# Landau gauge lattice gluon and ghost propagators in the infrared

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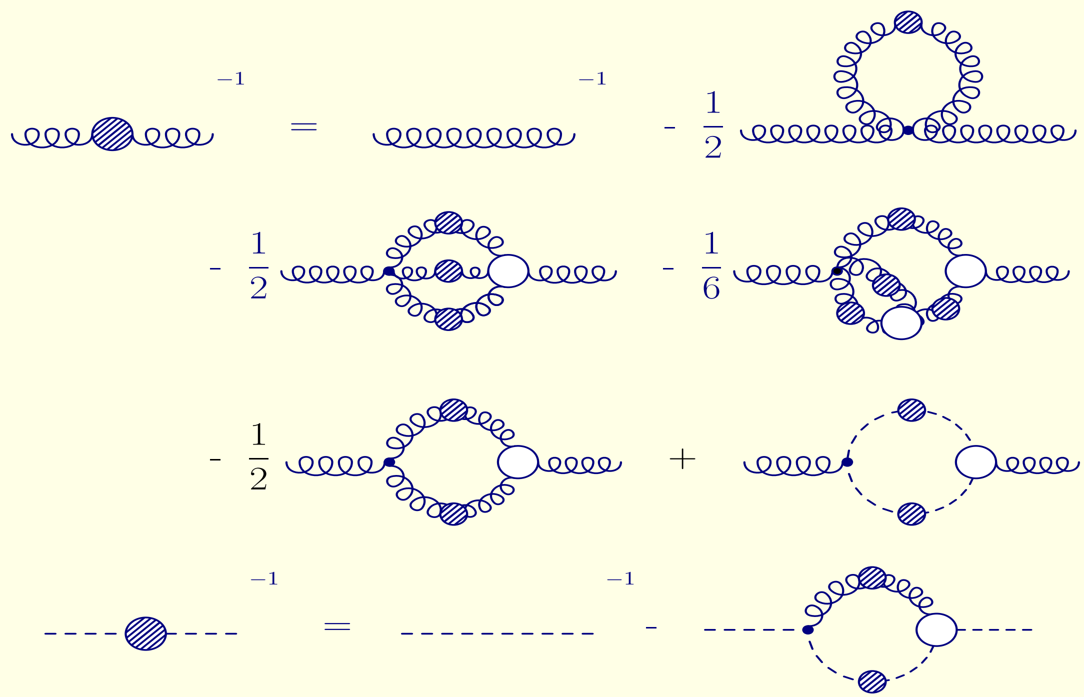
# Outline of the talk

1. Introduction, motivation
2. Gluon and ghost propagators in pure $SU(3)$ gauge theory and full QCD: lattice and DSE results at finite volume
3. The running coupling
4. Gluon and ghost propagators in pure $SU(3)$ gauge theory: recent lattice results on huge lattices
5. Improved gauge fixing: hope to solve the puzzle?
6. Conclusion and outlook

# 1. Introduction, Motivation

Landau gauge gluon and ghost propagators computed from non-perturbative (truncated) Dyson-Schwinger Equations (DSE)

[Alkofer, Fischer, Maas, Pawlowski, von Smekal, ..., Zwanziger ('97 - '07)]




Propagators and Vertex functions = input for hadron phenomenology:

Bethe-Salpeter eqs. for mesons, Faddeev eqs. for baryons.

In the infrared limit $q^2 \to 0$

DSE provide asymptotic power-like solutions:

$$D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z(q^2)}{q^2}\,, \qquad Z(q^2) \propto (q^2)^{\,\kappa_D}$$

$$G^{ab} = \delta^{ab}\frac{J(q^2)}{q^2}\,, \qquad J(q^2) \propto (q^2)^{-\kappa_G}$$

are claimed

- to be unique, when DSE combined with functional renormalization group,
- to hold without any DSE truncation,
- to be independent of the number of colors $N_c$,
- to look qualitatively the same in any dimension $d = 2, 3, 4$.

$\kappa_D = 2\,\kappa_G + (4-d)/2\,,$

$d = 4:$ $\kappa_G \simeq 0.59$ and $\kappa_D = 2\,\kappa_G$.

(Conflicting claims: Boucaud et al. ('05 -'07), Aguilar, Natale ('05-'07))

Running coupling from ghost-ghost-gluon vertex in MOM scheme assuming $Z_1 \equiv 1$:

$$\alpha_s(q^2) = \frac{g_0^2}{4\pi} \; Z(q^2) \cdot [J(q^2)]^2$$

$d = 4$: $\quad 0 < \alpha_s(q^2) < \infty\,,$ i.e. finite in the infrared $\quad q^2 \to 0$.

