Remarks on the gluon and ghost propagators in the MAG

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- *The maximal Abelian gauge*
- *The issue of the Gribov copies*
- *Dimension two condensates*
- *A look at propagators*
- *Conclusion: a possible framework*

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The SU(2) maximal Abelian gauge

The gauge field is decomposed into diagonal and off-diagonal components

$$A_\mu{}^3 \equiv A_\mu = \quad \text{diagonal} \quad \text{component}$$

$$A_\mu{}^a \quad a = 1,2 \quad \text{off} - \text{diag. components}$$

For the gauge fixing, we impose the following conditions

$$\partial_\mu A_\mu = 0$$

$$D_\mu{}^{ab} A^b{}_\mu \equiv \left(\delta^{ab}\partial_\mu - g\varepsilon^{ab} A_\mu\right) A_\mu{}^b = 0$$

The Faddeev-Popov operator is found to be

$$M^{ab} = -D^{ac}{}_{\mu} D^{cb}{}_{\mu} - g^2 \varepsilon^{ac} \varepsilon^{bd} A^{c}{}_{\mu} A^{d}{}_{\mu}$$

For the Faddeev-Popov quantization formula we have

$$\int DA\, \delta(\partial A)\, \delta(D^{ab} A^{b})\left(\det M^{ab}\right) e^{-\frac{1}{4}\int d^4x\, F^2}$$

As in the case of the Landau gauge, the Faddeev-Popov operator M^{ab} possesses zero modes, meaning that the MAG is plagued by the existence of Gribov copies.

The issue of the Gribov copies

The issue of the Gribov copies can be (partially) faced by restricting the domain of integration in the Faddeev-Popov formula to the so called Gribov region Ω, obtained by minimizing the functional

Capri et al., PRD 74 105007 (2006)

$$\int d^4x\, A^a{}_\mu\, A^a{}_\mu$$

For the region Ω we have

$$\Omega = \left\{ \partial A = 0,\ D^{ab} A^b = 0;\ M^{ab} = -D_\mu^{ac} D_\mu^{cb} - g^2 \varepsilon^{ac} \varepsilon^{bd} A^c_\mu A^d_\mu > 0 \right\}$$

Properties of the Gribov region Ω in the MAG

Private communications from D. Zwanziger

Till now, the following properties of Ω have been established

- *The origin in field space, $\{A_\mu=0,\ A_\mu^a=0\}$ belongs to Ω*
- *Ω is **bounded** in all off-diagonal directions*
- *Ω is **unboundend** in the diagonal direction*
- *Convexity of Ω : **open point !***

Restriction to Ω

As we learn form the work by Zwanziger, the restriction to the region Ω is achieved by introducing a non-local functional, known as the horizon term

$$Z = \int_{\Omega} DA\,\delta(\partial A)\,\delta(D^{ab}A^{b})\left(\det M^{ab}\right) e^{-\frac{1}{4}\int d^4x\, F^2}$$

$$\Big\downarrow$$

D. Zwanziger, NPB 399, 477 (1993)

$$Z = e^{-\Gamma} = \int DA\,\delta(\partial A)\,\delta(D^{ab}A^{b})\left(\det M^{ab}\right) e^{-(S_{YM}+S_H)}$$

$$S_H = \gamma^4 g^2 \int d^4x\, \varepsilon^{ab} A_\mu \left(M^{-1}\right)^{ab} A_\mu$$

$$\frac{\delta\Gamma}{\delta\gamma} = 0$$

Capri et al., PRD 74 105007 (2006)

