# IR fixed points in Landau gauge Yang-Mills Theory

Reinhard Alkofer, Markus huber &
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R. Alkofer, M. huber and K. Schwenzer, arXiv:0801.2762 [hep-th]

PROJECT

# Description of matter

- The Physical degrees of freedom of matter at low scales ...

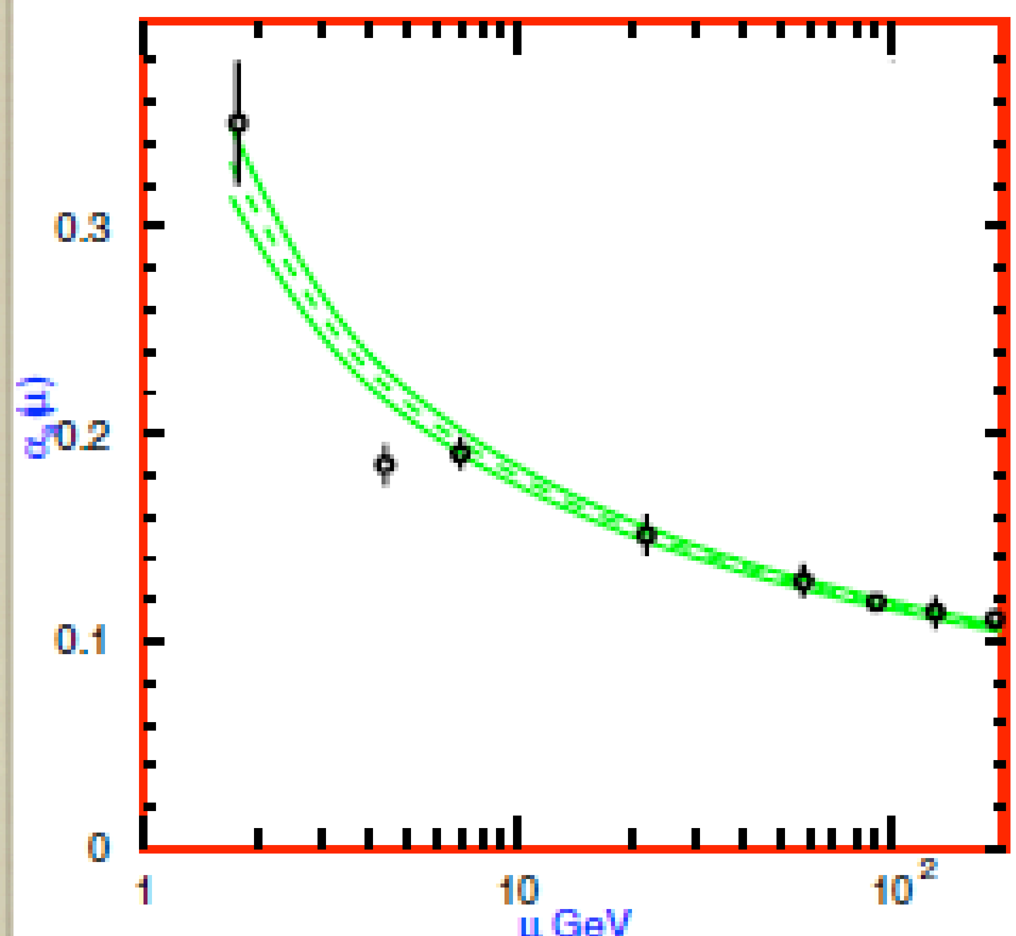
... are quite different from the underlying building blocks

- It would be very desirable to have a direct connection of the physical observables to the dynamics of the fundamental local constituents


- Some kind of Construction manual ...

# Degrees of freedom

- Quenched QCD Suited to study confinement
- Pert. RG breaks down
- Scale generation
- description in terms of initial local degrees of freedom could fail ...
  - but non-perturbative dynamics prevents this for long ranged gauge interactions



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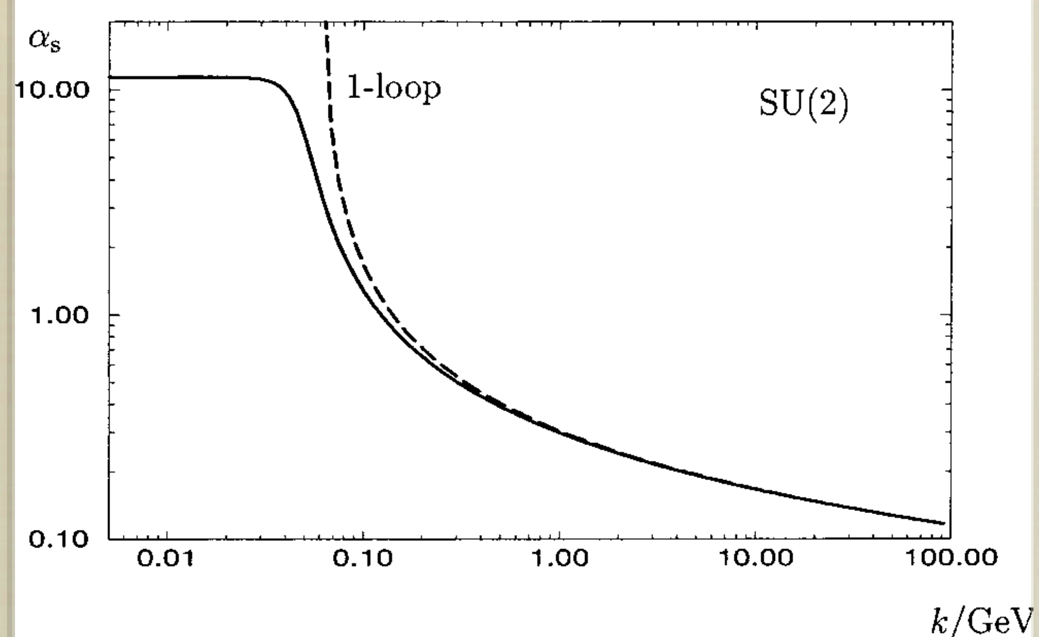
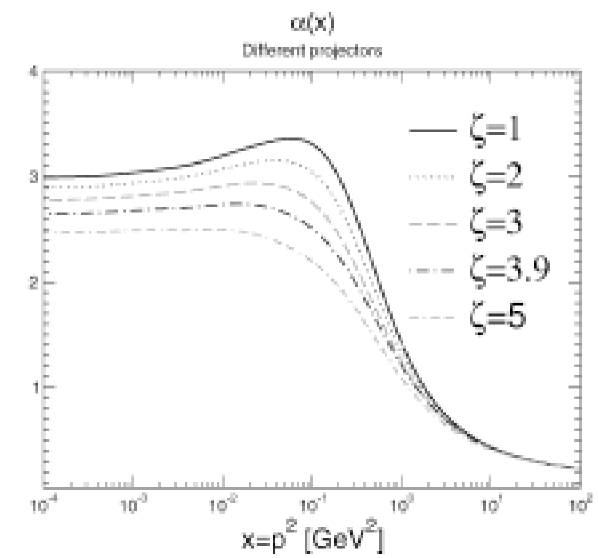