Compare also with analytic perturb. theory [D.V. Shirkov, I.L. Solovtsov ('97 - '02)].

Infrared power behavior of $Z$, $J$ in agreement with confinement scenarios:

- Kugo-Ojima confinement criterion [Ojima, Kugo ('78 - '79)]:
  absence of colored physical states $\Longleftrightarrow$ ghost (gluon) propagator more (less) singular than simple pole for $q^2 \to 0$.

- Gribov-Zwanziger confinement scenario
  [Gribov ('78), Zwanziger ('91 - ...)]:
  gauge fields within the Gribov region

$$\Omega = \Big\{ A_\mu(x) : \; \partial_\mu A_\mu = 0, \; M_{FP} \equiv -\partial D(A) \geq 0 \Big\}$$

  are accumulated at the Gribov horizon $\partial\Omega$ :

  non-trivial eigenvalues of $M_{FP}$: $\lambda_0 \to 0$.

$$\Longrightarrow \quad \begin{array}{ll} \text{Ghost:} & G(q^2) \;\to\; \infty \\ \text{Gluon:} & D(q^2) \;\to\; 0 \end{array} \qquad \text{for } q^2 \to 0.$$

Gribov problem:

- Existence of several gauge copies inside $\Omega$.
- What are the right copies?
  Restriction inside $\Omega$ to fundamental modular region (FMR) required?

$$\Lambda = \left\{ A_\mu(x) : \; F(A^g) < F(A) \;\; \text{for all} \;\; g \neq \mathbf{1} \right\} .$$

Answer in the limit of infinite volume [Zwanziger ('04)]:

Non-perturbative quantization requires only restriction to $\Omega$,

$$\text{i.e.} \quad \delta_\Omega(\partial_\mu A_\mu) \, \det(-\partial_\mu D_\mu^{ab}) e^{-S_{YM}[A]} .$$

Expectation values taken on $\Omega$ or $\Lambda$ should be equal in the thermodynamic limit.

- What happens on a (finite) torus?
- How Gribov copies influence finite-size effects?

# Questions to lattice QCD:

- Do propagators show the infrared behavior proposed by DSE ?
- What is the influence of Gribov copies on the propagators? Large-volume limit ?
- Full QCD versus quenched QCD ?
- Infrared limit of the MOM-scheme running coupling $\alpha_s(q^2)$?
- What lattice QCD can tell about various confinement criteria?
- What about the eigenvalues and eigenmodes of the Faddeev-Popov operator?

$$G(q) = \langle \sum_{i=1}^{n} \frac{1}{\lambda_i} \vec{\Phi}_i(k) \cdot \vec{\Phi}_i(-k) \rangle$$

# 2. Gluon and ghost propagators: lattice and DSE results at finite volume

A few technicalities:

i) Generate lattice discretized gauge fields $U = \{U_{x,\mu} \in SU(N_c)\}$ by MC simulation from path integral

$$Z_{\mathsf{Latt}} = \int \prod_{x,\mu} [dU_{x,\mu}] \; (\det\, Q(\kappa, U))^{N_f} \; \exp(-S_G(U))$$

– standard Wilson plaquette action

$$\begin{aligned} S_G(U) &= \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{N_c} \mathfrak{Re}\,\mathrm{Tr}\, U_{x,\mu\nu}\right), \\ U_{x,\mu\nu} &\equiv U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}, \qquad \beta = 2N_c/g_0^2 \end{aligned}$$

– (clover-improved) Dirac-Wilson fermion operator $Q(\kappa, U)$:

$N_f = 0$ – pure gauge case,

$N_f = 2$ – full QCD with equal bare quark masses

$ma = 1/2\kappa - 1/2\kappa_c$, $\quad a(\beta)$ – lattice spacing.

ii) $Z_{\text{Latt}}$ is simulated with (Hybrid) Monte Carlo method without any gauge fixing.

iii) Gauge fix each lattice field $U$:

$$U^g_{x\mu} = g_x \cdot U_{x\mu} \cdot g^\dagger_{x+\hat{\mu}}$$

standard gauge orbits: $\{g_x\}$ periodic on the lattice

Landau gauge: $$\mathcal{A}_{x+\hat{\mu}/2,\mu} = \frac{1}{2iag_0} \left( U_{x\mu} - U^\dagger_{x\mu} \right)\Big|_{\text{traceless}}$$

$$(\partial\mathcal{A})_x = \sum_{\mu=1}^{4} \left( \mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu} \right) = 0$$

equivalent to maximizing the gauge functional

$$F_U(g) = \sum_{x,\mu} \frac{1}{N_c} \mathfrak{Re}\ \text{Tr}\, U^g_{x\mu} = \text{Max.}$$

Maximization: by various iterative techniques possible:

overrelaxation, simulated annealing, Fourier acceleration,...

