Bosons in 1D Optical Superlattices: Computing the Phase Diagram from Experimental Parameters

Felix Schmitt Markus Hild Robert Roth

Institut für Kernphysik
TU Darmstadt

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Q 7.3 Quantengase: Bosonen im Gitter I

Bose-Hubbard Model (1D)

- bosons on a 1D optical lattice at $T = 0\,K$
- fixed particle number $\sum_i n_i = N$, I lattice sites

number-basis representation

$$|\,\psi\,\rangle = \sum_{\alpha=1}^{dim} C_\alpha \,|\,\{n_1, n_2, \ldots, n_I\}_\alpha\,\rangle$$

Hamiltonian

$$\hat{\mathrm{H}} = \sum_{i=1}^{I} \left\{ -J_{i,i+1}\,(\hat{\mathrm{a}}^\dagger_{i+1}\,\hat{\mathrm{a}}_i + \hat{\mathrm{a}}^\dagger_i\,\hat{\mathrm{a}}_{i+1}) + \frac{1}{2} U_i\,(\hat{\mathrm{n}}_i - 1)\,\hat{\mathrm{n}}_i + \epsilon_i\,\hat{\mathrm{n}}_i \right\}$$

$J_{i,i+1}$ hopping energy, U_i on-site interaction energy, ϵ_i local potential

Hubbard parameters are functions of the optical potential,
i.e., wavelength and amplitude, and the s-wave scattering length

Strategy

experimental parameters

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Hubbard parameters

QM many-body problem

▼

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▼

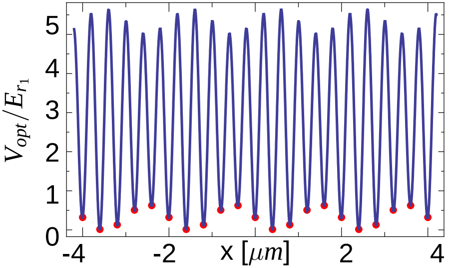
phase diagram

Two-Color Superlattice

- two standing laser fields with wavelength $\lambda_1,\ \lambda_2$
- natural energy scale is the *recoil energy* $E_{r_i} = \frac{h^2}{2m\lambda_i^2}$
- experimental knobs are the amplitudes $s_i = \frac{V_i}{E_{r_i}}$

optical potential

$$V_{opt}(x) = s_1 E_{r_1} \sin^2\left(\frac{2\pi}{\lambda_1}x + \phi\right) + s_2 E_{r_2} \sin^2\left(\frac{2\pi}{\lambda_2}x\right)$$



- $\lambda_2 = 800\,nm$
  $s_2 = 5$
- $\lambda_1 = 1000\,nm$
  $s_1 = 1$
- $\phi = \pi/4$

Strategy

experimental parameters

▼

band-structure calculation

▼

Hubbard parameters

QM many-body problem

▼

?

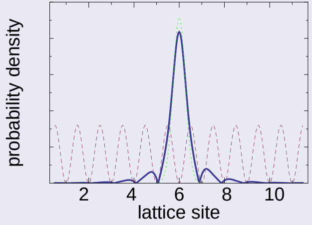
▼

phase diagram

Single-Particle Band-Structure Calculation (1D)

1. experimental parameters
   - laser wavelength: λ_1 ,λ_2
   - laser amplitudes: s_1 ,s_2
   - scattering length: a_s
   - transverse trapping frequency: $\omega_\perp$
2. obtain localized Wannier function $w_i(x)$ from 1D bandstructure calculation
3. calculate Hubbard parameters for each site

Wannier function $w_i(x)$



$$-J_{i,i+1} = \int dx\, w_i^*(x) \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_{opt}(x) \right) w_{i+1}(x)$$

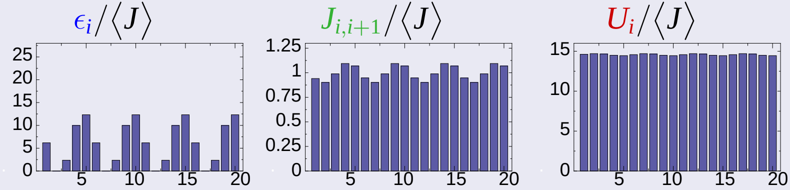
$$\epsilon_i = \int dx\, w_i^*(x) \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_{opt}(x) \right) w_i(x)$$

$$U_i = 2\,\omega_\perp \hbar\, a_s \int dx\, |w_i(x)|^4$$

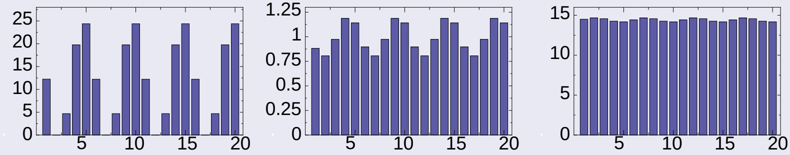
Site-Dependent Hubbard Parameters

$\lambda_2 = 800\,nm,\ \lambda_1 = 1000\,nm,\ \phi = \pi/4,\ \omega_\perp = 30E_r,\ {}^{87}$Rb

