Astrophysics and Nuclear Structure / International Workshop XXXIV on Gross Properties of Nuclei and Nuclear Excitations / Hirschegg, January 15 - 21, 2006

Nuclear Ground State and Collective Excitations based on Correlated Realistic NN Interactions

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REALISTIC AND EFFECTIVE NUCLEON-NUCLEON INTERACTIONS

Realistic nucleon-nucleon interactions are determined from the phase-shift analysis of nucleon-nucleon scattering

ARGONNE V18 $\longrightarrow$ Wiringa et al., PRC 51, 38 (1995)

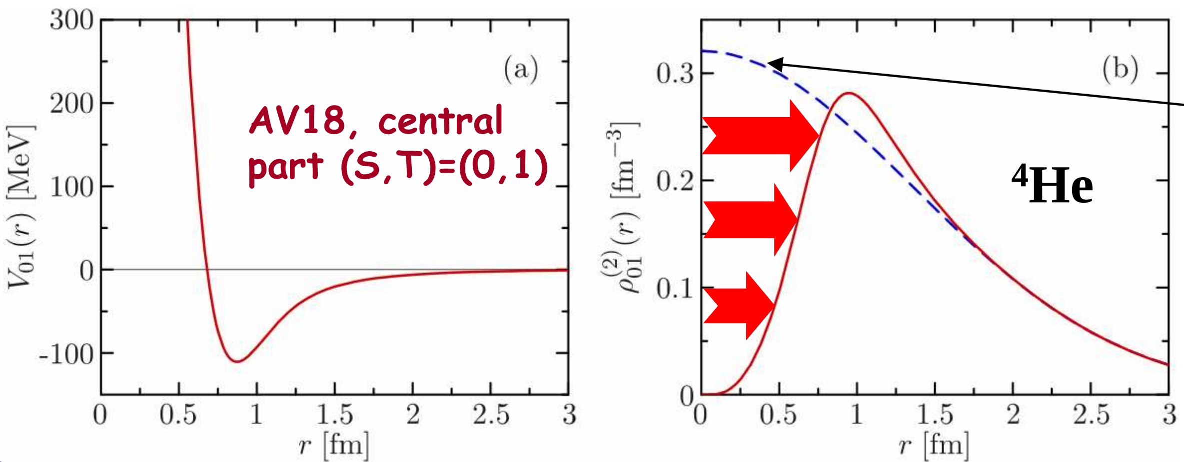
$$V(NN) = V^{EM}(NN) + V^{\pi}(NN) + V^{R}(NN)$$

$$V^{\pi}(pp) = f_{pp}^2 \tfrac{1}{3} m_{\pi} [Y(r)\boldsymbol{\sigma}_i \boldsymbol{\sigma}_j + T(r) S_{ij}]$$ $\longrightarrow$ One-pion exchange

$$V^{R}(NN) = V^{R} + V^{l2} L^2 + V^{t} S_{12} + V^{ls} \mathbf{L}\cdot\mathbf{S} + V^{ls2} (\mathbf{L}\cdot\mathbf{S})^2$$ $\longrightarrow$ intermediate and short-range phenomenological terms

NN interactions in the nuclear structure models $\rightarrow$ truncation of the full many-body Hilbert space to a subspace of tractable size (e.g. Slater det.) is problematic

Effective interactions!



Very large matrix elements of interaction (relative wave functions penetrate the core)

SHORT-RANGE NN INTERACTION-INDUCED CORRELATIONS

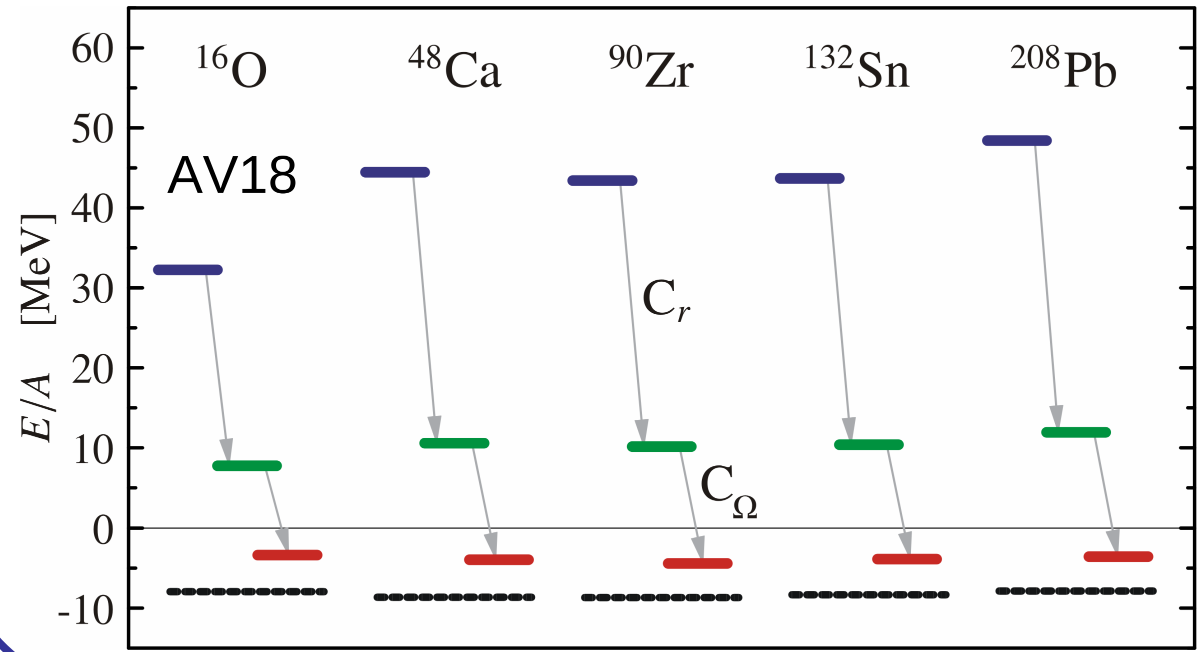
- *Short-range central* and *tensor* correlations are included in the simple many body states via *unitary transformation*

CORRELATED MANY-BODY STATE

$$|\hat{\Psi}\rangle = \mathrm{C}\ |\Psi\rangle\ = \mathrm{C}_{\Omega}\mathrm{C}_{r}\ |\Psi\rangle$$

UNCORRELATED MANY-BODY STATE

- Both the *central* and *tensor* correlations are necessary to obtain a bound nuclear system:



Expectation value of the Hamiltonian for Slater determinant of harmonic oscillator states

THE UNITARY CORRELATION OPERATOR METHOD (UCOM)

FELDMEIER, NEFF, ROTH (GSI&TUD)

$$C = C_\Omega C_r = e^{-i\sum_{i<j} g_{\Omega,ij}} e^{-i\sum_{i<j} g_{r,ij}}$$

Argonne V18 Potential — 40

$$g_r = \frac{1}{2}[s(r)q_r + q_r s(r)]$$ → Central Correlator C_r — 12

$$g_\Omega = \vartheta(r)\, s_{12}(\mathbf{r}, \mathbf{q}_\Omega)$$ → Tensor Correlator C_Ω — 6

Correlation functions are constrained by the energy minimization in the two-body system

Additional constraints necessary to restrict the ranges of the correlation functions

$$\int \vartheta(r) r^2 dr = I_\vartheta$$

TWO-NUCLEON SYSTEM

Two-Body Approximation

$$\hat{O} = C^\dagger O C = \hat{O}^{[1]} + \hat{O}^{[2]} + \cancel{\hat{O}^{[3]} + \dots}$$

→ V_{UCOM}

3-Nucleon Interaction ?