Gribov problem: large number of local maxima of $F_U(g)$.

Practical solution: Initial random gauges

$\Longrightarrow$ best copies (bc) from subsequent maximizations,

$\Longrightarrow$ compared with first copies (fc)).

iv) Compute propagators

- Gluon propagator:

$$D^{ab}_{\mu\nu}(q) = \left\langle \widetilde{A}^a_\mu(k)\widetilde{A}^b_\nu(-k)\right\rangle \equiv \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu\ q_\nu}{q^2}\right) D(q^2)$$

for lattice momenta

$$q_\mu(k_\mu) = \frac{2}{a}\sin\left(\frac{\pi k_\mu}{L_\mu}\right), \qquad k_\mu \in \left(-L_\mu/2, L_\mu/2\right]$$

- Ghost propagator:

$$G^{ab}(q) = \frac{1}{V^{(4)}} \sum_{x,y} \left\langle \mathrm{e}^{-2\pi i\, k\cdot(x-y)} [M^{-1}]^{ab}_{xy} \right\rangle \equiv \delta^{ab} G(q)\,.$$

$M \sim \partial_\mu D_\mu$ - Landau gauge Faddeev-Popov operator

$$M^{ab}_{xy}(U) = \sum_\mu A^{ab}_{x,\mu}(U)\,\delta_{x,y} - B^{ab}_{x,\mu}(U)\,\delta_{x+\hat{\mu},y} - C^{ab}_{x,\mu}(U)\,\delta_{x-\hat{\mu},y}$$

$$\begin{aligned} A^{ab}_{x,\mu} &= \mathfrak{Re}\,\mathrm{Tr}\left[\{T^a,T^b\}(U_{x,\mu}+U_{x-\hat{\mu},\mu})\right], \\ B^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{Tr}\left[T^b T^a\, U_{x,\mu}\right], \\ C^{ab}_{x,\mu} &= 2\cdot\mathfrak{Re}\,\mathrm{Tr}\left[T^a T^b\, U_{x-\hat{\mu},\mu}\right], \qquad \mathrm{Tr}\,[T^a T^b] = \delta^{ab}/2\,. \end{aligned}$$

$M^{-1}$ from solving

$$M^{ab}_{xy}\phi^b(y) = \psi^a_c(x) \equiv \delta^{ac}\exp(2\pi i\, k\cdot x)$$

with (preconditioned) conjugate gradient algorithm.

# Lattice Landau gauge ghost and gluon propagators:

SU(2):

Cucchieri, Maas, Mendes ('96-'07); Gattnar, Langfeld, Reinhardt,... ('02 - '03); Bloch, Cucchieri, Mendes, Langfeld ('04).

Finite-size and Gribov copy effects:

Bakeev, Ilgenfritz, Mitrjushkin, M.-P., PRD 69, 074507 (2004), hep-lat/0311041;
Bogolubsky, Burgio, Mitrjushkin, M.-P., PRD 74, 034503 (2006), hep-lat/0511056;
Bogolubsky et al., arXiv:0707.3611 [hep-lat], poster contr. LATTICE '07.

SU(3):

Suman, Schilling ('96); Bonnet, Leinweber, Williams,... ('99 - '06);
Furui, Nakajima ('03 - '06); Boucaud et al. ('98-'05); Oliveira, Silva ('05 - '07).

Finite-size and Gribov copy effects:

Sternbeck, Ilgenfritz, M.-P., Schiller, PRD 72, 014507 (2005), hep-lat/0506007;

Sternbeck, Ilgenfritz, M.-P., PRD 73, 014502 (2006), hep-lat/0510109;

Bogolubsky, Ilgenfritz, M.-P., Sternbeck, poster contr. LATTICE '07.

## SU(3) study: pure gauge theory versus full QCD

- Pure gauge case $N_f = 0$:
  bare coupling $\beta = 5.7, 5.8, 6.0, 6.2$;
  lattice sizes $12^4, \ldots, 56^4$ , and recently $64^4, \ldots, 96^4$.

