Dynamic Response of Ultracold Bose Gases in Optical Lattices

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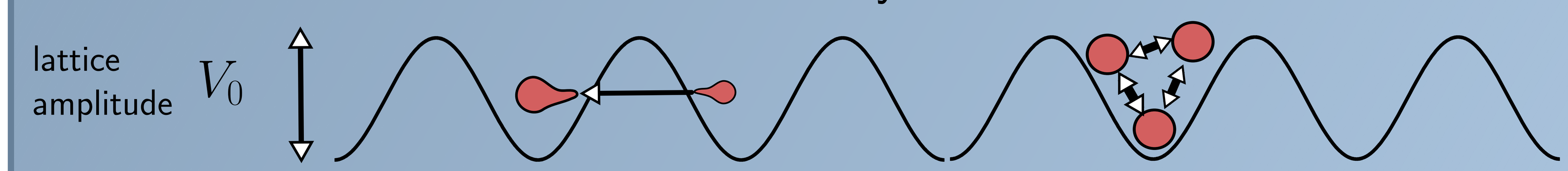
Summary & Motivation

- we investigate dynamic properties of strongly correlated ultracold Bose gases in 1D optical lattices within the Bose-Hubbard model (BHM) [1,2]
- we use a weak periodic modulation of the lattice amplitude to excite the system (Bragg spectroscopy)
- exact time evolution yields precise description of the resonance structure [3,4] but is feasible for moderate system sizes only
- the results are in excellent agreement with the prediction from a linear response analysis [4]
- random phase approximation (RPA) allows for studies of the resonance structure at the level of particle-hole excitations and beyond [5]
- in combination with the coupling constant from the linearization it is possible to obtain response functions similar to the results from exact time evolution

Bose-Hubbard Model (BHM) & Lattice Modulation

- N bosons on I lattice sites described by the Bose-Hubbard Hamiltonian



$$\mathbf{H} = \overbrace{-J\sum_{i=1}^{I}\left(\mathbf{a}_i^\dagger \mathbf{a}_{i+1} + \mathbf{a}_{i+1}^\dagger \mathbf{a}_i\right)}^{\text{kinetic term ("tunneling")}} + \overbrace{\frac{U}{2}\sum_{i=1}^{I}\mathbf{n}_i(\mathbf{n}_i - 1)}^{\text{on-site interaction}} = -J\mathbf{H}_J + U\mathbf{H}_\mathrm{U}$$

- eigenstates in number basis representation

$$|\nu\rangle = \sum_{j=1}^{D} c_j^{(\nu)} \left|\{n_1 \cdots n_I\}_j\right\rangle$$

modifications due to the amplitude modulation

- potential modified by a time-dependent factor : $V_0 \longrightarrow \tilde{V}_0(t) = V_0[1 + F\sin(\omega t)]$
- parameters J, U become time dependent

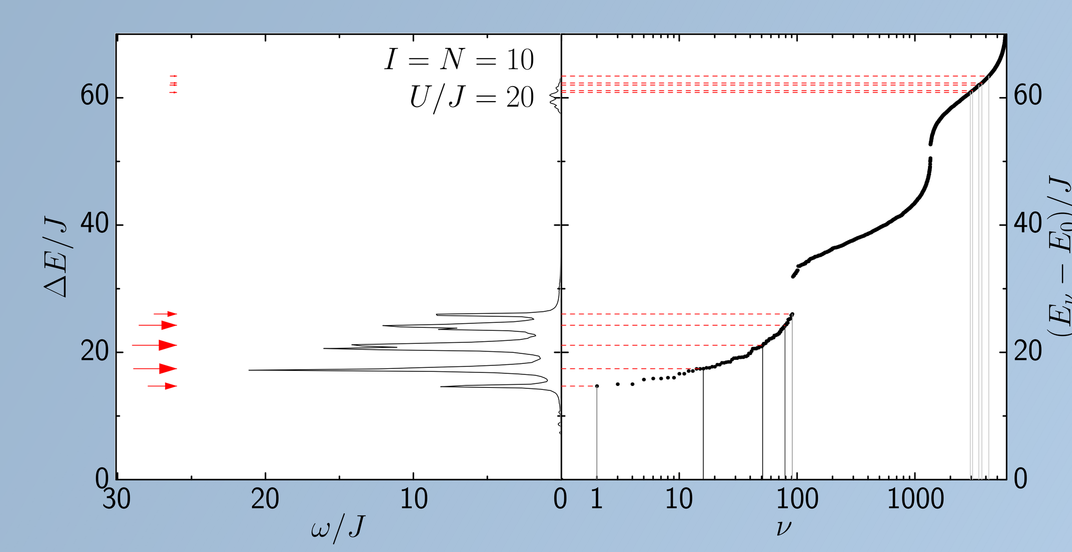
Linear Response Analysis

- which excited states couple to ground state?

