4) for the coefficient $R_\nu(Li)$ and, based on the theorem of residues, changing Σ_ν to $\int \mathrm{d}\eta$ (Soloviev *et al* 1977, Malov and Soloviev 1976), we get for $b_2(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta)$ the final expression

$$b_2(\mathrm{d}\sigma/\mathrm{d}\Omega, \eta) = \frac{1}{\pi}\,\mathrm{Im}\left(\frac{\Sigma M_{ii'}(\eta+\frac{1}{2}\mathrm{i}\Delta)N_i^{1/2}N_{i'}^{1/2}F_{Li}(q^2)F_{Li'}(q'^2)}{\mathscr{F}(\eta+\frac{1}{2}\mathrm{i}\Delta)}\right). \tag{8}$$

Thus, for the calculation of the (e, e′) scattering cross section with excitation of the states, described by the wavefunction (2) in a certain excitation energy interval, we should calculate the function (8) at a number of points. From the computational point of view this task turns out to be much simpler than the calculation for each state of the $(\mathrm{d}\sigma/\mathrm{d}\Omega)_{L\nu}$ value. The parameter Δ is the value of the excitation energy interval over which the value of $(\mathrm{d}\sigma/\mathrm{d}\Omega)$ for a given state is 'smeared'. The choice of the value of Δ and the form of the weight function, which we have chosen as the Lorentz form, have been discussed in detail by Malov (1981).

The model Hamiltonian parameters, including the parameters of the single-particle Saxon–Woods potential, the constants of the proton–proton and neutron–neutron interactions G_Z and G_N and the constants of the isoscalar and isovector multipole and spin-multipole forces are taken to be the same as those of Ponomarev *et al* (1979) and Vdovin *et al* (1979). The (e, e′) scattering cross sections have been calculated using the DWBA (Tuan *et al* 1968). The current transition densities comprise the convective and magnetic components (Vdovin *et al* 1980, Lee 1975), the values of the effective gyromagnetic factors $g_\mathrm{s}^* = 0.8g_\mathrm{s}^{\mathrm{free}}$ and $g_\uparrow^* = g_\uparrow^{\mathrm{free}}$ coinciding with those we have used earlier. The value of Δ is 0.1 MeV.

Lindgren *et al* (1976) have investigated the excitation energy interval $5 < E_\mathrm{x} < 11$ MeV in ^{58}Ni. On the basis of the behaviour of the form factors of the individual, most strongly excited states, one can divide this interval into two parts: (1) $5.5 < E_\mathrm{x} < 9.3$ MeV in which the excited states have, as a rule, the quantum numbers $L^\pi = 2^-$ and (2) $9.3 < E_\mathrm{x} < 11.2$ MeV in which the 1^+ states are mainly excited. Table 1 shows the summed

Table 1. Experimental and theoretical values of the sum of the differential excitation cross sections $\Sigma_{i\in\Delta E_x}$ $(d\sigma/d\Omega)_i$ (nb sr^{-1}) of the 1^+ and 2^- states in ^{58}Ni for two excitation energy intervals ΔE_x and different values of the incident electron energy E_0. Scattering angle $\theta = 163°$.

	$\Delta E_x = 5.5$–9.3 MeV					$\Delta E_x = 9.3$–11.2 MeV				
		Theory					Theory			
		$L^\pi = 1^+$		$L^\pi = 2^-$			$L^\pi = 1^+$		$L^\pi = 2^-$	
E_0 (MeV)	Experiment	RPA	$Q^+ + Q^+Q^+$	RPA	$Q^+ + Q^+Q^+$	Experiment	RPA	$Q^+ + Q^+Q^+$	RPA	$Q^+ + Q^+Q^+$
40	43.4 ± 8.4	3.1	44.0	0.0	47.3	47.8 ± 4.0	84.7	34.2	48.6	18.6
50	39.8 ± 5.7	1.1	21.2	0.0	54.6	21.9 ± 3.3	41.1	16.4	56.7	21.3
60	35.0 ± 3.7	0.3	9.2	0.0	51.3	16.8 ± 2.5	18.1	7.1	53.7	19.8
75	26.6 ± 3.6	0.04	0.4	0.0	34.0	10.0 ± 2.7	0.6	0.2	35.0	14.6

cross section for all states studied by Lindgren *et al* (1976) in these intervals at different energy values E_0 of the incident electrons.