- Full QCD case $N_f = 2$:
  thanks: configurations provided by QCDSF - collaboration,
  bare coupling $\beta = 5.29, 5.25$; mass parameter
  $\kappa = 0.135, ..., \; 0.13575$; lattice size $16^3 \times 32, \;\; 24^3 \times 48$.

- Gauge fixing:
  start with random gauge copies and apply
  standard over-relaxation (OR)
  compare first (fc) and best gauge copies (bc)

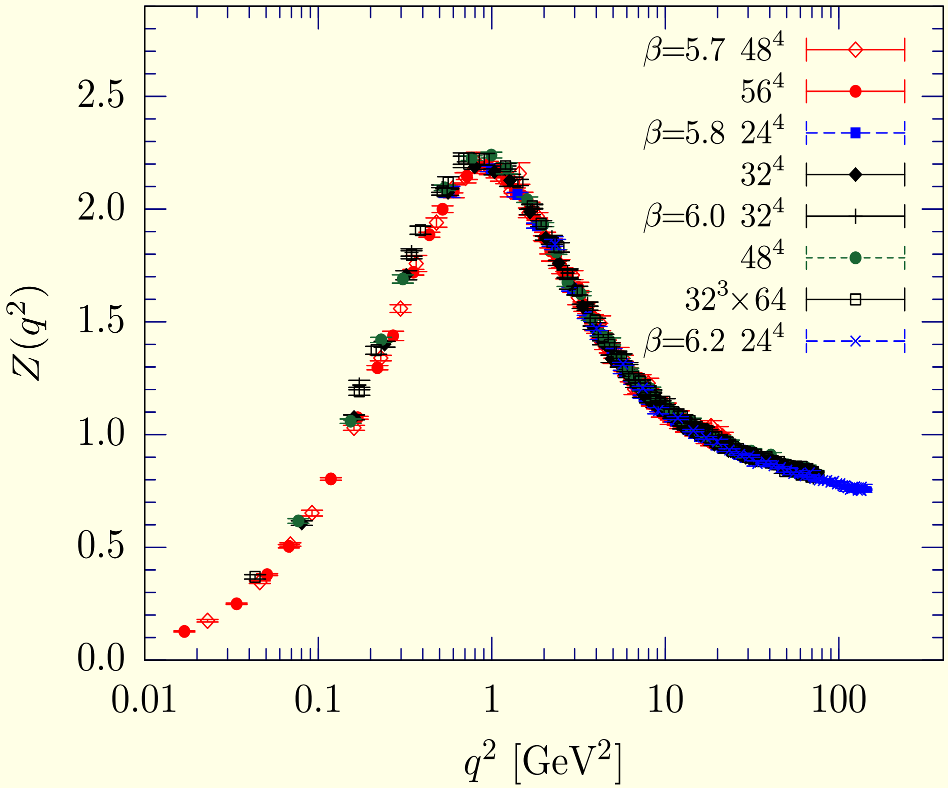
- Dressing functions:

$$\text{Gluon} \quad Z(q^2) \equiv q^2 D(q^2), \qquad \text{Ghost} \quad J(q^2) \equiv q^2 G(q^2)$$

