Short range unitary two-body correlations

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Due to the short-ranged repulsive core in the nucleon-nucleon potential V the many-body state is depleted as a function of the relative distance $x_{ij} = |\vec{x}_i - \vec{x}_j|$ for each pair (ij) when they are close to each other. These short range correlations cannot be described by shell model states. The most common procedure to remedy this problem is Brueckner's G-matrix method which replaces the bare V by an effective interaction G. The second method is the Jastrow approach where the correlated ground state of the nucleus is assumed to be of the form $\prod_{i<j} f(x_{ij})|\Phi\rangle$ where $f(x_{ij})$ is a correlation factor which diminishes the probability to find two nucleons at small distances x_{ij}.

We propose a third new method where the correlated state $|\Psi\rangle$ is obtained by a *unitary* transformation $e^{-i\mathbf{S}}$ (not to be confused with the $\exp\mathbf{S}$ method [1])

$$|\Psi\rangle = e^{-i\mathbf{S}}|\Phi\rangle \ ,$$

where $\mathbf{S}$ is a hermitean two-body operator which depends in general on the relative distance $\vec{\mathbf{x}}_{12}$, the relative momentum $\vec{\mathbf{p}}_{12}$, the spins and isospins of the two nucleons. The aim of $e^{-i\mathbf{S}}$ is to push two nucleons away from each other whenever they get too close. The most simple ansatz which does that is

$$\mathbf{S} = \frac{1}{2}\sum_{i<j}\left(\vec{\mathbf{p}}_{ij}\vec{\nabla}s(\mathbf{x}_{ij}) + \vec{\nabla}s(\mathbf{x}_{ij})\vec{\mathbf{p}}_{ij}\right) \ .$$

$|\vec{\nabla}s|(x)$ is roughly speaking the distance which the particles are moved away from each other by $e^{-i\mathbf{S}}$ if they are found at a distance x in $|\Phi\rangle$. $|\vec{\nabla}s|(x)$ is largest if x lies inside the hard core and $|\vec{\nabla}s|(x) \to 0$ if x is outside the repulsive interaction.

Fig. 1 displays the radial dependence of the correlated and uncorrelated deuteron wave function together with the Afnan Tang S3 potential [2]. The functional form of $s(x)$ is adjusted such that the correlated state $|\psi_d\rangle$ for short distances equals the exact solution.

Once the correlator $e^{-i\mathbf{S}}$ is adjusted to reproduce the two-body system at low energies (long wave length) we calculate the ground state energies of nuclei with many particles by minimizing

$$E = \langle\Psi|\mathbf{H}|\Psi\rangle = \langle\Phi|e^{i\mathbf{S}}\mathbf{H}e^{-i\mathbf{S}}|\Phi\rangle$$

with respect to $|\Phi\rangle$ which is a single Slater determinant composed of localized Gaussians (FMD [3]). Unlike $\mathbf{H} = \mathbf{T} + \mathbf{V}$

$$\begin{aligned}\mathbf{H}_{eff} &\equiv e^{i\mathbf{S}}\mathbf{H}e^{-i\mathbf{S}} = e^{i\mathbf{S}}\mathbf{T}e^{-i\mathbf{S}} + e^{i\mathbf{S}}\mathbf{V}e^{-i\mathbf{S}} \\ &= \mathbf{T} + \mathbf{T}_{eff}^{[2]} + \mathbf{V}_{eff}^{[2]} + \mathbf{T}_{eff}^{[3]} + \mathbf{V}_{eff}^{[3]} + \cdots\end{aligned}$$

is not a one- plus two-body operator but contains three-body and higher interactions.

We calculate $\mathbf{T}_{eff}^{[2]}$ and $\mathbf{V}_{eff}^{[2]}$, which are then functionals of $s(x)$, analytically and approximate the energy by neglecting three-body and higher terms in the cluster expansion. This turns out to be a very good approximation at typical nuclear densities. Estimations of the three-body terms give corrections less than 5% of binding energy for the α-particle. Fig. 2 compares uncorrelated (left column) an correlated (right column) energies. The correlated potential energy $\langle\Phi|\mathbf{V}_{eff}^{[2]}|\Phi\rangle$ (grey bars at negative values) is about twice the uncorrelated $\langle\Phi|\mathbf{V}|\Phi\rangle$ in all nuclei. This gain in binding is counteracted by an increase in the kinetic energies (grey bars at positive values). Both together yield binding energies (black bars) which are within 8% deviation from results of Yakubovski calculations [4] for ^{4}He and FHNC calculations [5] for ^{16}O and ^{40}Ca. This suprises since $\mathbf{H}_{eff}$ is the same for all nuclei and not density dependent. In addition one may easily conceive improved shell model states $|\Phi\rangle$.

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[2] I.R. Afnan, Y.C. Tang: Phys. Rev. 175 (1968) 1337
[3] H. Feldmeier, K. Bieler, J. Schnack: Nucl. Phys. A586 (1995) 493
[4] H. Kamada, W. Glöckle: Nucl. Phys. A548 (1992) 205
[5] G. Co' et al.: Nucl. Phys. A549 (1992) 439

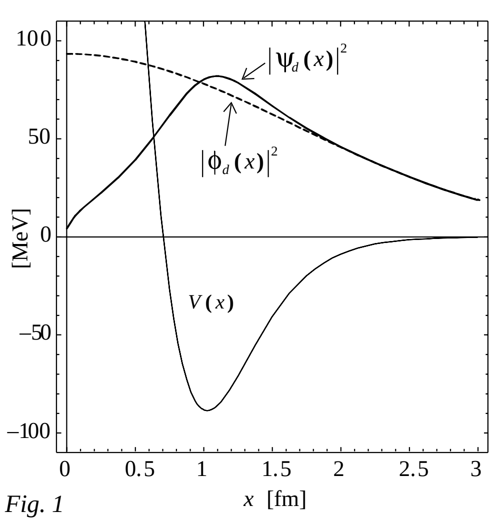

Fig. 1

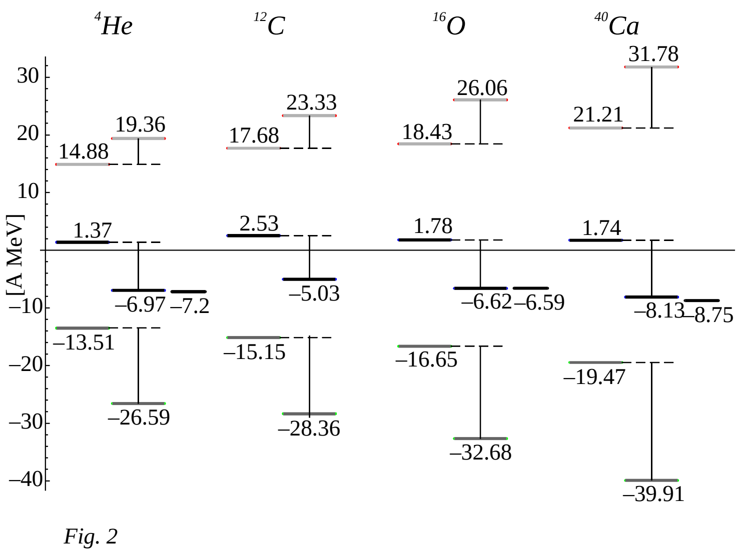

Fig. 2