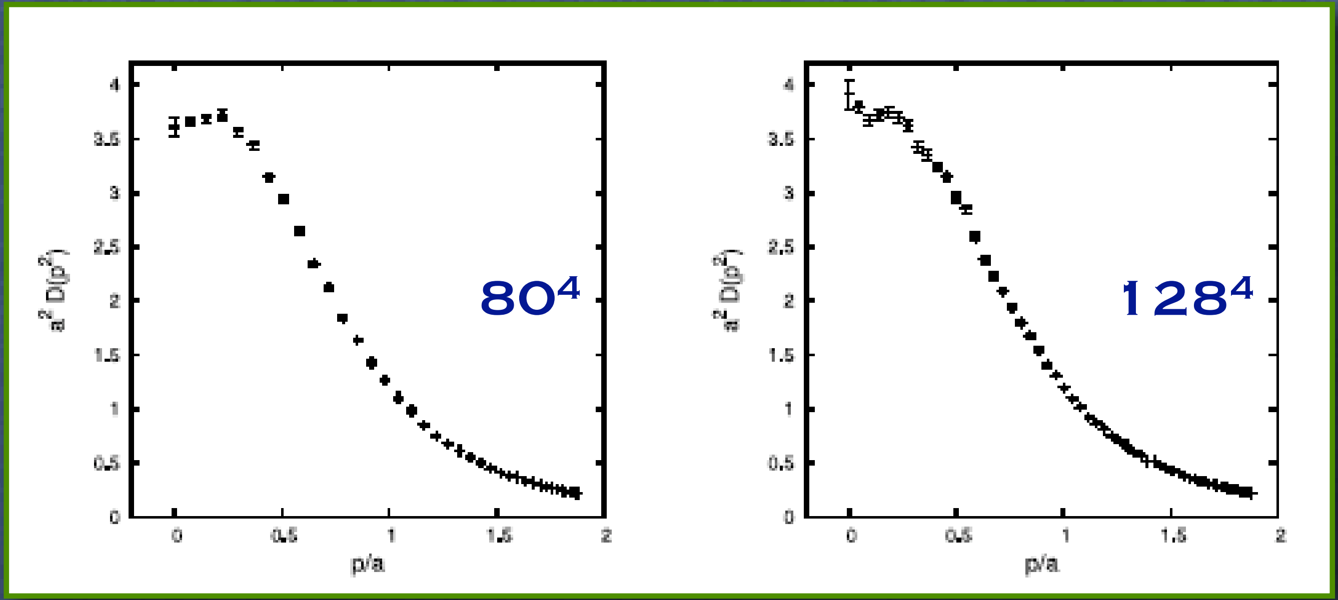

H. Gies, Phys. Rev. D 66 (2002) 025006




C.S.Fischer, R.Alkofer, Phys. Lett.B 536 (2002) 177

# Recent lattice data

- Challenging recent data on large lattices




A. Cucchieri, T. Mendes, 0710.0412 [hep-lat]

- Gluon seems to become IR finite and the ghost roughly bare
  - problems with Gribov copies? --> Axel Maas´s talk
  - Problems with the gauge definition? --> Lorenz von Smekal´s talk

**Where is the confinement?**

# Dyson-Schwinger eq´s

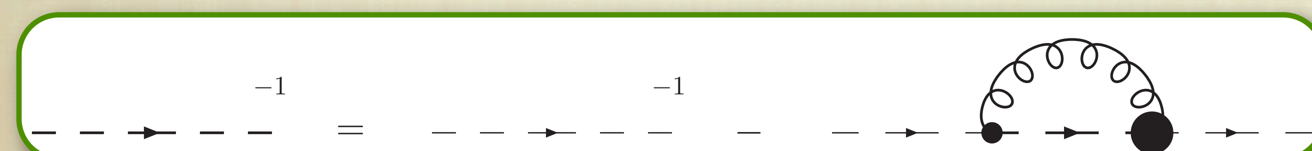
- Idea: An average should not depend on the way the sum is performed:
  (in YM theory $\phi \equiv (A, \bar{c}, c)$ )

$$\delta \left\langle \mathrm{e}^{J\cdot\phi} \right\rangle = \int D\phi \frac{\delta}{\delta\phi} \mathrm{e}^{-S[\phi]+J\cdot\phi} = 0$$

- Formulation in terms of the effective action $\Gamma$:

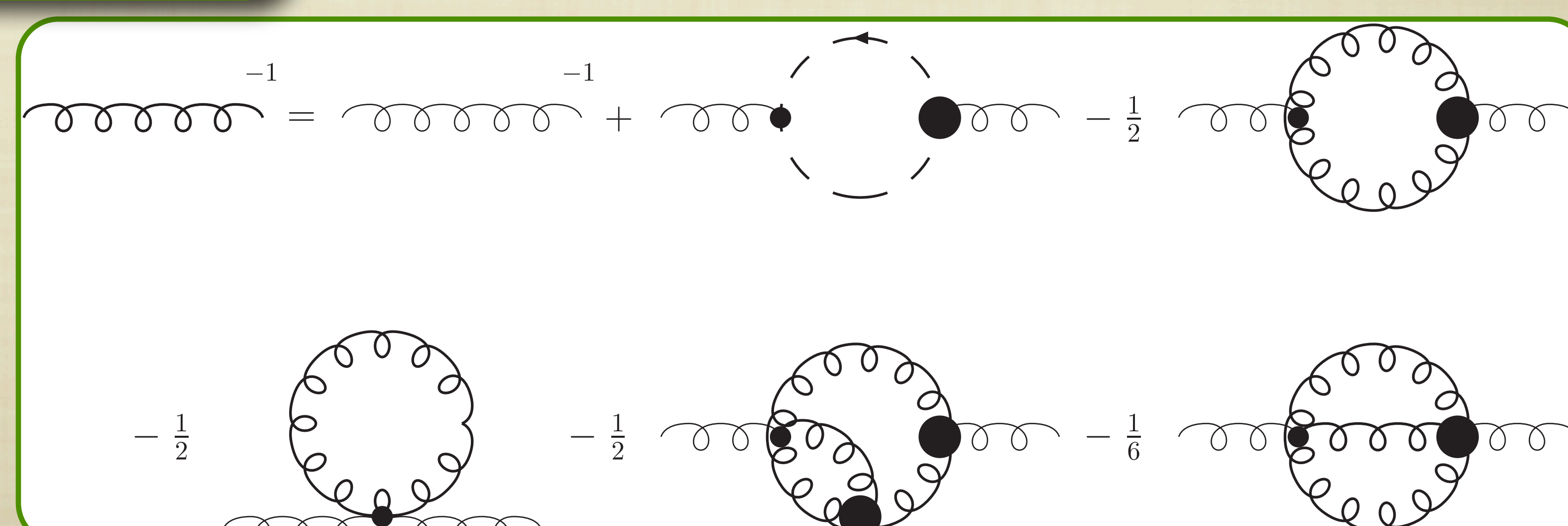
$$\frac{\delta\Gamma}{\delta\Phi_i} = \frac{\delta S}{\delta\phi_i}\bigg|_{\phi_i \to \Phi_i + \Delta_{ij}\frac{\delta}{\delta\Phi_j}}$$

- Higher Greens functions via further functional differentiation ...




