Quantum Phases of Bose and Fermi Gases in Optical Lattices

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Overview & Summary

- experiments on the Mott-insulator transition for bosonic atoms in optical lattices [1] reveal a huge potential for the study of the complex mechanisms behind quantum phase transitions
- we explore which other quantum phases can be produced with ultracold atoms in optical lattices, how they can be characterised, and which experimental signatures can be expected
- we utilise the Bose-(Fermi-)Hubbard model to describe pure Bose gases [2] and boson-fermion mixtures [3] at zero temperature via an exact numerical solution of the eigenvalue problem
- the rigidity of the system under phase variations is used to obtain information on the superfluid density of the bosonic species and the conductivity of the fermionic component [4,5]
- bosons in two-colour superlattice reveal a rich phase diagram with additional insulating phases (like the quasi Bose-glass) governed by the competition between on-site energies and two-body interaction [4,6]
- boson-fermion mixtures exhibit different insulating phases which can be characterised using the superfluid fraction of the bosonic and the conductivity of the fermionic component

Bose-Fermi-Hubbard Model

- one-dimensional lattice with I sites, N_B bosons, and N_F fermions
- single-band **Bose-Fermi-Hubbard Hamiltonian** with nearest neighbour hopping, on-site two-body interactions, and on-site single-particle energies [3]:

$$\hat{H} = -J_B \sum_{i=1}^{I} (\hat{a}^\dagger_{i+1}\hat{a}_i + \text{h.a.}) - J_F \sum_{i=1}^{I} (\hat{c}^\dagger_{i+1}\hat{c}_i + \text{h.a.}) \qquad \textbf{hopping terms}$$

$$+ \frac{V_{BB}}{2} \sum_{i=1}^{I} \hat{n}^B_i(\hat{n}^B_i - 1) + V_{BF} \sum_{i=1}^{I} \hat{n}^B_i \hat{n}^F_i \qquad \textbf{two-body interaction}$$

$$+ \sum_{i=1}^{I} \epsilon^B_i\, \hat{n}^B_i + \sum_{i=1}^{I} \epsilon^F_i\, \hat{n}^F_i \qquad \textbf{on-site energy}$$

$\hat{a}^\dagger_i$, $\hat{c}^\dagger_i$	creation operators for boson/fermion at site i
$\hat{n}^B_i$, $\hat{n}^F_i$	boson/fermion occupation number operators for site i
J_B, J_F	tunnelling matrix element between adjacent sites
V_{BB}, V_{BF}	on-site boson-boson/boson-fermion interaction strength
ϵ^B_i, ϵ^F_i	on-site single-particle energies

- states represented in a **complete basis of Fock states** $|n^B_1, ..., n^B_I\rangle \otimes |n^F_1, ..., n^F_I\rangle$ with all allowed sets of occupation numbers with $\sum_i n^B_i = N_B$ and $\sum_i n^F_i = N_F$

$$|\Psi_0\rangle = \sum_{\alpha=1}^{D_B} \sum_{\beta=1}^{D_F} C_{\alpha\beta}\, |\{n^B_1, ..., n^B_I\}_\alpha\rangle \otimes |\{n^F_1, ..., n^F_I\}_\beta\rangle$$

- **exact solution** of large-scale eigenvalue problem for a few eigenstates with Lanczos-type algorithm; basis dimensions up to $D = D_B D_F \approx 10^6$ feasible

Bosonic Superfluidity & Fermionic Conductivity

- beyond simple quantities — like mean occupation numbers $\bar{n}_i$, number fluctuations σ_i, and condensate fraction — the **rigidity under phase twists** is an important indicator for fundamental dynamical properties of the system [4,5]
- we impose a linear phase variation on either the bosonic or the fermionic component through Peierls phase factors in the respective hopping term

$$\hat{a}^\dagger_{i+1}\hat{a}_i \to e^{-i\Theta_B/I}\, \hat{a}^\dagger_{i+1}\hat{a}_i \qquad \hat{c}^\dagger_{i+1}\hat{c}_i \to e^{-i\Theta_F/I}\, \hat{c}^\dagger_{i+1}\hat{c}_i$$

- the phase twist causes an increase of the ground state energy; the energy change is connected to the kinetic energy of the flow generated by the phase gradient
- **boson twist**: the energy change resulting from a phase twist for the bosons is a measure for the superfluid density of the bosonic component; the rigidity can be identified with the **superfluid fraction** f^B_s (neglecting the suppression of the superfluid flow by the lattice itself) [4,5]

