Pairing with Correlated Realistic NN Interactions

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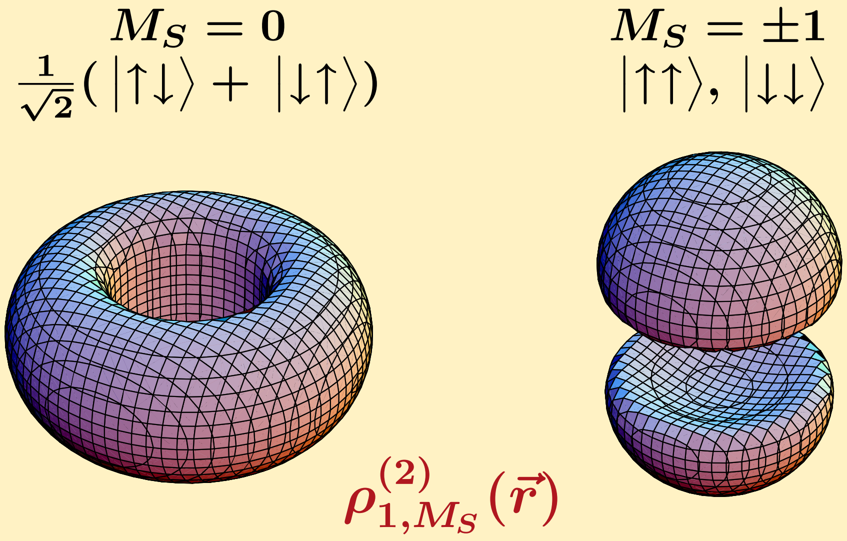
Overview

- The Unitary Correlation Operator Method
- Hartree-Fock-Bogoliubov
  - Particle-Number Projection
  - Inclusion of $3N$-Forces
- Summary & Outlook

Motivation

Argonne V18 Deuteron Solution



- **central correlations:** two-body density is suppressed at low distances
- **tensor correlations:** angular distribution depends on the relative spin alignments

