Ultracold Bose Gases In Optical Lattices

A Bloch-Representation Approach

Ilona Türschmann, Felix Schmitt, Markus Hild, and Robert Roth

Institut für Kernphysik, TU Darmstadt, Schloßgartenstraße 9, 64289 Darmstadt

Summary & Motivation

- ultracold, dilute atomic gases in optical lattices are well described by the single-band Bose-Hubbard Hamiltonian[2]
- these are perfect laboratories to investigate the rich physics of strongly correlated quantum systems [1,5,6,7]
- the Bose-Hubbard Hamiltonian is transformed from the Wannier to the Bloch representation
- the ground state is obtained by an exact diagonalisation of the corresponding Hamilton matrix using Lanczos algorithms
- for reasons of symmetry only a small fraction of the coefficients of the ground state is non-zero
- with this reduced amount of number-states the static observables are accessible with the same precision as with the full basis [3,4]

Bose-Hubbard Model

- 1D optical lattice with I lattice sites and N bosonic particles
- nearest neighbour hopping, and on-site two-particle interactions

$$\hat{H} = -J\sum_{l=1}^{I}\left(\hat{a}_{l+1}^{\dagger}\hat{a}_l + h.a.\right) \quad \text{tunneling term}$$
$$+\frac{U}{2}\sum_{l=1}^{I}\hat{n}_l\,(\hat{n}_l - 1) \quad \text{interaction term}$$

$\hat{a}_l^{\dagger},\ \hat{a}_l,\ \hat{n}_l$	creation, annihilation, occupation-number operators
J	tunnelling matrix element
U	two particle interaction energy

Transformation to Bloch Basis

- operators transformed into Bloch basis $\qquad q_j = \frac{2\pi}{I}j$

$$\hat{c}_{q_j}^{\dagger} = \frac{1}{\sqrt{I}}\sum_{l=1}^{I} e^{-iq_jl}\hat{a}_l^{\dagger}, \qquad \hat{c}_{q_j} = \frac{1}{\sqrt{I}}\sum_{l=1}^{I} e^{iq_jl}\hat{a}_l,$$

$\Rightarrow$ Bose-Hubbard Hamiltonian in Bloch basis

$$\hat{H} = \quad -J\sum_{j=0}^{I-1} 2\cdot cos(q_j)\hat{n}_{q_j} + \frac{U}{2I}\sum_{j=0}^{I-1}\sum_{k=0}^{I-1}\sum_{m=0}^{I-1}\hat{c}_{q_j}^{\dagger}\hat{c}_{q_k}^{\dagger}\hat{c}_{q_m}\hat{c}_{(q_j+q_k-q_m)}$$

- states are represented in a quasimomentum occupation number basis with dimension D

$$\mid \psi^{(0)} \,\rangle = \sum_{\alpha=1}^{D} C_{\alpha}^{(0)} \mid \{n_{q_0}, \ldots, n_{q_{I-1}}\}_{\alpha} \,\rangle$$

