The IR Behavior of the Gluon and the Ghost Propagator in the Landau Gauge within the Gribov-Zwanziger Framework.

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Overview

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Formulation of a problem

Recent lattice data show [e.g. Cucchieri et al. (2007, 2008)]:

- A gluon propagator $\mathcal{D}(p)$ which is:
  - infrared suppressed:

$$\mathcal{D}(p) \overset{IR}{\propto} \frac{1}{p^{\alpha}} \quad \text{with} \quad \alpha < 2$$

  - non-vanishing at zero momentum:

$$\mathcal{D}(0) \neq 0$$

  - positivity violating

- A ghost propagator $\mathcal{G}(p)$ which is:
  - not enhanced:

$$\mathcal{G}(p) \overset{IR}{\propto} \frac{1}{p^2}$$

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Formulation of a problem

How to solve it

- Within the Gribov-Zwanziger approach [*Dudal et al. (2007); paper in prep*]
- Within the Schwinger-Dyson approach [*Aguilar et al. (2004), Pène et al. (2007-2008)*]

Overview

The Gribov-Region

Equivalent fields

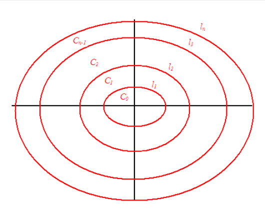
- In the Landau gauge:

$$\text{two fields are equivalent} \Updownarrow \partial\widetilde{A} = \partial A$$

with $\widetilde{A}_\mu = S^\dagger \partial_\mu S + S^\dagger A_\mu S$

Exclude Gribov copies

- Restriction of the domain of integration to the first horizon



- The Faddeev Popov operator:
  $$\mathrm{FP} = -\partial_\mu(\partial_\mu \cdot + [A_\mu, \cdot])$$
- $\widetilde{A}_\mu$ close to $A_\mu \Rightarrow 0$ eigenmode of FP
- In C_0: FP $\rightarrow$ 0 negative eigenvalues
  At ℓ_1: FP $\rightarrow$ 1 zero eigenvalue

The Gribov-Zwanziger action: non-local

The Gribov-Zwanziger action is given by

$$S_h = S_{YM} + S_{gf} + \gamma^4 \int d^4x \, h(x)$$

with

- The classical Yang-Mills action

$$S_{YM} = \frac{1}{4} \int d^4x F^a_{\mu\nu} F^a_{\mu\nu}$$

- The Landau gauge fixing

$$S_{gf} = \int d^4x \, \left(b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right)$$

- The horizon function

$$\begin{aligned} h(x) &= g^2 f^{abc} A^b_\mu \left(\mathcal{M}^{-1} \right)^{ad} f^{dec} A^e_\mu \\ \mathcal{M}^{ab} &= -\partial_\mu \left(\partial_\mu \delta^{ab} + g f^{acb} A^c_\mu \right) \end{aligned}$$

The Gribov-Zwanziger action: non-local

The Gribov-Zwanziger action is given by

$$S_h = S_{YM} + S_{gf} + \gamma^4 \int \mathrm{d}^4x\, h(x)$$

The parameter γ is determined by the horizon condition

$$\langle h(x) \rangle = d(N^2 - 1)$$

The Gribov-Zwanziger action: local

The Gribov-Zwanziger action can be localized through a suitable set of extra fields

$$S_{GZ} = S_{\mathrm{YM}} + S_{gf} + S_{\varphi\overline{\varphi}\omega\overline{\omega}} + S_{\gamma}$$

with

$$\begin{aligned} S_{\varphi\overline{\varphi}\omega\overline{\omega}} &= \int d^4x \Big(\overline{\varphi}_\mu^{ac} \partial_\nu \left(\partial_\nu \varphi_\mu^{ac} + g f^{abm} A_\nu^b \varphi_\mu^{mc} \right) - \overline{\omega}_\mu^{ac} \partial_\nu \left(\partial_\nu \omega_\mu^{ac} + g f^{abm} A_\nu^b \omega_\mu^{mc} \right) - g \left(\partial_\nu \overline{\omega}_\mu^{ac} \right) f^{abm} \left(D_\nu c \right)^b \varphi_\mu^{mc} \Big) \\ S_\gamma &= -\gamma^2 g \int d^4x \left(f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \overline{\varphi}_\mu^{bc} + \frac{4}{g} \left(N^2 - 1 \right) \gamma^2 \right) \end{aligned}$$

whereby

- $\left(\overline{\varphi}_\mu^{ac}, \varphi_\mu^{ac} \right)$ are a pair of complex conjugate bosonic fields
- $\left(\overline{\omega}_\mu^{ac}, \omega_\mu^{ac} \right)$ are a pair of complex conjugate anticommuting fields