Localization

As in the case of the Landau gauge, the horizon term can be cast in local form through the addition of a set of auxliary fields

$$e^{-S_H} = \int D\varphi D\bar{\varphi} D\omega D\bar{\omega}\; e^{-S_{Loc}}$$

$$S_{Loc} = \int d^4x \left(\bar{\varphi}^{ab}{}_{\mu} M^{ac} \varphi^{cb}{}_{\mu} - \bar{\omega}^{ab}{}_{\mu} M^{ac} \omega^{cb}{}_{\mu} + \gamma^2 g \varepsilon^{ab} \left(\varphi^{ab}{}_{\mu} - \bar{\varphi}^{ab}{}_{\mu} \right) A_{\mu} \right)$$

$$\frac{\delta\Gamma}{\delta\gamma} = 0 \quad \longrightarrow \quad \left\langle \varepsilon^{ab} \left(\varphi^{ab}{}_{\mu} - \bar{\varphi}^{ab}{}_{\mu} \right) A_{\mu} \right\rangle \neq 0$$

BRST breaking

After the localization of the horizon term, we get a local action

$$S = \frac{1}{4}\int d^4x\, F^2 + S_{FP} + S_{Loc}$$

$$S_{FP} = \int d^4x \left(i b^a D_\mu^{\ ab} A_\mu^{\ b} + \bar{c}^a M^{ab} c^b \right)$$

$$sS = 0 \quad ?$$

$$s\,S = \gamma^2\, g\, \Delta$$

$$s\, A_\mu^{\ a} = -\left(D_\mu^{\ ab} c^b + g\varepsilon^{ab} A_\mu^{\ b} c \right)$$

$$s\, A_\mu = -\partial_\mu c + g\varepsilon^{ab} A_\mu^{\ a} c^b$$

$$s\, c^a = g\,\varepsilon^{ab} c^b c$$

$$s\, c = \frac{g}{2}\varepsilon^{ab} c^a c^b$$

$$s\, \bar{c}^a = i\, b^a \qquad sb^a = 0$$

$$s\, \bar{c} = ib \qquad sb = 0$$

$$s\varphi^{ab}{}_\mu = \omega^{ab}{}_\mu \qquad s\,\omega^{ab}{}_\mu = 0$$

$$s\,\bar{\omega}^{ab}{}_\mu = \bar{\varphi}^{ab}{}_\mu \qquad s\,\bar{\varphi}^{ab}{}_\mu = 0$$

$$s^2 = 0$$

$$\Delta = \int d^4x \left(\varepsilon^{ab}\omega^{ab}{}_\mu A_\mu - \varepsilon^{ab}\left(\varphi^{ab}{}_\mu - \bar{\varphi}^{ab}{}_\mu \right)\left(\partial_\mu c + g\varepsilon^{mn} A_\mu^{\ m} c^n \right)\right)$$

Dimension two operators

Despite the presence of the soft breaking term Δ, the resulting action enjoys renormalizability. As in the case of the Landau gauge, also in the MAG it is possible to write down Slavnov-Taylor identities in the presence of the beaking Δ, ensuring the renormalizability of the theory.

As already observed, the gap equation for the Gribov parameter is equivalent to the introduction of a nonvanishing dimension two condensate, namely

$$\left\langle \varepsilon^{ab}\left(\varphi_{\mu}{}^{ab}-\overline{\varphi}_{\mu}{}^{ab}\right)A_{\mu}\right\rangle \neq 0$$

It seems therefore natural to look for other dimension two operators which can be introduced in the theory.

Since the BRST invariance is softly broken by the horizon function, these dimension two operators do not need to be necessarily BRST invariant. The only requirement is that their introduction has to be done in a way which preserves renormalizability

*It turns out that, in addition of the horizon term, **three dimension two operators** can be consistently introduced, so that renormalizability is preserved. These are:*

Dudal et al., PRD 70, 114038 (2004)

$$A^a{}_\mu A^a{}_\mu$$

*The condensation of this operator gives rise to a dynamical mass **m** for off-diagonal gluons, in agreement with the hypothesis of Abelian dominance*

The second dimension two operator is given by

$$O_{\text{ghost}} = \varepsilon^{ab}\bar{c}^{a}c^{b}$$

It gives rise to the ghost condensate $< O_{ghost} >$ responsible for the spontaneous symmetry breaking of the SL(2,R) symmetry present in the ghost sector of the MAG,

Capri et al., JHEP 0801: 006, 2008

Finally, there is a third dimension two operator, namely

$$O_{\varphi} = \left(\bar{\varphi}^{ab}_{\mu}\varphi^{ab}_{\mu} - \bar{\omega}^{ab}_{\mu}\omega^{ab}_{\mu} - \bar{c}^{a}c^{a}\right)$$

which reflects the nontrivial dynamics of the interacting auxiliary localizing fields. arXiv:0801.0566

It is a remarkable feature that all these dimension two operators can be simultaneously introduced in the action, in a way which preserves renormalizability.

