Prediction for a four-neutron resonance

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We utilize various ab initio approaches to search for a low-lying resonance in the four-neutron (4n) system using the JISP16 realistic NN interaction. Our most accurate prediction is obtained using a J-matrix extension of the No-Core Shell Model and suggests a 4n resonant state at an energy near $E_r = 0.8$ MeV with a width of approximately $\Gamma = 1.4$ MeV.

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With interest sparked by a recent experiment [1] on the possibility of a resonant four neutron (4n) structure (see also [2] for a recent communication) and while awaiting for forthcoming experiments on the same system [3–5], we search for 4n (tetraneutron) resonances using the high precision nucleon-nucleon interaction JISP16 [6]. The experiment has found a candidate 4n resonant state with an energy of 0.83 ± 0.65 (stat) ± 1.25 (syst) MeV above the 4n disintegration threshold and with an upper limit of 2.6 MeV for the width. The 4n system was probed by studying the reaction between the bound 4He nucleus and the weakly bound Helium isotope, $^8$He. It has been shown [7] that the four neutrons in $^8$He form a relatively compact geometry. Hence the experimental study of the $^4$He+$^4$He collisions is a promising avenue for the isolation of the 4n subsystem.

The experimental quest for the very exotic 4n structure started almost fifteen years ago when the possibility of a bound 4n (or tetraneutron) was proposed [8] in $^{14}$Be breakup reactions ($^{14}$Be → $^{10}$Be + 4n). This experimental result however has not been confirmed. Early calculations of the 4n system in a small basis [9] found it unbound by about 18.5 MeV. More recent state-of-the-art theoretical calculations have concluded that without altering fundamental characteristics of the nuclear forces [10], the tetraneutron should not be bound. More theoretical calculations were performed [11, 12], all of them agreeing that a bound tetraneutron is not supported by theory. Calculations performed in the complex energy plane to search of multi-neutron resonances within the Complex Scaling Method [13–15] give quantitatively similar results and point to the fact that the 4n resonance, if it exists, would have a very large width (∼15 MeV), likely prohibitive for experimental detection. The tetraneutron could however exist if confined in a strong external field. In Nature, this would be the case of $^8$He, where the nuclear mean-field is strong enough to confine the tetraneutron around the tightly bound α-core. Once the field is suddenly removed by knocking out $^4$He, it is expected that the tetraneutron will disintegrate very fast due to its anticipated large width.

There is also a work [16] where the continuum response of the tetraneutron was studied. The outcome was that there exists a resonant-like structure at around 4–5 MeV above threshold, however this structure depends on the tetraneutron production reaction mechanism represented by the source term in this study, and the conclusion was that the 4n probably cannot be interpreted as a well-defined resonance but most probably as a few-body continuum response in a reaction.

Nevertheless, our current knowledge of nuclear interactions and many-body methods provide new opportunities to probe exotic states above thresholds. We are further motivated by the conclusion in Ref. [10] that even though the existence of a bound tetraneutron is ruled out, extrapolations of (artificially) bound state results to the unbound regime, imply that there may be a 4n resonance at about 2 MeV above the four-neutron threshold.

A complete investigation of the tetraneutron as a resonant state, would consist of performing calculations of the actual experimental reaction $^4$He($^4$He,$^8$Be). However, such a realistic calculation is currently out of reach, though we are witnessing the first steps for such theoretical calculations to become a reality [17, 18].

We treat the 4n system with a realistic non-relativistic Hamiltonian which consists of the kinetic energy and the realistic inter-neutron potential defined by the JISP16 interaction [6]. We solve for the 4n energies by employing basis expansion techniques for the Hamiltonian. Specifically, we employ the No-Core Shell Model (NCSM) [19] and artificially bind the 4n system by scaling the interaction to track its lowest state as a function of that scaling. We also employ the No-Core Gamow Shell Model (NCGSM) [20, 21] which provides resonant parameters directly in the complex energy plane. Finally, we extend NCSM using the Single-State Harmonic Oscillator Representation of Scattering Equations (SS-HORSE) for-

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nalism [22, 23] for calculations of the $S$-matrix resonant parameters.