Fermionic Molecular Dynamics

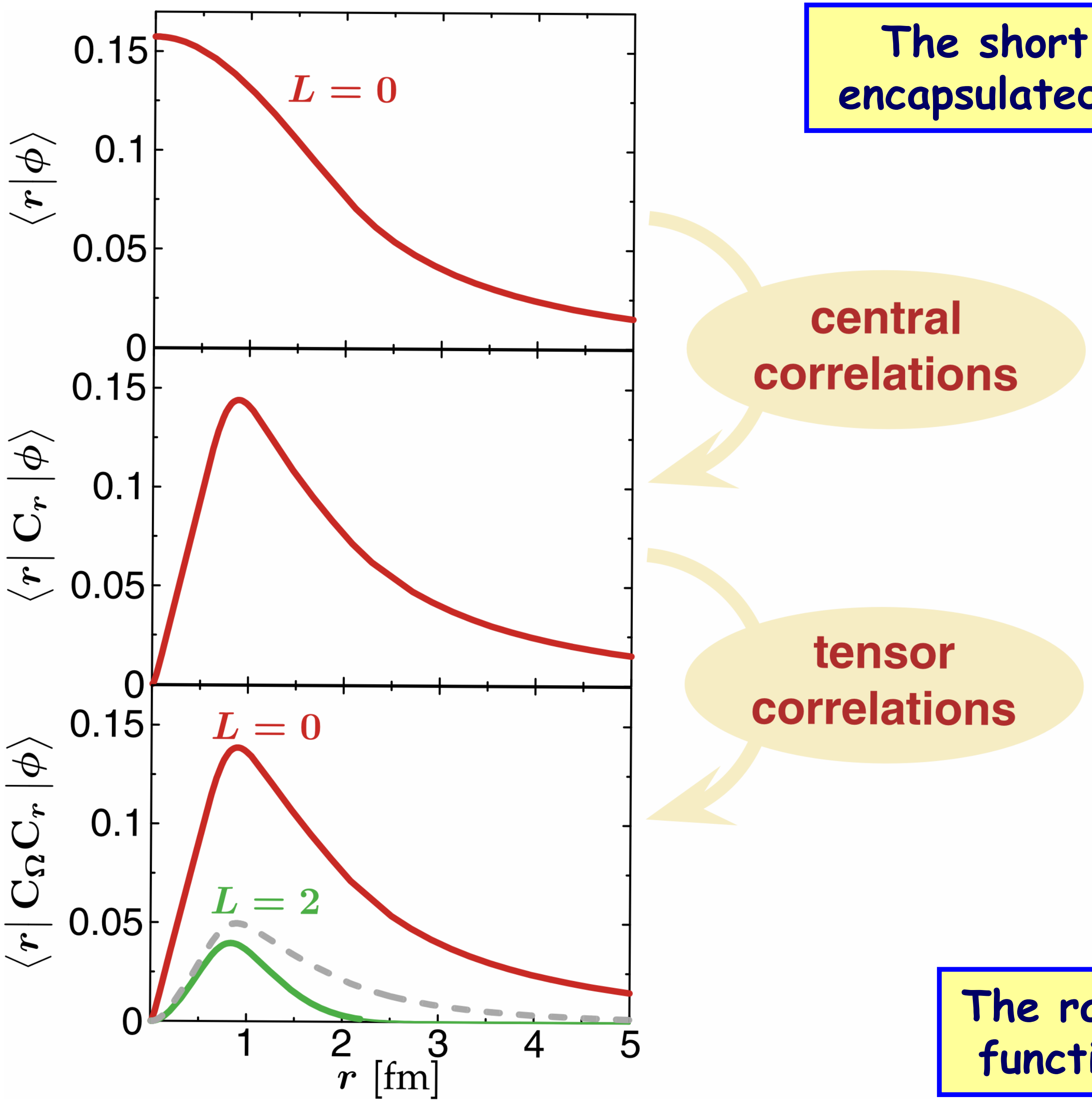
→*T. Neff*

No-core Shell Model

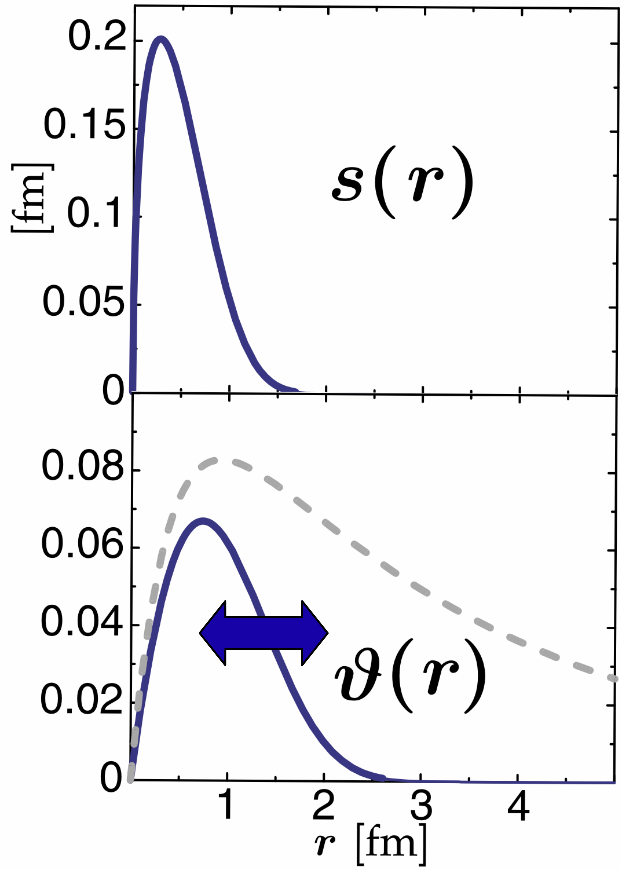
Hartree-Fock → Random-Phase Approximation

FINITE NUCLEI

CORRELATED WAVE FUNCTIONS - DEUTERON



The short-range correlations are encapsulated in correlation functions:



The range of tensor correlation function becomes a parameter

CORRELATED OPERATORS IN THE UCOM SCHEME

- Instead of correlated states with uncorrelated operators, the correlated operators are employed

$$\langle\hat{\Psi}|\,\mathrm{O}\,|\hat{\Psi}'\rangle = \langle\Psi|\,\mathrm{C}_r^\dagger \mathrm{C}_\Omega^\dagger \mathrm{O}\,\mathrm{C}_\Omega \mathrm{C}_r\,|\Psi'\rangle = \langle\Psi|\,\hat{\mathrm{O}}\,|\Psi'\rangle$$

- All observables need to be transformed consistently !