Central Correlator C_r

radial distance-dependent shift in the relative coordinate of a nucleon pair

$$C_r = \exp(-i \sum_{i,j}^{A} g_{r,ij}[s(r_{ij})])$$

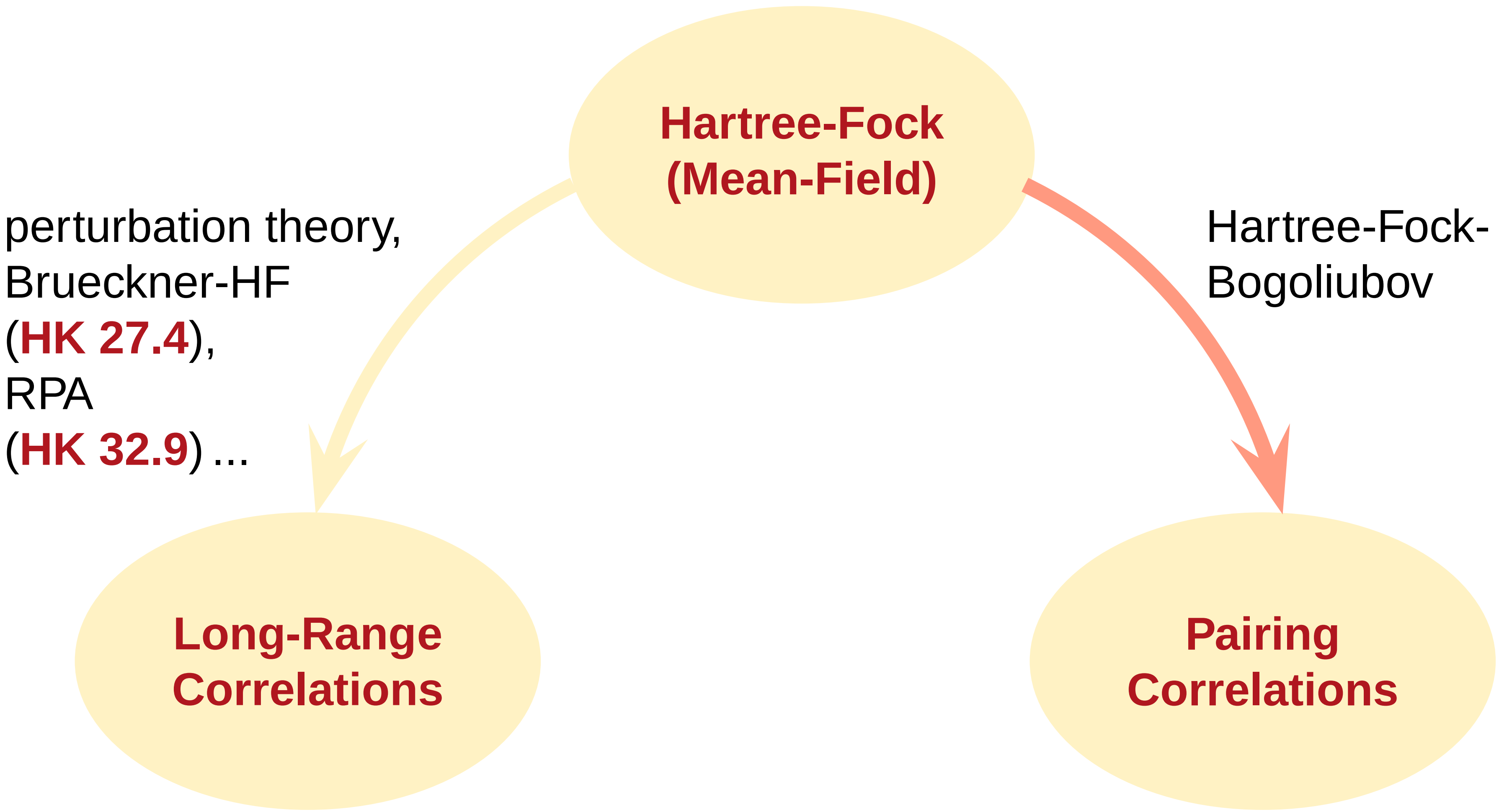
Tensor Correlator C_Ω

angular shift, depending on orientation of spin and relative coordinate

$$C_\Omega = \exp(-i \sum_{i,j}^{A} g_{\Omega,ij}[\vartheta(r_{ij})])$$

$s(r)$ and $\vartheta(r)$ encapsulate the physics of short-range correlations

Beyond Hartree-Fock



HFB Theory Overview

Bogoliubov Transformation

$$\beta_k^\dagger = \sum_q U_{qk} c_q^\dagger + V_{qk} c_q$$

$$\beta_k = \sum_q U_{qk}^* c_q + V_{qk}^* c_q^\dagger$$

where

$$\{\beta_k, \beta_{k'}\} \stackrel{!}{=} \{\beta_k^\dagger, \beta_{k'}^\dagger\} \stackrel{!}{=} 0$$

$$\{\beta_k, \beta_{k'}^\dagger\} \stackrel{!}{=} \delta_{kk'}$$

HFB Densities & Fields

$$\rho_{kk'} \equiv \langle\Psi| c_{k'}^\dagger c_k |\Psi\rangle = (V^* V^T)_{kk'}$$

$$\kappa_{kk'} \equiv \langle\Psi| c_{k'} c_k |\Psi\rangle = (V^* U^T)_{kk'}$$

$$\Gamma_{kk'} = \sum_{qq'} \left(\frac{2}{A}\bar{t}_{\text{rel}} + \bar{v}\right)_{kq',k'q} \rho_{qq'}$$

$$\Delta_{kk'} = \sum_{qq'} \left(\frac{2}{A}\bar{t}_{\text{rel}} + \bar{v}\right)_{kk',qq'} \kappa_{qq'}$$

Energy

$$E[\rho, \kappa, \kappa^*] = \frac{\langle\Psi| H |\Psi\rangle}{\langle\Psi|\Psi\rangle} \equiv \frac{1}{2}\left(\text{tr}\ \Gamma\rho - \text{tr}\ \Delta\kappa^*\right)$$

HFB Equations

$$(\mathcal{H} - \lambda\mathcal{N})\begin{pmatrix} U \\ V \end{pmatrix} \equiv \begin{pmatrix} \Gamma - \lambda & \Delta \\ -\Delta^* & -\Gamma^* + \lambda \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix} = E\begin{pmatrix} U \\ V \end{pmatrix}$$

Particle Number Projection

Variation of Projected Energy

$$\delta E(N_0) = \frac{1}{2\pi\langle \mathrm{P}_{N_0}\rangle} \int_0^{2\pi} d\phi \, \langle e^{i\phi(\mathrm{N}-N_0)}\rangle \left\{ \delta\langle \mathrm{H}\rangle_\phi - \left(E(N_0) - \langle \mathrm{H}\rangle_\phi\right) \delta \log \langle e^{i\phi \mathrm{N}}\rangle \right\}$$

$$\langle \mathrm{H}\rangle_\phi \equiv \langle \mathrm{H} e^{i\phi\mathrm{N}}\rangle / \langle e^{i\phi\mathrm{N}}\rangle$$

Lipkin-Nogami + PAV

- power series expansion
- expansion coefficients **not varied**
- indeterminate / numerically unstable at shell closures
- exact PNP after variation

VAP

- higher (but managable) computational effort
- implement with care: **subtle cancellations between divergences of direct, exchange, and pairing terms**

☞ Structure of **HFB equations is preserved** by both methods!

Flocard & Onishi, Ann. Phys. 254, **275** (1997, approx. PNP); Sheikh et al., Phys. Rev. **C66**, 044318 (2002, exact PNP)