We now study the results of the calculations. The strength functions $b_1(d\sigma/d\Omega, \eta)$ and $b_2(d\sigma/d\Omega, \eta)$ for the M1 states (broken curves) and M2 states (full curves) are shown in figures 1(*a*) and (*b*). A comparison of figures 1(*a*) and (*b*) clearly shows that the interaction of one- and two-phonon states changes the dependence of the strength function on the excitation energy sharply. There is a considerable difference between the functions $b_1(\eta)$ and $b_2(\eta)$ for the states with $L^\pi = 2^-$: the energy centroid of the lowest group of 2^- levels with large excitation probabilities is lowered by 3 MeV; the one-phonon 2^- state with energy $E_x = 22$ MeV and with a large value of $(d\sigma/d\Omega)$ disappears completely by decomposing over complex states. The one-phonon states with $L^\pi = 1^+$ are also fragmented due to the interaction with the two-phonon states, but not as strongly as the 2^- states. These results completely coincide qualitatively with those obtained earlier by Ponomarev *et al* (1979) in analysing the distributions of B(M2) and B(M1) values.

The result of the changes described above in the distributions of the magnetic-state excitation probabilities is much better agreement between theoretical and experimental cross sections. The experimental data of Lindgren *et al* (1976) obtained for the excitation energies $E_x \leqslant 11$ MeV are shown schematically in figure 1(*c*) as a histogram with a summation interval of 200 keV. Such a representation of the experimental data stems from the fact that one cannot use the exact description with the wavefunction (2) for individual

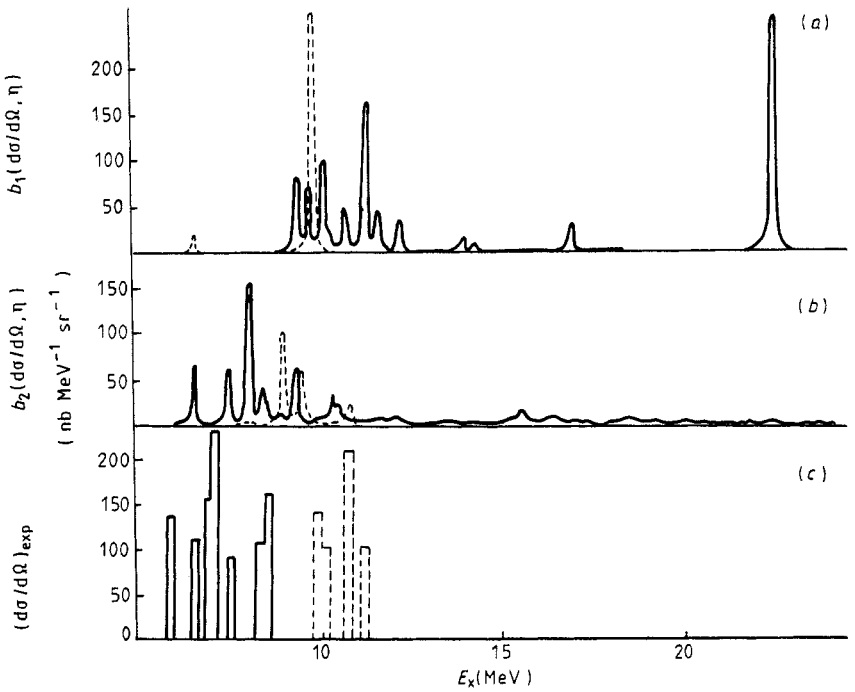

Figure 1. The inelastic scattering cross section of electrons with $E_0 = 50$ MeV at $\theta = 163°$ in ^{58}Ni with excitation of the 1^+ states (broken curves) and 2^- states (full curves). (*a*) Strength function $b_1(d\sigma/d\Omega, \eta)$ (see equation (5)), calculated in the RPA; (*b*) strength function $b_2(d\sigma/d\Omega, \eta)$ (see equation (8)), calculated taking the interaction of one- and two-phonon states into account; (*c*) the experimental cross section (Lindgren *et al* 1976) represented as a histogram (the summation interval is 200 keV).

states observed experimentally. The comparison of figures 1(*a*), (*b*) and (*c*) shows that the data of Lindgren *et al* (1976) can be explained by taking only the interaction of one- and two-phonon states into account (the same conclusion was reached by Ponomarev *et al* 1979). Now we consider the theoretical values of $d\sigma/d\Omega$ calculated in the RPA for different incident electron energies E_0 (see table 1). In the first excitation energy interval ($5.5 < E_x < 9.3$ MeV) the theoretical value of the summed cross section turns out to be less than the experimental value by more than an order of magnitude. At the same time in the second interval ($9.3 < E_x < 11.2$ MeV) the theoretical cross section is 3–5 times as large as the experimental one. The interaction of one- and two-phonon states redistributes the M1 and M2 strengths within the region studied ($5.5 < E_x < 11.2$ MeV), so that in both the intervals the summed value of the 1^+- and 2^--level excitation cross sections becomes rather close to the experimental value, though somewhat larger than it. This difference is obviously due to the fact that the experimental cross sections are obtained by summation of the most strongly excited 1^+ and 2^- states, rather than all the states.

