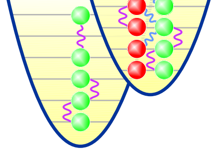

Trapped Ultracold Fermi Gases
using Effective s- and p-Wave Contact Interactions

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Abstract

Effective Contact Interaction

- zero-range potential for each partial wave
- reproduces the exact two-body energy spectrum when applied in mean-field fashion
- energy-density for trapped multi component Fermi gases in Thomas-Fermi approximation including s- and p-wave interactions ($T = 0\,\mathrm{K}$)

One Component Fermi Gas

- only p-wave interactions contribute, since s-wave is suppressed by the Pauli principle
- attractive p-wave interactions cause a mean-field instability against collapse to a high-density state
- critical densities and particle numbers as function of p-wave scattering length

Two Component Fermi Gas

- interplay between s- and p-wave interactions determines stability
- novel effects like absolute stabilization due to p-wave repulsion and a p-wave stabilized high-density phase
- repulsive s-wave interactions can cause spatial separation of both components

Conclusions

- p-wave interaction can have significant influence on the structure and stability of ultracold single- and multi-component Fermi gases

It is not appropriate to neglect the p-wave interaction from the outset!

Formalism

Effective Contact Interaction (ECI)

What is a proper effective interaction to describe properties of dilute ultracold quantum gases?

- system is dilute and cold
  $$\rho^{-1/3} \gg \text{interaction range}$$
  $$q^{-1} \gg \text{interaction range}$$
- particles experience an average interaction and do not probe details of the potential
- the many-body problem can be treated in a mean-field approach with an proper effective interaction instead of the full atom-atom potential

Concept of the ECI

➡ consists of a non-local zero-range interaction for each partial wave

➡ interaction strength is chosen such that the expectation value of the ECI in the two-body system (calculated with free angular-momentum eigenstates) equals the energy-shift induced by the full interaction

$$\langle nlm_l |\, \mathrm{v}^{\mathrm{ECI}}\, | nlm_l \rangle \stackrel{!}{=} \Delta E_{nl} \qquad (\star)$$

Construction of the ECI

- two particles interacting via a potential $v(r)$ with phase shifts $\eta_l(q)$ for the l-th partial wave
- impose a boundary condition $R_{nl}(\Lambda) = 0$ for the relative wave-function
- energy-shift ΔE_{nl} of the n-th positive energy state of the interacting spectrum $\bar{E}_{nl}$ compared to the n-th level of the free spectrum E_{nl}
  $$\frac{\Delta E_{nl}}{E_{nl}} = -\frac{2}{q_{nl}\Lambda}\,[\eta_l(q_{nl}) - \pi n_{\mathrm{bs}}]$$
- ansatz for the ECI for the l-th partial wave
  $$\mathrm{v}_l^{\mathrm{ECI}} = \int d^3r\; |\vec{r}\rangle \frac{\overleftarrow{\partial}^l}{\partial r^l}\, g_l\, \frac{\delta(r)}{4\pi r^2}\, \frac{\overrightarrow{\partial}^l}{\partial r^l} \langle \vec{r}|$$
- interaction strength g_l obtained from evaluation of the energy-shift condition ($\star$)
  $$g_l = -\frac{4\pi}{m}\left[\frac{(2l+1)!!}{l!}\right]^2 \frac{[\eta_l(q) - \pi n_{\mathrm{bs}}]}{q^{2l+1}}$$
- parameterization in terms of scattering lengths
  $$g_l \approx \frac{4\pi}{m}\frac{(2l+1)!}{(l!)^2}\, a_l^{2l+1} + \mathcal{O}(q^2)$$

Energy-Density for Trapped Fermions

- assume a gas of Ξ different fermionic components at $T = 0\,\mathrm{K}$ interacting via the s-wave and the p-wave part of the ECI
- the energy-density of the inhomogeneous Fermi gas is calculated in a mean-field approach using the Thomas-Fermi approximation
- groundstate is described by the density that minimizes the energy under the constraint of given particle numbers N_ξ
- functional variation leads to the extremum condition that has to be solved for each point $\vec{x}$