A look at the propagators

- ***Off-diagonal gluon propagator: Yukawa type***

$$\left\langle A^{a}{}_{\mu}(k)A^{b}{}_{\nu}(-k)\right\rangle = \delta^{ab}\,\frac{1}{k^{2}+m^{2}}\left(\delta_{\mu\nu}-\frac{k_{\mu}k_{\nu}}{k^{2}}\right)$$

Mmm

Where m is the dynamical mass originating from ***< $A^a_\mu A^a_\mu$>***

- ***Diagonal gluon propagator: Gribov-Stingl type***

$$\left\langle A_{\mu}(k)A_{\nu}(-k)\right\rangle = \frac{k^{2}+\mu^{2}}{k^{4}+\mu^{2}k^{2}+4\gamma^{4}}\left(\delta_{\mu\nu}-\frac{k_{\mu}k_{\nu}}{k^{2}}\right)$$

Where μ originates from ***<O_φ>.*** *Notice that the diagonal propagator is suppressed in the IR and does not vanish at the origin. Its behavior looks similar to that recently reported in the case of the Landau gauge, see* ***Cucchieri et al., arXiv:0712.3517***

- *The symmetric off-diagonal ghost propagator*

$$\left\langle \bar{c}^{a}(k)\,c^{b}(-k)\right\rangle = \delta^{ab}\,\frac{k^{2}+\mu^{2}}{k^{4}+2\mu^{2}k^{2}+(\mu^{4}+v^{4})}$$

Where v originates from the ghost condensate ***< O_{ghost} > ~ v***

- *For the anti-symmetric off-diagonal ghost propagator*

$$\left\langle \bar{c}^{a}(k)\,c^{b}(-k)\right\rangle = \varepsilon^{ab}\,\frac{v^{2}}{k^{4}+2\mu^{2}k^{2}+(\mu^{4}+v^{4})}$$

The above propagators turns out to be in very good agreement with the recent lattice results obtained by ***Cucchieri-Mendes-Mihara, arXiv: hep-lat/0611002.***

Conclusion: a possible framework

- *Start with the Faddeev-Popov quantization formula*

- ***Face the Gribov issue**, for example, by suitably restricting the Feynman path integral to the Gribov region Ω. Introduction of the horizon function (see Landau, Coulomb, MAG)*

- *Localize the horizon function. Gribov parameter is determined by a gap equation.*
  Gap equation ⟷ ***dimension two condensate***

- ***BRST versus the restriction to the Gribov region**. The original BRST invariance of the Faddeev-Popov action is broken by soft terms proportional to the Gribov parameter. The presence of this soft breaking does not jeopardize the renormalizability of the theory. **Very important point!***

- *Look at other dimension two operators which can be multaneously added to the theory.*

- *It should be noticed that, since the BRST is softly broken by the restriction to the Gribov horizon, these dimension two operators do not need to be necessarily BRST invariant. The only requirement is that they have to be compatible withthe renormalizabiltiy of the theory.*

- *The presence of those soft breaking terms might signal that physics in the IR cannot be defined in the same way as in the deep ultraviolet, where we recover exact BRST invariance as well as the notion of BRST cohomology in order to define the physical subspace and the local observables.*

- *This framework has been successfully worked out in the Landau gauge, giving rise to propagators in good agreement with the recent lattice data,* ***Dudal et al., arXiv:0711.4496***

Thank you for your attention