## Gluon Propagator

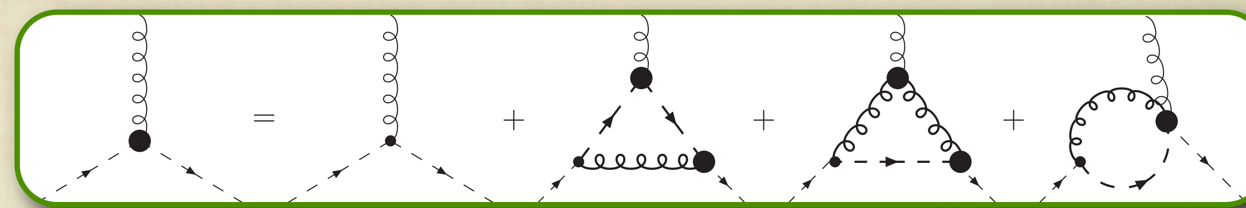
## Ghost Propagator



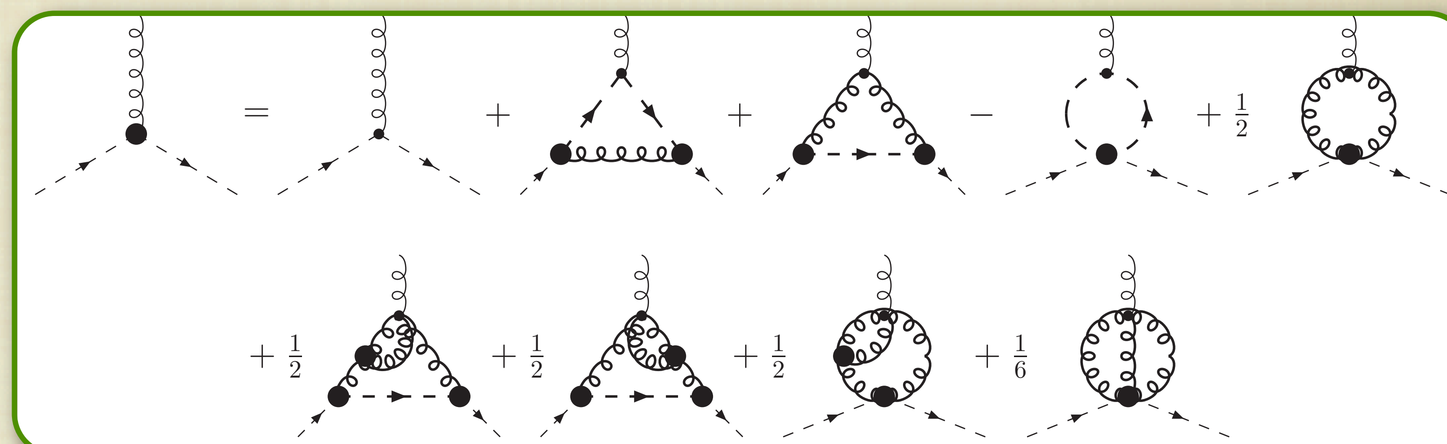

**Coupled Nonlinear integral equations for the propagators**

# Vertex DSEs

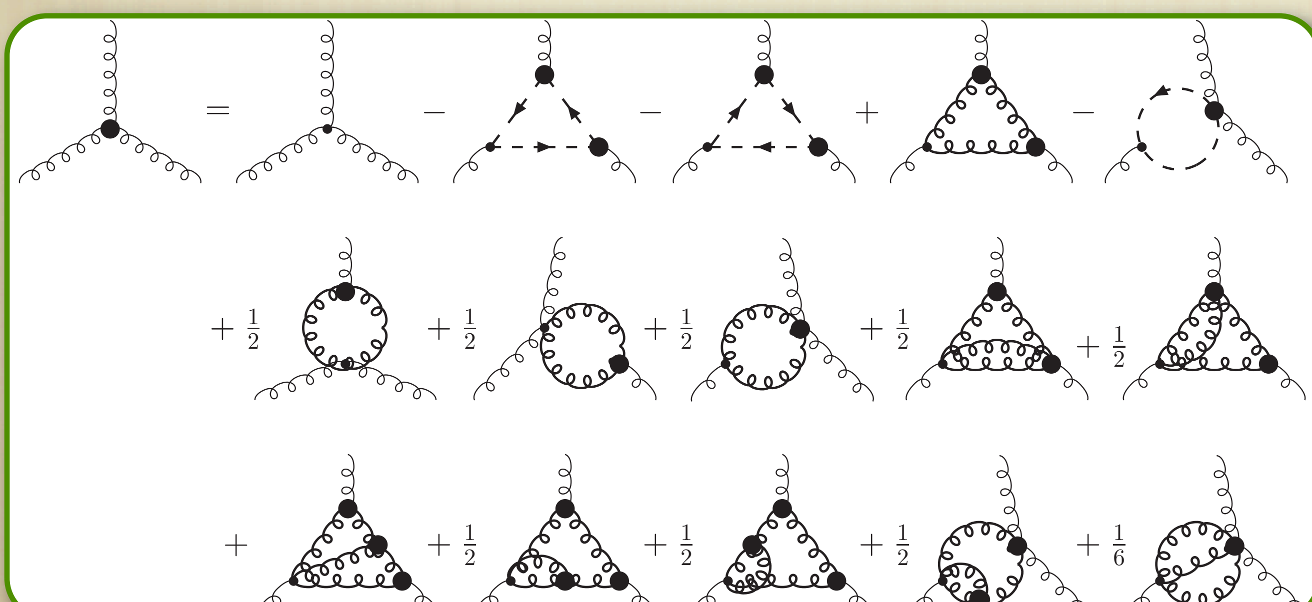
- Propagator DSEs involve the vertices
- Infinitely coupled system of equations
- Approximation scheme required!