One Component

$$\mathcal{E}[\kappa](\vec{x}) = \frac{1}{6\pi^2}\, U(\vec{x})\, \kappa^3(\vec{x}) \quad \leftarrow \textbf{trap} \rightarrow$$
$$+ \frac{1}{20\pi^2 m}\, \kappa^5(\vec{x}) \quad \leftarrow \textbf{kinetic} \rightarrow$$
$$✗ \quad \leftarrow \textbf{s-wave} \rightarrow$$
$$+ \frac{a_1^3}{30\pi^3 m}\, \kappa^8(\vec{x}) \quad \leftarrow \textbf{p-wave} \rightarrow$$

Two Components

$$\mathcal{E}[\kappa_1, \kappa_2](\vec{x}) = \frac{1}{6\pi^2}\,[U_1(\vec{x})\,\kappa_1^3(\vec{x}) + U_2(\vec{x})\,\kappa_2^3(\vec{x})]$$
$$+ \frac{1}{20\pi^2 m}\,[\kappa_1^5(\vec{x}) + \kappa_2^5(\vec{x})]$$
$$+ \frac{a_0}{9\pi^3 m}\,\kappa_1^3(\vec{x})\,\kappa_2^3(\vec{x})$$
$$+ \frac{a_1^3}{30\pi^3 m}\,[\kappa_1^8(\vec{x}) + \kappa_2^8(\vec{x}) + \tfrac{1}{2}\kappa_1^3(\vec{x})\kappa_2^5(\vec{x}) + \tfrac{1}{2}\kappa_1^5(\vec{x})\kappa_2^3(\vec{x})]$$

- local Fermi momentum
  $$\kappa_\xi(\vec{x}) = [6\pi^2\, \rho_\xi(\vec{x})]^{1/3}$$
- energy
  $$E = \int d^3x\; \mathcal{E}[\kappa_1, \ldots, \kappa_\Xi](\vec{x})$$
- particle number
  $$N_\xi = \frac{1}{6\pi^2}\int d^3x\; \kappa_\xi^3(\vec{x})$$

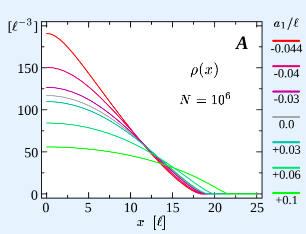
Application & Results

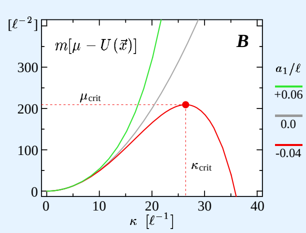
One Component — Stability

- s-wave contact interaction does not contribute due to the Pauli principle; p-wave is the leading interaction term
- the groundstate densities [➡ figure A] are obtained by point-wise solution of the extremum condition
  $$m[\mu - U(\vec{x})] = \frac{1}{2}\kappa^2(\vec{x}) + \frac{8\, a_1^3}{15\pi}\kappa^5(\vec{x})$$
- for attractive p-wave interactions ($a_1 < 0$) the r.h.s of the extremum condition exhibits a maximum at $(\kappa_{\mathrm{crit}}, \mu_{\mathrm{crit}})$ [➡ figure B]
  $$\kappa_{\mathrm{crit}} = \frac{(3\pi)^{1/3}}{2\,|a_1|} \qquad \rho_{\mathrm{crit}} = \frac{1}{16\pi\,|a_1|^3}$$
  $$\mu_{\mathrm{crit}} = \frac{3(3\pi)^{3/2}}{40 m\,|a_1|^2} \qquad N_{\mathrm{crit}} = \frac{(0.445\,\ell)^6}{|a_1|^6}$$
  ℓ: mean oscillator length

➡ the metastable state exist only for densities, chemical potentials and particle numbers below the critical values

➡ beyond the critical values the mean-field attraction is not stabilized by the kinetic energy contribution any more and the system collapses to a self-bound state