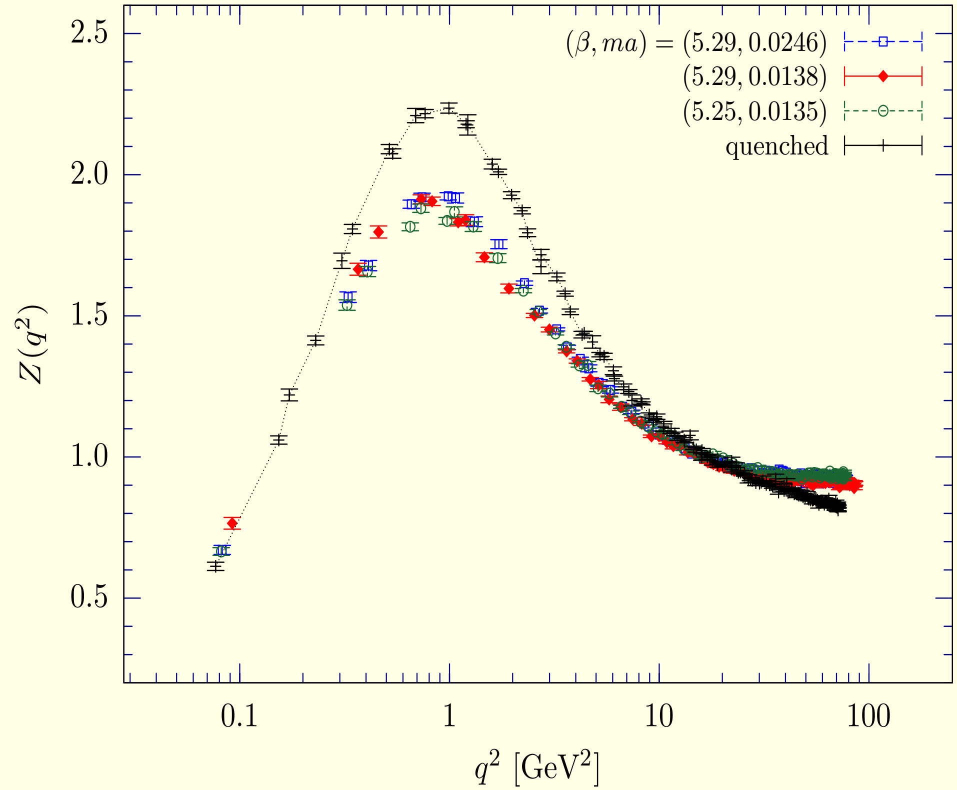
# Gluon dressing functions from first copies:

renormalization point: $q = \mu = 4\text{GeV}$

$N_f = 0$ Sternbeck et al. PRD 72, IRQCD 06



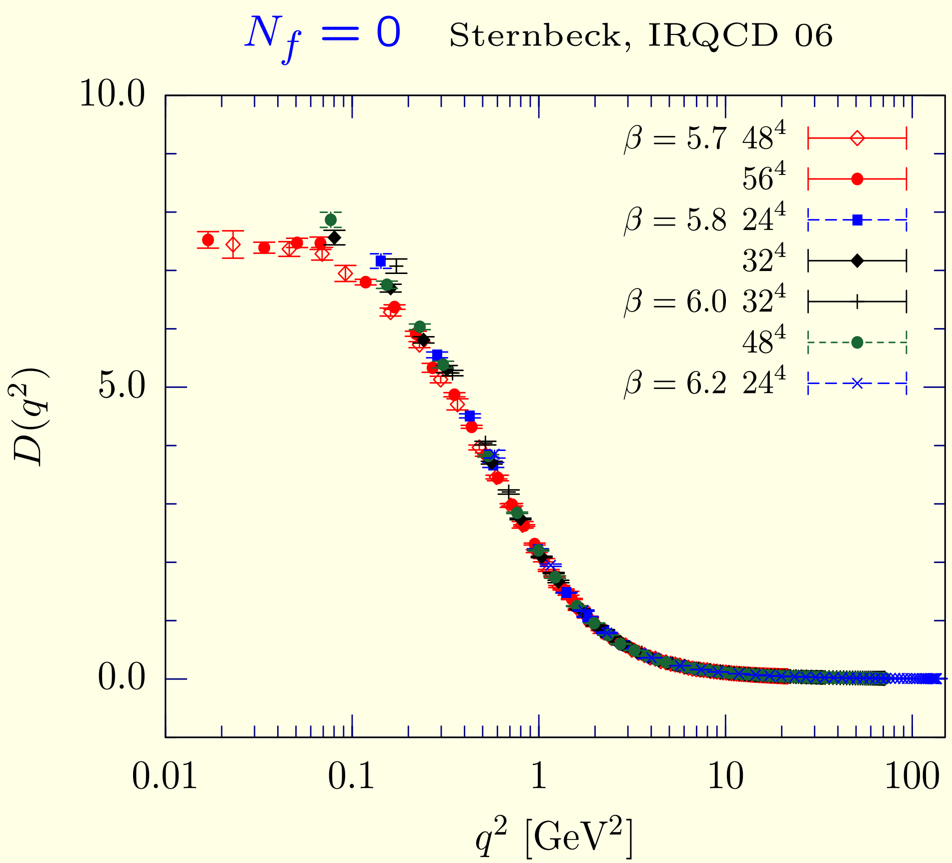

$N_f = 2$ Ilgenfritz et al. '06




$\Longrightarrow$ Influence of virtual quark loops clearly visible.

$\Longrightarrow$ $D(q^2) = Z(q^2)/q^2$ vanishing in the infrared ?