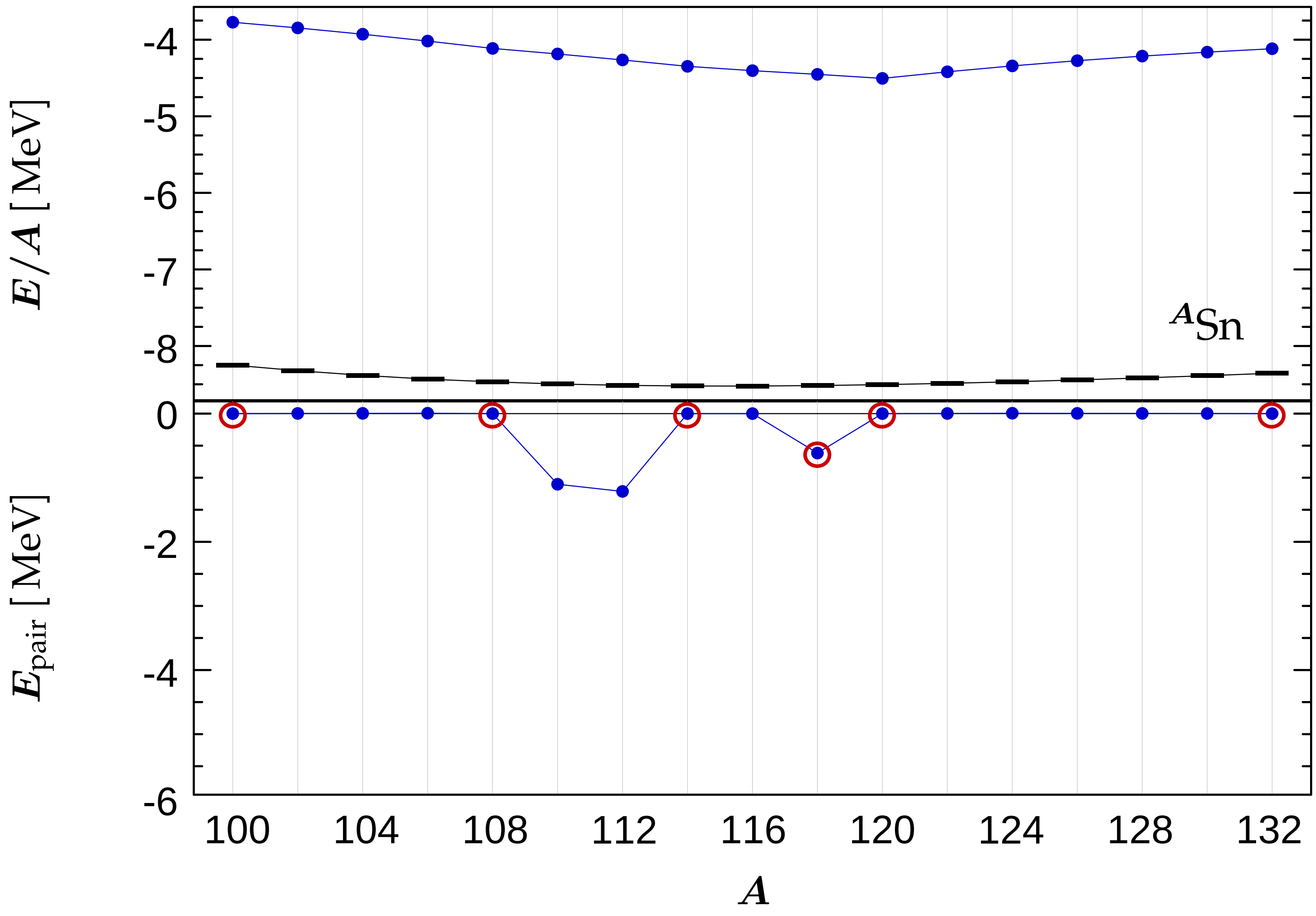
Implementation

- vary **intrinsic energy** $\mathbf{H}_{\text{int}} = \mathbf{H} - \mathbf{T}_{\text{cm}}$
- project on proton and neutron numbers simultaneously: $\mathbf{P}_{N_0 Z_0} = \mathbf{P}_{N_0} \mathbf{P}_{Z_0}$

consistent treatment of direct, exchange & pairing terms

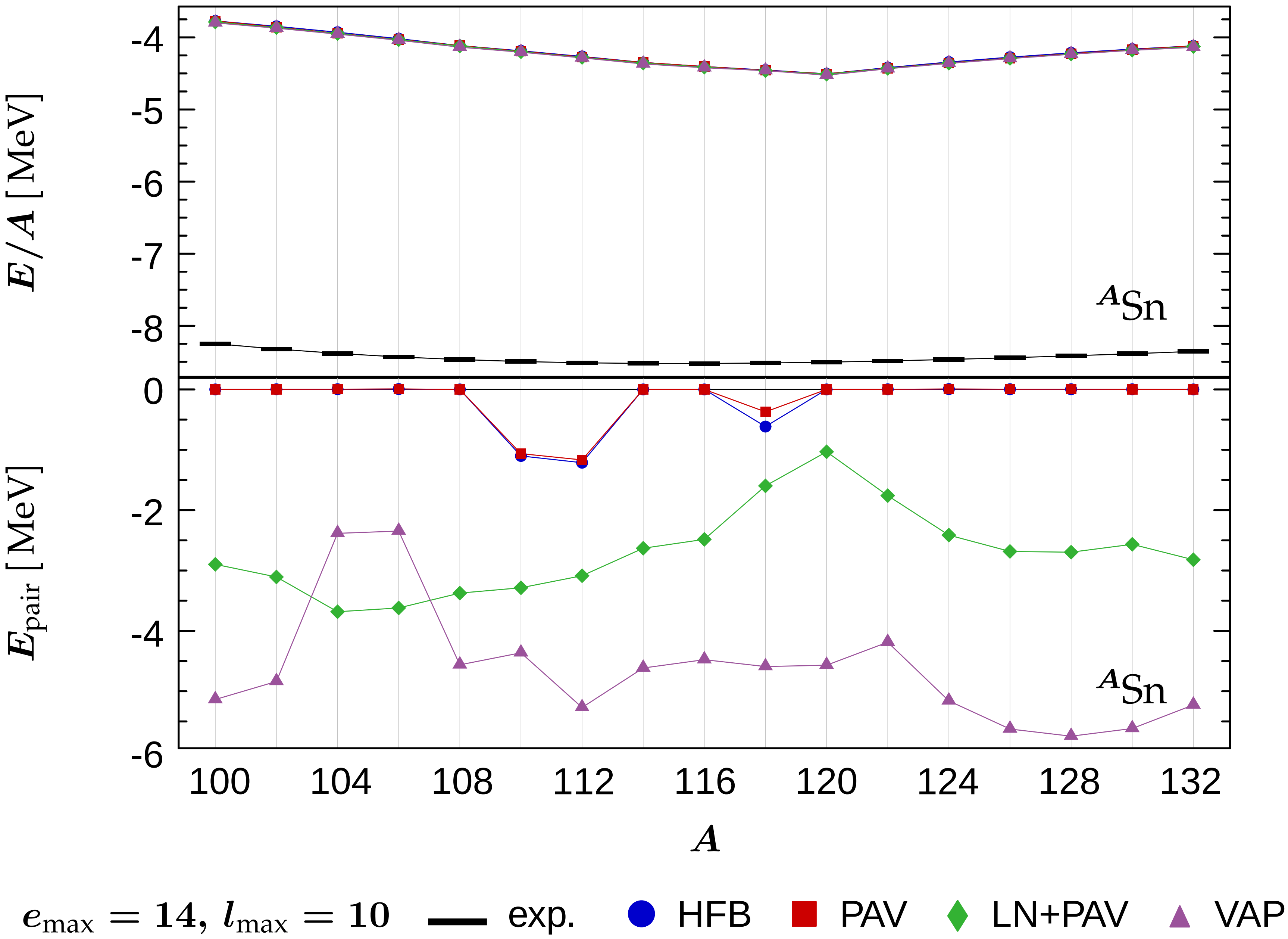
- $\mathbf{V}_{\text{UCOM}}$ in **ph- and pp-channel** !
- include (anti-)pairing effects from intrinsic kinetic energy and Coulomb interaction
- this level of consistency is **crucial for particle-number projection**

