Systematics of 2^+ states in C isotopes from the *ab initio* no-core shell model

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Abstract. We study low-lying states of even-even carbon isotopes in the range $A = 10 - 20$ within the large-scale *ab initio* no-core shell model (NCSM). Using several accurate nucleon-nucleon (NN) as well as NN plus three-nucleon (NNN) interactions, we calculate excitation energies of the lowest 2^+ state, the electromagnetic $B(\mathrm{E2};\ 2_1^+ \rightarrow 0_1^+)$ transition rates, the 2_1^+ quadrupole moments as well as selected electromagnetic transitions among other states. Recent experimental campaigns to measure 2^+-state lifetimes indicate an interesting evolution of nuclear structure that pose a challenge to reproduce theoretically from first principles. Our calculations do not include any effective charges or other fitting parameters. Overall, we find a good agreement with the experimentally observed trends, although our calculated $B(\mathrm{E2};\ 2_1^+ \rightarrow 0_1^+)$ value for ^{16}C is lower compared to the most recent measurements. Relative transition strengths from higher excited states are investigated and the influence of NNN forces is discussed. In particular for ^{16}C we find a remarkable sensitivity of the transition rates from higher excited states to the details of the nuclear interactions.

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1. Introduction

Electric quadrupole (E2) matrix elements are important quantities in probing nuclear structure. In particular, they are very sensitive to nuclear deformation, the decoupling of proton and neutron degrees of freedom, and they are often affected by small components of the nuclear wave functions. In this paper we perform systematic studies of observables obtained from diagonal and non-diagonal E2 matrix elements for even carbon isotopes, from ^{10}C to the very neutron rich ^{20}C. Quadrupole moments, corresponding to diagonal E2 matrix elements, are inherently difficult to measure for excited 2^+ states. Off-diagonal matrix elements, however, have recently been studied for several unstable carbon isotopes using lifetime measurements [1, 2, 3, 4, 5, 6]. In this way, the reduced transition probability, $B\left(\mathrm{E2}; 2_1^+ \rightarrow 0_1^+\right)$, can be extracted since it's inversely proportional to the lifetime of the 2^+ state. As a result of these experimental studies, different claims have been made on the nuclear structure in this chain of isotopes. Initial excitement was triggered by the observation of a strongly quenched E2 transition in ^{16}C [2]. Based on the liquid-drop model, which predicts the B(E2) to be inversely proportional to the 2^+ excitation energy, Imai *et al.* [2] claimed an anomalous reduction of the E2 strength when comparing 2^+ lifetimes for ^{14}C ($E_{2^+} = 7.01$ MeV) and ^{16}C ($E_{2^+} = 1.77$ MeV). However, the ^{16}C(2^+) lifetime was remeasured by Wiedeking *et al.* [3] providing a much shorter value, thus indicating a larger B(E2) strength. Their results were analyzed in terms of shell-model calculations. Adjusting the effective neutron charge to reproduce their measured lifetimes they made the claim that the results for ^{16}C are "normal" to this region. Lifetime measurements of ^{16,18}C were reported by Ong *et al.* [4]. The presented results for ^{16}C came from a reanalysis of the original data [2], now giving a larger but still quenched B(E2) strength, while the new ^{18}C data indicated the persistence of the quenching of E2 strengths in heavy carbon isotopes. Possible explanations were put forward in terms of the decoupling of protons and neutrons resulting in very low values for the neutron effective charges and/or the appearance of a new proton magic number $Z = 6$ in this region. Some of these statements were backed up by new shell-model calculations by Fujii *et al.* [7] reproducing the ^{16,18}C results employing exceptionally small effective charges. An alternative explanation in terms of core polarization effects was recently proposed by Ma *et al.* [8]. They used a microscopic particle-vibration approach to compute core polarization effects on valence nucleons. In contrast with empirical effective charges, usually employed in shell-model calculations, they noted a very strong quenching from core polarization on *sd*-shell neutrons for heavy carbon isotopes.

These various developments provide a strong motivation to perform *ab initio* calculations, without fitting parameters, to study the evolving nuclear structure in the carbon chain of (even) isotopes with particular focus on 2^+ states and quadrupole moments. We have, therefore, carried out large-scale *ab initio* no-core shell model (NCSM) [9, 10] calculations for low-lying states of the even-even carbon isotopes with $A = 10 - 20$. These calculations are performed starting from realistic

Hamiltonians without adjustable parameters. In particular, since our many-body scheme does not involve an inert-core approximation we use bare charges when evaluating electromagnetic observables. In addition, particular efforts are made to quantify the uncertainties of the calculated results.

