Time Evolution of Bosonic Gases in 1D Two-Colour Superlattices

Markus Hild   Felix Schmitt   Robert Roth

Institut für Kernphysik • Technische Universität Darmstadt

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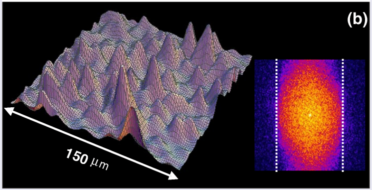
Overview

- Introduction
  - Framework : Bose-Hubbard Model
  - Two-Colour Superlattice
  - Zero-Temperature Phase Diagramm
- Approach
  - Eigenproblem / Time Evolution
  - Adaptive Basis Truncation
  - Excitation by Temporal Modulation
- Results
- Summary

Motivation

- bosonic gases in optical lattices are well suited for studying quantum phase transitions
- all relevant parameters can be tuned
- effect of an irregular lattice potential ?
- dynamical signatures as a probe for quantum phases ?

2D-Random Lattice Potential



J.E. Lye *et al.*, PRL **95**,070401 (2005)

Framework

Bose-Hubbard Hamiltonian

$$\mathbf{H} = \underbrace{-J \sum_{i=1}^{I} \left(\mathbf{a}_i^\dagger \mathbf{a}_{i+1} + \mathbf{a}_{i+1}^\dagger \mathbf{a}_i \right)}_{\text{Hopping}} + \underbrace{\sum_{i=1}^{I} \epsilon_i \, \mathbf{n}_i}_{\text{Ext. Potential}} + \underbrace{\frac{U}{2} \sum_{i=1}^{I} \mathbf{n}_i \left(\mathbf{n}_i - 1 \right)}_{\text{Interaction}}$$

tunneling strength J interaction strength U on-site potential ϵ_i

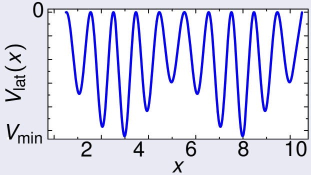
Basis Representation

$$|\Psi\rangle = \sum_{\alpha}^{D} c_\alpha |\{n_1 n_2 \dots n_I\}_\alpha\rangle$$

- states are defined by coefficients c_α
- coefficients $c_\alpha^{(0)}$ of the groundstate are obtained by diagonalisation of the Hamilton matrix

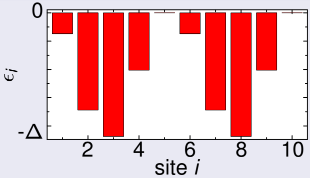
Two-Colour Superlattice

A Sinusoidal Modulated Lattice...



- superposition of two standing waves
- detuning of the wavelengths leads to a sinusoidal modulation of the optical lattice

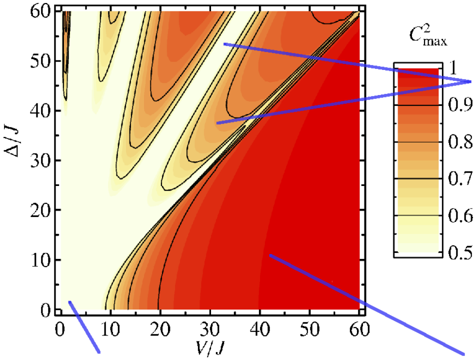
...Forms a Chain of Supercells



- Bose-Hubbard model: corresponding on-site energies ϵ_i with modulation amplitude Δ

Zero-Temperature Phase Diagram

10 Bosons / 10 Lattice Sites



(Quasi-) Bose Glass

rearrangements driven by the competition between interaction strength U and spatial modulation amplitude Δ

Superfluid Phase

in the region of weak interaction strengths / ext. potential all number states contribute to the groundstate

Homogenous Mott Insulator

the interaction strength exceeds the modulation amplitude ($U > \Delta$) $\Longrightarrow$ groundstate is dominated by a single number state