Sn Isotopes: Binding & Pairing Energies

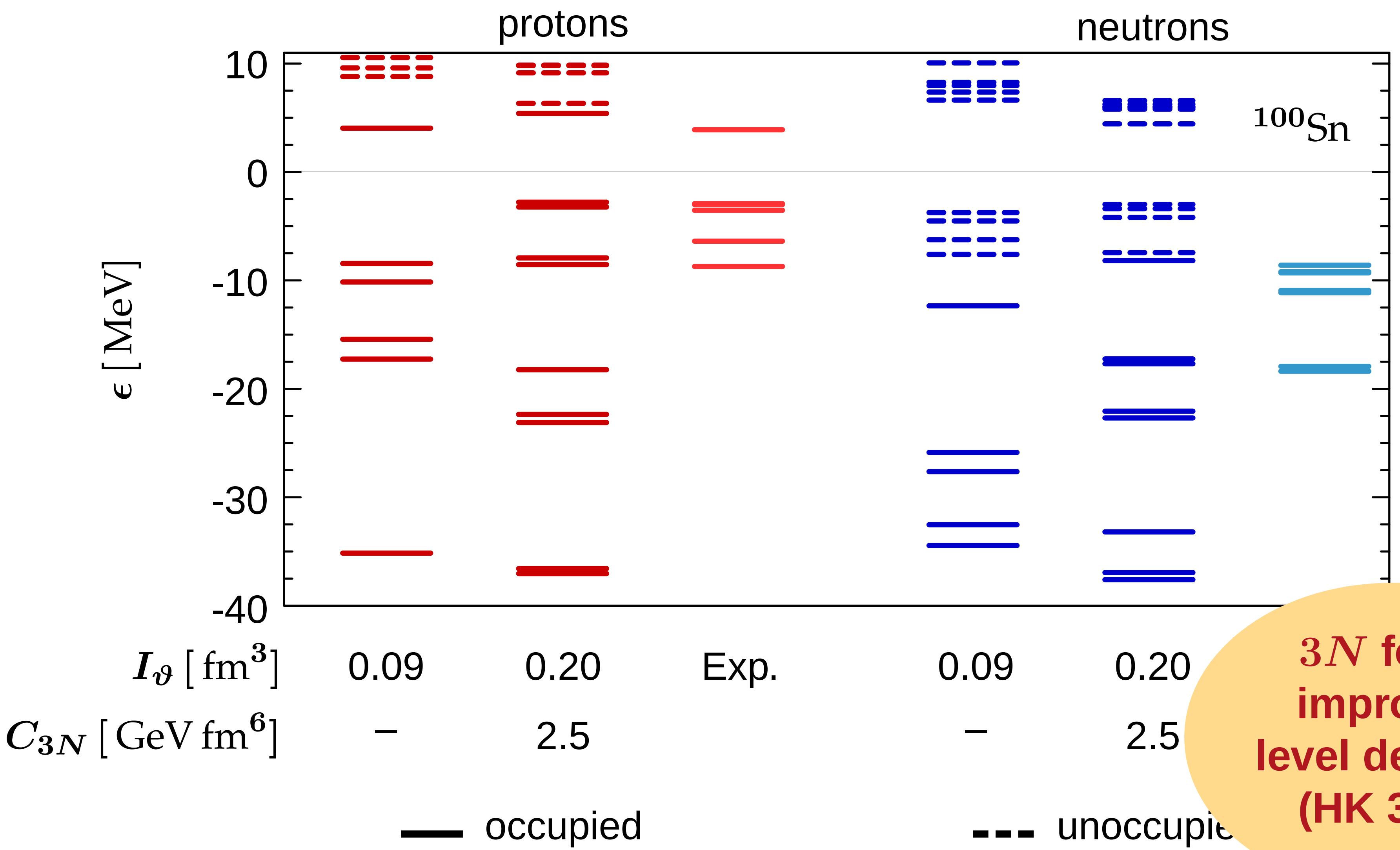


$e_{\text{max}} = 14,\ l_{\text{max}} = 10$ — exp. ● HFB

Sn Isotopes: Binding & Pairing Energies

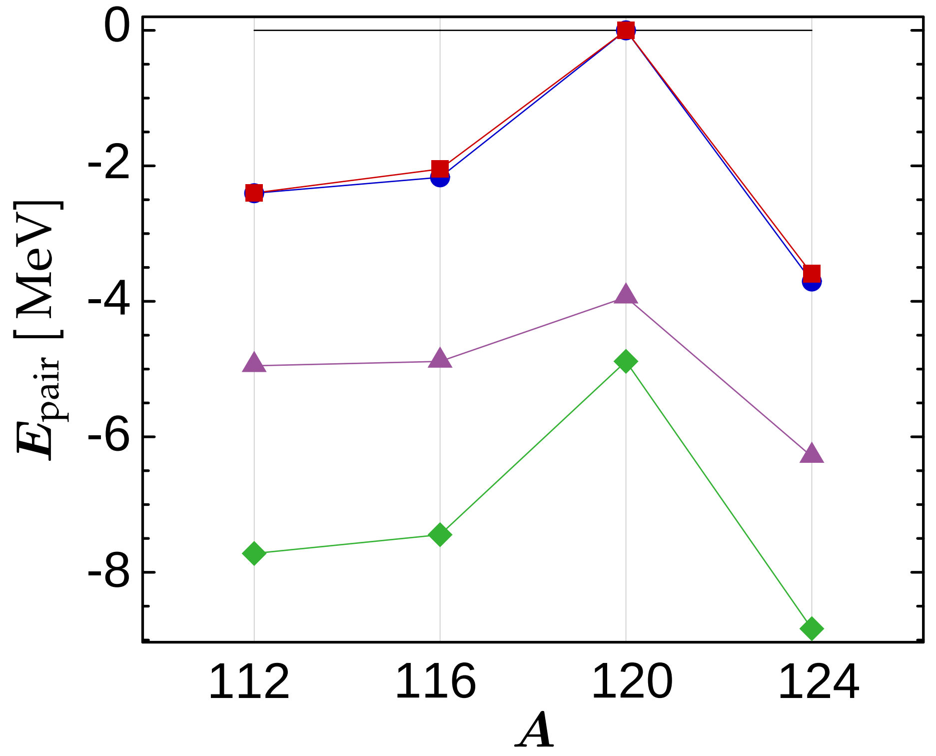


$3N$ Forces: HF Single-Particle Energies



$3N$ force improves level density! (HK 32.8)

$3N$ Forces: Pairing



$$\bar{v}^{[2]}_{kk',qq'} \;\longrightarrow\; \bar{v}^{[2]}_{kk',qq'} + f \cdot \sum_{rr'} \bar{v}^{[3]}_{kk'r,qq'r'} \rho^{HF}_{r'r}\,, \qquad f = \begin{cases} \frac{1}{3} & \text{expect. values} \\ \frac{1}{2} & \text{fields} \end{cases}$$

phenomenological VAP calculations: $E_{\text{pair}} \simeq 10 - 20\ \text{MeV}$

(Stoitsov et al., nucl-th/0610061; Anguiano et al., Phys. Lett. **B545** (2002), 62)