1.1. Theoretical formalism

The NCSM method has been described in great detail in several papers, see e.g., Ref. [10]. Here, we just outline the approach as it is applied in the present study. We start from the intrinsic Hamiltonian for the A-nucleon system $H_A = \mathcal{T}_{\rm rel} + \mathcal{V}$, where $\mathcal{T}_{\rm rel}$ is the relative kinetic energy and $\mathcal{V}$ is the sum of nuclear and Coulomb interactions. The potential term will always contain two-body operators, but we can also include three-body terms originating from an initial *NNN* force, or three-body terms induced by a unitary transformation of the Hamiltonian. This transformation, further described below, is employed to soften the Hamiltonian for use in a truncated many-body basis.

In this work we have used several different nuclear Hamiltonians: the pure *NN* interactions CD-Bonn 2000 [11] (CDB2k), based on one-boson exchange theory, and INOY [12] that introduces a nonlocality to include some effects of three-nucleon forces. The latter is fitted also to three-nucleon obervables. In addition, we have used the most recent chiral *NN* plus *NNN* interaction, i.e. the N^{3}LO *NN* interaction of Ref. [13] and a local chiral N^{2}LO *NNN* potential with low-energy constants determined entirely in the three-nucleon system [14]. All three Hamiltonians reproduce *NN* phase shifts with very high precision.

We solve the many-body problem in a large but finite harmonic-oscillator (HO) basis truncated by a maximal total HO energy of the A-nucleon system. The many-body model space is usually characterized by the truncation parameter $N_{\rm max}$, giving the maximum number of HO excitations above the unperturbed A-nucleon ground state. The diagonalization of the Hamiltonian in this many-body basis is a highly non-trivial problem because of the very large dimensions that is encountered. To solve this problem, we have used a specialized version of the shell model code Antoine [15], adapted to the NCSM [16]. For the runs involving explicit *NNN* interactions we used the NCSD code [17] as well as the NSuite package [18, 19], which is also capable of performing the importance-truncated NCSM calculations described below.

Due to the strong short-range correlations generated by the *NN* potentials, we usually compute an effective interaction to speed up the convergence. Two different similarity transformations have been used to construct the effective interactions: For CDB2k and INOY as initial *NN* interactions we compute two-body effective interactions appropriate to the low-energy basis truncation by a unitary transformation in the two-nucleon HO basis (Lee-Suzuki effective interaction [9, 20]). For the chiral $NN+NNN$ Hamiltonian we employ the similarity renormalization group (SRG) with the initial and induced three-body terms included consistently [19, 21].

Our results exhibit dependence on $N_{\rm max}$ and the HO frequency Ω that should

disappear once a complete convergence is reached. This implies that $N_{\max}$-sequences obtained at different HO frequencies should all converge to the same result. This feature can be utilized to perform a constrained fit to multiple sequences [22, 23]. To this end, we use as large an $N_{\max}$ basis as feasible for a wide range of HO frequencies, and extrapolate calculated observables to infinite space. Results obtained for a range of frequencies are used in the fits. We find that the convergence behaviour for energy observables, as a function of $N_{\max}$, can be well fitted by: $x = x_\infty + c_0 \exp(-c_1 N_{\max})/N_{\max}$, and for EM observables by $x = x_\infty + c_0/N_{\max} + c_1/N_{\max}^2$. The parameters c_0 and c_1 are allowed to vary for each $N_{\max}$-sequence, while the single parameter x_∞ gives the extrapolated result at $N_{\max} \to \infty$. Typically we use a range of five HO frequencies for these constrained fits. Finally, an error estimate is made based on repeating the constrained fit keeping various subsets (pairs and triples) of frequencies in the selected range.

In this work, the limit on full-$N_{\max}$-space calculations with two-body Hamiltonians is $N_{\max} = 6$ for ^{18}C with a dimension of 1.4×10^9. To reach still larger $N_{\max}$-spaces we employ the importance-truncated (IT) NCSM scheme [18, 24]. It makes use of the fact that many of the basis states are irrelevant for the description of a set of low-lying states. Based on many-body perturbation theory, one can define a measure for the importance of individual basis states and discard states with an importance measure below a threshold value, thus reducing the dimension of the matrix eigenvalue problem. Through a sequence of IT calculations for different thresholds and an a posteriori extrapolation of all observables to vanishing threshold, we can recover the full NCSM results up to extrapolation errors [18]. For ^{16,18}C (^{12}C) the IT scheme allows us to extend our calculations to $N_{\max} = 8(6)$ and to improve the reliability of our extrapolations.