$s_2 = 10,\ s_1 = 0.5$



$s_2 = 10,\ s_1 = 1$



Strategy

experimental parameters

▼

band-structure calculation

▼

Hubbard parameters

QM many-body problem

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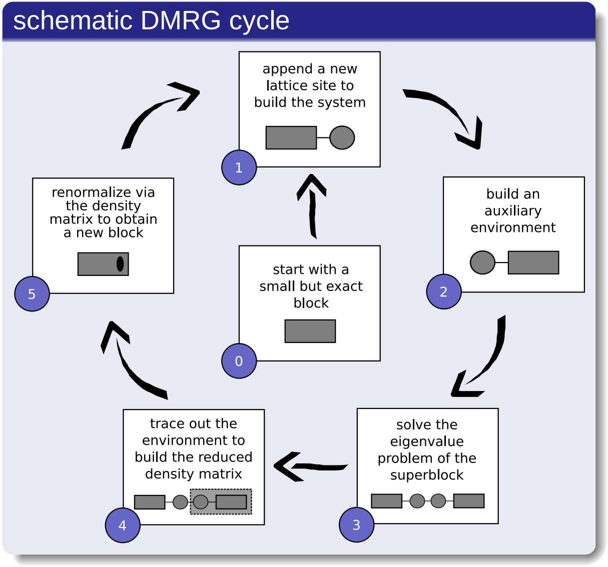
DMRG

▼

phase diagram

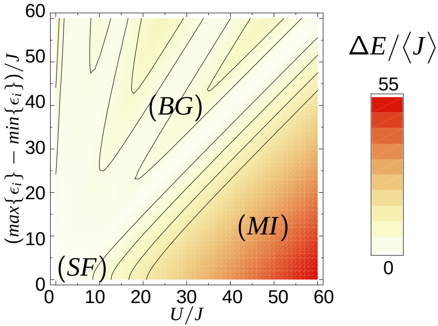
Density-Matrix Renormalisation Group (DMRG)



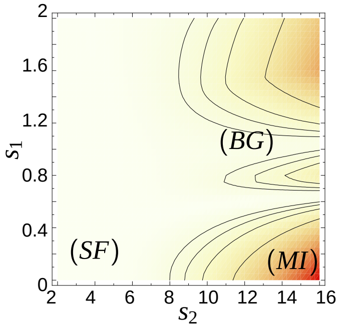
- most efficient ”exact” method for these types of 1D systems
- ”in-situ” basis truncation
- lattice grows successively to arbitrary size

Phase Diagrams

Hubbard parameters[1]



optical lattice parameters



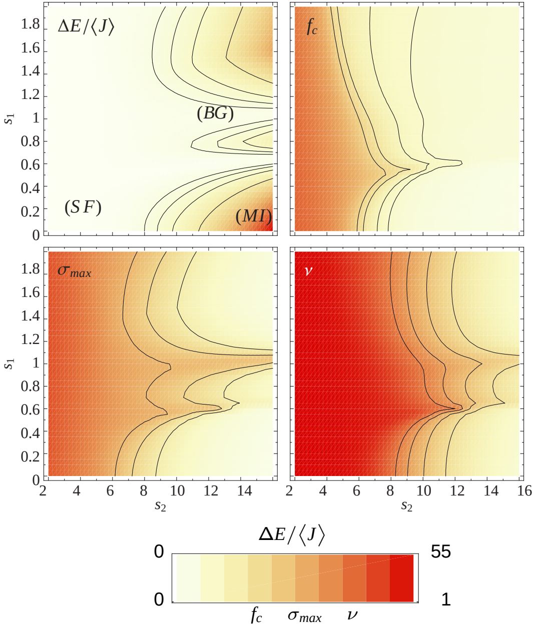
all phases accessible via s_1 and s_2

- superfluid phase (SF)
- quasi Bose-glass phase (BG)
- homogeneous Mott insulator (MI)

20 particles on 20 lattice sites
$\lambda_2 = 800\,nm$, $\lambda_1 = 1000\,nm$
s-wave scattering of ^{87}Rb
trans. trapping $\omega_\perp = 30\,E_{r_1}$

[1] Roth et al. *Phys. Rev. A* **68**, 023604 (2003), Rapsch et al. *Europhys. Lett.* **46**, 559 (1999)

Observables



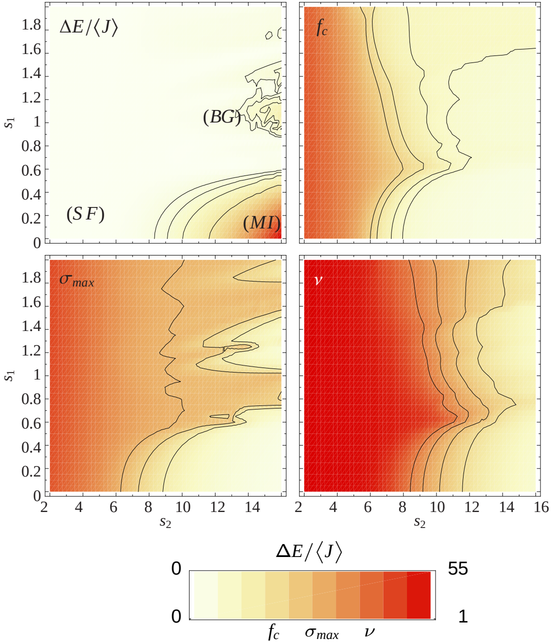
commensurate lattice

- $\Delta E/\langle J\rangle$ energy gap
- f_c condensate fraction
- σ_{max} max. number fluctuation
- ν fringe visibility

transition to quasi Bose-glass

go to MI $s_2 > 10$ $(U/J > 8)$ and crank up the second laser

Observables



incommensurate lattice

- $\lambda_2 = 830\,nm$
  $\lambda_1 = 1076\,nm$
- s-wave scattering length of ^{87}Rb
- transverse trapping frequency $\omega_\perp = 30\,E_{r_1}$

more "randomness"

fragmentation of the substructures in the quasi Bose-glass phase

Summary & Outlook

- summary
  - mapping of the phase diagram to experimental parameters via band-structure calculations
  - all phases are accessible by tuning s_1 and s_2
  - substructure of observables in the quasi Bose-glass phase depend on the lattice topology
- outlook
  - systematic study of the effect of different lattice topologies on the observables