Two Components — Stability

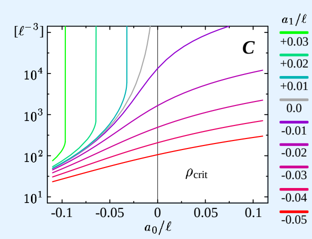
- s-wave and p-wave terms of the ECI contribute, their interplay has a strong influence on the stability
- extremum condition under the assumption of equal $[\mu - U(\vec{x})]$ and equal $\kappa(\vec{x})$ for both components
  $$m[\mu - U(\vec{x})] = \frac{1}{2}\kappa^2(\vec{x}) + \frac{2\, a_0}{3\pi}\kappa^3(\vec{x}) + \frac{4\, a_1^3}{5\pi}\kappa^5(\vec{x})$$
- metastable states exist only for densities below ρ_{crit} given by [➡ figure C]
  $$-(6\pi^2)^{1/3}\, a_0\, \rho_{\mathrm{crit}}^{1/3} - 12\pi^2\, a_1^3\, \rho_{\mathrm{crit}} = \pi/2$$

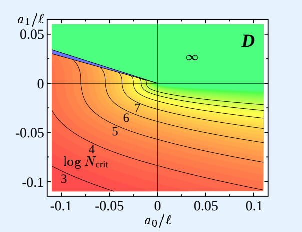
➡ the collapse can occur if the s-wave and/or the p-wave interaction is attractive

➡ if one of the interactions is repulsive then it stabilizes the system, i.e., it increases the critical density

➡ a repusive p-wave interaction leads to an absolute stabilization ($\rho_{\mathrm{crit}} \to \infty$) of the Fermi gas against s-wave induced collapse if
$$a_1/|a_0| > 2/(3\pi^{2/3})$$

- assuming a harmonic trap with mean oscillator length ℓ one obtains the particle number N_{crit} where the critical density is reached [➡ figure D]





Two Components — p-Wave Stabilized High-Density Phase

- given a system with attractive s-wave and repulsive p-wave interaction, such that
  $$0.274 < a_1/|a_0| < 0.311$$
- the r.h.s. of the extremum condition shows two separated branches at low and high densities, respectively [➡ figure E]
- for $[\mu - U(\vec{x})] > \mu_{\mathrm{trans}}$ the high-density solution is energetically favored otherwise the low-density solution is preferred (Maxwell construction)

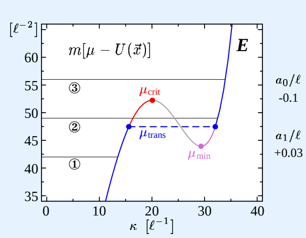
➡ depending on the value of $\mu(N)$ the solution shows different density profiles [➡ figure F]

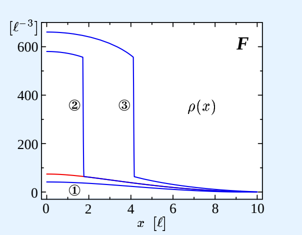
① $\mu < \mu_{\mathrm{trans}}$: smooth low density

② $\mu_{\mathrm{trans}} < \mu < \mu_{\mathrm{crit}}$: the energetically preferred configuration exhibits a region of high density in the center of the trap; the smooth low-density solution may occur as metastable state

③ $\mu_{\mathrm{crit}} < \mu$: no low-density solution exists for the central region of the trap; high-density phase must occur

➡ central density is increased by at least one order of magnitude; life-time is reduced significantly but may be still sufficient to observe the phenomenon





Two Components — Component Separation

- for strong repulsive s-wave interactions it is energetically favorable to separate the two components spatially, i.e., $\kappa_1(\vec{x})\kappa_2(\vec{x}) \equiv 0$
- comparison of the energies of overlapping and separated configurations leads to a critical density ρ_{sep} beyond which the separated configuration is preferred [➡ figure G]
  $$2.01\, a_0\, \rho_{\mathrm{sep}}^{1/3} - 24.9\, a_1^3\, \rho_{\mathrm{sep}} = 1$$

➡ separation occurs for repulsive s-wave interactions only; p-wave repulsion alone does not lead to component separation

➡ repulsive p-wave interactions increase the critical density for separation and lead to an absolute stabilization ($\rho_{\mathrm{sep}} \to \infty$) if
$$a_1/a_0 > 0.349$$

➡ attractive p-wave interactions lead to an interference with the collapse; the separated configuration collapses immediately if
$$|a_1|/a_0 > 0.394$$

- critical particle number N_{sep} for component separation can be obtained from ρ_{sep} for a harmonic trap with mean oscillator length ℓ [➡ figure H]