2. Results

2.1. Convergence and error estimates

For our detailed studies of observables we are looking for the regions in which the $N_{\max}$-convergence is the fastest and the dependence on $\hbar\Omega$ is the smallest. This optimal frequency range can vary between different observables and different isotopes. We will use the $N_{\max}$-dependence of the binding energy and the first 2^+ excitation energy in the largest model spaces as our primary criterion for selecting the optimal frequency range. The largest model spaces that we are able to reach in the full-space NCSM calculations span from $N_{\max} = 10$ in ^{10}C, via $N_{\max} = 8$ in ^{12,14}C, to $N_{\max} = 6$ in ^{16,18}C and $N_{\max} = 4$ in ^{20}C. The largest matrix dimension was $D = 1.4 \times 10^9$ for ^{18}C. However, using the IT-NCSM scheme we are able to obtain results also with $N_{\max} = 8$ for ^{16,18}C and $N_{\max} = 6$ for ^{12}C. From plots such as Fig. 1, focusing in particular on the trend for large model spaces, we find that the $\hbar\Omega$-range 10-14 MeV is optimal for all considered observables using the CDB2k-interaction and for the whole range of carbon isotope.

Figure 2 shows several examples of the constrained-fit procedure for quadrupole

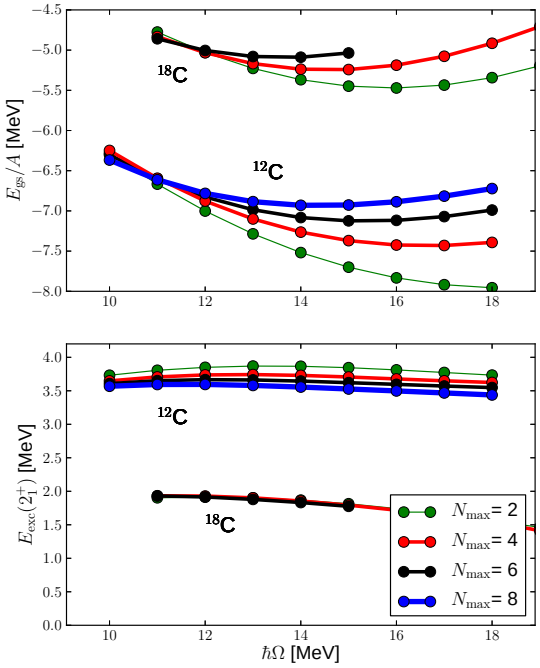

Figure 1. (Colour online) $\hbar\Omega$-dependence for the ground-state energy (presented as E_{gs}/A) and the first 2^+ excitation energy for ^{12,18}C. Each curve corresponds to a particular model space represented by the truncation parameter N_{max} (see text for details).

moments and B(E2) strengths. The results are plotted as a function of $1/N_{max}$ for the selected range of HO frequencies. Infinite model space corresponds to $1/N_{max} \to 0$. We note that the use of a range of frequencies usually include sequences that converge from above and from below. This allows a more precise determination of the extrapolated, final result. A particular exception to this behaviour is the B(E2) strength of ^{16}C, for which all sequences converge from below. This will be further commented below.

2.2. Binding energies and 2^+ *state properties*

In Figs. 3 and 4 we compare the calculated and experimental trends for binding energies, 2_1^+ excitation energies, and E2 observables for the carbon chain of isotopes. It is clear that the CDB2k interaction underbinds these isotopes by 10-20 % while the INOY interaction provides additional binding. The positive two-neutron separation energy for ^{16}C is not reproduced with any of these two realistic *NN* interactions. However, the many-body HO basis still provides a bound-state approximation to these states, and the additional binding provided, e.g., by *NNN* interactions will not necessarily change their structure (see the discussion in Sec. 2.3). We note that excitation energies are well converged, and we find a very good agreement with the experimental trend. The possible exception is the large 2_1^+ excitation energy of ^{14}C that is overpredicted with the