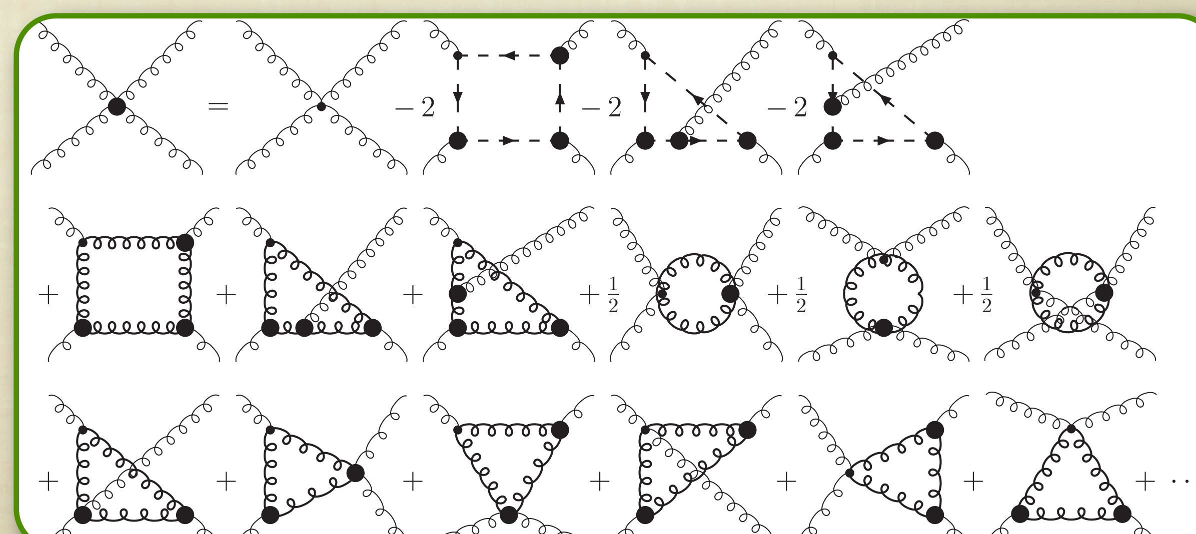


Ghost-gluon vertex (V1 & 2)



3-gluon vertex





4-gluon vertex

# IR-Analysis

- Confinement is a long range / IR phenomenon
- Classical Yang-Mills theory is "conformal" but quantum Fluctuations induce a scale $\Lambda_{QCD}$
- Renormalization group:
  - far below this scale Greens functions should be described by some kind of scaling solution

$$\sim c \cdot \left( \frac{p^2}{\Lambda_{QCD}^2} \right)^{\delta}$$

anomalous IR-exponent

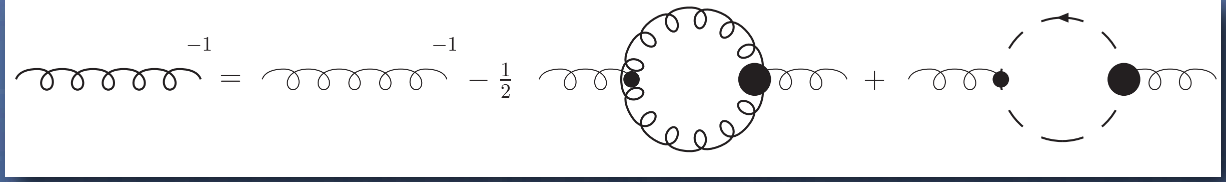
characteristic momentum

- After the tensor decomposition the Integrals within the DSEs are dominated by the poles of the integrands

$$I_3(p,q) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k+p)^{2\alpha}} \frac{1}{(k-q)^{2\beta}} \frac{1}{k^{2\gamma}}$$

# Power counting

- The parametric IR-dependence of the integrals on the external scale can be obtained via a power counting analysis --> Christian Fischer´s talk
  - Without numerically solving the DSEs
- Leading loop correction & leading tensor structure dominates and determines scaling of the vertex --> algebraic equations for exponents
- E.g. Gluon DSE



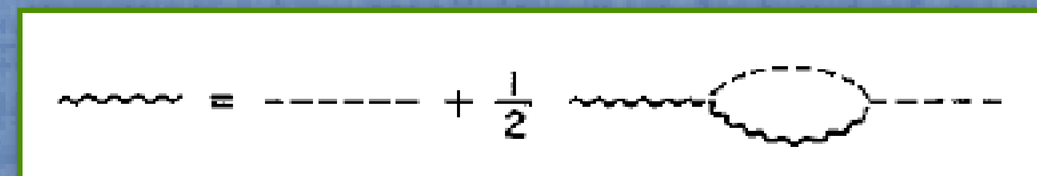
$$(p^2)^{1-\delta_{gl}} = \max\left(p^2, p^4\left(p\left(p^2\right)^{\delta_{3g}}\right) p\left(p^{-2}\left(p^2\right)^{\delta_{gl}}\right)^2, p^4\left(p\left(p^2\right)^{\delta_{gg}}\right) p\left(p^{-2}\left(p^2\right)^{\delta_{gh}}\right)^2\right)$$

$$\Rightarrow\ -\delta_{gl}+1 = \min\left(1, 2+\left(\delta_{3g}+\tfrac{1}{2}\right)+\tfrac{1}{2}+2\left(\delta_{gl}-1\right), 2+\left(\delta_{gg}+\tfrac{1}{2}\right)+\tfrac{1}{2}+2\left(\delta_{gh}-1\right)\right)$$

- Solvable system of such algebraic equations

# Mandelstam solution

- Simplest self-consistent DSE truncation in Landau gauge S. Mandelstam, Phys. Rev. D 20 (1979) 3223
  - Only gluon DSE solved



- IR-enhanced Gluon Propagator $1/q^4$
  - Confining Forces - "infrared slavery"

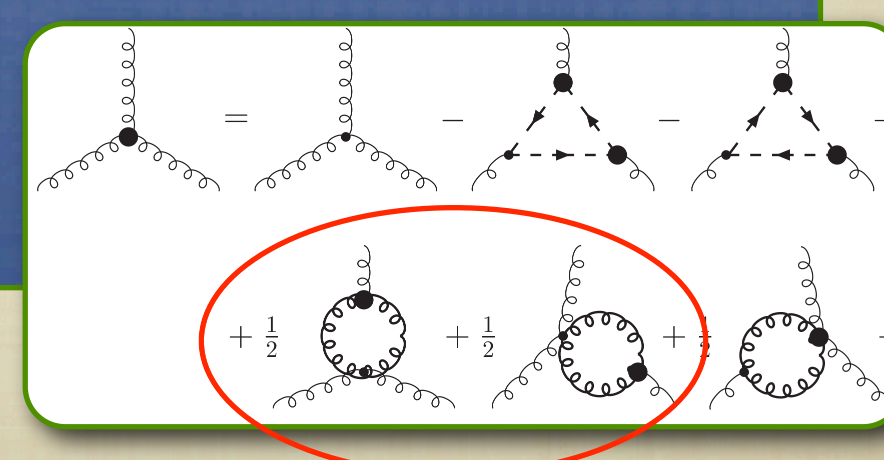
G. West, Phys. Lett. B 115 (1983) 468

- But ... inconsistent with loop graphs in 3g DSE!