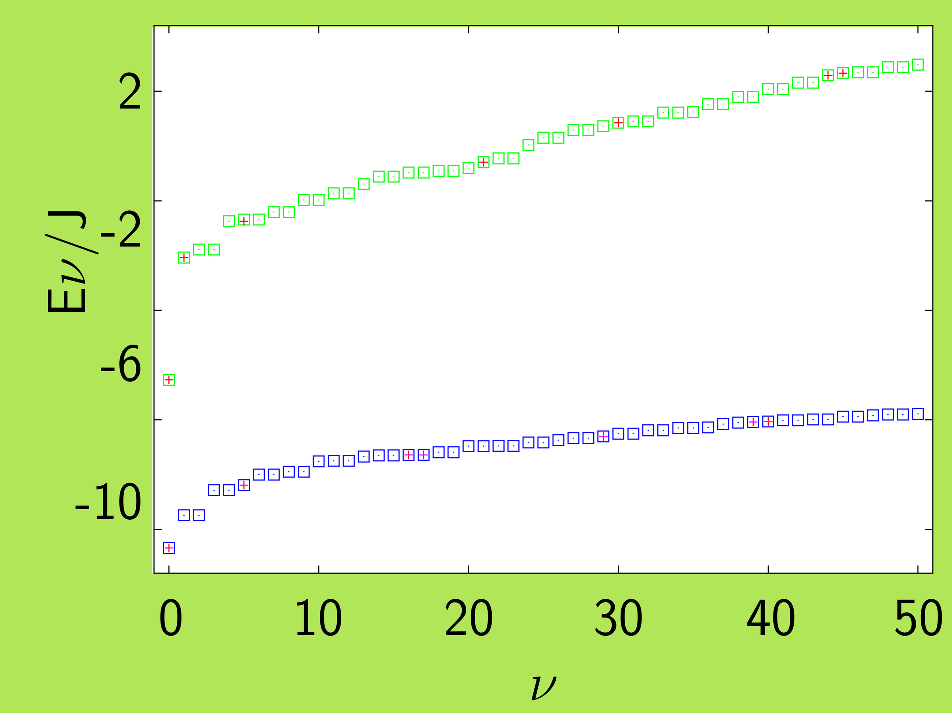
Reduced Basis

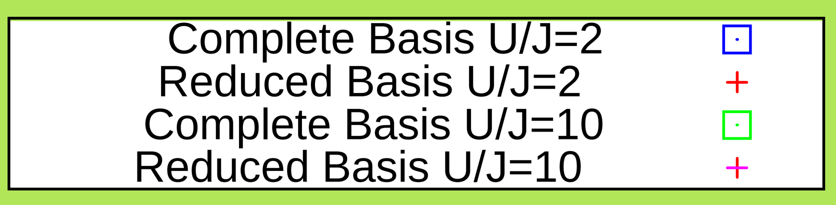
- in many cases only the ground state observables are of interest
- to calculate those we only have to take into account the Fock-states with quasimomentum equal zero
- thus we can reduce the size of the Hilbert space without any loss of information

$$\left\langle\, \psi_{reduced}^{(0)} \mid \psi_{full}^{(0)} \,\right\rangle = 1$$

Number of Particles	Dimension	Reduced Dimension
3	10	4
5	126	26
7	1716	246
8	6435	810
9	2431	2704
10	92378	9252
11	352716	32066
12	1352078	112720