linearization of $\mathbf{H}$ based on the Taylor expansion in the modulation amplitude F:

$$\mathbf{H}_{\text{lin}}(t) = \mathbf{H}_0 + F\sin(\omega t)\,[\lambda \mathbf{H}_0 + \kappa \mathbf{H}_J]$$

- in first order only $\mathbf{H}_J$ couples ground and excited states!
- look for finite matrix elements $\langle 0|\mathbf{H}_J|\nu\rangle$
- right panel: lower eigenspectrum with matrix elements; left panel: resonance structure (time evolution)
- matrix elements predict the resonances and their fragmentation



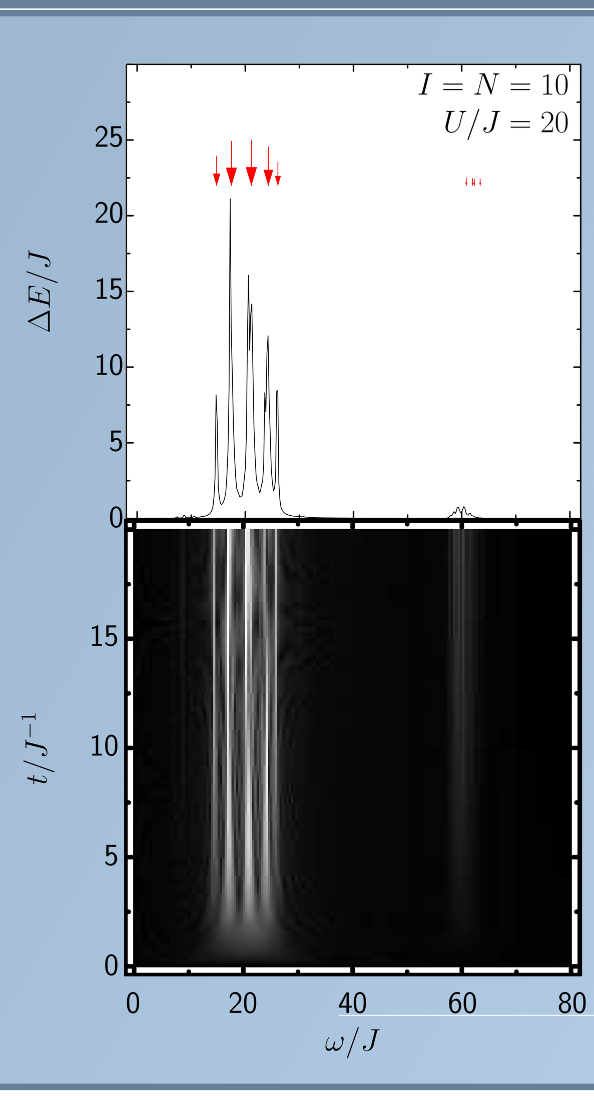
Exact Time Evolution

procedure

1. ground state of the unperturbed Hamiltonian $\mathbf{H}(t{=}0)$ as initial state
2. construct the time evolution operator $\mathbf{U}(t, \Delta t)$ (Crank-Nicholson scheme) for current time t
3. do finite time step Δt and evaluate the observables

features

- yields the systems state $|\psi, t\rangle$ at each time t
- plot on the right: energy transfer ΔE over time and modulation amplitude ω (lower panel) and the time averaged values (upper panel)
- not feasible for realistic system sizes



Generalized Random Phase Approximation (RPA)

- RPA ground state approximated by $|\text{RPA}\rangle \approx |1, \cdots, 1\rangle$ for $N/I{=}1$
- excitations generated by phonon operator $\mathbf{Q}_\nu^\dagger$ (obeying $\mathbf{Q}_\nu|\text{RPA}\rangle = 0$) :

$$\mathbf{Q}_\nu^\dagger = \sum_{ijk} X_{ijk}^\nu \,\mathbf{a}_k^\dagger \mathbf{a}_i^\dagger \mathbf{a}_j \mathbf{a}_k - \sum_{ijk} Y_{ijk}^\nu \,\mathbf{a}_k^\dagger \mathbf{a}_j^\dagger \mathbf{a}_i \mathbf{a}_i ,$$