$$\Delta\Gamma^{abc}_{\mu\nu\rho}(q_1,q_2) \xrightarrow[p^2\to 0]{} p^4\left(\left(p^2\right)^{-1+\delta_{gl}}\right)^2\left(p^2\right)^{\frac{1}{2}+\delta^u_{3g}} \sim \left(p^2\right)^{\frac{1}{2}+\delta^u_{3g}+2\delta_{gl}}$$

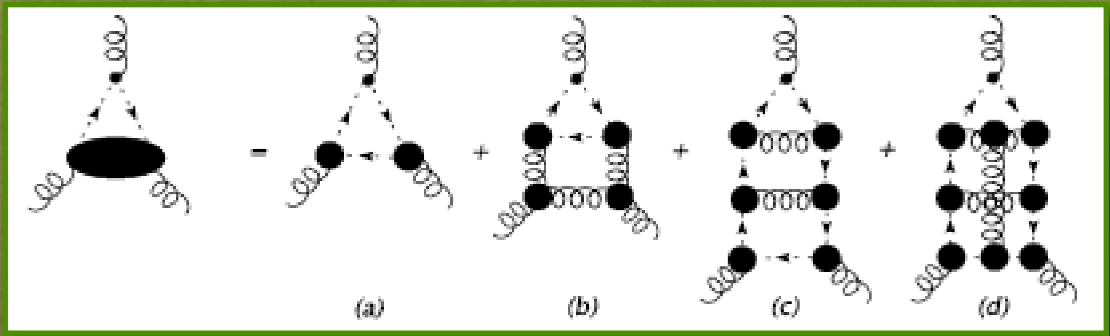
- Solution explicitly excluded by vertex DSE!

$$\delta^u_{3g} \leq \delta^u_{3g} + 2\delta_{gl} \quad \Rightarrow \quad \delta_{gl} \geq 0$$



# Skeleton expansion

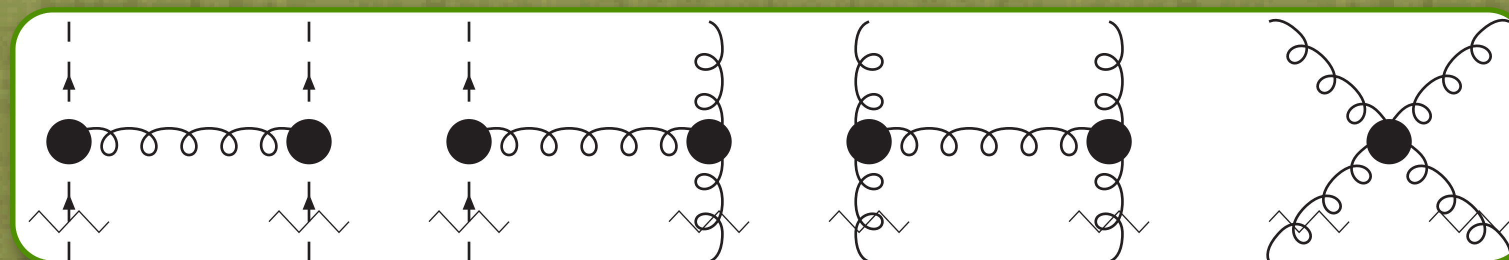
- Expand higher order n-point functions in terms of dressed primitively divergent propagators and vertices
- Example:
  ghost-gluon scattering kernel




- Minimal assumption in order to describe the physics in terms of local degrees of freedom:
  - Skeleton expansion should not explicitly diverge!
  - But no convergence assumed ...

# Skeleton constraints

- The skeleton expansion can be generated by replacement rules

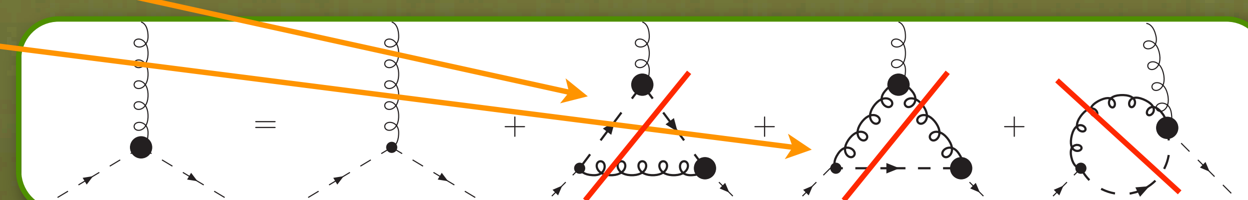


- They should not increase the IR-divergence

- This yields powerful constraints:

$$
\begin{aligned}
2\delta_{gg} + 2\delta_{gh} + \delta_{gl} &\geq 0\,,\\
\delta_{3g} + \delta_{gg} + \delta_{gh} + 2\delta_{gl} &\geq 0\,,\\
2\delta_{3g} + 3\delta_{gl} &\geq 0\,,\\
\delta_{4g} + 2\delta_{gl} &\geq 0
\end{aligned}
$$

e.g.: ghost-gluon vertex:




- These cancel basically all non-linearities in the IR DSEs!

# Skeleton reduction

- The DSE system for the IR exponents ...

$$
\begin{aligned}
-\delta_{gh} &= \min\left(0, \delta_{gg} + \delta_{gh} + \delta_{gl}\right) \ , \\
-\delta_{gl} &= \min\left(0, \delta_{3g} + 2\delta_{gl}, \delta_{gg} + 2\delta_{gh}, 2\delta_{3g} + 4\delta_{gl}, \delta_{4g} + 3\delta_{gl}\right) \\
\delta_{gg} &= \min\left(0, 2\delta_{gg} + 2\delta_{gh} + \delta_{gl}, \delta_{3g} + \delta_{gg} + \delta_{gh} + 2\delta_{gl}\right) \\
\delta_{3g} &= \min\left(0, 2\delta_{gg} + 3\delta_{gh}, 2\delta_{3g} + 3\delta_{gl}, \delta_{3g} + 2\delta_{gl}, \delta_{4g} + 2\delta_{gl}, 3\delta_{3g} + 5\delta_{gl}, \delta_{4g} + \delta_{3g} + 4\delta_{gl}\right) \ , \\
\delta_{4g} &= \min\left(0, 3\delta_{gg} + 4\delta_{gh}, 3\delta_{3g} + 4\delta_{gl}, \delta_{4g} + 2\delta_{gl}, 2\delta_{3g} + 3\delta_{gl}, \delta_{4g} + \delta_{3g} + 3\delta_{gl}, 4\delta_{3g} + 6\delta_{gl}, \delta_{4g} + 2\delta_{3g} + 5\delta_{gl}
\end{aligned}
$$

$$
\begin{aligned}
-\delta_{gh} &= \min\left(0, \delta_{gh} + \delta_{gl}\right) \ , \\
-\delta_{gl} &= \min\left(0, 2\delta_{gh}\right) \ , \\
\delta_{gg} &= 0 \ , \\
\delta_{3g} &= \min\left(0, 3\delta_{gh}\right) \ , \\
\delta_{4g} &= \min\left(0, 4\delta_{gh}, 3\delta_{3g} + 4\delta_{gl}\right)
\end{aligned}
$$