First, to get an estimate of whether JISP16 can provide a $4n$ resonant state, we exploit the technique suggested in Ref. [10] and perform pure NCSM calculations by constructing an artificially bound $4n$ system by scaling up the NN interaction. Our extrapolations to the unbound regime are in quantitative agreement with Ref. [10] that predicts a resonance at around 2 MeV above threshold but without any indication of the width. We tried also a much more elaborate technique of Analytic Continuation in the Coupling Constant (ACCC) [24, 25]. The ACCC requires exact results for the $4n$ energy with scaled interactions while NCSM provides only variational energy upperbounds; extrapolations to the infinite basis space appear to lack the precision needed for a definite prediction of the resonance energy and width.

In order to shed further light on a possible $4n$ resonance, we solve the NCGSM with the JISP16 interaction. In the NCGSM one employs a basis set that is spanned by the Berggren states [26] which includes bound, resonant and non-resonant states; they correspond to solutions of the single particle (s.p.) Schrödinger equation obeying outgoing (bound-resonant states) and scattering (non-resonant states) boundary conditions. In this basis the Hamiltonian matrix becomes complex symmetric and its eigenvalues acquire both real and imaginary parts. The real part is identical to the position of the resonant state above the threshold and the imaginary part is related to its width $\Gamma = -2 \text{Im}(E)$. We adopt the basis provided by a Woods–Saxon (WS) potential for a neutron in relative motion with a $3n$ system. We modify the WS parameters in a way that it extends the WS parameterizations and independent of the $h\Omega$ truncation, i.e., it is nearly independent of the WS parameterizations and independent of the frequency of the HO basis.

At the same time, we observe that the resonance energy decreases together with the width as the NCGSM basis increases. Getting the converged resonance pole position in this approach requires the NCGSM basis spaces beyond our current reach.

Finally, following the $J$-matrix formalism in scattering theory [27] as represented in the HORSE method [28], we extend the finite NCSM Hamiltonian matrix in the harmonic oscillator (HO) basis into the continuum by appending to it the infinite kinetic energy matrix.

For the kinetic energy extension of the NCSM Hamiltonian, we use the democratic decay approximation (also known as true four-body scattering or $4 \rightarrow 4$ scattering suggested [29, 30]) and first applied to the tetraneutron problem [31–33] by Jibuti et al. Later it was exploited in other tetraneutron studies (see, e.g., Refs. [13, 16, 34, 35]). Democratic decay implies a description of the continuum using a complete hyperspherical harmonics (HH) basis. In practical applications, a limited set of HH is selected which is adequate for the systems like the $4n$ which has no bound subsystems.

The general theory of the democratic decay within the HORSE formalism was proposed in Ref. [36]. We use here the minimal approximation for the four-neutron decay mode, i.e., only HH with hyperspherical momentum $K = K_{\text{min}} = 2$ are retained in the kinetic energy extension to the NCSM. This approximation relies on the fact that the decay in the hyperspherical states with $K > K_{\text{min}}$ is strongly suppressed by a large hyperspherical centrifugal barrier $\frac{\rho^2}{\rho^2+1}$ where the effective momentum $\mathcal{L} = K + 3$ and the hyperradius $\rho^2 = \sum_{i=1}^{4}(r_i - \mathbf{R})^2$, $\mathbf{R}$ is the tetraneutron center-of-mass coordinate and $r_i$ are the coordinates of individual neutrons. Note, all possible HH are retained in the NCSM basis. The accuracy of this approximation was confirmed in studies of democratic decays in cluster models [37–40].

Realistic $NN$ interactions require large NCSM basis spaces and extensive computational resources. For computational economy, we also adopt the SS HORSE approach [22, 23] where we calculate the $4 \rightarrow 4$ $S$-matrix $S(E)$ at one of the positive eigenenergies of the NCSM Hamiltonian, $E = E_{\lambda}$. In this case, the general HORSE formula for the $S$-matrix simplifies: expressing $S(E)$ through the $4 \rightarrow 4$ phase shifts $\delta(E)$,

$$S(E) = e^{2i\delta(E)},$$

we obtain for the phase shifts [22, 23]

$$\delta(E_{\lambda}) = -\tan^{-1} \frac{S_{N_{\text{tot}}_{\text{max}} + 2, \mathcal{L} = 12} (E_{\lambda})}{C_{N_{\text{tot}}_{\text{max}} + 2, \mathcal{L} = 12} (E_{\lambda})}.$$  