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S_{GZ} displays a global symmetry, therefore we can simplify the notation:

$$\left(\overline{\varphi}_{\mu}^{ac}, \varphi_{\mu}^{ac}, \overline{\omega}_{\mu}^{ac}, \omega_{\mu}^{ac} \right) = \left(\overline{\varphi}_{i}^{a}, \varphi_{i}^{a}, \overline{\omega}_{i}^{a}, \omega_{i}^{a} \right)$$

The Gribov-Zwanziger action: local

The Gribov-Zwanziger action becomes then

$$S_{GZ} = S_{\text{YM}} + S_{gf} + S_{\varphi\overline{\varphi}\omega\overline{\omega}} + S_{\gamma}$$

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$$S_{\varphi\overline{\varphi}\omega\overline{\omega}} = \int \mathrm{d}^4x \left(\overline{\varphi}_i^a \partial_\nu \left(D_\nu \varphi_i\right)^a - \overline{\omega}_i^a \partial_\nu \left(D_\nu \omega_i\right)^a - g \left(\partial_\nu \overline{\omega}_i^a\right) f^{abm} \left(D_\nu c\right)^b \varphi_i^m \right)$$

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The horizon condition is translated as

$$\frac{\partial \Gamma}{\partial \gamma^2} = 0$$

with Γ the quantum action defined as

$$\mathrm{e}^{-\Gamma} = \int [D\Phi] \mathrm{e}^{-S} ,$$

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The Gribov-Zwanziger action is renormalizable!

The gluon propagator

The tree level gluon propagator in the Gribov-Zwanziger model:

$$\left\langle A^a_\mu A^b_\nu \right\rangle_p \equiv \delta^{ab}\mathcal{D}(p^2)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) = \delta^{ab}\frac{p^2}{p^4+\frac{\lambda^4}{4}}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

Conclusion

The gluon propagator is:

- infrared suppressed
- positivity violating
- vanishing at the origin $\leftrightarrow$ lattice data

The ghost propagator

The ghost propagator in the Gribov-Zwanziger model (up to one loop):

$$\mathcal{G}(k)_{k\to 0} = \frac{512\pi\gamma^2}{21Ng^2}\frac{1}{k^4}$$

Conclusion

The ghost propagator is:

- enhanced $\leftrightarrow$ lattice data

Overview

The modified GZ action

How to modify the GZ action?

$$S_{GZ} = S_{\text{YM}} + S_{gf} + S_{\varphi\overline{\varphi}\omega\overline{\omega}} + S_{\gamma}$$

with

$$S_{\varphi\overline{\varphi}\omega\overline{\omega}} = \int \mathrm{d}^4x \left(\overline{\varphi}_i^a \partial_\nu \left(D_\nu \varphi_i\right)^a - \overline{\omega}_i^a \partial_\nu \left(D_\nu \omega_i\right)^a - g \left(\partial_\nu \overline{\omega}_i^a\right) f^{abm} \left(D_\nu c\right)^b \varphi_i^m\right)$$

$$S_\gamma = -\gamma^2 g \int \mathrm{d}^4x \left(f^{abc} A_\mu^a \varphi_\mu^{bc} + f^{abc} A_\mu^a \overline{\varphi}_\mu^{bc} + \frac{4}{g}\left(N^2 - 1\right)\gamma^2\right)$$

- An $A\varphi$-coupling at the quadratic level
- A non-trivial effect in the φ-sector $\rightarrow$ A-sector

Proposition:

We add the following term to the GZ action with $M^2 = J$ a new source:

$$S_M = \int \mathrm{d}^4x \left(-M^2 \left(\overline{\varphi}_i^a \varphi_i^a - \overline{\omega}_i^a \omega_i^a\right)\right)$$

Renormalizability

Is the action $S'_{GZ} = S_{GZ} + S_M$ renormalizable?

- We are going to follow the algebraic renormalization procedure
- We start with the action:

$$S'_{GZ} = S_{\text{YM}} + S_{gf} + S_{\varphi\overline{\varphi}\omega\overline{\omega}} + S_{\gamma} + S_M$$

Step one:

Adding extra sources

- We want to treat $f^{abc}A^a_\mu\varphi^{bc}_\mu$ and $f^{abc}A^a_\mu\overline{\varphi}^{bc}_\mu$ as composite operators
- Therefore we replace S_γ with

$$S'_\gamma = -\int \mathrm{d}^4x \left(M^{ai}_\mu \left(D_\mu\varphi_i\right)^a + V^{ai}_\mu \left(D_\mu\overline{\varphi}_i\right)^a + 4\gamma^4(N^2-1) \right)$$