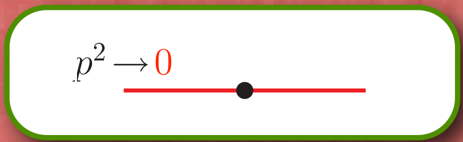
... simplifies considerably!

- Unique solution from the DSE system alone

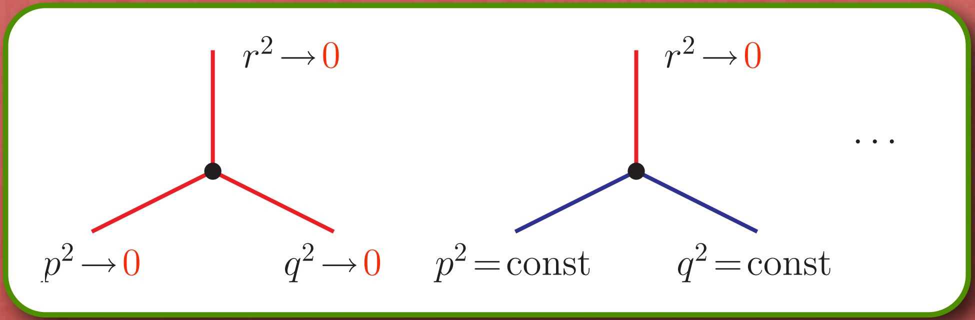
R.Alkofer, C.S.Fischer and F.J.Llanes-Estrada, Phys. Lett. B 611 (2005) 279,
C.S.Fischer and J.Pawlowski, Phys. Rev. D 75 (2007) 025012

# IR singularities

- Possible IR enhancement of Greens functions whenever momenta becomes small ...
- Unique IR-limit for the propagators

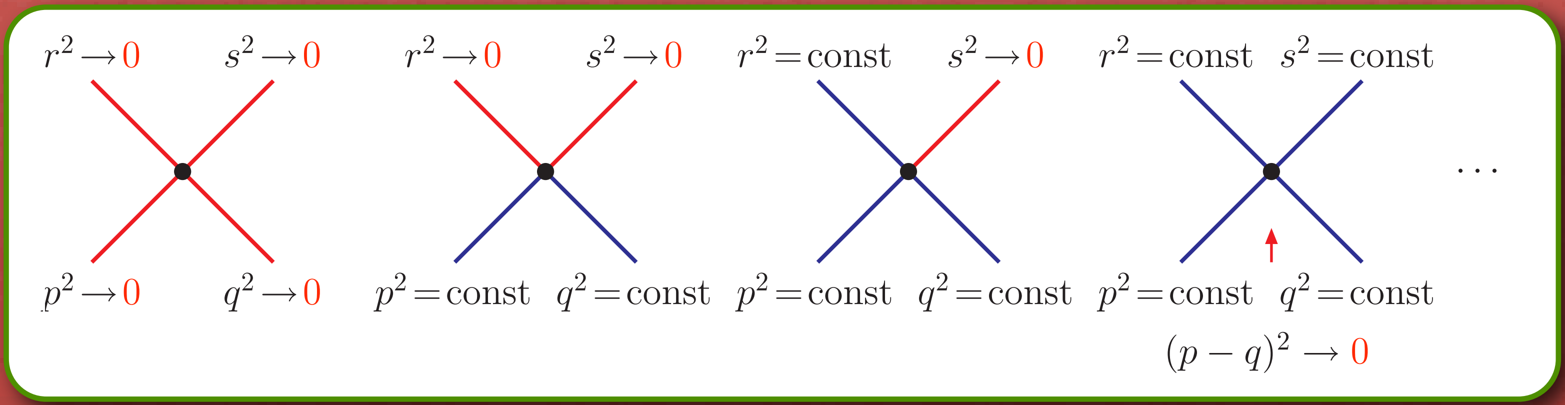



- possibly Additional kinematic Divergence for the vertices




uniform soft

- Even more possibilities for higher vertices ...



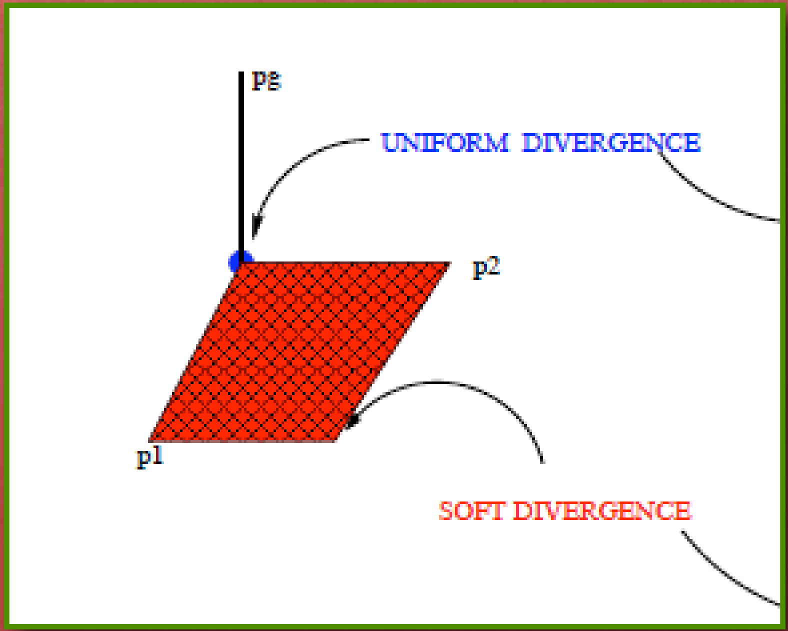
# Finite scales & masses

- For higher order Greens functions IR divergences can occur when any subset of the momenta vanishes ("soft / kinematic divergences")

$$\Gamma^{\mu_1\cdots\mu_m}(q_1,\cdots,q_n)=\sum_t\sum_i c_{i,t}\left(q_1^2/p_i^2,\cdots,q_n^2/p_i^2\right)\left(p_i^2\left(q_1^2,\cdots,q_n^2\right)/\Lambda_{QCD}^2\right)^{\delta_{i,t}}T_t^{\mu_1\cdots\mu_m}(q_1,\cdots,q_n)$$