Here the maximal total quanta in the NCSM basis $N_{\text{max}} = N_{\text{min}} + N_{\text{max}}$, $N_{\text{min}} = 2$ is the quanta of the
lowest possible oscillator state of the 4n system, $N_{\text{max}}$ is the maximal excitation quanta in the NCSM basis; analytical expressions for the regular $S_{N,L}(E)$ and irregular $C_{N,L}(E)$ solutions of the free many-body Hamiltonian in the oscillator representation can be found elsewhere [36]. Varying $N_{\text{max}}$ and $\hbar \Omega$ in the NCSM calculations, we obtain the phase shifts and S-matrix over an energy interval. Parametrizing the S-matrix in this energy interval, we obtain information about its nearby poles and hence resonances in the system.

The NCSM calculations were performed with $N_{\text{max}} = 2, 4, ..., 18$ using the code MFDn [41, 42] and with $\hbar \Omega$ values, $1 \text{ MeV} \leq \hbar \Omega \leq 40 \text{ MeV}$. The results for the $0^+$ tetraneutron ground state are shown in the upper panel of Fig. 1.

The convergence patterns of the NCSM SS-HORSE approach to the $4 \rightarrow 4$ phase shifts using Eq. (2) are shown in the lower panel of Fig. 1. We observe that the phase shifts tend to the same curve when $N_{\text{max}}$ is increased. The convergence is first achieved at the higher energies while larger $N_{\text{max}}$ yield converged phase shifts at smaller energies. We obtain nearly completely converged phase shifts at all energies with $N_{\text{max}} = 16$ and 18.

We need only phase shifts close to convergence for the phase shift parametrization. Our selected NCSM eigenenergies are enclosed by the shaded area on the top panel of Fig. 1 since their resulting phase shifts form a single smooth curve (see Figs. 2 and 3).

We will describe now how we utilize the NCSM solutions within the SS-HORSE method in order to obtain resonance positions. Due to the S-matrix symmetry...
property, \( S(k) = 1/S(-k) \), and Eq. (1), the \( 4 \rightarrow 4 \) phase shift \( \delta(E) \) is an odd function of momentum \( k \) and its expansion in Taylor series of \( \sqrt{E} \sim k \) includes only odd powers of \( \sqrt{E} \):

\[
\delta(E) = v_1 \sqrt{E} + v_3 (\sqrt{E})^3 + ... + v_{11} (\sqrt{E})^{11} + ... \quad (3)
\]

Furthermore, the \( 4 \rightarrow 4 \) phase shifts at low energies, i.e., in the limit \( k \rightarrow 0 \), should behave as \( \delta \sim k^{2L+1} \). Note, in our case, \( L = K_{\min} + 3 = 5 \), hence \( v_1 = v_3 = ... = v_9 = 0 \) and expansion (3) starts at the 11th power.

Supposing the existence of a low-energy resonance in the \( 4n \) system, we express the \( S \)-matrix as \( S(E) = \Theta(E) S_r(E) \), where \( \Theta(E) \) is a smooth function of energy \( E \) and \( S_r(E) \) is a resonant pole term. The respective phase shift is

\[
\delta(E) = \phi(E) + \delta_r(E), \quad (4)
\]

where the pole contribution \( \delta_r(E) \) takes the form

\[
\delta_r(E) = -\tan^{-1}(a\sqrt{E}/(E-b^2)). \quad (5)
\]

The resonance energy \( E_r \) and width \( \Gamma \) are expressed through parameters \( a \) and \( b \) entering Eq. (5) as

\[
E_r = b^2 - a^2/2, \quad \Gamma = 2a\sqrt{b^2-a^2}/4. \quad (6)
\]

We use the following expression for the background phase:

\[
\phi(E) = \frac{w_1 \sqrt{E} + w_3 (\sqrt{E})^3 + c(\sqrt{E})^5}{1 + w_2 \sqrt{E} + w_4 \sqrt{E} + w_6 \sqrt{E}^3 + d\sqrt{E}^4}. \quad (7)
\]

The parameters \( w_i, i = 1, 2, 3, 4, 6 \) are uniquely defined through the parameters \( a \) and \( b \) and guarantee the cancellation of the terms of powers up to 9 in the expansion (3).

