Correlated Realistic Two-Body Interactions and the Tjon Line

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With the advent of realistic nucleon-nucleon interactions, which reproduce the experimental NN scattering phase shifts and deuteron properties with high precision, the importance of three-body interactions became evident. Ab initio solutions of the nuclear many-body problem for p-shell nuclei, e.g. in the framework of Green's Function Monte Carlo or no-core shell model approaches, have demonstrated that the conventional realistic two-nucleon forces, e.g. the Argonne V18 or the Nijmegen I/II potentials are not sufficient to reproduce experimental binding energies and low-lying excited states. Typically, those potentials underestimate the binding energy of ⁴He by approx. 4 MeV. In order to compensate for this deficiency, phenomenological three-nucleon forces are introduced, which depend on the underlying two-nucleon force. Some of the realistic potentials, e.g. the CD Bonn interaction, require significantly weaker three-body forces than the Argonne V18 potential.

Starting from one of the realistic two-nucleon forces we have employed the Unitary Correlation Operator Method (UCOM) to transform the original potential into a phaseshift equivalent correlated interaction V_{UCOM} [1]. Formally, this is achieved by a unitary transformation of the manybody Hamiltonian involving kinetic energy T and bare twobody potential V using unitary operators C_r and C_{Ω}

$$C_r^{\dagger} C_{\Omega}^{\dagger} (T + V) C_{\Omega} C_r = T + V_{UCOM} + V_{UCOM}^{[3]} + \cdots$$

The correlation operators C_r and C_{Ω} are tailored to describe short-range central and tensor correlations, respectively. Details on the construction of the correlation operators are given in [1]. The tensor correlator C_{Ω} has one parameter I_{ϑ} , which controls the range of the tensor correlations included via the transformation. In addition to the correlated two-body interaction V_{UCOM} the transformation generates a three-body interaction $V_{\text{UCOM}}^{[3]}$ as well as higher-order terms. We focus on the application of V_{UCOM} in exact no-core shell model calculations for ³H and ⁴He using the code described in [3]. In this way we can asses the role of the omitted three-body terms $V_{\text{UCOM}}^{[3]}$ and the range of the tensor correlation functions.

Figure 1 summarizes exact results for the ground state energies of ³H and ⁴He. Each symbol corresponds to a different interaction. The labeled symbols in the upper right quadrant show the results of exact calculations for different bare realistic NN interactions, which deviate significantly from the experimental values (Exp.). All realistic potentials seem to fall on a line in the $E({}^{3}\text{H})-E({}^{4}\text{He})$ -plane known as Tjon line. If supplemented by a phenomenological threenucleon force, fitted to the experimental binding energies, a good agreement can be achieved (cross symbols) [4].



Figure 1: Exact ³H and ⁴He binding energies for different interactions in comparison with experiment (see text).

Now we add the results of no-core shell model calculations with V_{UCOM} to this picture. Using the range of the tensor correlators as a parameter, we can generate a manifold of phase-shift equivalent two-body interactions. The resulting ground state energies for the correlated Argonne V18 and Nijmegen I and II for $I_{\vartheta} = 0.05, 0.06, ..., 0.12 \text{fm}^3$ (from upper right to lower left) are depicted by the connected symbols. Obviously, one can choose the tensor correlator range such that the experimental binding energies are reproduced without employing an explicit three-body force and independent of the original interaction. This is a result of a cancellation between induced three-body interactions $V_{\text{UCOM}}^{[3]}$ and genuine three-body forces usually used to supplement the bare interaction.

The UCOM-transformation thus provides a way to construct phase-shift equivalent two-body potentials V_{UCOM} which require only minimal (or no) additional three-body components. Those are an ideal starting point for different many-body methods such as shell model, Hartree-Fock, perturbation theory, RPA, and Fermionic Molecular Dynamics, as discussed elsewhere (see [2] and references therein). Detailed no-core shell model calculations throughout the p-shell are presently in progress.

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References

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