Yang-Mills: $\delta_{gl},\ \delta_{gh},\ \delta^u_{gg},\ \delta^u_{3g},\ \delta^u_{4g},\ \delta^{gh}_{gg},\ \delta^{gl}_{gg},\ \delta^{gl}_{3g}\ldots$

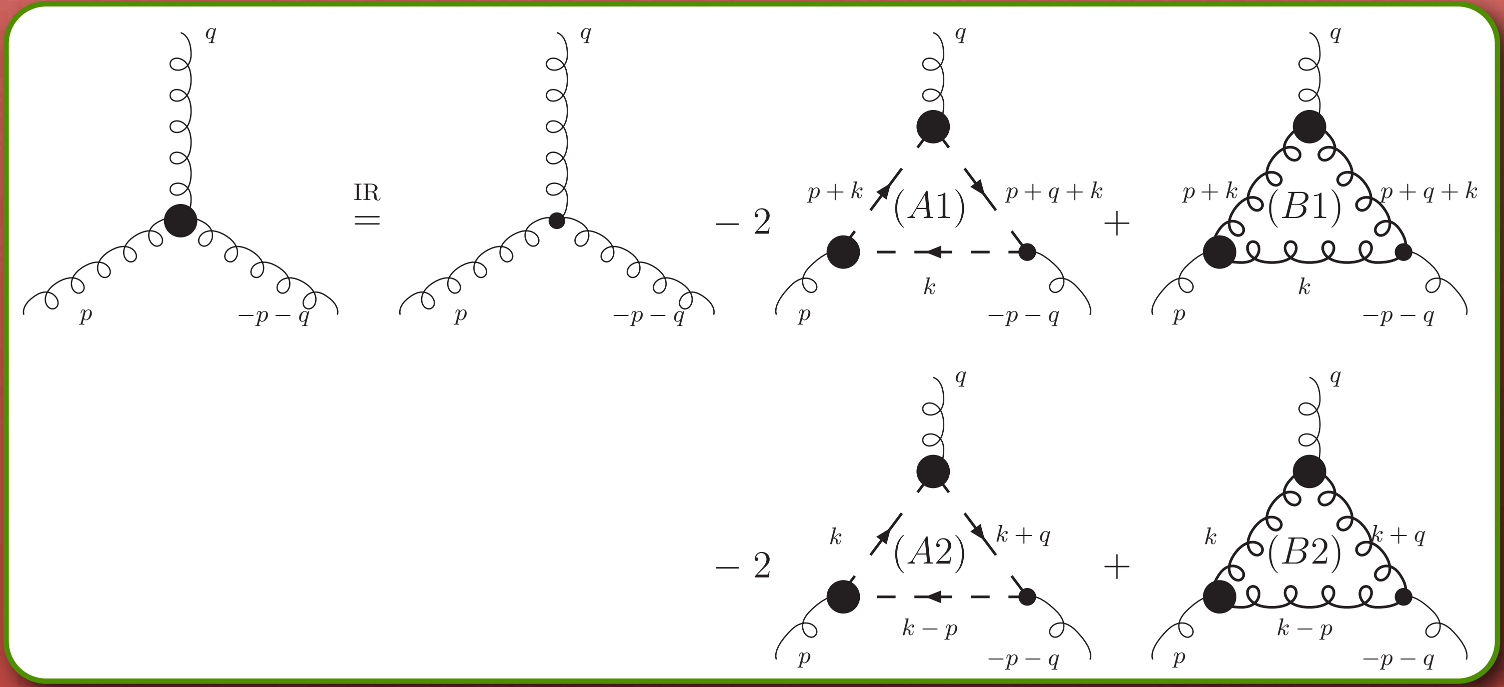
- Soft singularities have a larger support and can have quantitative impact




- fluctuations on hard scales can generate finite masses in the IR

# IR sensitive regions

- When both hard $p_h$ and soft $p_s$ external momenta are present ($p_s \ll p_h < \Lambda_{QCD}$)...
  - Loop momenta $k$ of the order of all external scales contribute $I_3(p,q) = \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k+p)^{2\alpha}} \frac{1}{(k-q)^{2\beta}} \frac{1}{k^{2\gamma}}$
  - Different regions of the integral are IR sensitive and could dominate:
    - Soft region
      $k \sim p_s$
    - Mixed region
      $k \sim p_s$ & $k \sim p_h$
    - Hard region
      $k \sim p_h, \Lambda_{QCD}$



# IR analysis

- Neglect 4-point vertices and 2-loop graphs
- Skeleton constraints strongly simplify the leading IR dynamics

$$
\begin{aligned}
-\delta_{gh}+1 &= \min\left(1, \delta^{u}_{gg}+\delta_{gh}+\delta_{gl}+1, \delta^{gh}_{gg}+1\right) \ , \\
-\delta_{gl}+1 &= \min\left(1, \delta^{u}_{gg}+2\delta_{gh}+1, \delta^{u}_{3g}+2\delta_{gl}+1, \delta^{gl}_{3g}\right) \ , \\
\delta^{u}_{gg}+\frac{1}{2} &= \min\left(\frac{1}{2}, \delta^{gl}_{3g}+\delta^{gh}_{gg}+1\right) \ , \\
\delta^{u}_{3g}+\frac{1}{2} &= \min\left(\frac{1}{2}, 2\delta^{u}_{gg}+3\delta_{gh}+\frac{1}{2}, 2\delta^{gl}_{3g}\right) \ , \\
\delta^{gh}_{gg} &= \min\left(0, 2\delta^{gh}_{gg}+\delta_{gh}+2\right) \ , \\
\delta^{gl}_{3g} &= \min\left(0, \delta^{u}_{gg}+\delta^{gh}_{gg}+2\delta_{gh}+1\right)
\end{aligned}
$$