Our phase shift parametrization is given by Eqs. (4), (5) and (7) with fitting parameters \( a, b, c \) and \( d \). For each parameter set, we solve Eq. (2) to find the values of the energies \( E^{a,b,c,d}_\lambda \) and search for the parameter set \( (a, b, c, d) \) minimizing the rms deviation of \( E^{a,b,c,d}_\lambda \) from the selected set of NCSM eigenenergies \( E_\lambda \). Following this route, we obtain an excellent description of the selected \( E_\lambda \) with an rms deviation of 5.8 keV with \( a = 0.724 \text{ MeV}^{-1/2}, b^2 = 0.448 \text{ MeV}, c = 0.941 \text{ MeV}^{-2}, \) and \( d = -9.1 	imes 10^{-4} \text{ MeV}^{-4} \). The resulting predictions for the NCSM eigenenergies are shown by solid lines in the upper panel of Fig. 1 where we also describe well NCSM energies with large enough \( N_{\text{max}} \) and/or \( h\beta \) not included in the minimization fit. We obtain also an excellent description of NCSM-SS-HORSE predicted phase shifts as is shown by solid line in Fig. 2.

However the resonance parameters describing the location of the \( S \)-matrix pole obtained by this fit, are surprisingly small: the resonance energy \( E_r = 0.186 \text{ MeV} \) and the width \( \Gamma = 0.815 \text{ MeV} \). Note, looking at the phase shift in Fig. 2, we would expect the resonance at the energy of approximately 0.8 MeV corresponding to the maximum of the phase shift derivative and with the width of about 1.5 MeV. The contribution of the pole term (5) to the phase shifts is shown by the dashed line in Fig. 2. This contribution is seen to differ considerably from the resulting phase shift due to substantial contributions from the background phase (7) which is dominated by the terms needed to fulfill the low-energy theorem \( \delta \sim k^{2L+1} \) and to cancel low-power terms in the expansion of the resonant phase \( \delta_r(E) \). Such a sizable contribution from the background in the low-energy region, impels us to search for additional poles or other singularities giving rise to a strong energy dependence which would be separate from the background phase.

After we failed to find a reasonable description of the NCSM-SS-HORSE phase shifts with a low-energy virtual state, we found the resolution of the strong background phase problem by assuming that the \( S \)-matrix has an additional low-energy false pole at a positive imaginary momentum [43]. We add the false term contribution

\[
\delta_f(E) = -\tan^{-1}\sqrt{E/|E_f|} \quad (8)
\]

to the phase shift to obtain the equation

\[
\delta(E) = \phi(E) + \delta_r(E) + \delta_f(E) \quad (9)
\]

replacing Eq. (4). This parametrization involves an additional fitting parameter \( E_f \). We obtain nearly the same quality description of the selected \( 4n \) ground state energies with the rms deviation of 6.2 keV with the parameters \( a = 0.701 \text{ MeV}^{-1/2}, b^2 = 1.089 \text{ MeV}, c = -27.0 \text{ MeV}^{-2/3}, d = 0.281 \text{ MeV}^{-4}, \) and a low-lying false pole at energy \( E_f = -54.9 \text{ keV} \). The respective \( 4n \) resonance at \( E_r = 0.844 \text{ MeV} \) and width \( \Gamma = 1.378 \text{ MeV} \) appears consistent with what is expected from directly inspecting the \( 4n \) phase shifts. The parametrized phase shifts are shown by solid line in Fig. 3 together with separate contributions from the resonant and false pole terms. We note that corrections introduced by this new parametrization to the solid lines in Figs. 1 and 2 are nearly unseen in the scales of these figures.

Conclusions. Our results with the realistic JISP16 interaction and the SS-HORSE technique show there is a resonant structure near 0.8 MeV above threshold with a width \( \Gamma \) of about 1.4 MeV. This is the first theoretical calculation that predicts such a low energy \( 4n \) resonance, without altering any of the properties of the realistic \( NN \) interaction. Our result is compatible with the recent experiment which found a resonant structure near 0.8 MeV, 378 MeV, and a low-lying false pole at energy \( E_f = -54.9 \text{ keV} \). The respective \( 4n \) resonance, nearly unseen in the scales of these figures.
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[43] A. I. Baz’, Ya. B. Zel’dovich, and A. M. Perelomov, Scattering, Reactions and Decay in Non-Relativistic Quan-