# Gluon propagator from first copies:

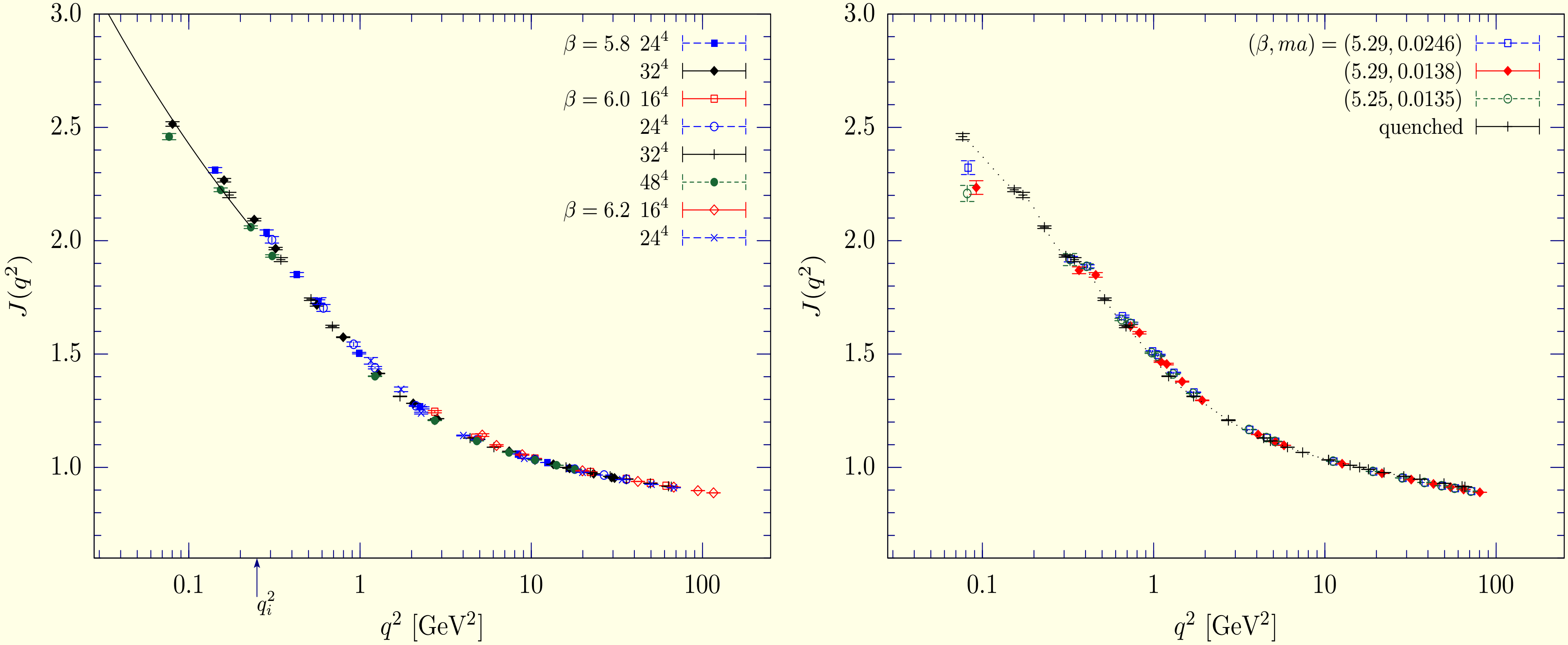



$\Longrightarrow$ $D(q^2) = Z(q^2)/q^2$ shows a plateau at small $q^2$ ??.

# Ghost dressing functions from first copies:

renormalization point: $q = \mu = 4\text{GeV}$

$N_f = 0$ Sternbeck et al. PRD 72 $N_f = 2$ Ilgenfritz et al. '06




$\Longrightarrow$ no finite-volume effects?