- If we set the sources in the end:

$$M^{ab}_{\mu\nu}\Big|_{phys} = V^{ab}_{\mu\nu}\Big|_{phys} = \gamma^2\delta^{ab}\delta_{\mu\nu}$$

$\Rightarrow$ We didn't change the theory

Action:

$$S'_{GZ} = S_{\mathrm{YM}} + S_{gf} + S_{\varphi\overline{\varphi}\omega\overline{\omega}} + S'_\gamma + S_M$$

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Step two:

BRST invariance

- Algebraic renormalization procedure: we want the action to be BRST invariant
- Therefore we replace S'_γ with

$$S_s = s \int d^4x \left(-U_\mu^{ai} \left(D_\mu \varphi_i\right)^a - V_\mu^{ai} \left(D_\mu \overline{\omega}_i\right)^a - U_\mu^{ai} V_\mu^{ai} \right)$$

whereby

$$\begin{array}{llll} sA_\mu^a = -\left(D_\mu c\right)^a , & sc^a = \frac{1}{2} g f^{abc} c^b c^c , & s\overline{c}^a = b^a , & sb^a = 0 , \\ s\varphi_i^a = \omega_i^a , & s\omega_i^a = 0 , & s\overline{\omega}_i^a = \overline{\varphi}_i^a , & s\overline{\varphi}_i^a = 0 , \\ sU_\mu^{ai} = M_\mu^{ai} , & sM_\mu^{ai} = 0 , & sV_\mu^{ai} = N_\mu^{ai} , & sN_\mu^{ai} = 0 \end{array}$$

- If we set the sources in the end:

$$\left. U_\mu^{ai} \right|_{phys} = \left. N_\mu^{ai} \right|_{phys} = 0$$

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Step three:

Adding an extra term S_{ext}

- An extra term is needed to define the nonlinear BRST transformations of the gauge and ghost fields

$$S_{\rm ext} = \int d^4x \left(-K^a_\mu \left(D_\mu c\right)^a + \frac{1}{2} g L^a f^{abc} c^b c^c \right)$$

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Action:

$$S'_{\text{GZ}} = S_{\text{YM}} + S_{gf} + S_{\varphi\overline{\varphi}\omega\overline{\omega}} + S_s + S_M + S_{ext}$$

Step Four:

Ward identities

- $U(f)$ invariance
- The Slavnov-Taylor identity
- Landau gauge condition and anti ghost equation
- The ghost Ward identity
- The lineary broken constraints
- The exact $\mathcal{R}_{ij}$ symmetry

Step Five:

The counterterm

- At quantumlevel:

$$S'_{GZ} \to S'_{GZ} + \Sigma_c$$

- Counterterm
  - The Ward identities $\Rightarrow$ constraints on the counterterm
  - Most general
  - Integrated local polynomial in the fields and sources
  - Dimension 4

$$\Rightarrow \Sigma^c = a_0 S_{YM} \quad + \quad a_1 \int d^4x \left(A^a_\mu \frac{\delta S_{YM}}{\delta A^a_\mu} + \widetilde{K}^a_\mu \partial_\mu c^a + \widetilde{V}^{ai}_\mu \widetilde{M}^{ai}_\mu - \widetilde{U}^{ai}_\mu \widetilde{N}^{ai}_\mu \right)$$

- Counterterm has to be reabsorbed through renormalization of fields and sources

Conclusion

- S'_{GZ} is renormalizable!
- Only two independent renormalization factors (Z_A and Z_c)

Remark

S'_{GZ} is not BRST invariant

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The tree level gluon propagator in the extended GZ model:

$$\left\langle A^a_\mu A^b_\nu \right\rangle_p \equiv \delta^{ab}\mathcal{D}(p^2)\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

with

$$\mathcal{D}(p^2) = \frac{p^2 + M^2}{p^4 + M^2 p^2 + 2g^2 N\gamma^2}$$

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The gluon propagator is:

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The ghost propagator

The ghost propagator in the extended GZ model (up to one loop):

$$\mathcal{G}(k) \equiv \frac{1}{k^2}(1+\sigma)$$

with for $k^2 \approx 0$

$$\sigma \sim 1 + g^2 M^2 \frac{1}{2\sqrt{M^4 - 8g^2 N\gamma^4}} \ln \frac{\left(M^2 + \sqrt{M^4 - 8g^2 N\gamma^4}\right)}{\left(M^2 - \sqrt{M^4 - 8g^2 N\gamma^4}\right)} + \mathrm{order}(k^2)$$

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Similar results are found in 3 dimensions!

work in preparation

Overview

What is the variational principle?