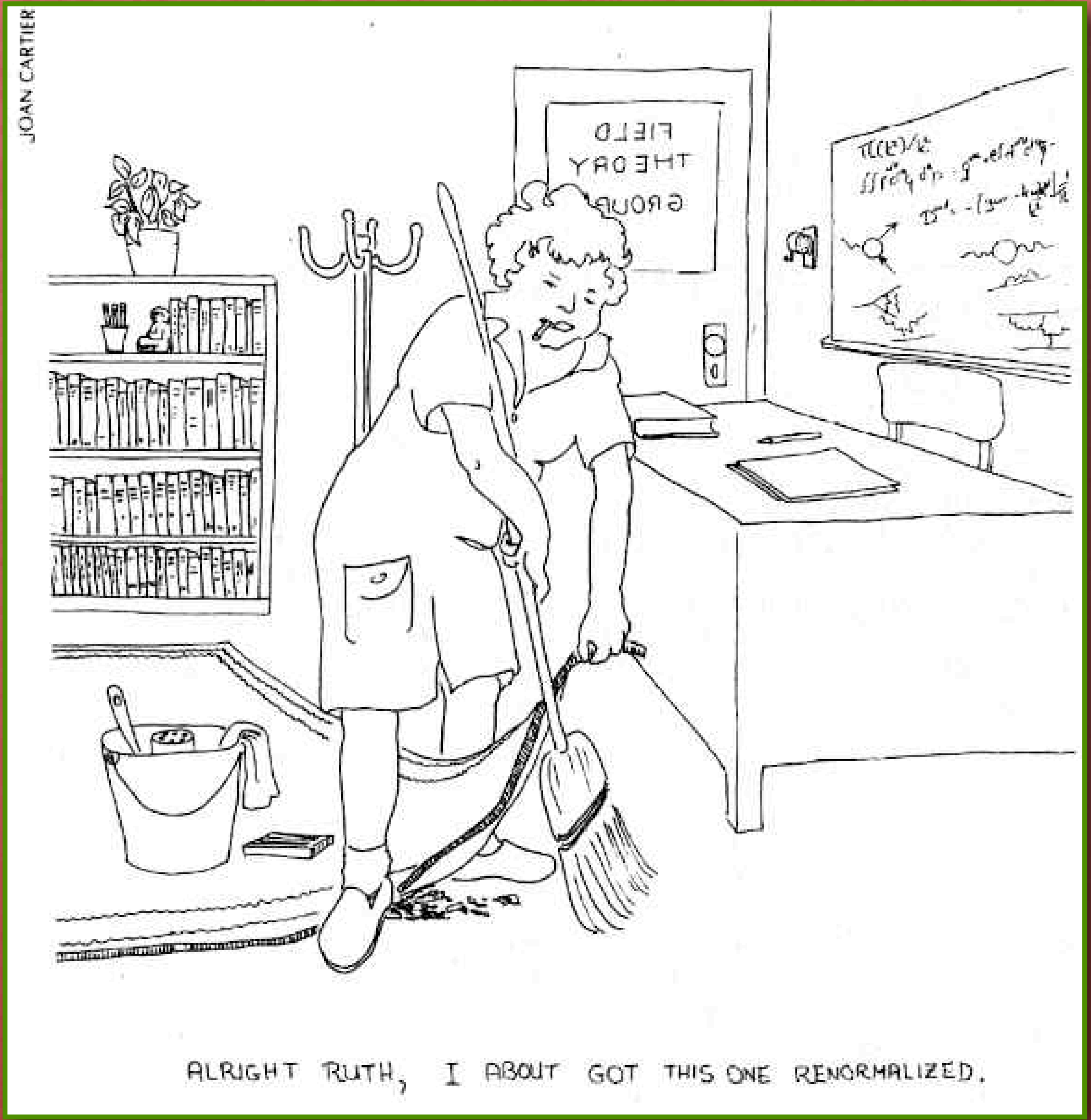
- Additional contributions from hard momenta in the loop integral due to soft divergences

# Renormalization

- Loop graphs require renormalization
- Simple due to asymptotic freedom
- Ghost renormalization:

  $$G^{-1}(p^2=0)=0$$

  - Cancels tree level ghost term exactly!
    L. v. Smekal, A. Hauck & R. Alkofer, Phys. Rev, Lett. 79 (1997) 3591
  - But not for a generic renormalization prescription
- Two qualitatively different fixed points



# IR fixed points

- Two qualitatively different ("unique") solutions depending on the renormalization prescription

| | $\delta_{gh}$ | $\delta_{gl}$ | $\delta^{u}_{gg}$ | $\delta^{u}_{3g}$ | $\delta^{u}_{4g}$ | $\delta^{gh}_{gg}$ | $\delta^{gl}_{gg}$ | $\delta^{gl}_{3g}$ |
|---|---|---|---|---|---|---|---|---|
| scaling | $-\kappa$ | $2\kappa$ | $0$ | $-3\kappa$ | $-4\kappa$ | $0$ | $1-2\kappa$ | $1-2\kappa$ |
| decoupling | $0$ | $1$ | $0$ | $-1/2$ | $0$ | $0$ | $0$ | $0$ |

- Decoupling scenario

C.S.Fischer, private communication
P.Boucaud, et.al., arXiv:0801.2721 [hep-ph]
A.C.Aguilar, D.Binosi, J.Papavassiliou, arXiv:0802.1870 [hep-ph]

  - Vertices are not IR-enhanced
  - IR regime is entirely finite
  - No description of quark confinement
  - No area law behavior of the Wilson loop

# IR fixed points

- Two qualitatively different ("unique") solutions depending on the renormalization prescription

| | $\delta_{gh}$ | $\delta_{gl}$ | $\delta^{u}_{gg}$ | $\delta^{u}_{3g}$ | $\delta^{u}_{4g}$ | $\delta^{gh}_{gg}$ | $\delta^{gl}_{gg}$ | $\delta^{gl}_{3g}$ |
|---|---|---|---|---|---|---|---|---|
| scaling | $-\kappa$ | $2\kappa$ | $0$ | $-3\kappa$ | $-4\kappa$ | $0$ | $1-2\kappa$ | $1-2\kappa$ |
| decoupling | $0$ | $1$ | $0$ | $-1/2$ | $0$ | $0$ | $0$ | $0$ |

- Scaling solution

  L. v. Smekal, A. Hauck & R. Alkofer, Phys. Rev, Lett. 79 (1997) 3591

  - Ghost dominance picture basically unchanged
  - supplemented by mild soft-gluon singularities
    - Confirmed via explicit computation

      --> Markus Huber´s Poster
  - induces strong IR singularities in the quark-gluon vertex and confines quarks via IR slavery

    R. Alkofer, C. Fischer, F. Llanes-Estrada, K. Schwenzer, in preparation

# Conclusion

- Unique scaling fixed point in Landau gauge Yang-Mills theory
  - IR regime is dominated by Ghost dynamics
  - More structure due to kinematic singularities
  - provides a coherent picture of the strongly interacting vacuum
    - chiral symmetry breaking & confinement

      --> Christian Fischer´s talk
    - spontaneous & anomalous mass generation

      --> Richard Williams´s talk
- Decoupling solution cannot be excluded
  - BUT ... the IR regime is not enhanced at all!