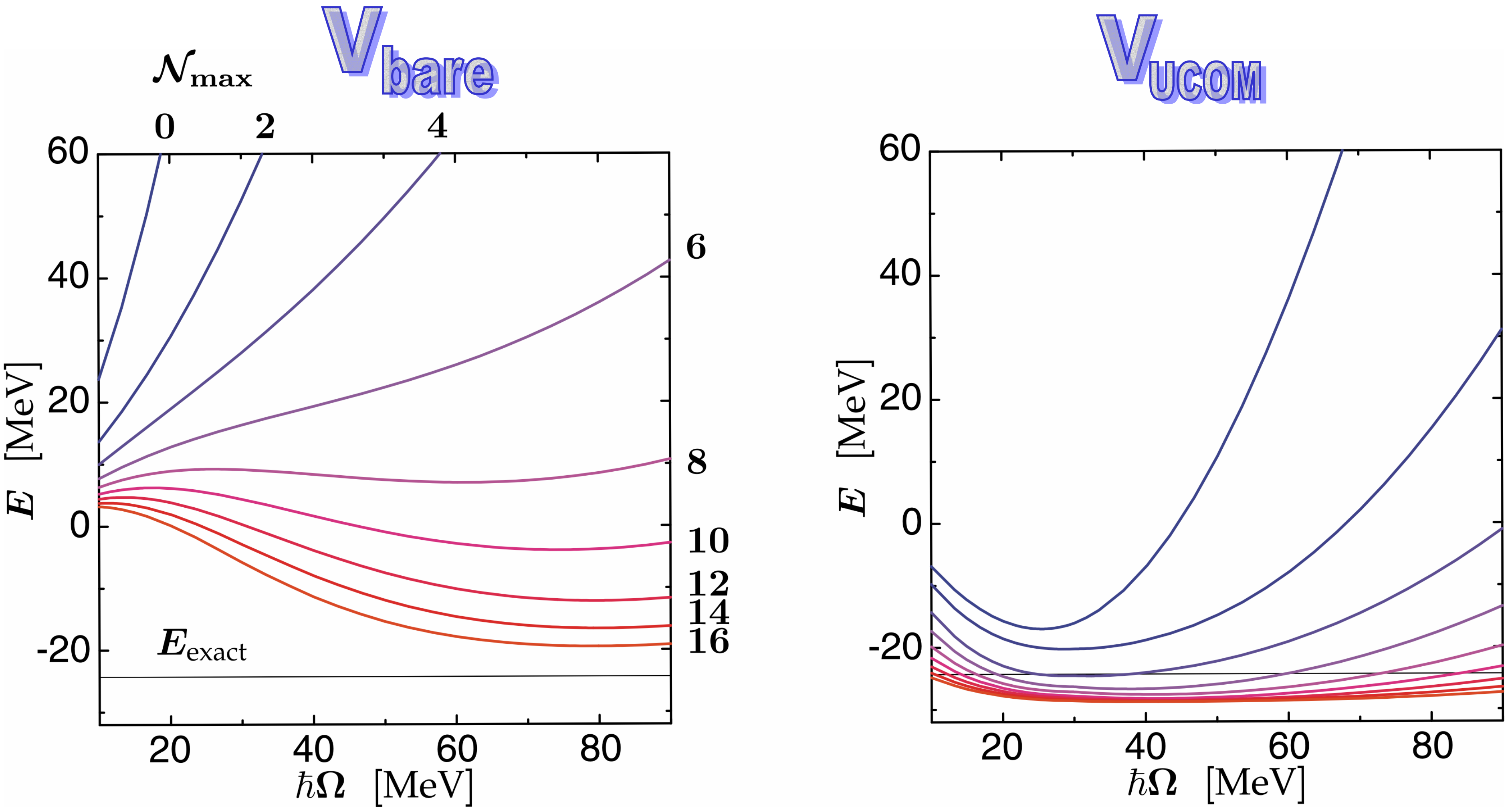
- Correlated Hamiltonian in two-body approximation

$$\hat{\mathrm{H}}^{C2} = \mathrm{C}_r^\dagger \mathrm{C}_\Omega^\dagger \mathrm{H} \mathrm{C}_\Omega \mathrm{C}_r = \mathrm{T}^{[1]} + \mathrm{V}_{\mathrm{UCOM}}$$

- Correlated realistic NN interaction is given in an explicit operator form

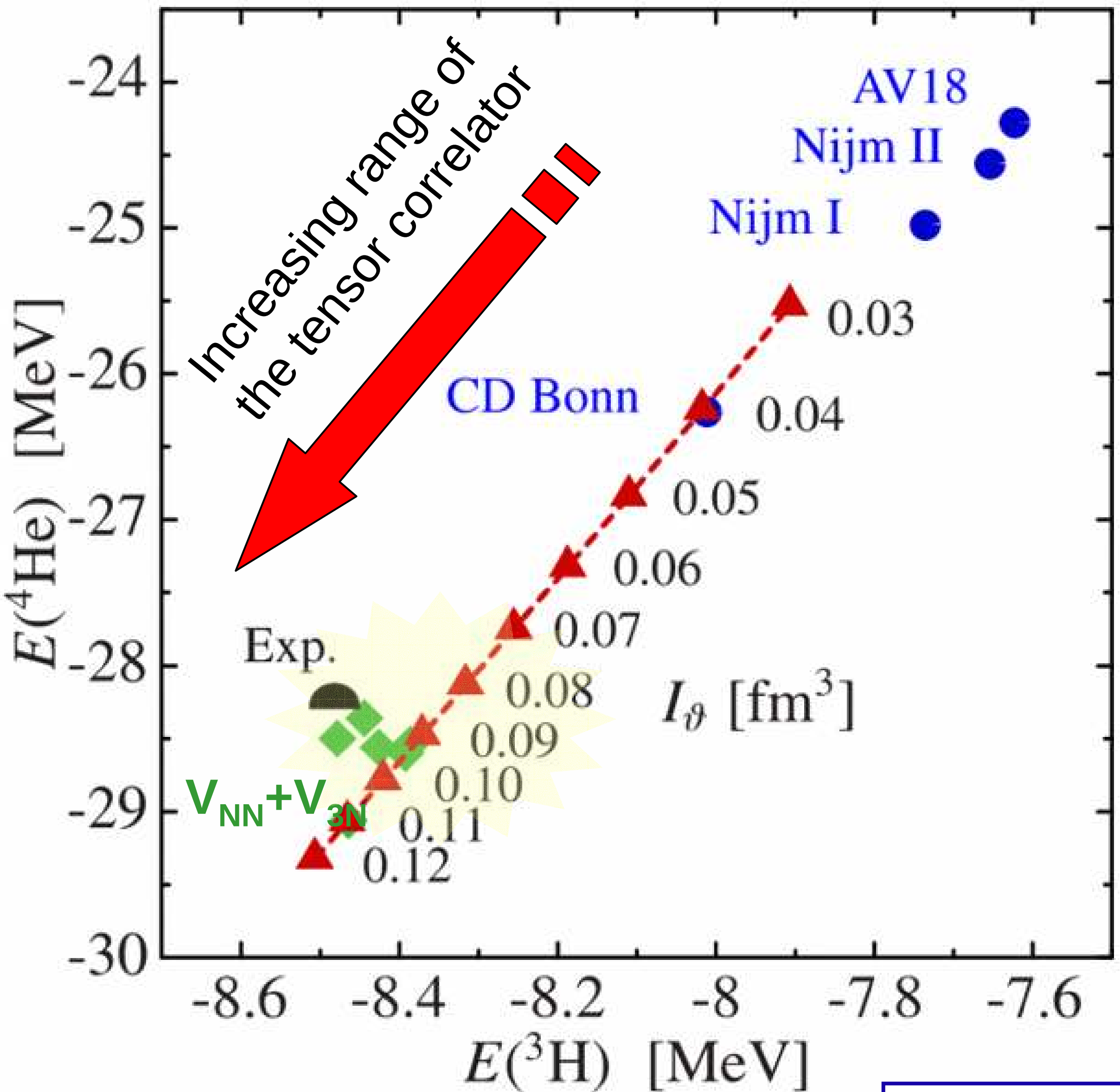
- Momentum-space matrix elements of the correlated interaction V_{UCOM} and low-momentum interaction $V_{low\text{-}k}$ are similar

TEST OF CONVERGENCE FOR THE V_{UCOM} POTENTIAL: ^4He



NCSM code by P. Navrátil, Phys. Rev. C 61, 044001 (2000)

