Dynamic Response of Ultracold Bose Gases in 1D Optical Lattices

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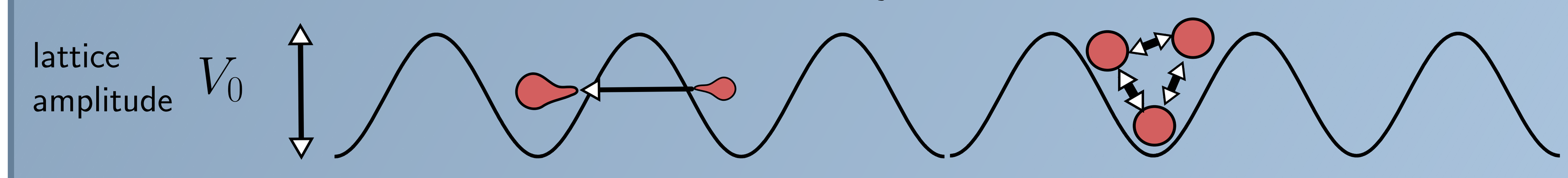
Summary & Motivation

- we investigate dynamic properties of strongly correlated ultracold Bose gases in 1D optical lattices within the Bose-Hubbard model (BHM) [1,2]
- we use a weak periodic modulation of the lattice amplitude to excite the system (Bragg spectroscopy)
- exact time evolution yields precise description of the resonance structure [3,4] but is feasible for moderate system sizes only
- the results are in excellent agreement with the prediction from a linear response analysis [4]
- random phase approximation (RPA) allows for studies of the resonance structure at the level of particle-hole excitations and beyond [5]
- transition amplitudes derived from the linear response analysis lead to response functions

Bose-Hubbard Model (BHM) & Lattice Modulation

- N bosons on I lattice sites described by the Bose-Hubbard Hamiltonian



$$\mathbf{H} = \overbrace{-J\sum_{i=1}^{I}\left(\mathbf{a}_i^\dagger\mathbf{a}_{i+1} + \mathbf{a}_{i+1}^\dagger\mathbf{a}_i\right)}^{\text{kinetic term ("tunneling")}} + \overbrace{\frac{U}{2}\sum_{i=1}^{I}\mathbf{n}_i(\mathbf{n}_i - 1)}^{\text{on-site interaction}} = -J\mathbf{H}_J + U\mathbf{H}_\mathrm{U}$$

- eigenstates in number basis representation $|\nu\rangle = \sum_{j=1}^{D} c_j^{(\nu)} \left|\{n_1\cdots n_I\}_j\right\rangle$

modifications due to the amplitude modulation

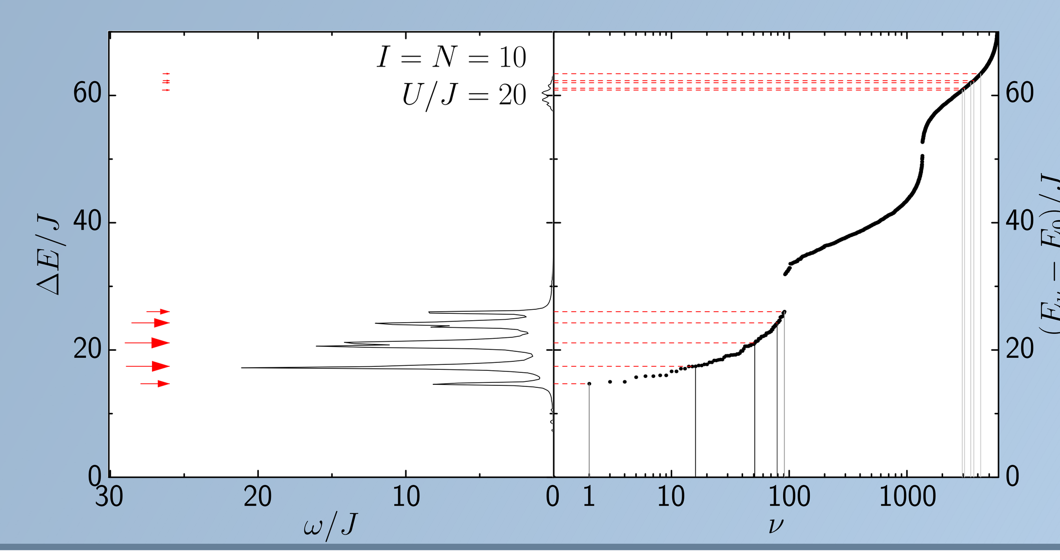
- potential modified by a time-dependent factor : $V_0 \longrightarrow \tilde{V}_0(t) = V_0[1 + F\sin(\omega t)]$
- parameters J, U become time dependent