- generalized eigenproblem to solve

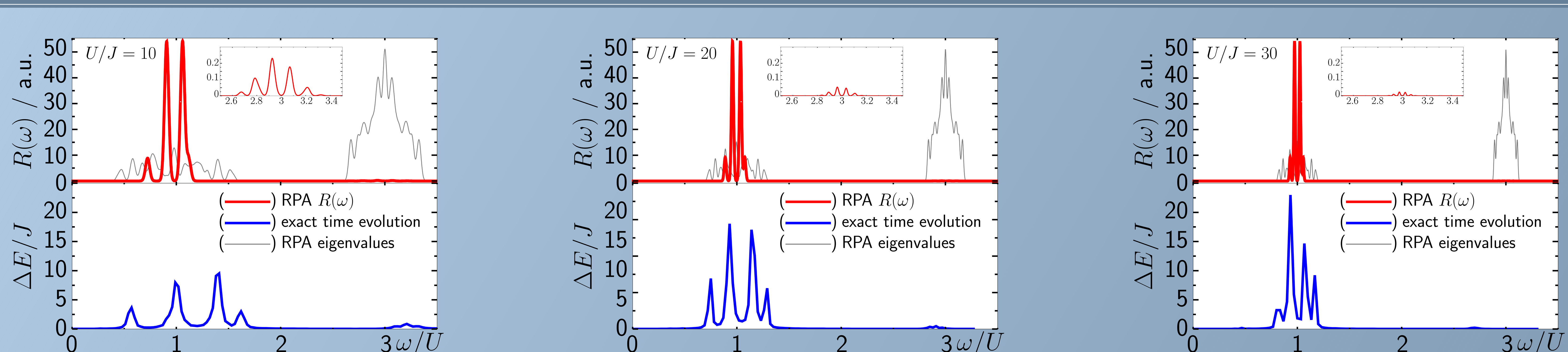
$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix}\begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} S & -T \\ -T & S \end{pmatrix}\begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

$A_{ijki'j'k'} = \langle 1,\cdots,1|\,[\mathbf{a}_k^\dagger \mathbf{a}_j^\dagger \mathbf{a}_i \mathbf{a}_i, \mathbf{H}, \mathbf{a}_{i'}^\dagger \mathbf{a}_{i'}^\dagger \mathbf{a}_{j'} \mathbf{a}_{k'}]\,|1,\cdots,1\rangle$, $S_{ijkij'k'} = \langle 1,\cdots,1|\,[\mathbf{a}_k^\dagger \mathbf{a}_j^\dagger \mathbf{a}_i \mathbf{a}_i, \mathbf{a}_{i'}^\dagger \mathbf{a}_{i'}^\dagger \mathbf{a}_{j'} \mathbf{a}_{k'}]\,|1,\cdots,1\rangle$,
$B_{ijki'j'k'} = \langle 1,\cdots,1|\,[\mathbf{a}_k^\dagger \mathbf{a}_j^\dagger \mathbf{a}_i \mathbf{a}_i, \mathbf{H}, \mathbf{a}_{k'}^\dagger \mathbf{a}_{j'}^\dagger \mathbf{a}_{i'} \mathbf{a}_{i'}]\,|1,\cdots,1\rangle$, $T_{ijki'j'k'} = \langle 1,\cdots,1|\,[\mathbf{a}_k^\dagger \mathbf{a}_j^\dagger \mathbf{a}_i \mathbf{a}_i, \mathbf{a}_{k'}^\dagger \mathbf{a}_{j'}^\dagger \mathbf{a}_{i'} \mathbf{a}_{i'}]\,|1,\cdots,1\rangle$

- weighting the positive RPA spectrum $\{\omega_\nu\}$ by the strengths $\left|\langle 1,\cdots,1|\mathbf{H}_J|\omega_\nu\rangle\right|^2$ yields the response function

$$R(\omega) = \sum_\nu \delta(\omega - \omega_\nu)\left|\langle 1,\cdots,1|\mathbf{H}_J|\omega_\nu\rangle\right|^2$$

Exact Time Evolution vs. RPA for $I = N = 8$ Bosons



- plots show the $\omega = 1U$ and $\omega = 3U$ resonances for several interaction strengths in the strongly correlated regime excited by weak lattice modulations
- position of the resonances of both methods are in good agreement
- RPA reproduces the fragmentation of the resonances
- strengths $\left|\langle 1,\cdots,1|\mathbf{H}_J|\omega_\nu\rangle\right|^2$ scales the RPA eigenvalues according to the applied excitation
- resonances are more narrow for RPA-calculations - improvement by extending the phonon ansatz?

[1] D. Jaksch et al., *Phys. Rev. Lett.* **81**, 3108 (1998)
[2] M. Greiner et al., *Nature* **415**, 39 (2002)
[3] S.R. Clark and D. Jaksch, *New J. Phys.* **8**, 160 (2006)
[4] M. Hild et. al, *J. Phys. B.: At. Mol. Opt. Phys.* **39**, 4547 (2006)
[5] D.J. Rowe, *Rev. Mod. Phys.* **40**, 1 (1968)