TJON-LINE AND THE RANGE OF THE TENSOR CORRELATOR



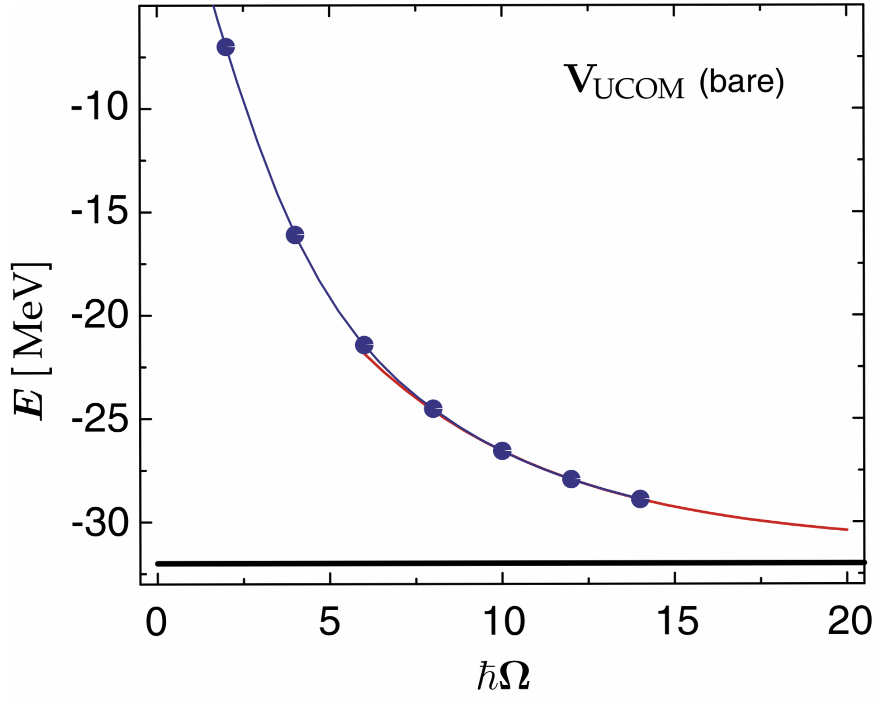
NO-CORE SHELL MODEL CALCULATIONS USING V_{UCOM}

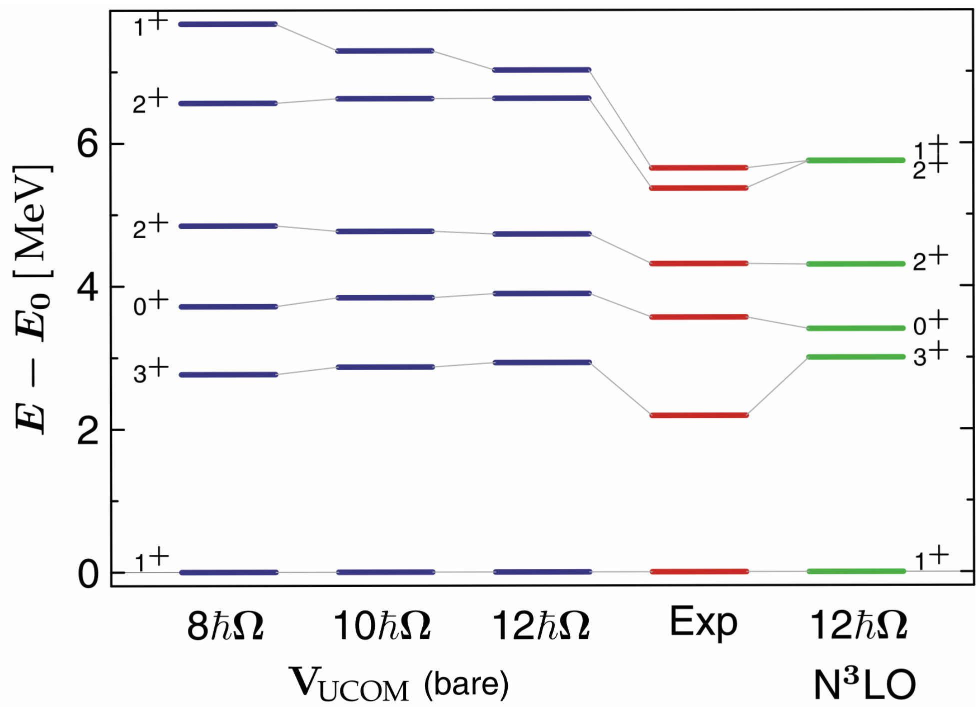
SELECT A CORRELATOR WITH ENERGY CLOSE TO EXPERIMENTAL VALUE

CANCELLATION OF OMMITED 3-BODY CONTRIBUTION OF CLUSTER EXPANSION AND GENUINE 3-BODY FORCE

R. Roth et al., Phys. Rev. C 72, 034002 (2005)

WORK IN PROGRESS : ^{6}Li





$\hbar\Omega$	8	10	12	14
E [MeV]	-24.522	-26.564	-27.938	-28.906
E [MeV] (extrapolation)				-31.226
E [MeV] (experiment)				-31.995

V_{UCOM}+Lee-Suzuki and more p-shell nuclei in progress...

NCSM calculations @ LLNL by P. Navratíl

HARTREE-FOCK BASED ON CORRELATED NN-POTENTIAL

- The nuclear ground state is described by a Slater determinant (short-range correlations are included in the correlated Hamiltonian)

- Expansion of the single-particle states in harmonic-oscillator basis

$$|\phi_{nljm}\rangle = \sum_\alpha D_{n\alpha}^{(lj)} |u_{\alpha ljm}\rangle$$

N_{MAX}=12

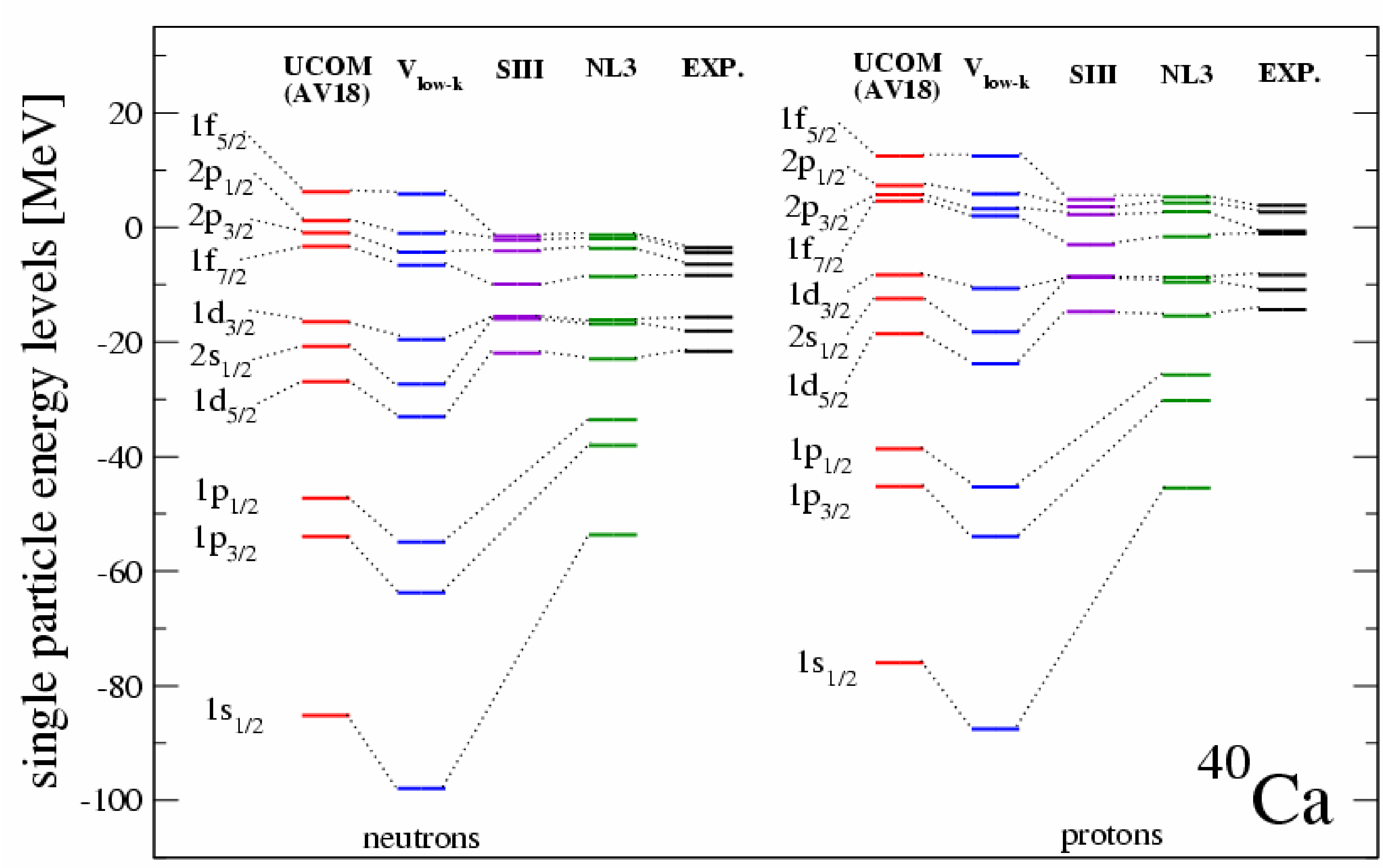
- Hartree-Fock matrix equations as a generalized eigenvalue problem