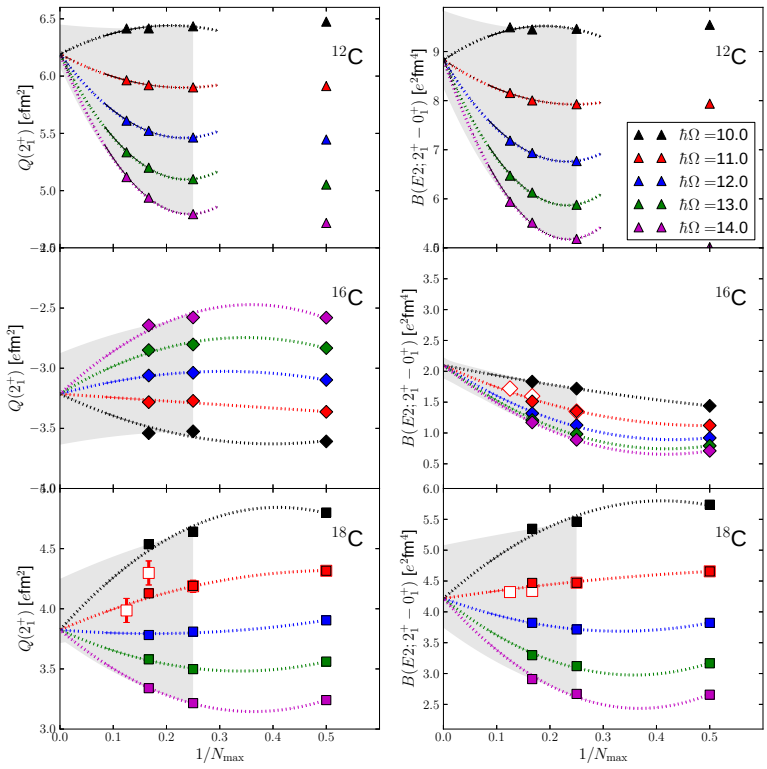

Figure 2. (Colour online) Model-space dependence of calculated E2 observables for ^{12,16,18}C in the NCSM. Results obtained with the CDB2k *NN* potential are presented as a function of $1/N_{\max}$. Filled (open) symbols correspond to full (importance-truncated) space results. Dotted lines correspond to constrained fits as described in the text. See also Table 1.

INOY interaction.

Numerical results for 2_1^+ excitation energies and E2 observables are presented in Table 1. We stress that no effective charges were used in our NCSM calculations and that there are no adjustable parameters when obtaining these results. Our calculated $B\left(\mathrm{E2}; 2_1^+ \to 0_1^+\right)$ agrees rather well with the most recent experimental data for the entire chain of isotopes. We note that the extrapolation of our ^{16}C $B\left(\mathrm{E2}; 2_1^+ \to 0_1^+\right)$ results is particularly difficult. Unlike the trends for other carbon isotopes, the B(E2) value increases with $N_{\max}$ in the whole investigated HO frequency range (see Fig 2). This makes the upper bound less constrained. Our extrapolated result is slightly below the most recent experimental results from LBNL [3, 26]. In addition, we note that our calculated quadrupole moment for the first 2^+ state of ^{16}C is $Q = -3.21 \pm 0.19$ $e$fm^2 while for $A = 12, 14, 18, 20$ we find the quadrupole moment of the 2_1^+ state to be positive.

A qualitative understanding of these findings can be obtained by studying the mean occupation numbers of different single-particle states in the NCSM wave functions. In

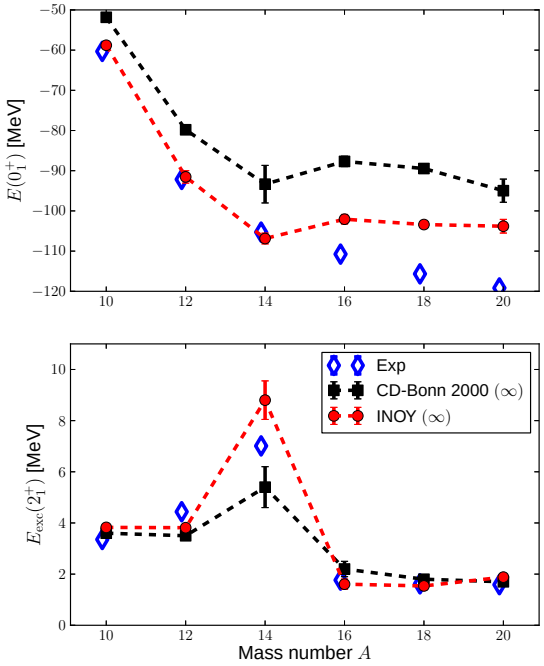

Figure 3. (Colour online) NCSM calculated binding energies and 2^+ excitation energies of $^{10-20}$C compared with experimental results. Theoretical NCSM results are obtained from constrained extrapolations (see text for details).

Table 1. NCSM calculated E2 observables of $^{10-20}$C compared with experimental results. NCSM results are obtained using the CDB2k interaction. The recommended values are obtained from constrained extrapolations (see text for details).

