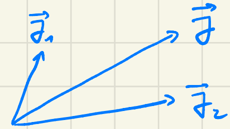
Announcement: Office hours for exam see webpage

Review: Addition of angular momenta



$\hat{\vec{L}} + \hat{\vec{S}} = \hat{\vec{J}}$
$\hat{\vec{S}}_1 + \hat{\vec{S}}_2 = \hat{\vec{S}}$
$\hat{\vec{L}}_1 + \hat{\vec{L}}_2 = \hat{\vec{L}}$ $\leftarrow$

$$\hat{\vec{J}} = \hat{\vec{J}}_1 + \hat{\vec{J}}_2$$

Coupled $\qquad\qquad\qquad\qquad\qquad\qquad$ uncoupled

$$|(J_1 J_2) J J_z\rangle = \sum_{m_{J_1}, m_{J_2}} \boxed{C^{J J_z}_{J_1 m_{J_1}, J_2 m_{J_2}}} \, |J_1 m_{J_1}\rangle |J_2 m_{J_2}\rangle$$

$$J_z = m_{J_1} + m_{J_2}$$

Clebsch-Gordan coefficients, reell numbers

Example: $S_1 = S_2 = \frac{1}{2}$

Spin triplet: $S=1$, $2S+1=3$, symmetric

$$|S=1, M_S=1\rangle = \boxed{1}\, |\uparrow\uparrow\rangle$$

$$|S=1, M_S=0\rangle = \boxed{\frac{1}{\sqrt{2}}}\, |\uparrow\downarrow\rangle + \boxed{\frac{1}{\sqrt{2}}}\, |\downarrow\uparrow\rangle$$

$$|S=1, M_S=-1\rangle = \boxed{1}\, |\downarrow\downarrow\rangle$$

Spin singlet: antisymmetric under spin exchange

$$|S=0, M_S=0\rangle = \boxed{\frac{1}{\sqrt{2}}}\, |\uparrow\downarrow\rangle - \boxed{\frac{1}{\sqrt{2}}}\, |\downarrow\uparrow\rangle$$

$$C^{11}_{\frac{1}{2}\,-\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}} = 0$$

Addition von 3 (oder mehr) Drehimpulsen

durch sukzessives Koppeln

$$\hat{\vec{J}} = \hat{\vec{J}}_1 + \hat{\vec{J}}_2 + \hat{\vec{J}}_3 = \underbrace{\left(\hat{\vec{J}}_1 + \hat{\vec{J}}_2\right)}_{\hat{\vec{J}}_{12}} + \hat{\vec{J}}_3$$

$C^{J_{12} J_{12z}}_{J_1 m_{J_1} J_2 m_{J_2}}$

$C^{J J_z}_{J_{12} J_{12z} J_3 m_{J_3}}$

oder

$$= \hat{\vec{J}}_1 + \underbrace{\left(\hat{\vec{J}}_2 + \hat{\vec{J}}_3\right)}_{\hat{\vec{J}}_{23}}$$

$$= \hat{\vec{J}}_{13} + \hat{\vec{J}}_2$$

Wasserstoffatom mit Spin-Bahn Wechselwirkung → Feinstruktur-aufspaltung

$$\hat{H} = \hat{H}_0 + V_{LS}(r)\, \hat{\vec{L}} \cdot \hat{\vec{S}} \qquad V_{LS}(r) = \frac{\hbar^2}{2m^2c^2} \frac{1}{r} \frac{dV}{dr} > 0$$

$$\hat{\vec{L}} \cdot \hat{\vec{S}} = \frac{1}{2}\left(\hat{\vec{J}}^2 - \hat{\vec{L}}^2 - \hat{\vec{S}}^2\right)$$

$$= \frac{1}{2}\left(J(J+1) - \ell(\ell+1) - s(s+1)\right)$$ gekoppelte Eigenzustände

für $S = \frac{1}{2}$ $\qquad |\ell - \frac{1}{2}| \le J \le \ell + \frac{1}{2}$

mögliche $\quad J = J_> = \ell + \frac{1}{2}$ und $J = J_< = \ell - \frac{1}{2}$ für $\ell > 0$

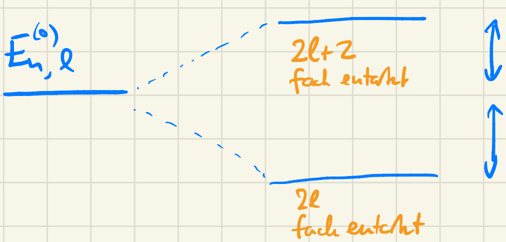
$$\Rightarrow \quad \hat{\vec{L}} \cdot \hat{\vec{S}} = \frac{1}{2} \begin{cases} l \,, & j_> = l + \frac{1}{2} \\ -(l+1) \,, & j_< = l - \frac{1}{2} \quad \text{für } l > 0 \end{cases}$$