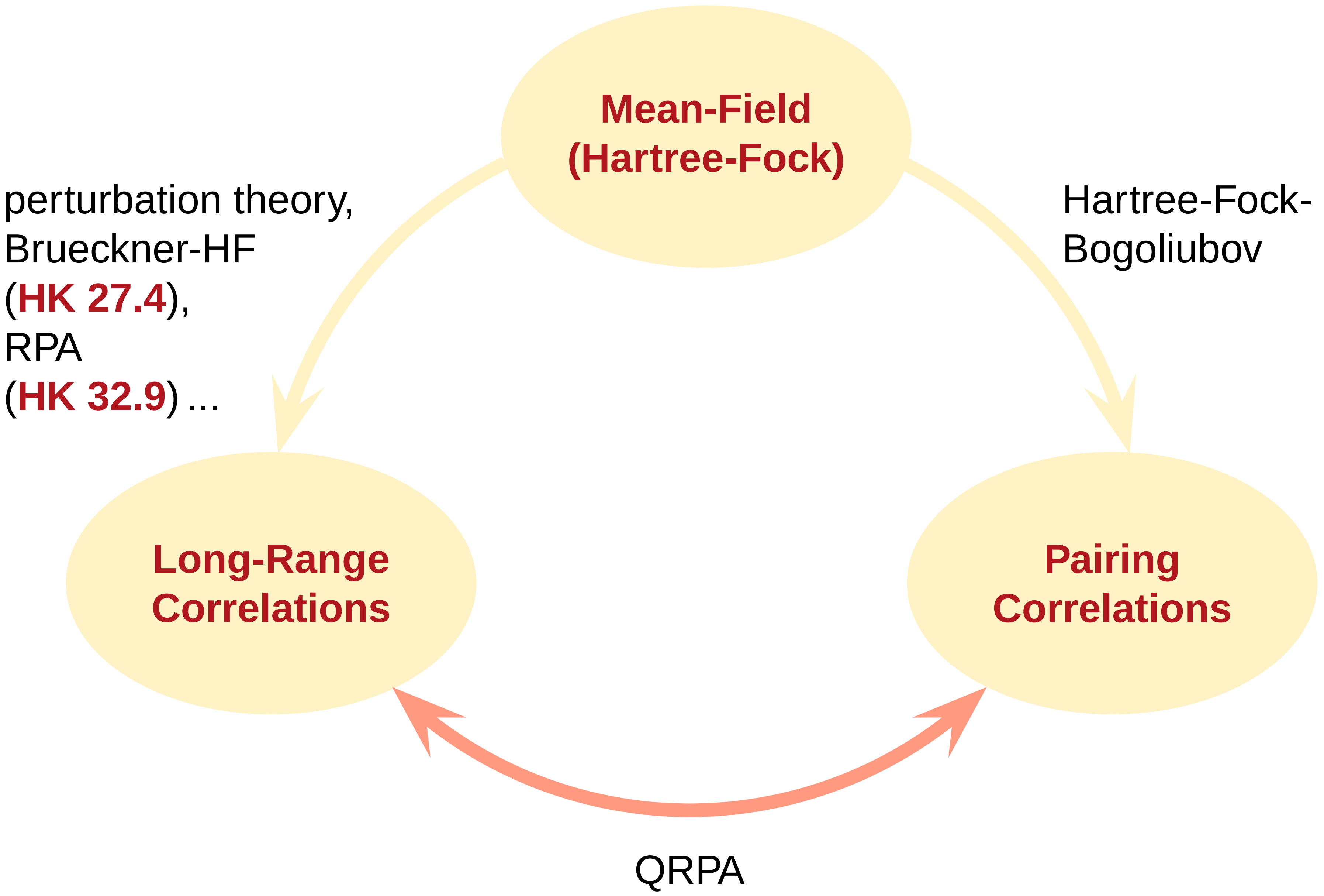
“correct” order of magnitude **with realistic NN int.**

Summary

- **fully consistent** HFB calculations with particle number projection, **based on a Hamiltonian**
- using $\mathbf{V}_{\mathrm{UCOM}}$ — a “universal” phase-shift equivalent $\boldsymbol{NN}$ interaction
- **inclusion of $3N$-forces** fixes the two-body $\mathbf{V}_{\mathrm{UCOM}}$’s problems with single-particle level density ($\longrightarrow$ A. Zapp, **HK 32.8**)
- HFB results are encouraging

$\Rightarrow$ contribute to a **consistent description of different aspects** of nuclear structure based on $\mathbf{V}_{\mathrm{UCOM}}$: HF(B), (Q)RPA, SRPA, MBPT, FMD, NCSM...