	$E_{\rm exc}\,(2_1^+)$ [MeV]		$Q\,(2_1^+)$ [$e$fm^2]		$B\,({\rm E2};\,2_1^+ \to 0_1^+)$[$e^2$fm^4]	
	CDB2k	Exp	CDB2k	Exp	CDB2k	Exp
^{10}C	3.6(1)	3.354	-1.1 ± 1.2 [a]	—	10 ± 2^a	$8.8(3)^b$
^{12}C	3.5(1)	4.439	+6.2(2)	+6(3)	8.8(7)	7.94(66)
^{14}C	5.4(8)	7.012(4)	+4.7(4)	—	5.3(7)	3.74(50)
^{16}C	2.2(3)	1.766(10)	-3.2(3)	—	2.2(6)	2.6(9), 4.15(73)^c
^{18}C	1.8(1)	1.620(20)	+3.8(2)	—	4.2(4)	4.3(1.2), $3.64^{+0.55}_{-0.61}$[d]
^{20}C	1.7(1)	1.588(22)	+4.3(6)	—	4.8(1.1)	< 5.7, $7.5^{+3.2}_{-1.8}$[e]

[a] Strong mixing of the first two 2^+ states. The estimate of ^{10}C E2 observables is obtained by studying the sums and ratios of results for both 2^+ states.
[b] From Ref. [1].
[c] From Refs. [4], [3], respectively.
[d] From Refs. [4], [25], respectively.
[e] From Refs. [5], [6], respectively.

Fig. 5 these occupancies are plotted for the ground- and first 2^+-state for the whole range of carbon isotopes. In particular for ^{14,16}C the excitation mechanisms are quite obvious. In ^{14}C the 2_1^+ state corresponds to a proton excitation within the p shell, while

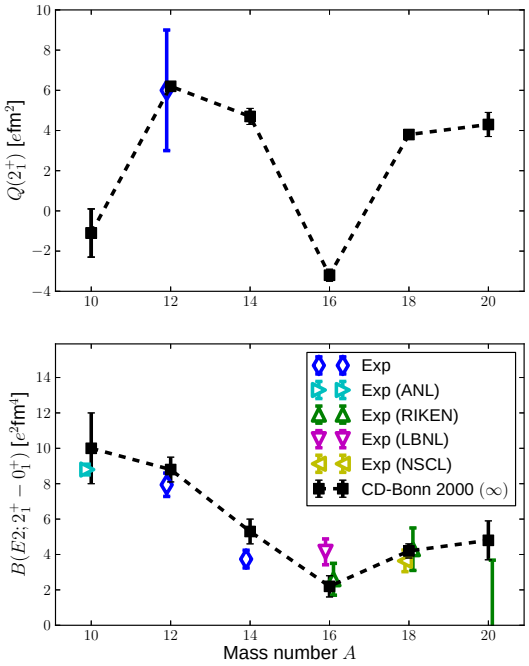

Figure 4. (Colour online) NCSM calculated E2 observables of $^{10-20}$C compared with experimental results. See also Table 1. NCSM results are obtained from constrained extrapolations (see text for details).

in ^{16}C the 2_1^+ state is obtained through a re-configuration of neutrons in the *sd* shell. The value of the B(E2), for this particular case, will be quite sensitive to the fine details of this re-configuration. Energy observables, on the other hand, are not as sensitive to these small nuclear-structure details and can therefore be expected to converge faster than the B(E2).

For ^{10}C we observe a very strong mixing of the first two 2^+ states using the CDB2k interaction at small frequencies. To get at least crude estimates of the E2 properties of the 2_1^+ state we used a slightly different extrapolation approach: The ratios of, e.g., $Q(2_1^+)$ and $Q(2_2^+)$ was plotted for larger frequencies where the mixing is not observed, while the sum was plotted for the full range of frequencies. From such plots, for Q and B(E2) observables, we can deduce estimates for $Q(2_1^+)$ and $B\left(\mathrm{E2}; 2_1^+ \to 0_1^+\right)$ and their uncertainties. These are included in Table 1 and Fig. 4.

We note that the different *NN* interactions used in this study give very similar isotopic trends for E2 observables, but with a consistently smaller magnitude for the INOY interaction. This observation is connected to the anomalously large nuclear density generated by this interaction found already in ^{4}He calculations [27, 28].