$$\boxed{\sum_\beta h_{\alpha\beta}^{(lj)} D_{n\beta}^{(lj)} = E_{nlj} \sum_\beta N_{\alpha\beta}^{(lj)} D_{n\beta}^{(lj)}}$$

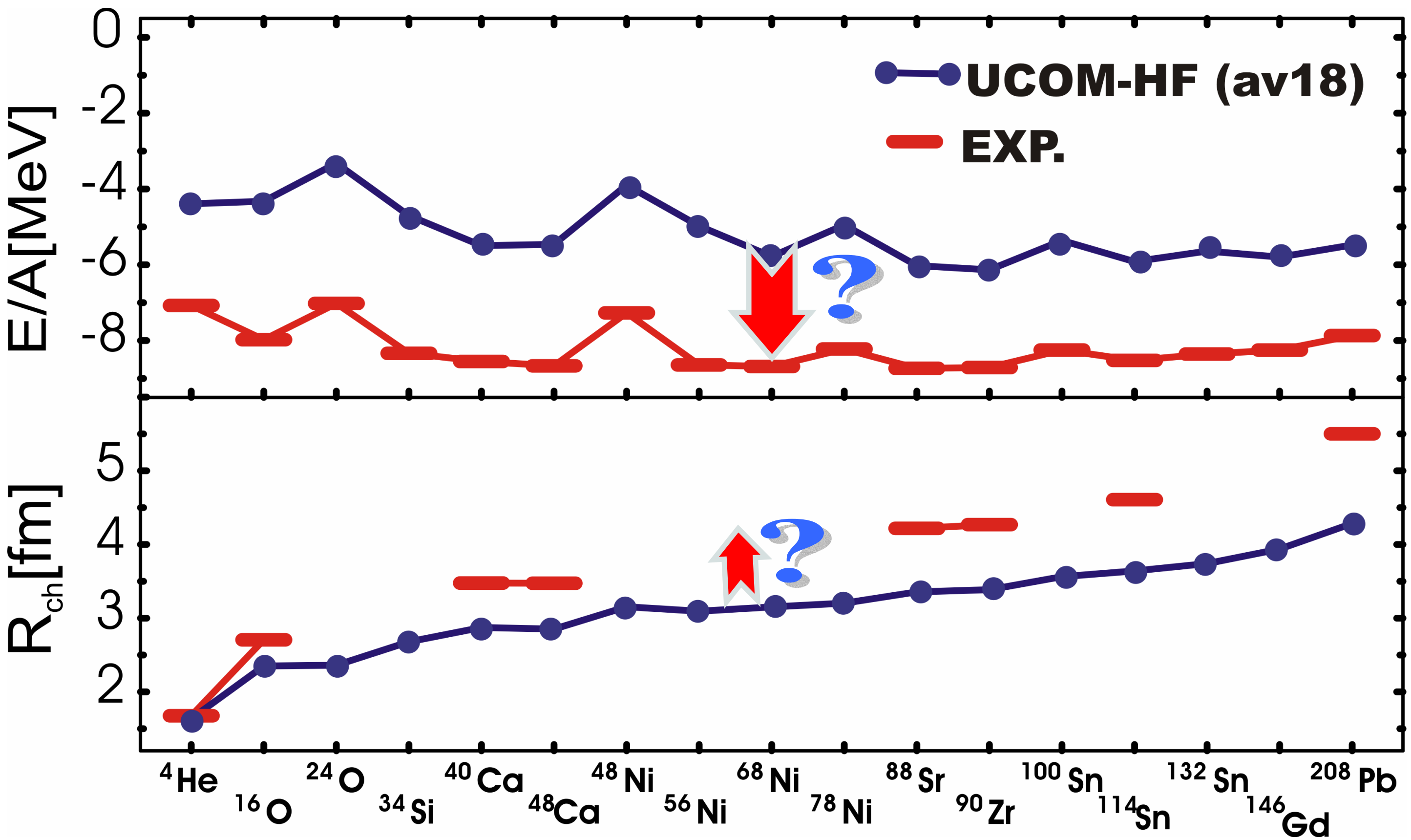
$$v_{\alpha\beta}^{(lj)} = \sum_{n'l'j'J\alpha'\beta'} \langle N_{n'l'j'}\rangle D_{n'\alpha'}^{(l'j')} D_{n'\beta'}^{(l'j')} \langle(\alpha lj, \alpha' l' j')J| \mathrm{V_{UCOM}} |(\beta lj, \beta' l' j')J\rangle_A$$

- Single-particle energies and wave functions are determined from iterative solution of the Hartree-Fock equations

UCOM HARTREE-FOCK SINGLE-PARTICLE SPECTRA



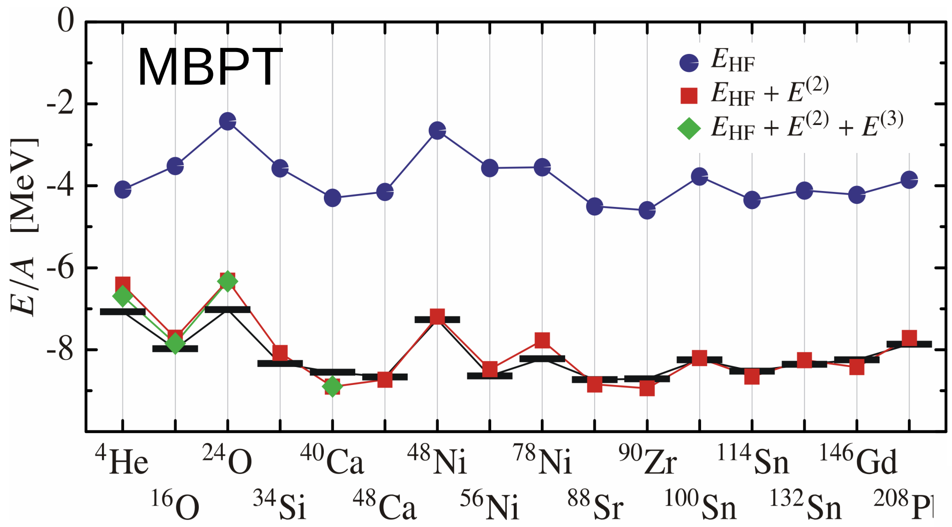
UCOM HARTREE-FOCK BINDING ENERGIES & CHARGE RADII



WHAT IS MISSING?