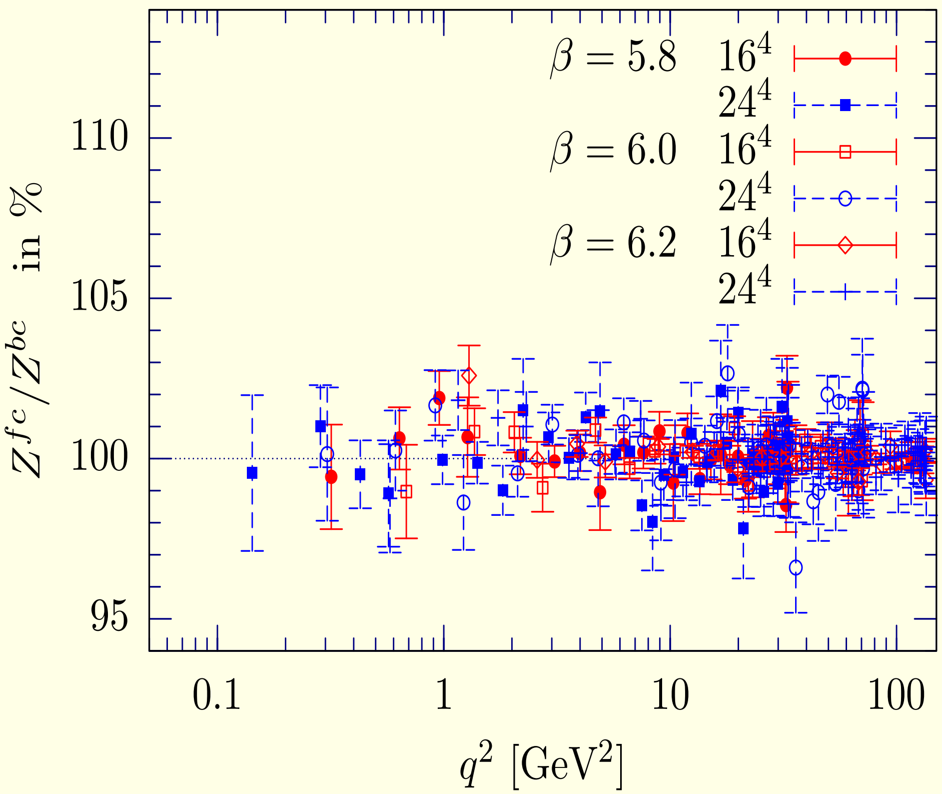
$\Longrightarrow$ no quenching effect!

(ghosts do not directly couple to quarks).

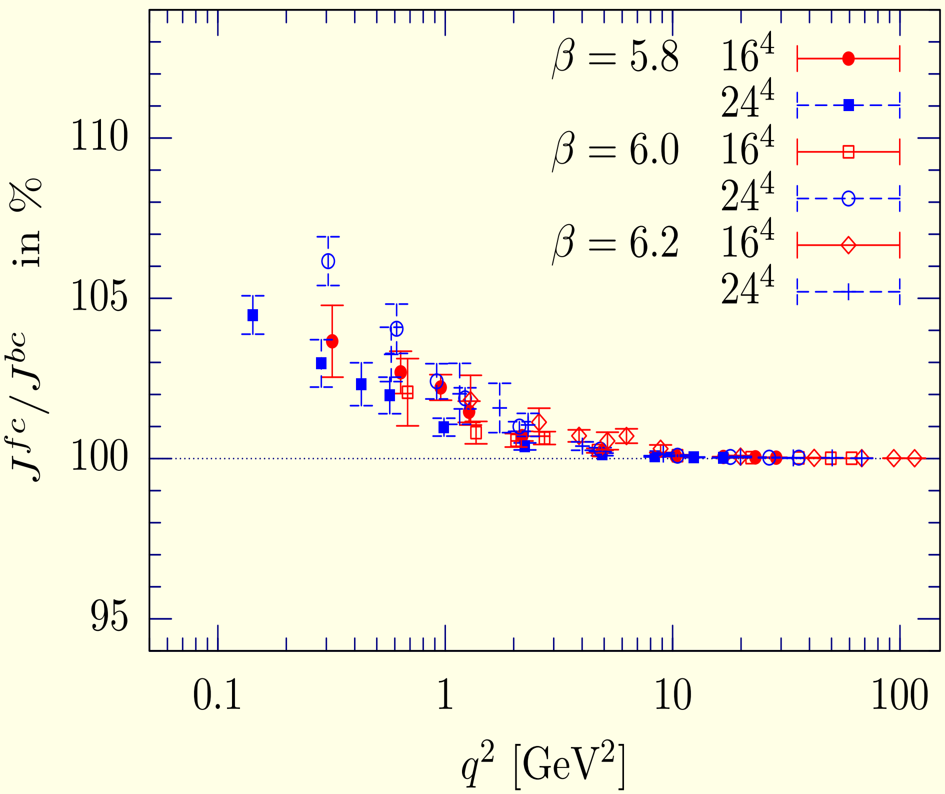
# Systematic effects: Gribov copies  fc / bc - ratios  ($N_f = 0$)

overrelaxation algorithm (OR), gauge transformations strictly periodic on the torus

gluon dressing fct.




ghost dressing fct.