Outlook


Mean-Field
(Hartree-Fock)
perturbation theory,
Brueckner-HF
(HK 27.4),
RPA
(HK 32.9) ...
Hartree-Fock-
Bogoliubov
Long-Range
Correlations
Pairing
Correlations
QRPA

Outlook

- **Quasiparticle RPA** (benchmark calculations in progress)
- investigation of pn **pairing** & pn**-QRPA**
  $\Rightarrow$ Isobaric Analog & Gamow-Teller Resonances
- **“deformed” UCOM-HFB** (in progress)
- **symmetry restoration** by projection (isospin, parity, angular momentum)

Epilogue...

My Collaborators

- R. Roth, P. Papakonstantinou, A. Zapp, P. Hedfeld

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  Gesellschaft für Schwerionenforschung (GSI)

- N. Paar

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References

- H. Hergert, R. Roth, nucl-th/0703006, submitted to Phys. Rev. C
- N. Paar, P. Papakonstantinou, H. Hergert, and R. Roth, Phys. Rev. C**74**, 014318 (2006)
- R. Roth, P. Papakonstantinou, N.Paar, H. Hergert, T. Neff, and H. Feldmeier, Phys. Rev. C**73**, 044312 (2006)
- http://crunch.ikp.physik.tu-darmstadt.de/tnp/

Optional

Correlated Interaction

Correlated Hamiltonian

$$\widetilde{\mathrm{H}} = \mathrm{T}^{[1]} + \mathrm{V}_{\mathrm{UCOM}} + \mathrm{V}_{\mathrm{UCOM}}^{[3]} + \dots$$

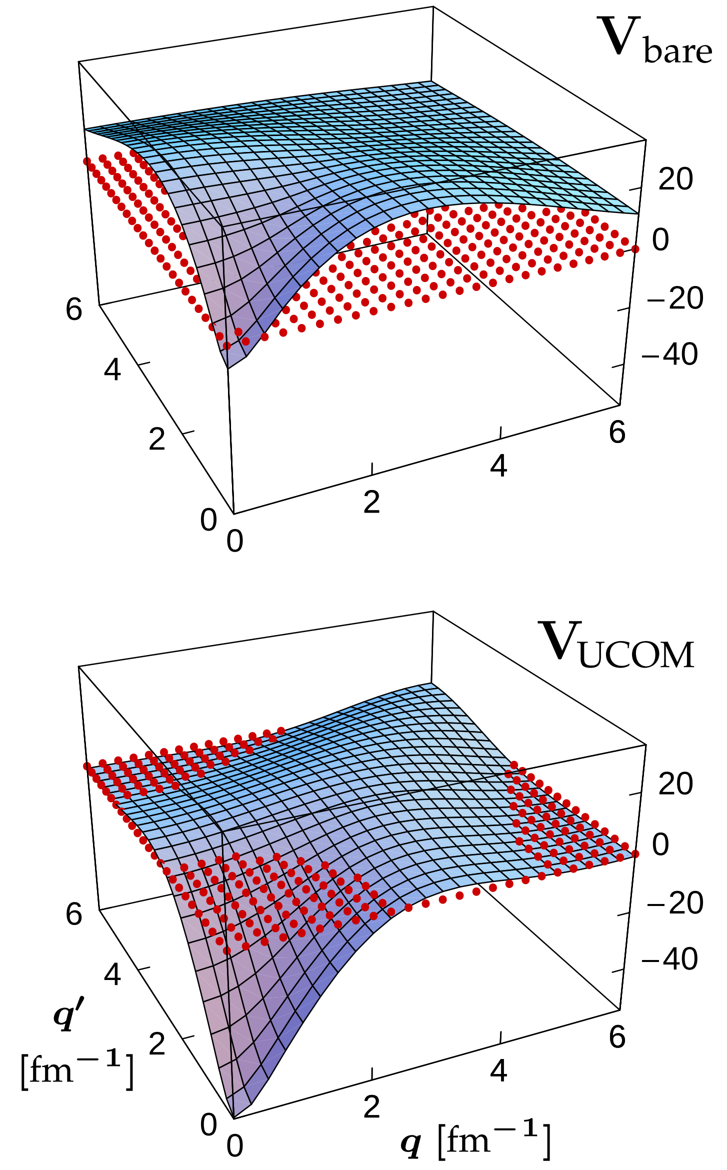
- **closed operator representation** of $\mathrm{V}_{\mathrm{UCOM}}$ in two-body approximation