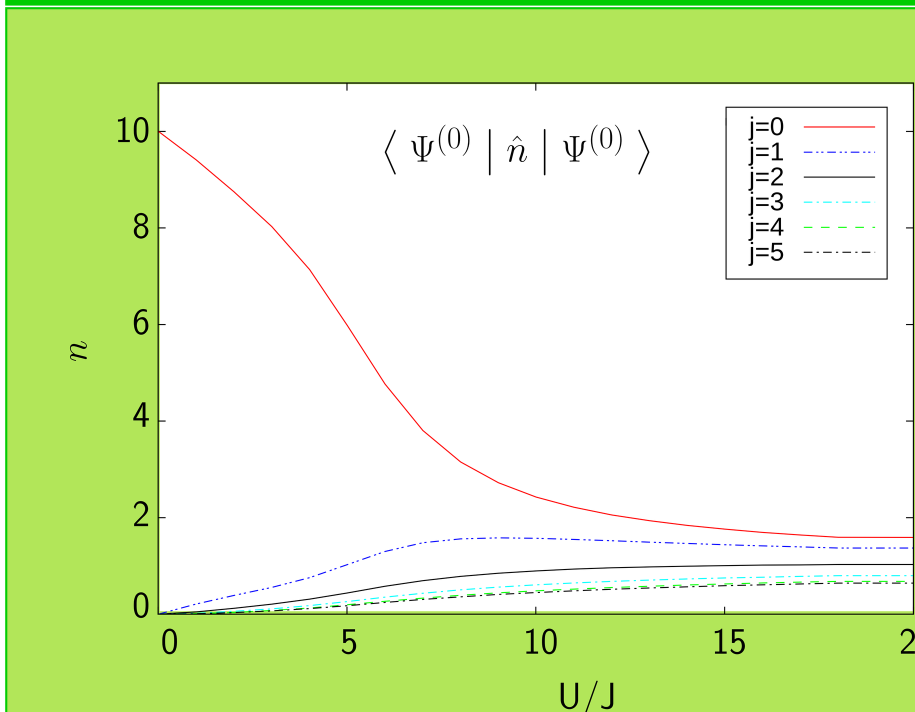
Energy Spectrum

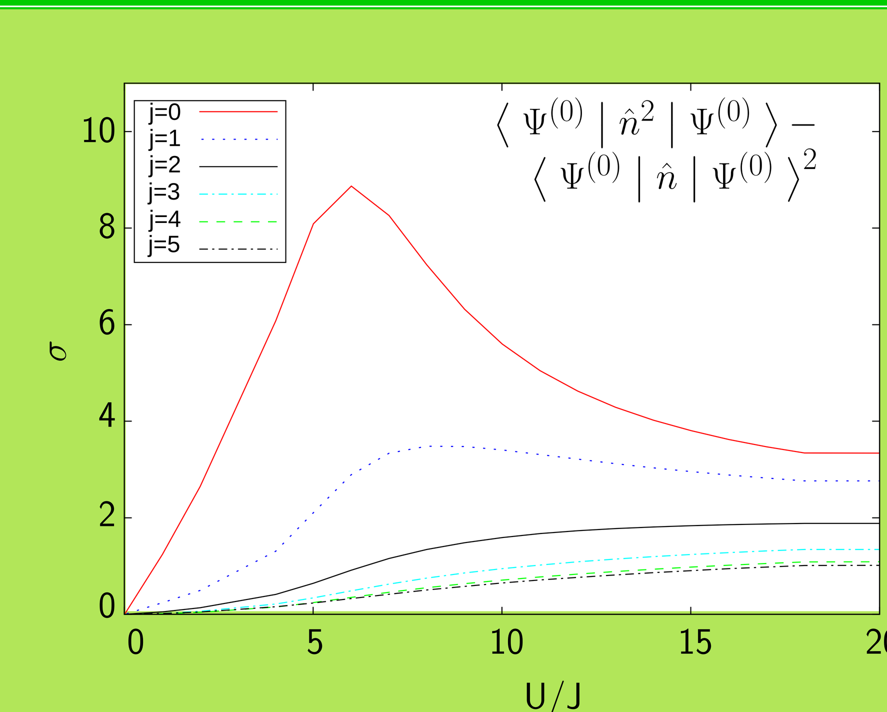




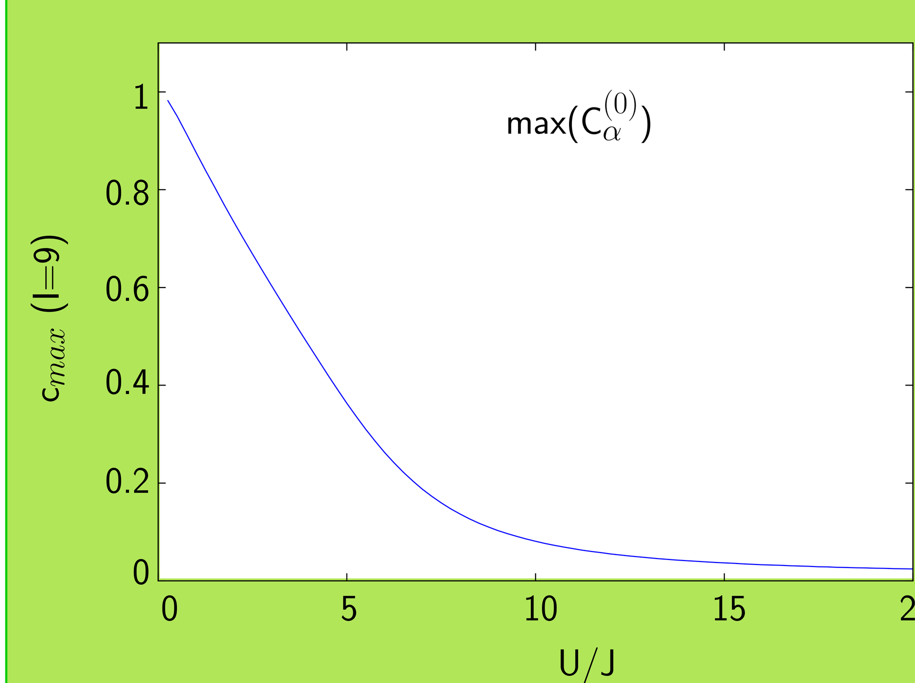
- energy-bands of eight particles and different interaction strengths
- the total quasimomentum of every non-degenerate eigenstate equals zero
- if the energy eigenvalue is degenerated, the states do not necessarily exhibit a total quasimomentum equal zero
- most eigenstates are superpositions of number states with different quasimomentum
- the groundstate is composed of number states with vanishing quasimomentum