Finally, a study of the characteristics of the second 2^+ state in these isotopes strengthens the conclusion of the prominence of ^{16}C in the structural evolution of the chain of even carbon isotopes. The sign of the quadrupole moment of this state, $Q(2_2^+)$, is reversed from $Q(2_1^+)$. I.e., it's negative for all isotopes except for ^{16}C. In addition,

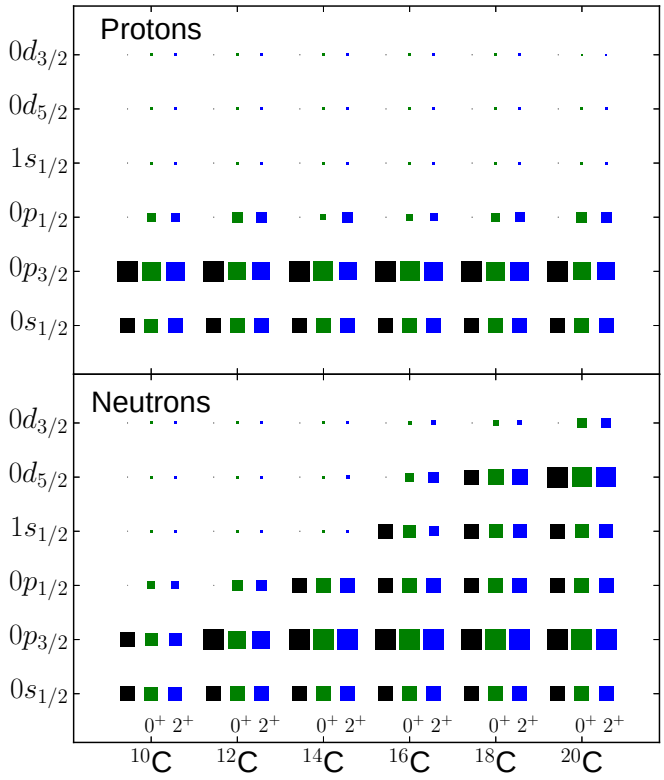

Figure 5. (Colour online) Occupation numbers for the ground- (middle, green squares) and first 2^+-state (right, blue squares) in $^{10-20}$C obtained with the CDB2k interaction. The area of the squares are proportional to the occupation numbers and can be compared with the unperturbed ground-state, shell-model occupation numbers (left, black squares). We can note in particular the proton (neutron) excitation character of the ^{14}C(^{16}C) 2^+ state.

as summarized in Table 2, the relative B(E2) strength from this second 2^+ state to the ground state is much smaller than that from the first 2^+ for all isotopes but ^{16}C. These findings are obtained with both *NN* Hamiltonians used in this study. However, the relative transitions from the second 2^+ in ^{16,18}C stand out with clear differences in the predictions of CDB2k and INOY, see Table 2. Note, however, that the convergence of the second 2^+ state is computationally more difficult, and therefore the statements on relative transition strengths are based on runs performed at a single HO frequency. For ^{18}C, in particular, there is a strong $\hbar\Omega$-dependence for the INOY results that makes the corresponding claim of a strong $2_2^+ \rightarrow 2_1^+$ E2 transition less robust. For ^{16}C, however, the interaction dependence is solid and intriguing. As the INOY interaction often hints to possible structural influence from *NNN* forces we continue our study in the next section with a more detailed investigation of the ^{16}C structure using Hamiltonians with realistic *NNN* terms.

Table 2. Relative B(E2) values for transitions among excited states of $^{14-20}$C. Results obtained at fixed HO-frequency with the CDB2k ($\hbar\Omega = 12$ MeV) and INOY ($\hbar\Omega = 17 - 18$ MeV) *NN* interactions are compared.

	$\frac{B(E2;2_2^+\to 0_1^+)}{B\left(\mathrm{E2};\, 2_1^+\to 0_1^+\right)}$		$\frac{B(E2;2_2^+\to 2_1^+)}{B\left(\mathrm{E2};\, 2_1^+\to 0_1^+\right)}$	
A	CDB2k	INOY	CDB2k	INOY
14	0.001	0.000	0.48	0.81
16	2.2	0.30	2.0	0.79
18	0.018	0.22[a]	0.047	1.7[a]
20	0.017	0.035	0.12	0.28

[a] For this particular interaction we observe considerable mixing between two 2^+ states, with different structure, for certain choices of the HO frequency. These results are for $\hbar\Omega = 18$ MeV.