1. Ordnung entartete Störungstheorie $\quad \hat{H}_0 + \underbrace{V_{LS}(r)\, \hat{\vec{L}} \cdot \hat{\vec{S}}}_{\text{Störung}}$

→ Diagonalisiere $V_{LS}(r)\, \hat{\vec{L}} \cdot \hat{\vec{S}}$ im Unterraum zu geg. n

$\hookrightarrow E_n^{(0)} = -\frac{m (Z\alpha c)^2}{2 n^2}$

in gekoppelten Zuständen $|(ls) j j_z\rangle$ ist $\hat{\vec{L}} \cdot \hat{\vec{S}}$ diagonal!



$E^{(1)}_{n,j} \sim l \,,\quad j_> = l + \frac{1}{2}$, $\quad 2j_> + 1$

$\sim -(l+1) \,,\quad j_< = l - \frac{1}{2}$, $\quad 2j_< + 1$

$$E^{(1)}_{n,j} = \langle (ls) j j_z |\, V_{LS}(r)\, \hat{\vec{L}} \cdot \hat{\vec{S}} \,| (ls) j j_z \rangle$$

$$= \underbrace{\left(\int_0^\infty r^2 dr \, |R_{nl}(r)|^2 \, V_{LS}(r) \right)}_{>0} \cdot \frac{1}{2} \begin{cases} l \,, & j_> \\ -(l+1) \,, & j_< \end{cases}$$

→ Feinstrukturaufspaltung in 2 Niveaus für $l > 0$

Dimensionscheck: ungestört $|nlm_l\rangle |sm_s\rangle$ $\quad (2l+1) \cdot (2s+1) = 2 \cdot (2l+1) \checkmark = 2l+2 + 2l$

größeres $j_>$ liegt aufgrund repulsiver V_{LS} höher in Energie

für $l = 0$ keine Aufspaltung und keine Verschiebung

$$\left(\int_0^\infty r^2 dr \, |R_{n\ell}(r)|^2 \, V_{LS}(r) \right) = \frac{\hbar^2}{2m^2c^2} \left\langle \frac{1}{r} \frac{dV}{dr} \right\rangle \qquad V(r) = -\frac{Z\alpha\hbar c}{r}$$

$$= \frac{\hbar^2}{2m^2c^2} Z\alpha\hbar c \underbrace{\left\langle \frac{1}{r^3} \right\rangle}_{\frac{Z^3}{n^3 r_0^3}}$$

$$= \frac{mc^2}{2n^3} (Z\alpha)^4$$

$$\frac{\Delta E_{LS}}{E^{(0)}_{n\ell}} = \frac{E^{(1)}_{nJ>} - E^{(1)}_{nJ<}}{E^{(0)}_{n\ell}} \sim \frac{(Z\alpha)^2}{n} \ll 1 \qquad E^{(0)}_{n\ell} = -\frac{m(Z\alpha c)^2}{2n^2}$$

Feinstrukturkonstante $\alpha = \frac{1}{137}$ → relativistische Korr.

$$\hat{H} = \underbrace{\frac{\vec{p}^{\,2}}{2m} - \frac{Z\alpha\hbar c}{r}}_{} + \overbrace{V_{\text{s}}(r) \underbrace{\vec{L}\cdot\vec{S}}_{}}^{}$$

nicht diag. $|n\,(\ell s)\,J J_z\rangle$

diagonal in $|n\,\ell m\rangle\,|s=\frac{1}{2} m_s\rangle$

$|n\,\ell m s m_s\rangle$

diagonal in $|n\,(\ell s)\,J J_z\rangle$

diagonal in $|(\ell s)\,J J_z\rangle$

nicht diag. in $|\ell m\, s m_s\rangle$

Hyperfeinstrukturaufspaltung

durch Kopplung des Gesamtspins des Elektrons $\hat{\vec{J}}_e = \hat{\vec{L}}_e + \hat{\vec{S}}_e$

mit Kernspin $\vec{I}_A$

$$\rightarrow \quad \vec{F} = \vec{J} + \vec{I} = \underbrace{(\hat{\vec{L}}_e + \hat{\vec{S}}_e)}_{\vec{J}} + \vec{I}$$

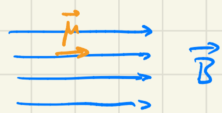
Wechselwirkung zwischen $\vec{J}_e \cdot \vec{I} \longrightarrow$ Hyperfeinstruktur

$$= \frac{1}{2}\left(\hat{\vec{F}}^2 - \hat{\vec{J}}^2 - \hat{\vec{I}}^2\right)$$