Adaptive Basis Truncation

Problem

- solving eigenproblem / time evolution
- basis dimension increases rapidly with number of atoms & lattice sites

Basis Truncation

- few number states contribute to low-lying eigenstates
- diagonal elements of Hamiltonian provide estimate for importance of basis states
- relevant number states $|\{n_1\, n_2\, \cdots\, n_I\}_\alpha\rangle$ satisfy the inequality

$$E_{trunc} \geq \sum_{i=1}^{I} \epsilon_i\, n_i + \frac{U}{2} \sum_{i=1}^{I} n_i\, (n_i - 1)$$

with the cut-off energy E_{trunc}

- precise description in the vicinity of the Mott insulating phase
- poster **Q30.2** : Tuesday, 16:30-18:30

Temporal Lattice Modulation

Probing the excitation spectrum...

... by lattice oscillation with amplitude $\mathcal{F}$ and frequency ω:

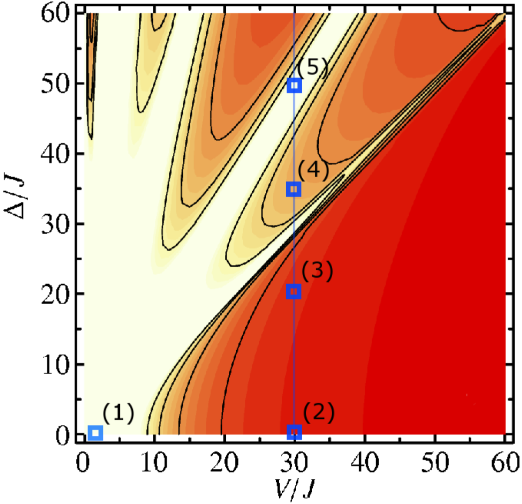
$$V_{\text{lattice}}(x,t) = V_{0,\text{lattice}}(x)\left(1+\mathcal{F}\sin\left(\omega t\right)\right)$$

↓

Time-Dependent Bose-Hubbard Parameters

$$\begin{aligned}
\text{tunneling strength} \quad J(t) &\approx J_0 \exp\left(-\mathcal{F}\sin\left(\omega t\right)\right) \\
\text{interaction strength} \quad U(t) &\approx U_0 \left(1+\mathcal{F}\sin\left(\omega t\right)\right)^{1/4} \\
\text{on-site energy} \quad \epsilon_i(t) &\approx \epsilon_{i,0}\left(1+\mathcal{F}\sin\left(\omega t\right)\right)
\end{aligned}$$

Roadmap



1) Superfluid Phase
2) Mott Insulator
3) Homogenous Mott Phase

4, 5) Bose Glass Phase

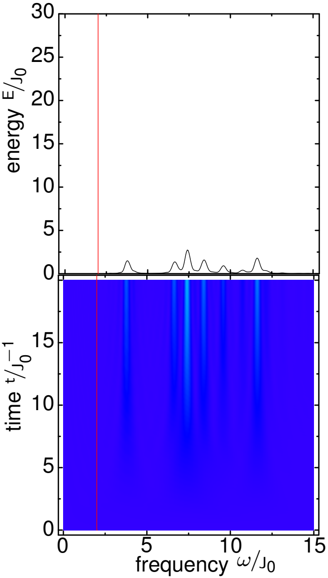
(1) Superfluid Phase

Setup

- $N = 6$ bosons, $I = 6$ sites
- interaction strength $\frac{U_0}{J_0} = 2$
- lattice modulation amplitude $\frac{\Delta}{J_0} = 0$

