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Structure and Stability of Binary Boson-Fermion Mixtures

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Mission Statement

Recent experiments [1-4] with ultracold dilute mixtures of a bosonic and a fermionic atomic species open exciting perspectives in the field of trapped degenerate quantum gases. Besides the unique possibilities to study basic quantum phenomena in systems with mixed statistics, an especially appealing prospect is the realization of BCS transition of the fermionic component to a superfluid state. For all these studies a detailed knowledge of the ground-state phase diagram is a prerequisite.

We investigate

- ground state properties of dilute interacting boson-fermion mixtures on the basis of a modified Gross-Pitaevskii equation for the bosonic component and a Thomas-Fermi treatment for the fermions
- influence of s-wave boson-boson and boson-fermion interactions: density profiles zero-temperature phase diagram, collapse and component separation

Summary of Results

- the interplay between the boson-boson and the boson-fermion interaction generates a rich zero-temperature phase diagram
- for attractive boson-boson interactions the collapse of the bosonic component dominates; the boson-fermion interaction has little effect
- attractive boson-fermion interactions induce a simultaneous collapse of both components; repulsive boson-boson interactions stabilize against this collapse
- repulsive boson-fermion interactions cause a spatial separation of the two components (in all relevant cases boson core + fermion shell); repulsive boson-boson interactions stabilize against this separation
- in general: repulsive (attractive) boson-fermion interactions reduce (enhance) the overlap of the two species
- important implications for the efficiency of sympathetic cooling schemes

Theoretical Model

- mean-field description of the ground state $|\Psi\rangle$ of a mixture of a bosonic (B) and a fermionic species (F)

$$|\Psi\rangle = |\Phi_B\rangle \otimes |\Psi_F\rangle$$

$$|\Phi_B\rangle = \bigotimes_{i=1}^{N_B} |\phi_B\rangle\,, \qquad |\Psi_F\rangle = \mathbf{A} \bigotimes_{i=1}^{N_F} |\psi_i\rangle$$

- Hamiltonian including s-wave boson-boson and boson-fermion contact interactions with scattering lengths a_{BB} and a_{BF}, resp.

$$\begin{aligned}\mathbf{H} = &\sum_{i=1}^{N}\Big[\frac{\vec{\mathbf{p}}_i^2}{2m_B} + U_B(\vec{\mathbf{x}}_i)\Big]\,\mathbf{\Pi}_i^B \\ &+ \sum_{i=1}^{N}\Big[\frac{\vec{\mathbf{p}}_i^2}{2m_F} + U_F(\vec{\mathbf{x}}_i)\Big]\,\mathbf{\Pi}_i^F \\ &+ \frac{4\pi a_{BB}}{m_B}\sum_{i<j=1}^{N}\delta^{(3)}(\vec{\mathbf{r}}_{ij})\,\mathbf{\Pi}_{ij}^{BB} \\ &+ \frac{4\pi a_{BF}}{m_{BF}}\sum_{i<j=1}^{N}\delta^{(3)}(\vec{\mathbf{r}}_{ij})\,\mathbf{\Pi}_{ij}^{BF}\end{aligned}$$

- fermion-fermion and boson-fermion p-wave interactions are neglected; however, in particular cases they can be very important [6]
- construct energy functional using the Thomas-Fermi approximation for the fermionic part only

$$\langle\Psi|\,\mathbf{H}\,|\Psi\rangle \to E[n_B, n_F]$$

- ground state density profiles $n_B(\vec{x})$ and $n_F(\vec{x})$ determined by minimization of the energy functional $E[n_B, n_F]$ under the constraint of fixed numbers of bosons N_B and fermions N_F
- leads to coupled Euler-Lagrange equations including chemical potentials μ_B and μ_F
  - Gross-Pitaevskii equation for $n_B(\vec{x})$

$$\Big[-\frac{1}{2m_B}\vec{\nabla}^2 + U_B(\vec{x}) + \frac{4\pi\,a_{BF}}{m_{BF}}\,n_F(\vec{x}) + \frac{4\pi\,a_{BB}}{m_B}\,n_B(\vec{x})\Big]\,n_B^{1/2}(\vec{x}) = \mu_B n_B^{1/2}(\vec{x})$$