2.3. Higher-lying states of ^{16}*C and the role of the NNN interaction*

Transitions from higher excited states of ^{16}C were studied in a recent experiment [26]. In particular, the transitions $2_2^+ \to 2_1^+$, $4_1^+ \to 2_1^+$ and $3_1^+ \to 2_1^+$ were observed. Interestingly, no transition from the 2_2^+ state to the ground state was seen. We performed additional calculations with different Hamiltonians to study higher excited states in ^{16}C and their electromagnetic transitions. In Fig. 6, we show the calculated and experimental energy levels of ^{16}C, and in Table 3 we summarize our calculated B(E2) values among excited states normalized to $B\left(\mathrm{E2};\, 2_1^+ \to 0_1^+\right)$. In particular, we compare results obtained with SRG-transformed chiral *NN* and chiral *NN* + *NNN* interactions (including the SRG-induced three-nucleon terms in both cases as discussed in Ref. [19]), to those obtained with the two-body effective CDB2k interaction. A striking feature is a strong suppression of the $2_2^+ \to 0_1^+$ transition when the initial *NNN* interaction is included. The sensitivity to the presence of the *NNN* interaction is remarkable. The $2_2^+ \to 0_1^+$ transition is suppressed by a factor of ~ 20 in the calculation with the *NNN*. Clearly, the calculation without the *NNN* interaction contradicts the new MSU experiment [26] where indeed the $2_2^+ \to 0_1^+$ transition was not observed. From Table 3 we observe that relative E2 transition strengths obtained with the chiral *NN* interaction are similar to the ones obtained with the CDB2k interaction. Furthermore, we see from Table 2 that the relative B(E2) calculated with the INOY interaction (that mimics some *NNN* effects) resemble results of the chiral *NN*+*NNN* Hamiltonian. The excitation energies of the five lowest ^{16}C excited states are also influenced by the *NNN* interaction as seen in Fig. 6. The agreement with the experimental spectrum is quite reasonable in all presented cases, although slightly improved in the calculation with the chiral *NN*+*NNN* Hamiltonian.

From Table 3, we also note a strong sensitivity of the $3_1^+ \to 2_1^+$ transition to the presence of the *NNN* interaction. The calculation with the chiral *NN*+*NNN* Hamiltonian predicts a strongly suppressed $B(\mathrm{E2};\, 3_1^+ \to 2_1^+)$ transition. A transition between these states is observed, however [26]. Our calculation with the *NNN* interaction predicts this

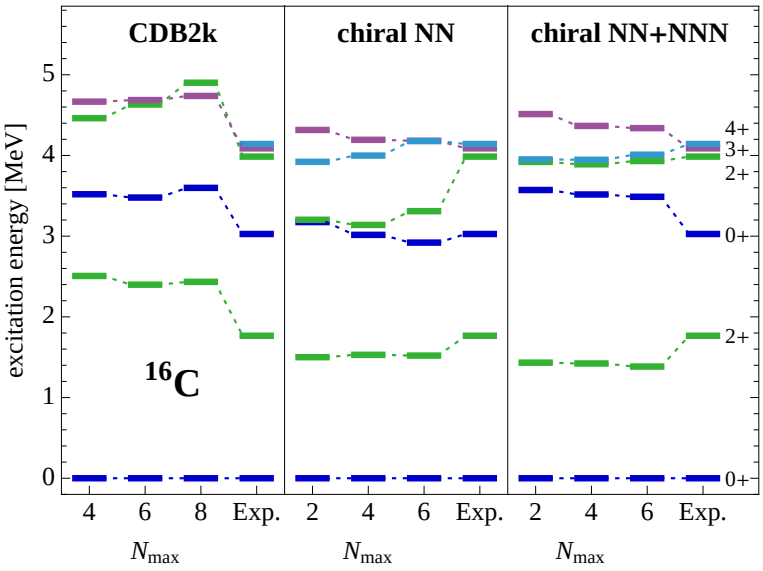

Figure 6. (Colour online) Excitation energies of the lowest states of ^{16}C. Calculations using the Lee-Suzuki-transformed CDB2k potential at $\hbar\Omega = 12$ MeV (left) and the SRG-evolved chiral *NN* and *NN*+*NNN* interactions with $\Lambda = 1.88\,\mathrm{fm}^{-1}$ for $\hbar\Omega = 16\,\mathrm{MeV}$ (middle and right) are compared to experiment for different values of $N_{\max}$. The SRG-evolved chiral interactions include induced *NNN* terms.

Table 3. Relative B(E2) values for transitions among excited states of ^{16}C. Results obtained with the CDB2k *NN* potential, the chiral *NN*, and the chiral *NN*+*NNN* interaction are compared. For CDB2k we use the Lee-Suzuki effective interactions ($\hbar\Omega = 12$ MeV, $N_{\max}$=6) and for the chiral interactions we use SRG-evolved interactions ($\Lambda = 1.88\,\mathrm{fm}^{-1}$, $\hbar\Omega = 16\,\mathrm{MeV}$, $N_{\max}$=6) including the induced three-nucleon terms.