Linear Response Analysis

- ? which excited states couple to ground state?
  linearization of $\mathbf{H}$ based on the Taylor expansion in the modulation amplitude F:

$$\mathbf{H}_{\text{lin}}(t) = \mathbf{H}_0 + F\sin(\omega t)\,[\lambda\mathbf{H}_0 - \kappa\mathbf{H}_J]$$

- ! in first order only $\mathbf{H}_J$ couples ground and excited states!
- look for finite matrix elements $\langle 0|\mathbf{H}_J|\nu\rangle$
- right panel: lower eigenspectrum with matrix elements; left panel: resonance structure (time evolution)



Generalized Random Phase Approximation (RPA) and Response Functions

- RPA ground state approximated by $|\text{RPA}\rangle \approx |1,\cdots,1\rangle$ for $N/I{=}1$
- excitations generated by phonon operator $\mathbf{Q}_\nu^\dagger$ (obeying $\mathbf{Q}_\nu|\text{RPA}\rangle = 0$) :

$$\mathbf{Q}_\nu^\dagger = \sum_{ij} X_{ij}^\nu\, \mathbf{a}_i^\dagger\mathbf{a}_i^\dagger\mathbf{a}_i\mathbf{a}_j - \sum_{ij} Y_{ij}^\nu\, \mathbf{a}_j^\dagger\mathbf{a}_i^\dagger\mathbf{a}_i\mathbf{a}_i,$$

- generalized eigenproblem to solve

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix}\begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} S & -T \\ -T & S \end{pmatrix}\begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

- matrix elements

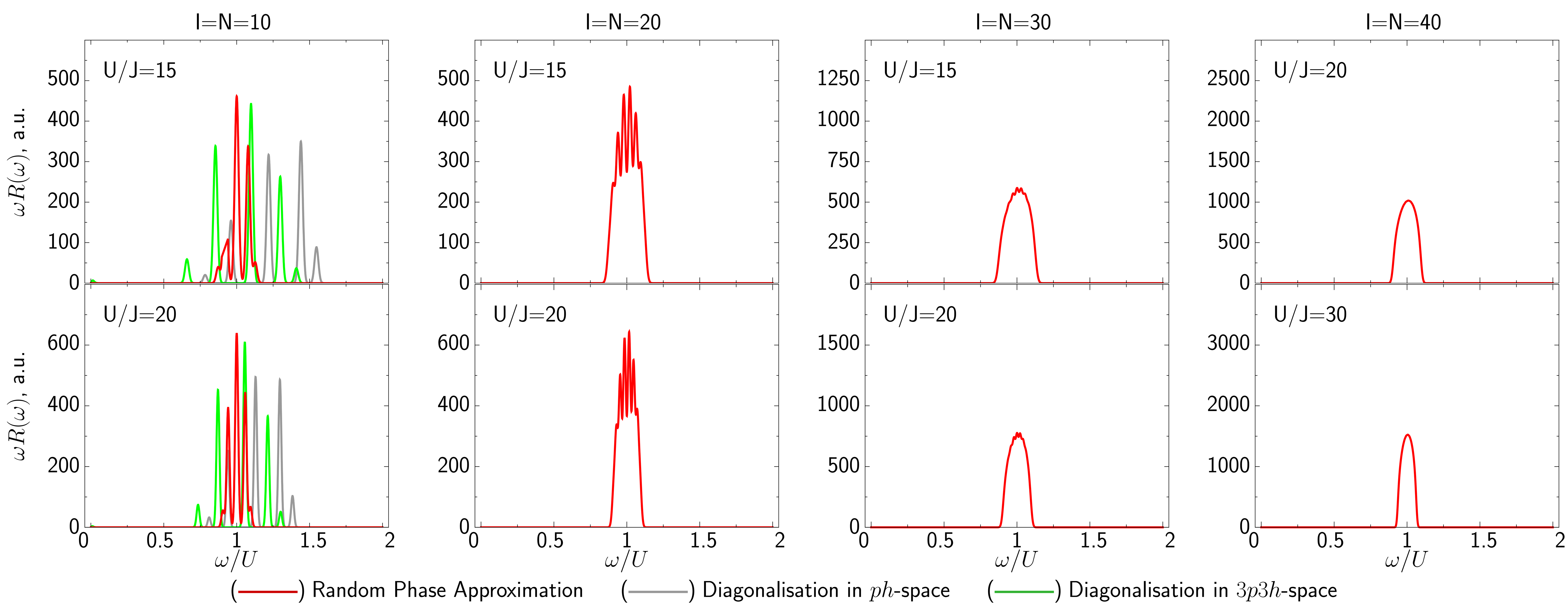
$$A_{iji'j'} = \langle 1,\cdots,1|\,[\mathbf{a}_j^\dagger\mathbf{a}_i^\dagger\mathbf{a}_i\mathbf{a}_i, \mathbf{H}, \mathbf{a}_{i'}^\dagger\mathbf{a}_{i'}^\dagger\mathbf{a}_{i'}\mathbf{a}_{j'}]\,|1,\cdots,1\rangle,\quad S_{ijij'} = \langle 1,\cdots,1|\,[\mathbf{a}_j^\dagger\mathbf{a}_i^\dagger\mathbf{a}_i\mathbf{a}_i, \mathbf{a}_{i'}^\dagger\mathbf{a}_{i'}^\dagger\mathbf{a}_{i'}\mathbf{a}_{j'}]\,|1,\cdots,1\rangle,$$