  - Thomas-Fermi solution for $n_F(\vec{x})$

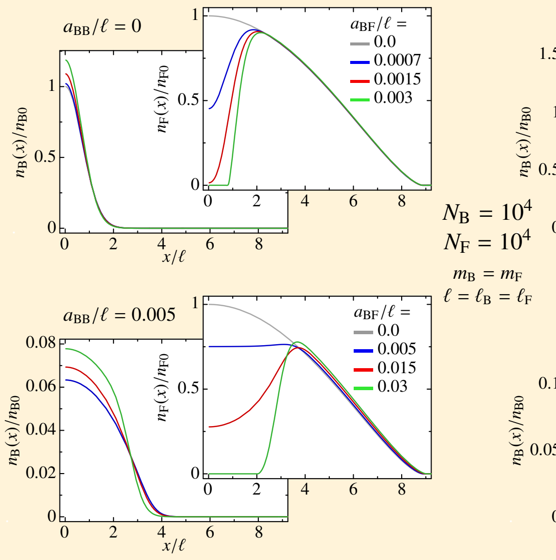
$$n_F(\vec{x}) = \frac{(2m_F)^{\frac{3}{2}}}{6\pi^2}\Big[\mu_F - U_F(\vec{x}) - \frac{4\pi\,a_{BF}}{m_{BF}}\,n_B(\vec{x})\Big]^{\frac{3}{2}}$$

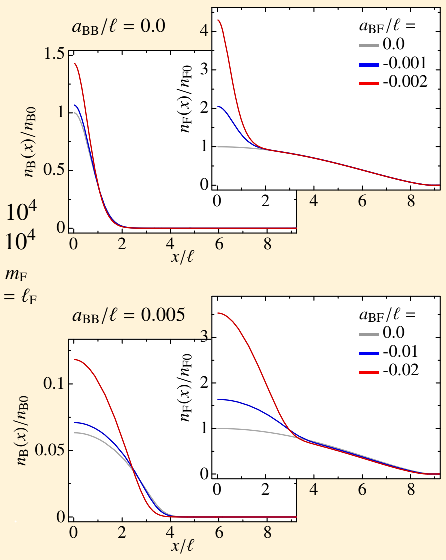
- self-consistent iterative solution of the coupled equations by an efficient imaginary time propagation algorithm
  - ○ initialization of $n_B(\vec{x})$
  - ① determine $n_F(\vec{x})$ from TF solution
  - ② perform a single imaginary time-step according to the GP equation for $n_B(\vec{x})$
  - ③ go to ① until convergence or divergence
- assuming spherical symmetric trapping potentials $U_B(x)$ and $U_F(x)$ with oscillator lengths $\ell_{B,F} = (m_{B,F}\,\omega_{B,F})^{-1/2}$

Density Profiles

■ *Repulsive Boson-Fermion Interactions*

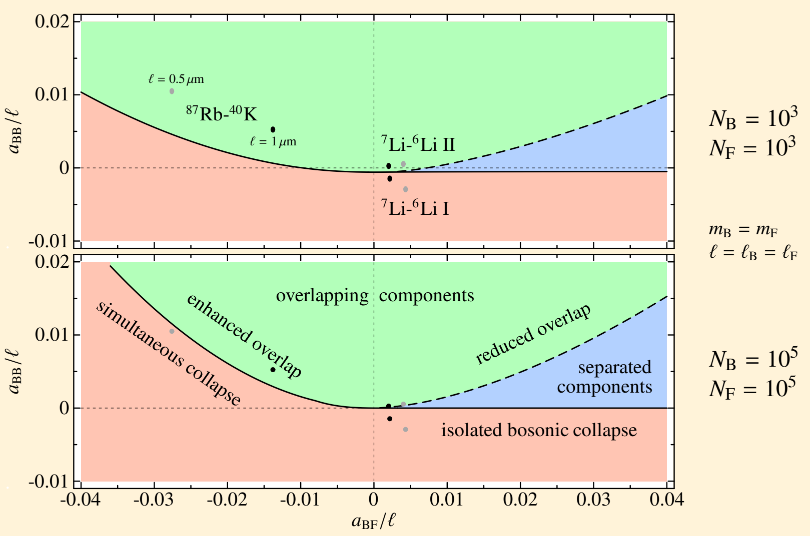
■ *Attractive Boson-Fermion Interactions*