Characterisics

- very weak excitation
- slow response



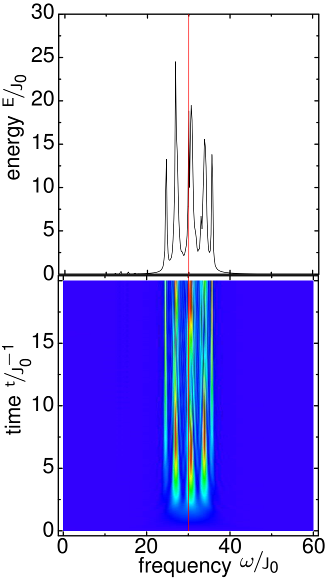
(2) Strongly Interacting Regime

Setup

- $N = 10$ bosons, $I = 10$ sites
- interaction strength $\frac{U_0}{J_0} = 30$
- lattice modulation amplitude $\frac{\Delta}{J_0} = 0$

Characteristics

- immediate response
- strong resonance at $\omega \approx U_0$
  $\Longrightarrow$ particle-hole excitations of the groundstate



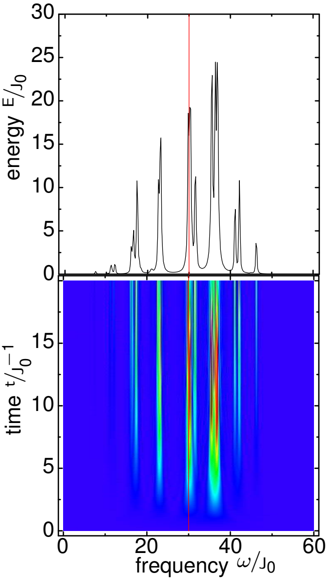
(3) Homogenous Mott Insulator & Superlattice

Setup

- $N = 10$ bosons, $I = 10$ sites
- interaction strength $\frac{U_0}{J_0} = 30$
- lattice modulation amplitude $\frac{\Delta}{J_0} = 20$

Characteristics

- resonance at frequency $\omega \approx U_0$ $\Longrightarrow$ particle-hole excitations of the groundstate
- broadening due to differences in the on-site energies



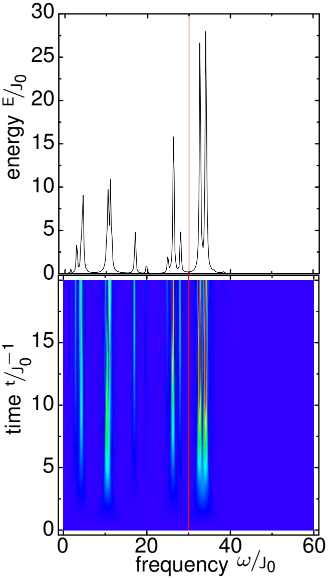
(4) Quasi-Bose Glass Phase

Setup

- $N = 10$ bosons, $I = 10$ sites
- interaction strength $\frac{U_0}{J_0} = 30$
- lattice modulation amplitude $\frac{\Delta}{J_0} = 35$

Characteristics

- rich resonance structure / resonances at low energies appear
- peaks agree with particle-hole excitations of the most propable number state



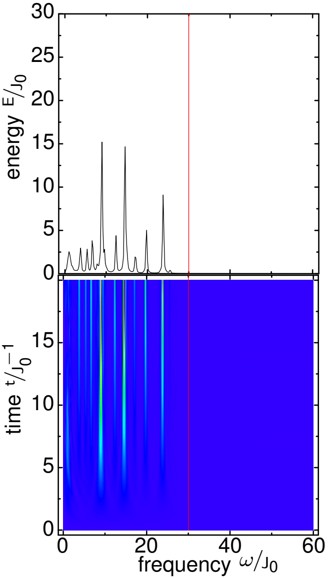
(5) Quasi-Bose Glass Phase

Setup

- $N = 10$ bosons, $I = 10$ sites
- interaction strength $\frac{U_0}{J_0} = 30$
- lattice modulation amplitude $\frac{\Delta}{J_0} = 50$

Characteristics

- resonances are "pushed" towards lower energies
- peaks agree with particle-hole excitations of the most propable number state



Summary

- strong resonance in the Mott insulator phase
- broadened by spatial lattice modulation
- larger modulation amplitude "pushes" the resonances towards lower energies

➜ dynamical signatures seem to be a promising tool to probe quantum phases