We should like to note the importance in calculations such as ours of using a sufficiently large space of two-phonon states. Analogous calculations for ^{58}Ni, though in a somewhat different formalism, have recently been performed by Goncharova *et al* (1981). The results differ somewhat from our results (and from those obtained by Ponomarev *et al* 1979). In particular, there is no pronounced shift of the M2 strength to the region of lower excitation energy. The analysis showed that this is due first of all to the fact that

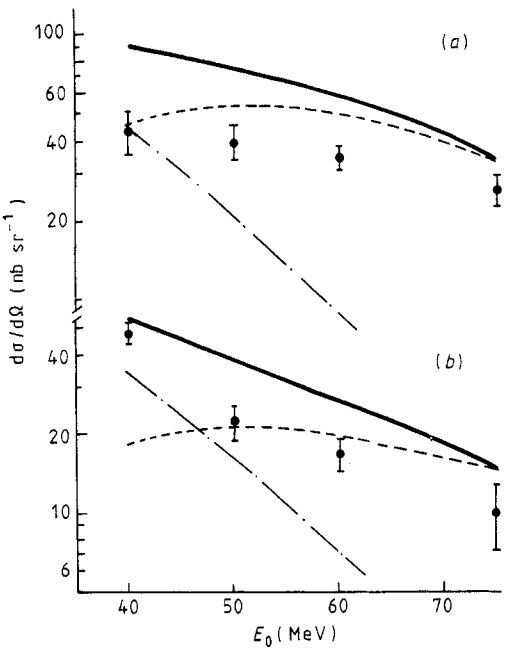

Figure 2. Dependence of the summed excitation cross sections of the states with $L^\pi = 1^+$ (chain curves) and $L^\pi = 2^-$ (broken curves) in different excitation energy intervals ΔE_x on the energy of incident electrons E_0. (*a*) $\Delta E_x = 5.5$–9.3 MeV; (*b*) $\Delta E_x = 9.3$–11.2 MeV. The full curves denote the sum of the 1^+ and 2^- excitation cross sections. The experimental points are obtained by summing over all the 1^+ and 2^- states found by Lindgren *et al* (1976) in the given interval ΔE_x.

Goncharova *et al* (1981) took into account the interaction with 2_1^+, 2_2^+ and 4_1^+ phonons only; this caused a sharp decrease in the space of complex states and an effective weakening of the interaction.

A more thorough analysis of the results illustrated in table 1 shows that in both intervals the relative contribution of the 2^- states to the summed cross section increases with increasing energy E_0. At $E_0 = 75$ MeV almost the whole summed cross section is exhausted by the 2^- states. This fact is seen clearly in figure 2. Thus, the presence of the 1^+ and 2^- states in both the intervals is important for the explanation of the behaviour of the summed cross section as a function of E_0. The contribution of the 1^+ states is especially important for small E_0, and for $E_0 > 60$ MeV the cross section is determined by the 2^- states. Referring to the conclusion made by Ponomarev *et al* (1979) about the predominant concentration of the 2^- states in the interval $5.5 < E_x < 9.3$ MeV and of the 1^+ states in the interval $9.3 < E_x < 11.2$ MeV, we should like to mention that they conform with the present results for small E_0.

Therefore, our calculations confirm once more the necessity of taking the interaction with complex configurations into account (Ponomarev *et al* 1979), in order to explain the results of Lindgren *et al* (1976), and the importance of taking as many complex components in the wavefunction (2) as possible into account. Our values for the backward (e, e′) scattering cross section are in agreement with the conclusions of Lindgren *et al* (1976) about the predominant localisation of the 2^- states in the interval $5.5 < E_x < 9.3$ MeV and of the 1^+ states in the interval $9.3 < E_x < 11.2$ MeV. However, they indicate that the presence of a certain number of 2^- states in the second interval allows one to explain the dependence of $(d\sigma/d\Omega)_{exp}$ on E_0.

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