Wasserstoffatom mit Spin in äußeren $\vec{B}$ Feld

$\vec{B} = (0, 0, B)$

Zeeman Effekt: $\hat{V}_B = -\hat{\vec{\mu}} \cdot \vec{B}$



$$\hat{\vec{\mu}} = \frac{e}{2mc}\hat{\vec{L}} = -\mu_B \frac{\vec{L}}{\hbar}$$

Bohr Magneton $\mu_B = \frac{|e|\hbar}{2mc}$

$$\hat{H} = \underbrace{\frac{\hat{\vec{p}}^2}{2m} - \frac{Z\alpha\hbar c}{r}} + \underbrace{\frac{\hbar^2}{2m^2c^2}\frac{1}{r}\frac{dV}{dr}\hat{\vec{L}}\cdot\hat{\vec{S}}} + g_l|\mu_B| B \hat{L}_z + g_s|\mu_B| B \hat{S}_z$$

dimlos.

g_l
g_s: gyromagnetische Faktor

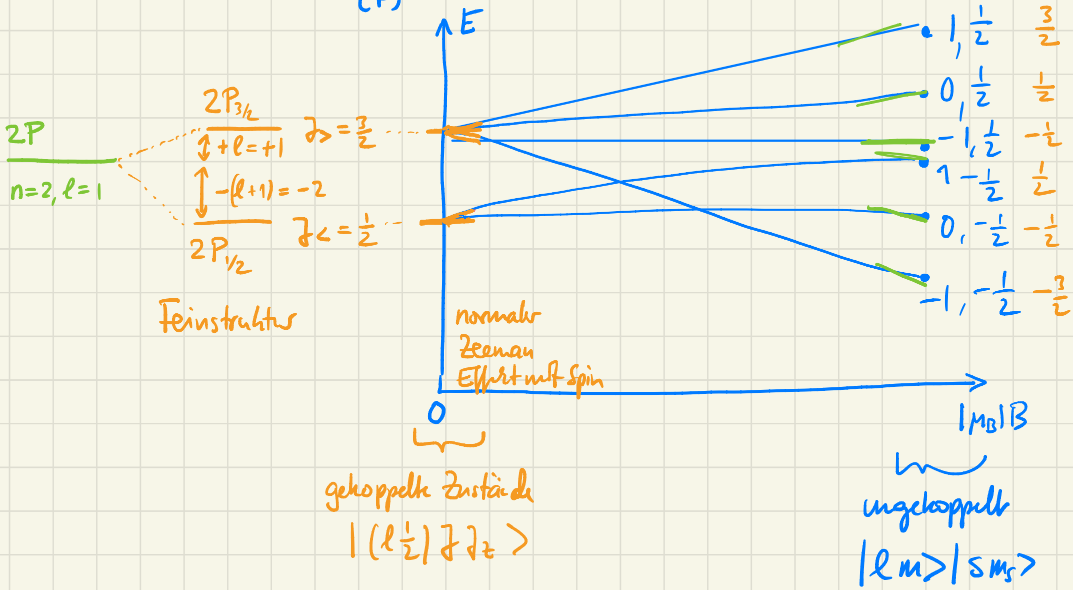
$g_l = 1$

$g_s = 2 + \frac{\alpha}{\pi} + \mathcal{O}(\alpha^2)$

Dirac Schwinger QED

$$g \approx 2{,}0023$$

Aufspaltung im $n=2,\ \ell=1$ (P) Unterraum 2P



große B $\quad \hat{H} = \dots \quad \underbrace{|\mu_B| B \hat{L}_z + g |\mu_B| B \hat{S}_z}$

$$|\mu_B| B \underbrace{\left(m + g m_s\right)}$$

$\ell = 1: \left.\begin{array}{l} 2\ell+1 = 3 \\ 2s+1 = 2 \end{array}\right\} 6$

m, m_s:	
$1, \frac{1}{2}$	$1 + \frac{g}{2} \approx 2.001$
$0, \frac{1}{2}$	$0 + \frac{g}{2} \approx 1.$
$-1, \frac{1}{2}$	$-1 + \frac{g}{2} \approx 0.001$
$1, -\frac{1}{2}$	$1 - g/2 = -0.001$
$0, -\frac{1}{2}$	$0 - g/2 \simeq -1.$
$-1, -\frac{1}{2}$	$-1 - g/2 \simeq -2.$

(rows $-1, \frac{1}{2}$ and $1, -\frac{1}{2}$): $\approx$ konst als Fkt. von B

$\rightarrow$ Entkopplung bei großen B-Feld: <u>Paschen-Back Effekt</u>

12. Zeitabh. Störungstheorie: Goldene Regel

QM:
$$\underbrace{[\hat{x}, \hat{p}_x] = i\hbar}_{\longrightarrow \text{ Heisenbergsche Unschärfer.}} \qquad \Delta x \Delta p_x \geq \hbar$$