- repulsive boson-fermion interactions reduce the overlap between the two species
- eventually the components are almost separated [5]; typically boson core surrounded by fermion shell
- repulsive boson-boson interactions stabilize the mixture against separation, i.e. shift the onset of separation towards larger a_{BF}
- attractive boson-fermion interactions enhance the boson and fermion density within the overlap region
- above a critical $a_{BF} < 0$ the mixture becomes instable and both components collapse simultaneously
- repulsive boson-boson interactions stabilize the mixture against collapse

Generic Phase Diagram



- overlapping phase: both components show stable overlapping density distributions; depending on the boson-fermion interaction the overlap can be enhanced or reduced
- mean-field instability: the mean-field generated by a sufficiently strong attractive interaction cannot be stabilized by the kinetic energy and the dilute system collapses towards high densities

  $a_{BF} < 0$: the mutual mean-field interaction drives a simultaneous collapse of both densities in the overlap region

  $a_{BB} < 0$: the bosonic mean-field interaction causes an isolate collapse of the bosons
- separated phase: sufficiently strong boson-fermion repulsions lead to a spatial separation of the species (phase boundary fixed by $n_F(0) = 0$, a small but finite overlap always remains)
- repulsive boson-boson interactions stabilize the overlapping phase against collapse and separation induced by the boson-fermion interaction
- but: the boson-fermion interaction cannot stabilize the system against collapse driven by a boson-boson attraction
- phase diagram more sensitive to a_{BB} and N_B than to a_{BF} and N_F

Experimental Implications

■ *^{7}Li-^{6}Li Mixture I* [1,2]

- $a_{BB} = -1.46\,\text{nm}$, $a_{BF} = 2.16\,\text{nm}$
- isolated collapse of the bosonic component driven by the boson-boson attraction
- severe limit on N_B which is influenced only marginally by the presence of and the interaction with the fermionic component

■ *^{7}Li-^{6}Li Mixture II* [2]

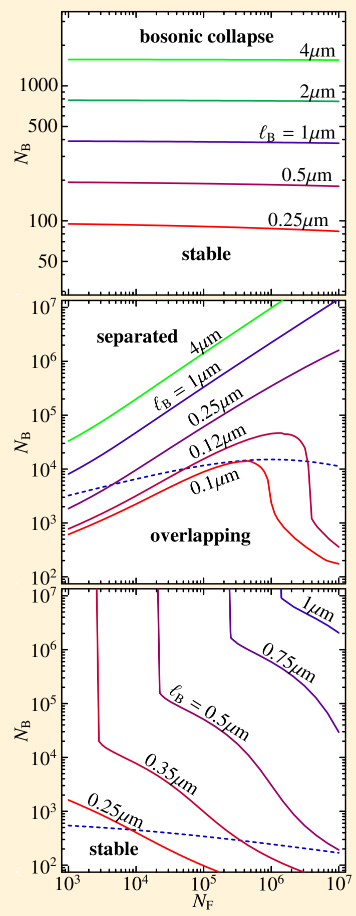
- $a_{BB} = 0.27\,\text{nm}$, $a_{BF} = 2.01\,\text{nm}$
- boson-fermion repulsion causes spatial separation of the two species
- boson-boson repulsion stabilizes the system although a_{BB} is small (dotted blue curve: no boson-boson interaction; compare to solid blue line)

■ *^{87}Rb-^{40}K Mixture* [3]

- $a_{BB} = 5.25\,\text{nm}$, $a_{BF} = -13.8\,\text{nm}$
- strong boson-fermion attraction induces simultaneous collapse of both densities
- dramatic stabilization due to the boson-boson repulsion (compare dotted blue with solid blue line); at small N_F absolute stabilization for all N_B

■ *^{23}Na-^{6}Li Mixture* [4]

- $a_{BB} = 2.75\,\text{nm}$, $a_{BF} =$ **???**
- large stable non-separated mixture produced experimentally [4]; from this observation one can estimate $-14.9\,\text{nm} < a_{BF} < 5.1\,\text{nm}$ (0.69 nm)



(calculations include different masses and magnetic moments of the two components but not trap deformation)

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