$\frac{B(\mathrm{E2};J_i\to J_f)}{B(\mathrm{E2};\,2_1^+\to 0_1^+)}$	CDB2k	chiral *NN*	chiral *NN*+*NNN*
$2_1^+ \to 0_1^+$	1	1	1
$2_2^+ \to 0_1^+$	2.2	0.75	0.11
$2_2^+ \to 2_1^+$	2.0	1.7	0.65
$3_1^+ \to 2_1^+$	0.36	0.31	0.02
$4_1^+ \to 2_1^+$	0.89	0.69	0.80

transition to be of M1 character as seen from Table 4. We also observe a sign change of the magnetic moments of both the 2_1^+ and the 2_2^+ states in calculations with the *NNN* interaction included. The magnetic moment of the 3_1^+ state is unaffected, however. The sensitivity of the 2_1^+ magnetic moment to the *NNN* interaction we also find in ^{20}C (see Table 4).

Overall, we find a strong sensitivity of the electromagnetic observables in ^{16}C to the details of nuclear Hamiltonian. More detailed experimental study of higher excited states and their transitions would be quite useful.

Table 4. Absolute values for magnetic dipole moments and B(M1) transition strengths of excited states in ^{16,20}C. Results obtained with the CDB2k *NN* potential and the SRG-evolved chiral *NN*+*NNN* interaction are compared. B(M1) in μ_N^2 and μ in μ_N. Parameters as in Table 3 with $N_{\max}$=4 for ^{20}C. The brackets indicate the uncertainties of the threshold extrapolation for the IT-NCSM.

	^{16}C		^{20}C	
		chiral		chiral
	CDB2k	*NN*+*NNN*	CDB2k	*NN*+*NNN*
B($M1$; $2_2^+ \to 2_1^+$)	0.013	0.063	0.015	
B($M1$; $3_1^+ \to 2_1^+$)	0.17	0.17	0.013	
$\mu(2_1^+)$	0.13	-0.42	0.22	0.001(8)
$\mu(2_2^+)$	1.3	-0.79	0.58	
$\mu(3_1^+)$	-3.2	-3.1	0.016	

3. Conclusion

In summary, we have computed low-lying states of even carbon isotopes with $A = 10-20$ within the *ab initio* NCSM. We have used several accurate nucleon-nucleon (*NN*) as well as *NN* plus *NNN* interactions and calculated excitation energies of the lowest 2^+ state, the electromagnetic $B\left(\mathrm{E2};\, 2_1^+ \to 0_1^+\right)$ transition strengths, the 2_1^+ quadrupole moments as well as selected electromagnetic transitions among higher excited states. Our main approximation is the use of a truncated many-body model space, which however can be systematically improved by increasing the cutoff. The calculations do not include effective charges or any other fitting parameters. Note that the truncation of the many-body basis used in the NCSM should in principle be followed by a transformation of the transition operator that is consistent with the renormalization of the Hamiltonian. For long-range operators, such as E2, this transformation is not expected to produce a very different end result for calculated observables [29, 30]. In addition, the small uncertainty associated with the approximation of using bare operators is partly built into the error estimates that we obtain from using several values of $\hbar\Omega$ and $N_{\max}$.

Overall, we have found a consistent NCSM description of the $B\left(\mathrm{E2};\, 2_1^+ \to 0_1^+\right)$ dependence on the mass number for the whole carbon isotopic chain from $A = 10$ to 20. However, our calculated $B\left(\mathrm{E2};\, 2_1^+ \to 0_1^+\right)$ values for ^{16}C, with different Hamiltonians, all underestimate the most recent experimental measurements. Our results are quite similar to the findings of Ma *et al.* [8], who used a microscopic particle-vibration approach to compute core polarization effects. In their picture the reduced B(E2) strength in heavy carbon isotopes can be traced back in particular to a strong quenching from core polarization on *sd*-shell neutrons. In our approach, however, there is no such separation into core and valence degrees of freedom.

In addition, we found a remarkable sensitivity of the transition rates from higher excited states in ^{16}C to the details of the nuclear interactions. The chiral *NN*+*NNN* interaction gives the excitation spectrum of ^{16}C in a slightly better agreement with experiment than the CDB2k *NN* potential and, furthermore, the former interaction

predicts the suppression of the $2_2^+ \to 0_1^+$ transition in agreement with experimental observations. We found a strong sensitivity of the magnetic moments of the 2_1^+ state to the nuclear interaction in ^{16}C and ^{20}C and even more so for the 2_2^+ state in ^{16}C.

The NCSM calculations predict sign changes of the 2_1^+ quadrupole moments between different carbon isotopes. In particular, we predict a negative quadrupole moment in ^{16}C, a very small quadrupole moment in ^{10}C and a $B\left(\mathrm{E2}; 2_1^+ \to 0_1^+\right)$ value in ^{10}C that is about the same as that in ^{10}Be. In ^{12}C, we obtain $Q(2_1^+)$=+6.2(2) $e$fm^2. It will be worth measuring these moments in the future.

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