  $\Rightarrow$ usable with **arbitrary many-body basis**

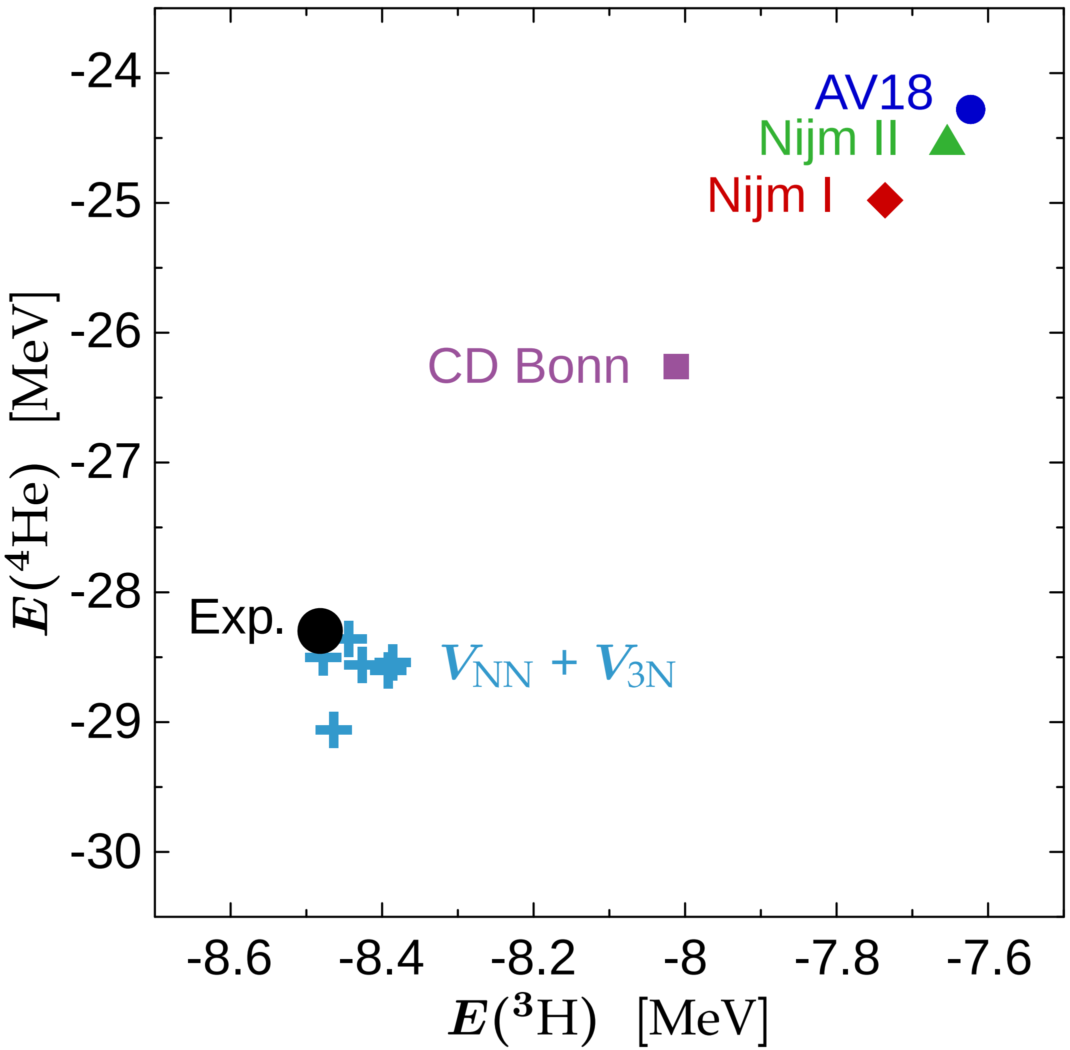
- $\mathrm{V}_{\mathrm{UCOM}}$ is **phase-shift equivalent** to the underlying bare nucleon-nucleon interaction

- $\mathrm{V}_{\mathrm{UCOM}}$ is pre-diagonalized in momentum space, i. e. **high-momentum components are decoupled** (similar to $\mathrm{V}_{\text{low-k}}$)