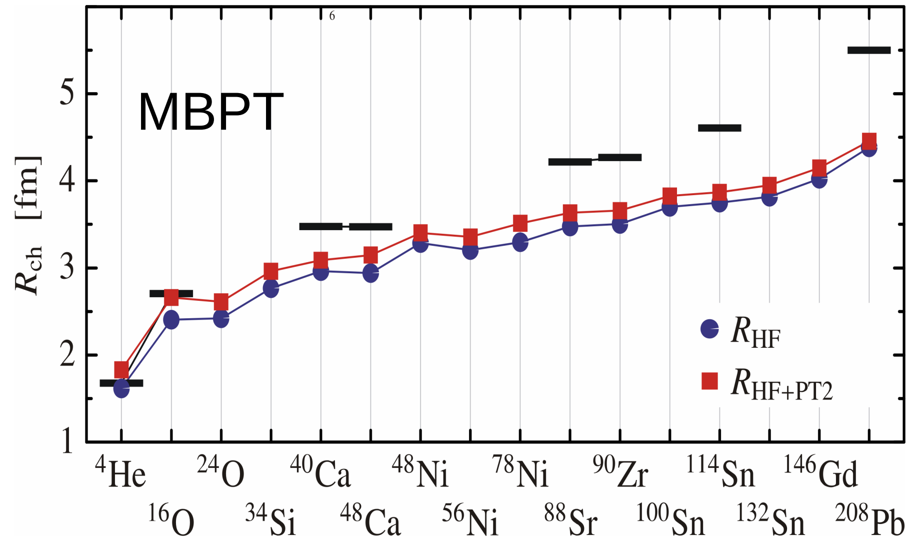
1. Long-range correlations
2. Genuine three-body forces
3. Three-body cluster contributions

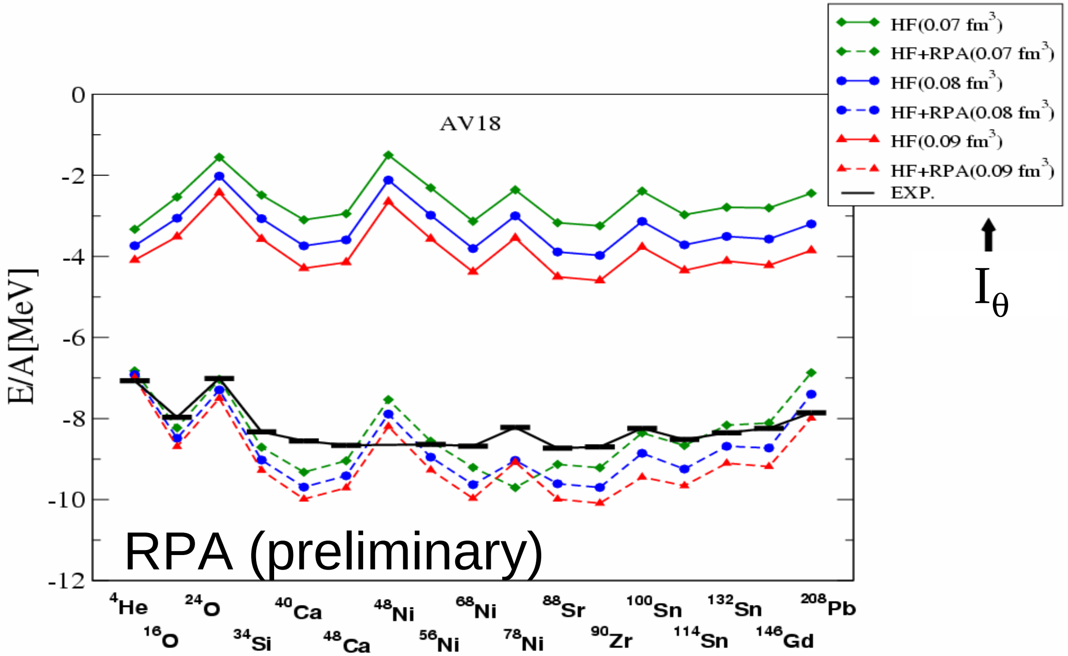
BEYOND THE HARTREE-FOCK



Long-range correlations can be recovered by the Many-Body Perturbation Theory (MBPT) or by evaluating RPA Correlations

$$E^{(2)} = -\frac{1}{4}\sum_{ij}^{occ}\sum_{ab}^{unocc}\frac{|\langle ij|\,T_{rel}+V_{UCOM}\,|ab\rangle|^2}{e_a+e_b-e_i-e_j}$$





$$\delta E = \sum_\lambda (2J+1)\hbar\omega_\lambda \sum_{ph}|Y^\lambda_{ph}|^2$$

UCOM RANDOM-PHASE APPROXIMATION

Low-amplitude collective oscillations

Vibration creation operator (1p-1h):

$$Q_\nu^+ = \sum_{mi} X^\nu_{mi} a^+_m a_i - \sum_{mi} Y^\nu_{mi} a^+_i a_m$$

$$Q_\nu |0\rangle = 0$$

$$Q_\nu^+ |0\rangle = |\nu\rangle$$

Equations of motion $\rightarrow$ RPA

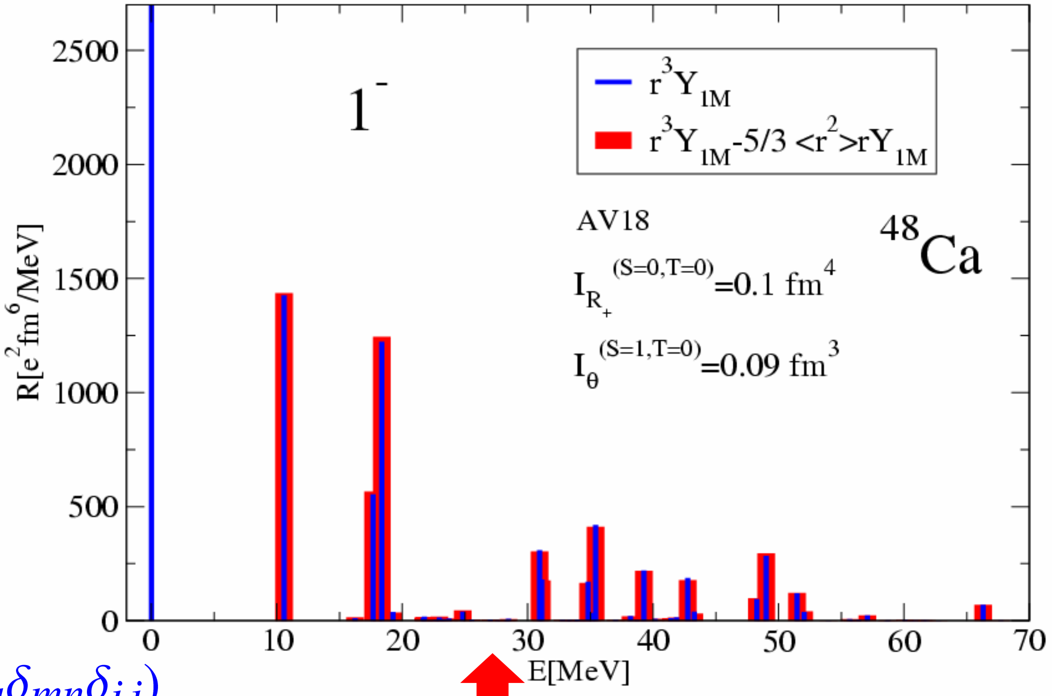
$(|0\rangle \approx |HF\rangle$ & $[Q_\nu(m,i), Q^+_\mu(n,j)] = \delta_{\nu\mu}\delta_{mn}\delta_{ij})$