$$B_{iji'j'} = \langle 1,\cdots,1|\,[\mathbf{a}_j^\dagger\mathbf{a}_i^\dagger\mathbf{a}_i\mathbf{a}_i, \mathbf{H}, \mathbf{a}_{j'}^\dagger\mathbf{a}_{i'}^\dagger\mathbf{a}_{i'}\mathbf{a}_{i'}]\,|1,\cdots,1\rangle,\quad T_{iji'j'} = \langle 1,\cdots,1|\,[\mathbf{a}_j^\dagger\mathbf{a}_i^\dagger\mathbf{a}_i\mathbf{a}_i, \mathbf{a}_{j'}^\dagger\mathbf{a}_{i'}^\dagger\mathbf{a}_{i'}\mathbf{a}_{i'}]\,|1,\cdots,1\rangle$$

- weighting the positive RPA spectrum $\{\omega_\nu\}$ by the transition amplitudes $\left|\langle 1,\cdots,1|\mathbf{H}_J|\omega_\nu\rangle\right|^2$ yields the response function

$$R(\omega) = \sum_\nu \delta(\omega - \omega_\nu)\left|\langle 1,\cdots,1|\mathbf{H}_J|\omega_\nu\rangle\right|^2$$

Response Functions : RPA vs. Diagonalisation



- plots show the $\omega = 1U$ resonances for several interaction strengths and systems sizes in the strongly correlated regime excited by weak lattice modulations
- diagonalization in $1p1h$-space fails to describe resonance position & shape
- RPA predicts correct centroid energy and resonance shape
- role of ground-state correlations on fine-structure & width to be investigated
- RPA allows for the computation of response functions for large systems with minimal numerical effort
- higher resonances and more complex lattice topologies are possible by extension of the phonon operator

[1] D. Jaksch et al., *Phys. Rev. Lett.* **81**, 3108 (1998)
[2] M. Greiner et al., *Nature* **415**, 39 (2002)
[3] S.R. Clark and D. Jaksch, *New J. Phys.* **8**, 160 (2006)
[4] M. Hild et. al, *J. Phys. B.: At. Mol. Opt. Phys.* **39**, 4547 (2006)
[5] D.J. Rowe, *Rev. Mod. Phys.* **40**, 1 (1968)