AV18 $^3S_1$



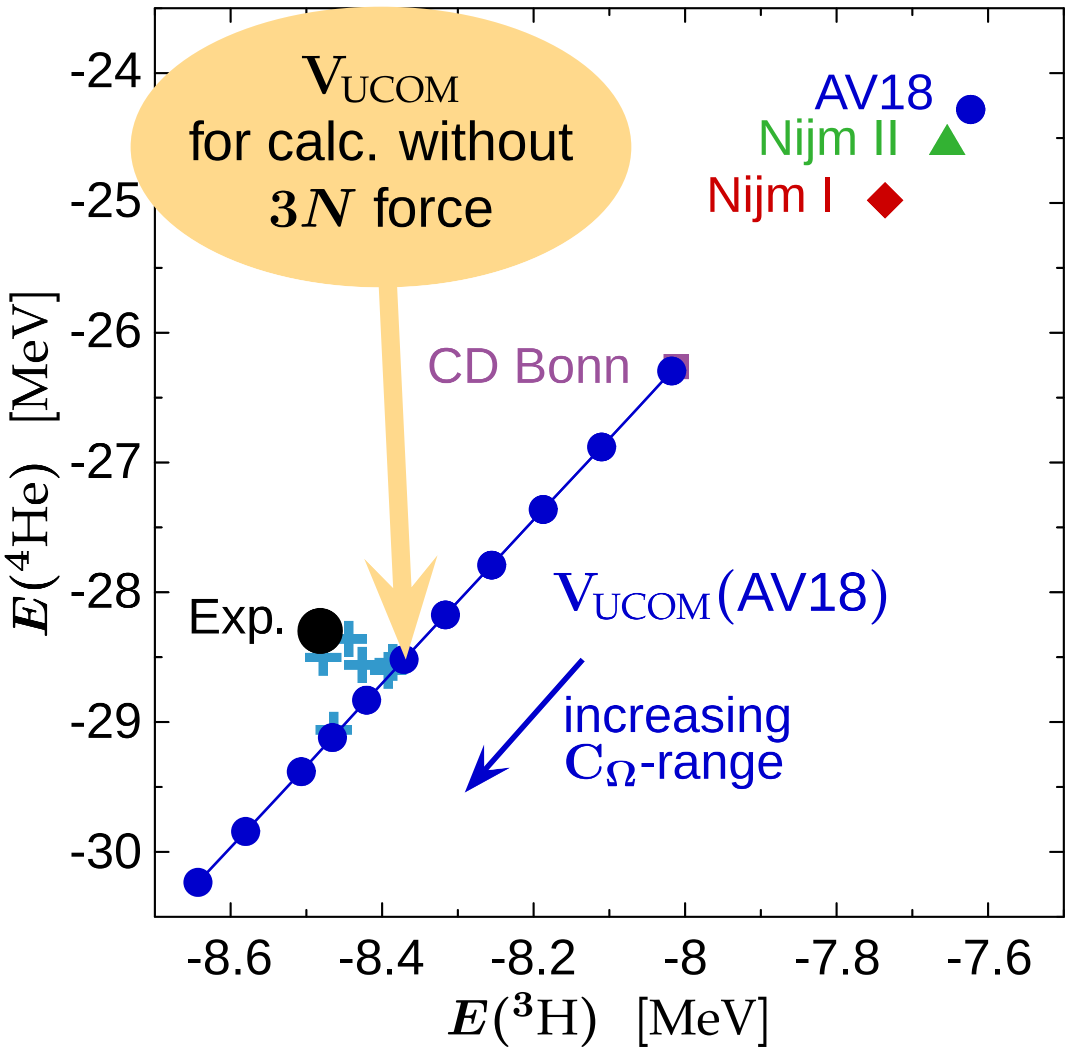
Tjon-Line and Correlator Range



- **Tjon-line**: $E(^4\mathrm{He})$ vs. $E(^3\mathrm{H})$ for phase-shift equivalent NN-interactions

Data points: A. Nogga et al., Phys. Rev. Lett. **85**, 944 (2000)

Tjon-Line and Correlator Range



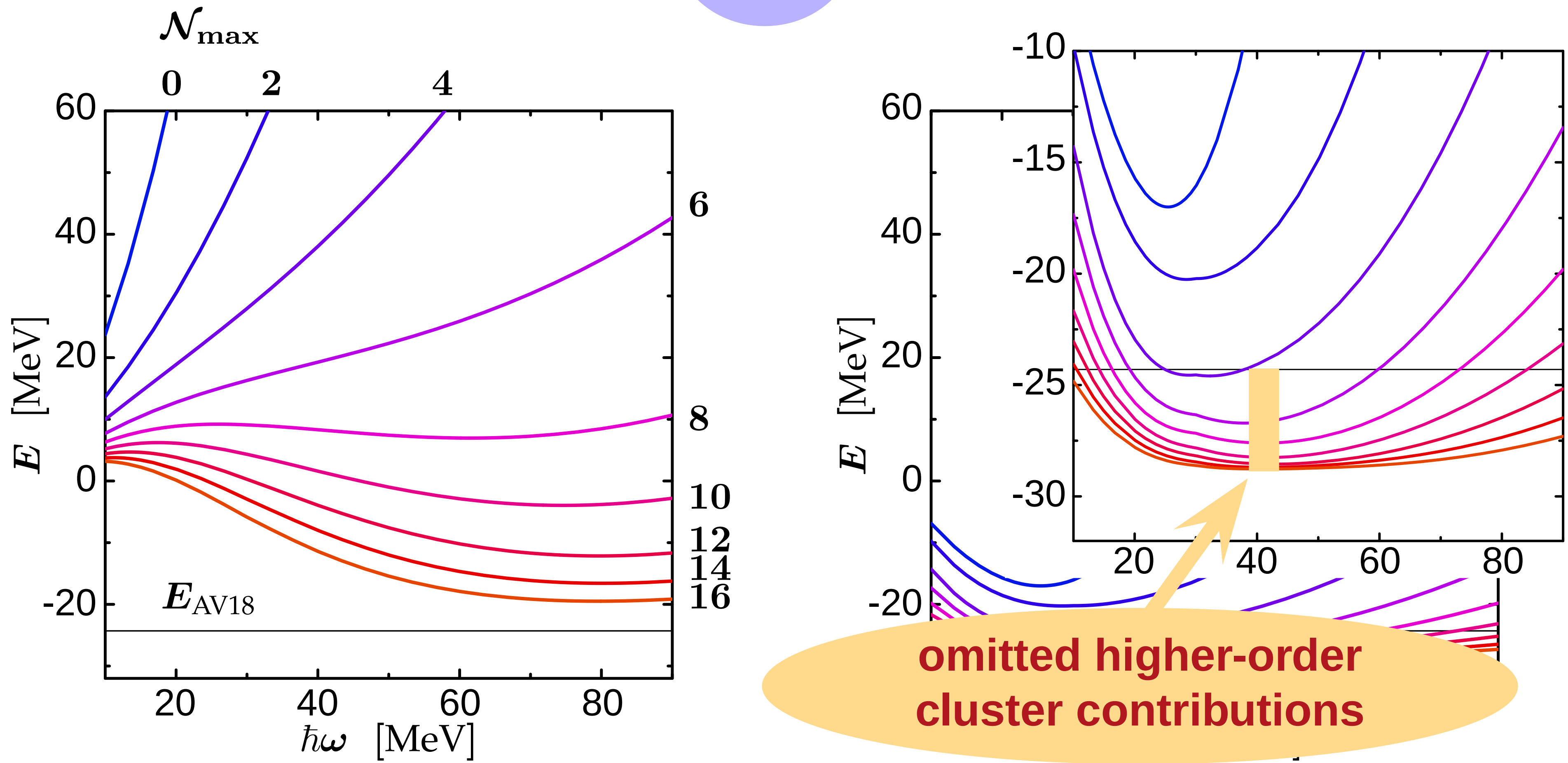
- **Tjon-line**: $E(^4\text{He})$ vs. $E(^3\text{H})$ for phase-shift equivalent NN-interactions
- change of C_Ω-correlator range results in shift along Tjon-line

minimize net three-body force
by choosing correlator with energies close to experimental value

Data points: A. Nogga et al., Phys. Rev. Lett. **85**, 944 (2000)

^{4}He: Convergence

AV18 | ^{4}He | V_{UCOM}



NCSM code by P. Navrátil [PRC 61, 044001 (2000)]