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

V_{UCOM}

EXCITATION ENERGIES

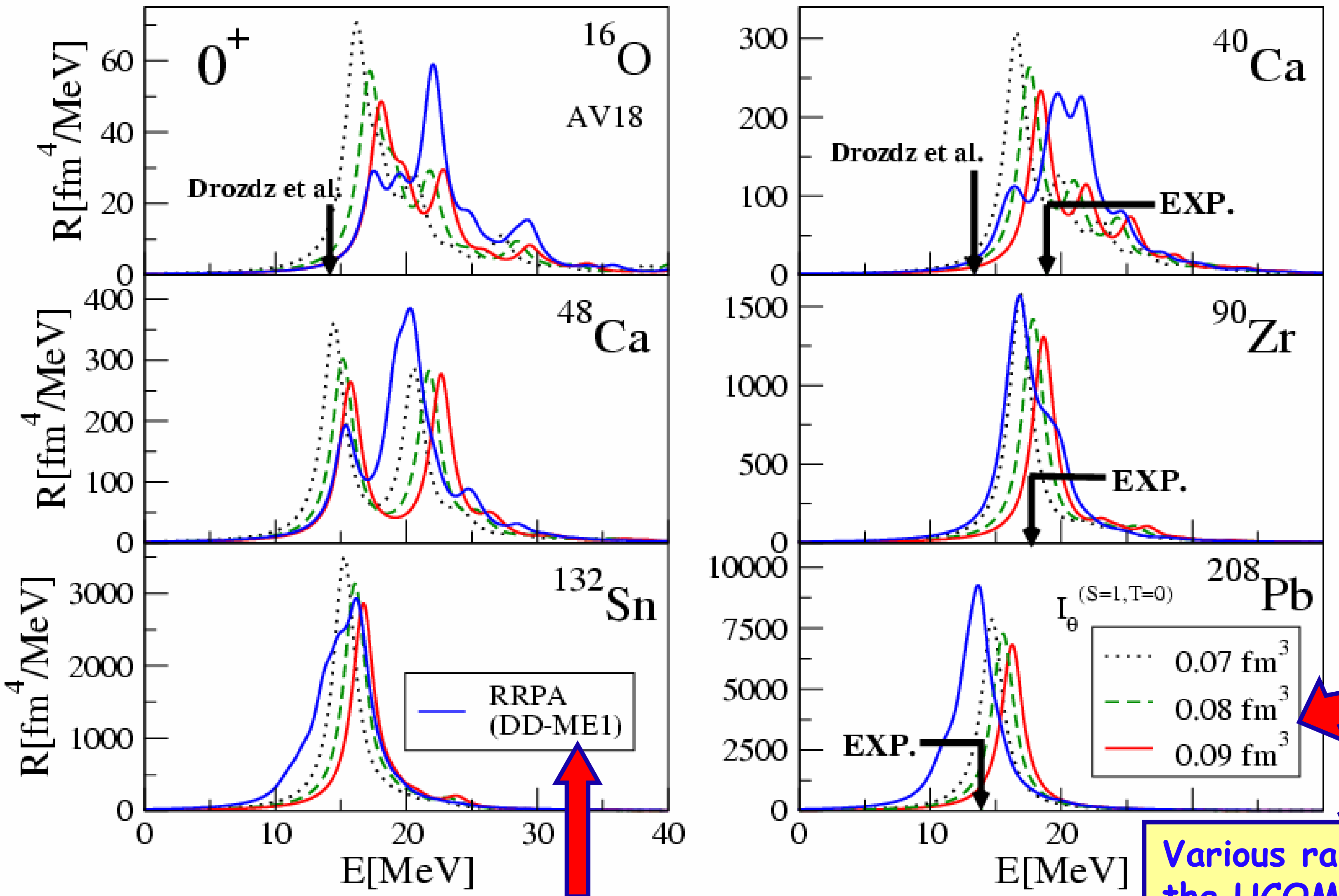
N. Paar et al., nucl-th/0511041 (2005)
nucl-th/0601026 (2006)



Fully-self consistent RPA model: there is no mixing between the spurious 1^- state and excitation spectra

Sum rules: $\epsilon = \pm 3\%$

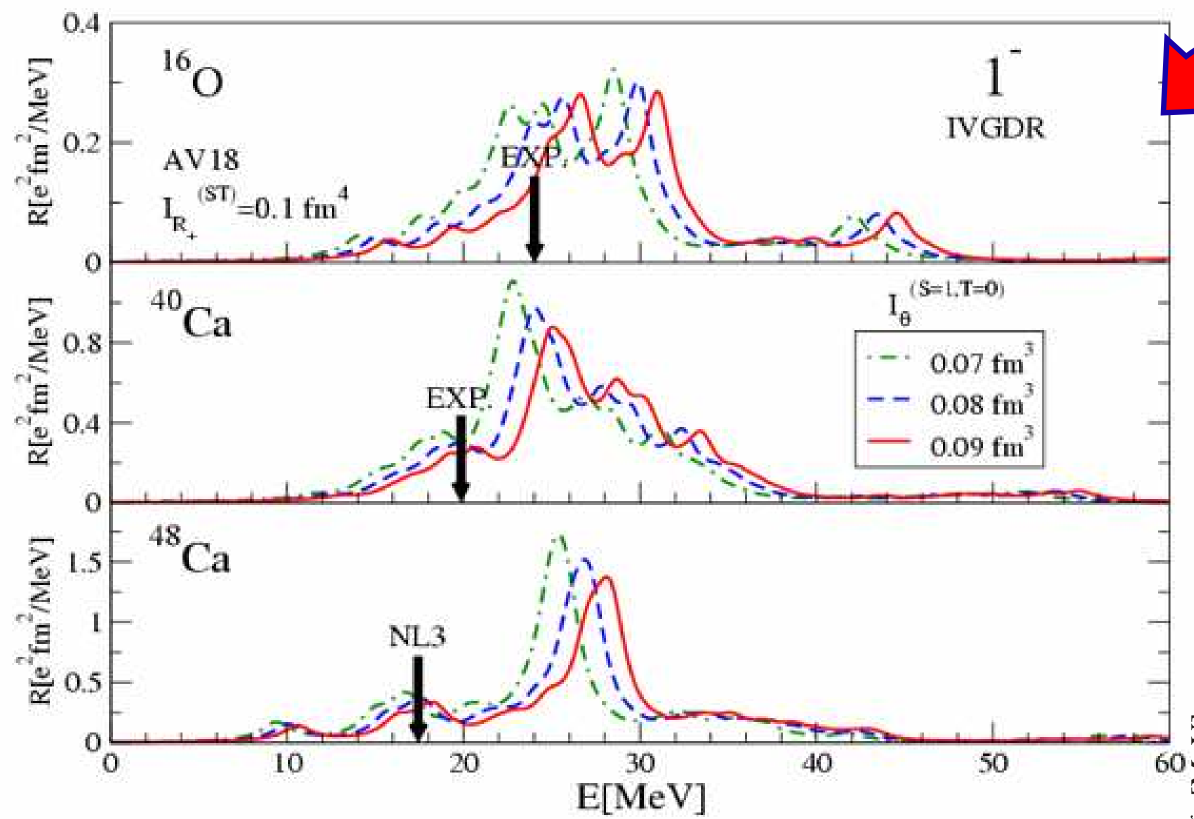
UCOM-RPA ISOSCALAR GIANT MONOPOLE RESONANCE