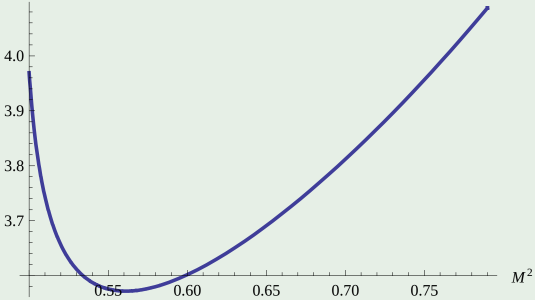
A dynamical value for the mass M^2

- So far, M^2 was put in by hand. How to obtain a dynamical value?
- New approach: variational principle:
  1. Assume we study a quantity $\mathcal{Q}$
  2. Write $S' = S_{M=0} + S_M - \ell S_M$
  3. We expand $\mathcal{Q}$ in powers of ℓ
  4. We set $\ell = 1$: we did not change the massless theory
  5. Minimal sensitivity approach : $\frac{\partial \mathcal{Q}}{\partial M^2} = 0 \Rightarrow M^2_{\text{min}} \Rightarrow \mathcal{Q}(M^2_{\text{min}})$
     FACC: We search for

$$\min \left| \frac{\mathcal{Q}^{(1)} - \mathcal{Q}^{(0)}}{\mathcal{Q}^{(0)}} \right|$$

More on the ghost propagator...

Applying the variational principle: results

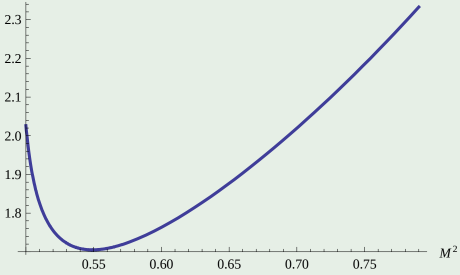


Applying the FACC gives

- $M^2 = 0.56\Lambda^2_{\overline{\text{MS}}}$
- $\frac{g^2N}{16\pi^2} = 0.90 \Rightarrow$ is smaller than one
- $\mathcal{G}(k)_{k^2\approx 0} = 3.57/k^2 \Rightarrow$ no enhancement!

More on the gluon propagator...

Applying the variational principle: results



Applying the FACC gives

- $M^2 = 0.55\Lambda^2_{\overline{\mathrm{MS}}}$
- $\frac{g^2 N}{16\pi^2} = 0.88 \Rightarrow$ is smaller than one
- $\mathcal{D}(0) = 34.72/k^2 \Rightarrow$ is not equal to zero!

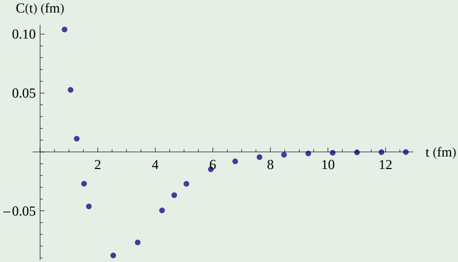
More on the gluon propagator...

Positivity violation of the gluon propagator

- The temporal correlator at tree level:

$$\mathcal{C}(t) = \int_0^{+\infty} \mathrm{d}M_p \rho(M_p^2) \mathrm{e}^{-M_p t} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \mathrm{e}^{-ipt} \mathcal{D}(p) \mathrm{d}p$$

- For each t, apply the principle of minimal sensitivity

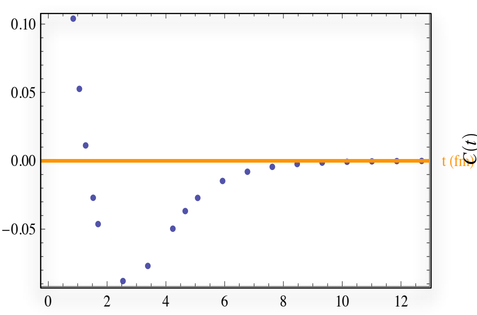


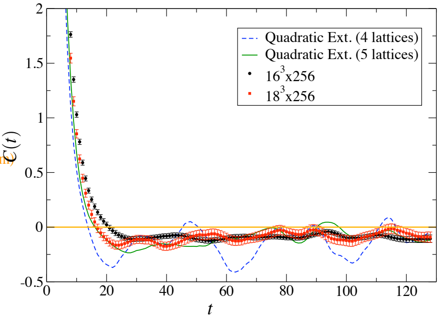
More on the gluon propagator...

Positivity violation of the gluon propagator

- If we compare lattice data [*P. J. Silva. and O. Oliveira (2006)*] and our results:

1. Same shape
2. Both show a positivity violation from $t \sim 1.5$ fm





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GZ adapted approach gives:

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  - ✓
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The End

Questions?