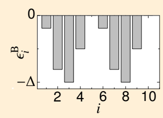
$$f^B_s = \frac{I^2}{N_B} \frac{E_{\Theta_B} - E_0}{J_B\, \Theta_B^2} \qquad \Theta_B \ll \pi$$

- **fermion twist**: the energy change resulting from fermionic phase twist is related to the conductivity of the fermionic component; the corresponding rigidity defines the **conducting fraction** f^F_c (equivalent to the well-known Drude weight)

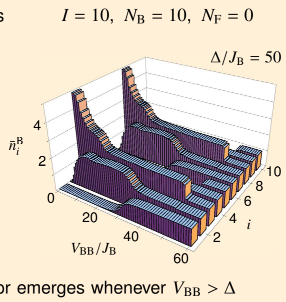
$$f^F_c = \frac{I^2}{N_F} \frac{E_{\Theta_F} - E_0}{J_F\, \Theta_F^2} \qquad \Theta_F \ll \pi$$

- an important further step is the distinction between normal- and superconductivity for the fermionic component (work in progress)

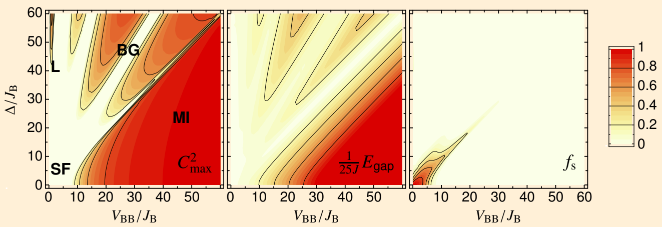
Bosons in Two-Colour Superlattices



- superposition of two standing wave lattices with different wavelengths generates spatial modulation of the well-depths ϵ^B_i ➔ **two-colour superlattice**
- competition between on-site energy (favours localisation), kinetic energy and repulsive interaction (delocalisation) generates rich phase diagram [4,6]

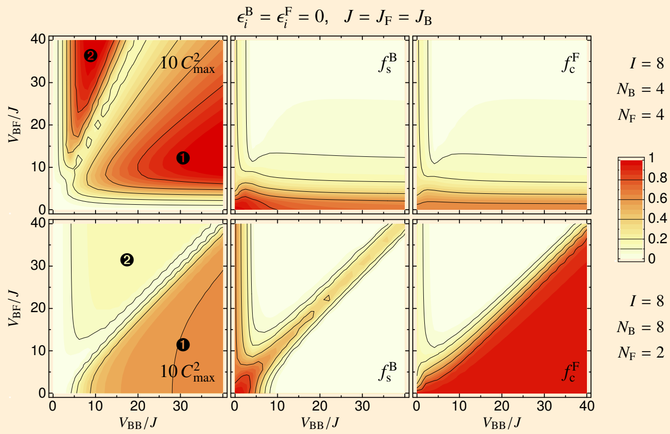


- **localised phase**: for small V_{BB}/J_B all particles are localised at the deepest well of each superlattice cell
- **quasi-Bose glass**: with increasing V_{BB}/J_B particles are redistributed gradually; plateaus of integer occupation with steplike rearrangements in between
- **Mott insulator**: homogeneous Mott insulator emerges whenever $V_{BB} > \Delta$
- **superfluid phase**: superfluid isle at small Δ and V_{BB}
- experimental distinction of the different insulating phases is possible, e.g., through Bragg diffraction of light (measures the static structure factor)



Boson-Fermion Mixtures in Lattices

- mixtures of bosons and fermions in lattices open a wide range of new quantum phases which are governed by the different quantum statistics and the competition between kinetic energy, boson-boson, and boson-fermion interaction
- on the basis of the boson superfluid fraction f^B_s and the fermion conducting fraction f^F_c one can distinguish several characteristic quantum phases
- $N_B/I = N_F/I = 1/2$: two **completely insulating phases** with different dominant intrinsic structures: ❶ alternating boson and fermion occupation and ❷ contiguous boson and fermion blocks
- $N_B/I = 1,\ N_F/I = 1/4$: ❶ **bosonic Mott-insulator** at $V_{BB} > V_{BF}$ with non-vanishing fermion conductivity and ❷ **completely insulating phase** with contiguous block structure



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Acknowledgements: This work was partially supported by the DFG, the UK EPSRC, and the EU via the "Cold Quantum Gases" network. K.B. thanks the Royal Society and Wolfson Foundation for support.