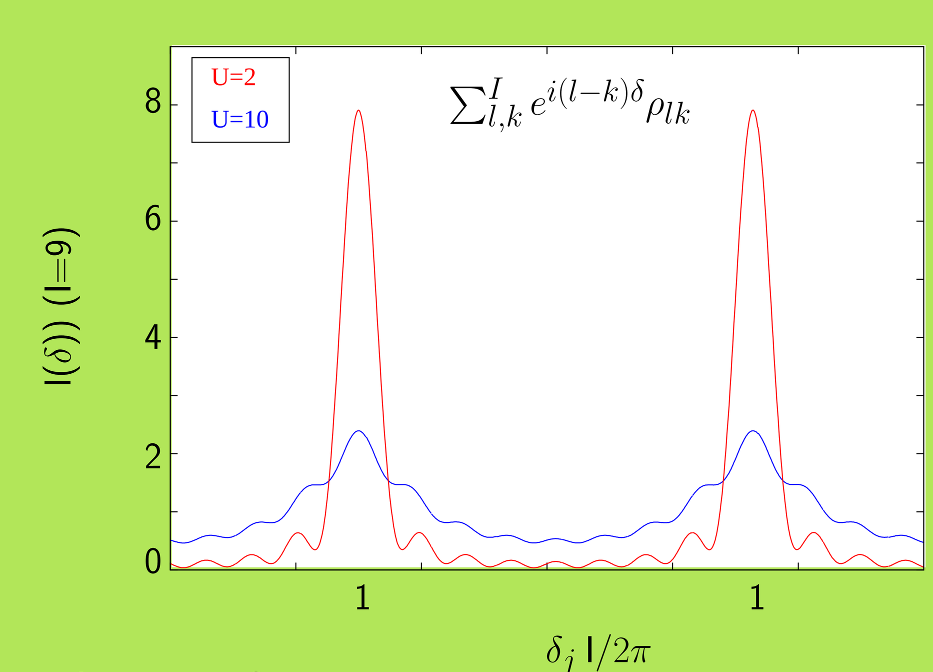
Some Observables





- mean occupation number n and fluctuation of the mean occupation number σ, for 10 particles with varying interaction strength U/J
- the mean occupaction number at q$_j = 0$ is equal to total particle number for U/J$= 0$ $\Rightarrow$ a perfect condensate
- for larger U/J the condensate is depleted





- the maximum coefficient C$_{max}$ decreases with increasing U/J
- C$_{max}$ is strongly correlated to the mean occupation number of the quasimomentum zero state
- the interference pattern is an experimentally accessible observable
- for strongly correlated systems the interference pattern forms a broad bump, whereas for small U/J on q$_j = 0$ there is a sharp peak

[1] Immanuel Bloch, Physics World (2004)
[2] D. Jacksch et al., Phys. Rev. Lett. 81, 3108-3111 (1998)
[3] Felix Schmitt et al. Q24.7
[4] Markus Hild et al. Q24.6
[5] Felix Schmitt, et al. J.Phys.B 39(2006)4547-4562
[6] Markus Hild, et al. J.Phys.B 40(2007)371-385
[7] Greiner et al. 2002 Nature (London)415 39