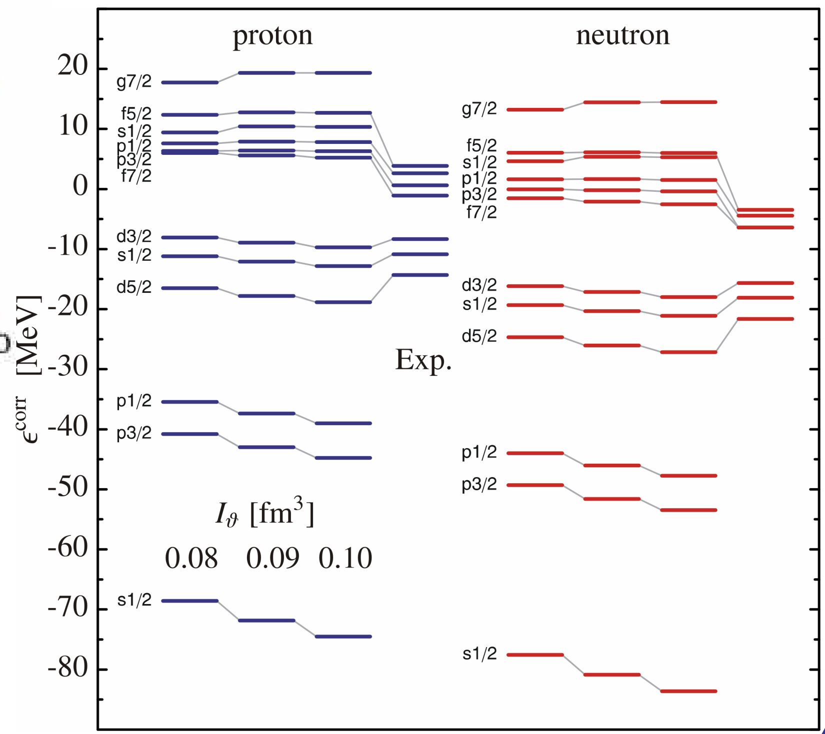
Relativistic RPA (DD-ME1 interaction)

Various ranges of the UCOM tensor correlation functions

UCOM-RPA ISOVECTOR GIANT DIPOLE RESONANCE

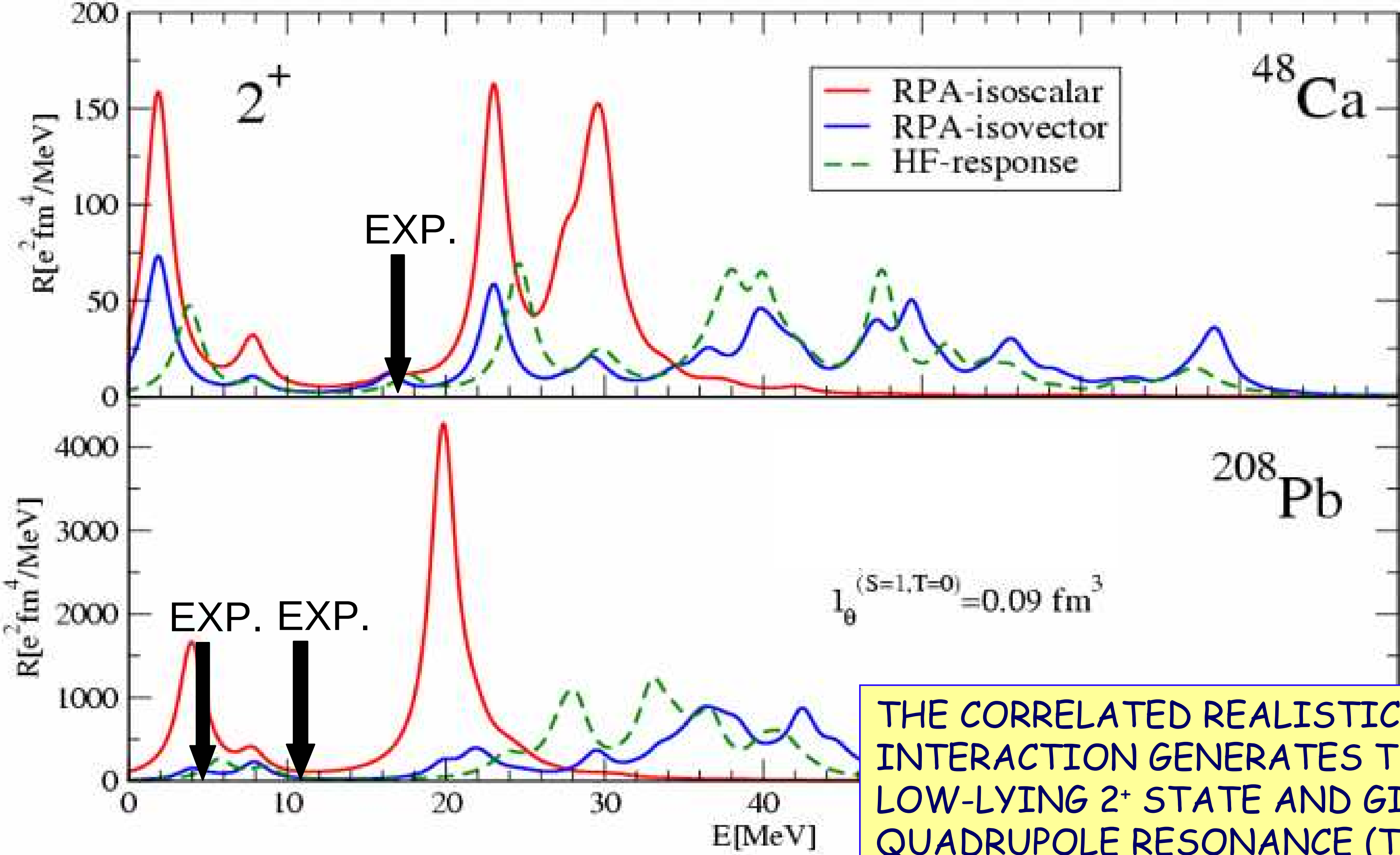


Various ranges of the UCOM tensor correlation functions



Improved description of the single-particle spectra (smaller range of the tensor correlator) pushes the IVGDR to lower energies.

UCOM-RPA GIANT QUADRUPOLE RESONANCE



THE CORRELATED REALISTIC NN INTERACTION GENERATES THE LOW-LYING 2^{+} STATE AND GIANT QUADRUPOLE RESONANCE (THE EXCITATION ENERGY TOO HIGH)

SUMMARY

- The correlated realistic nucleon-nucleon interaction (AV18) is employed in different nuclear structure methods: NCSM, HF, RPA

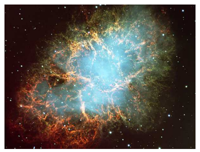
  - UCOM Hartree-Fock results in underbinding and small radii
    → *long-range correlations* are recovered by many-body perturbation theory and RPA

- Fully self-consistent UCOM Random-Phase Approximation (RPA) is constructed in the Hartree-Fock single-nucleon basis

  - Correlated realistic NN interaction generates collective excitation modes, however it overestimates the energies of giant resonances

⇒ Optimization of the ranges of correlators

⇒ Three-body interaction, complex configurations in RPA



WHAT ARE PERSPECTIVES FOR NUCLEAR ASTROPHYSICS?