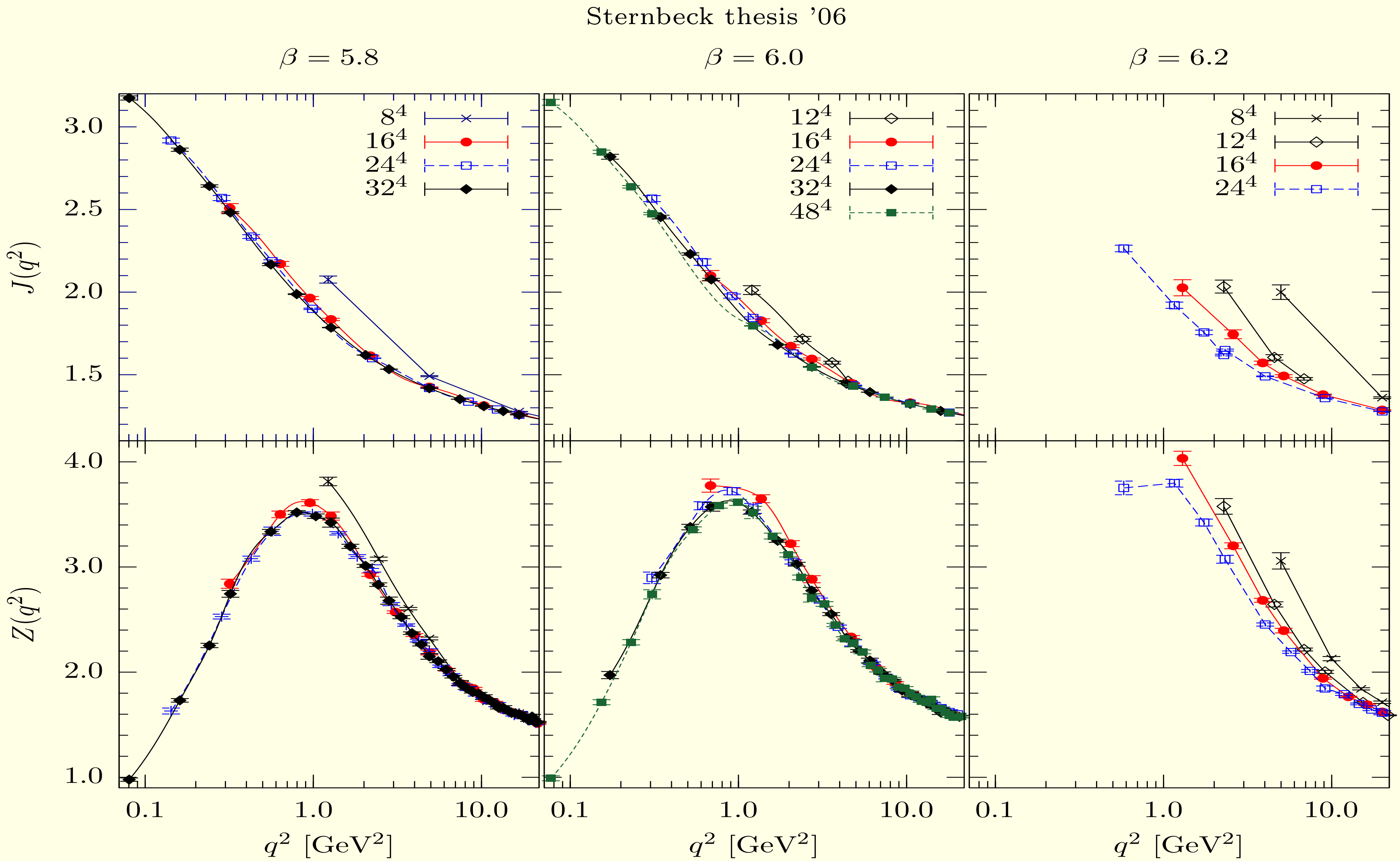



⇒ Gribov problem visible in the infrared for the ghost ( $O(5\%)$ effect), still not visible for the gluon propagator,

⇒ seems slightly to weaken as the volume increases (in acc. with Zwanziger),

⇒ more thorough studies under way for $SU(2)$.

# Systematic effects: finite-size dependence $(N_f = 0)$



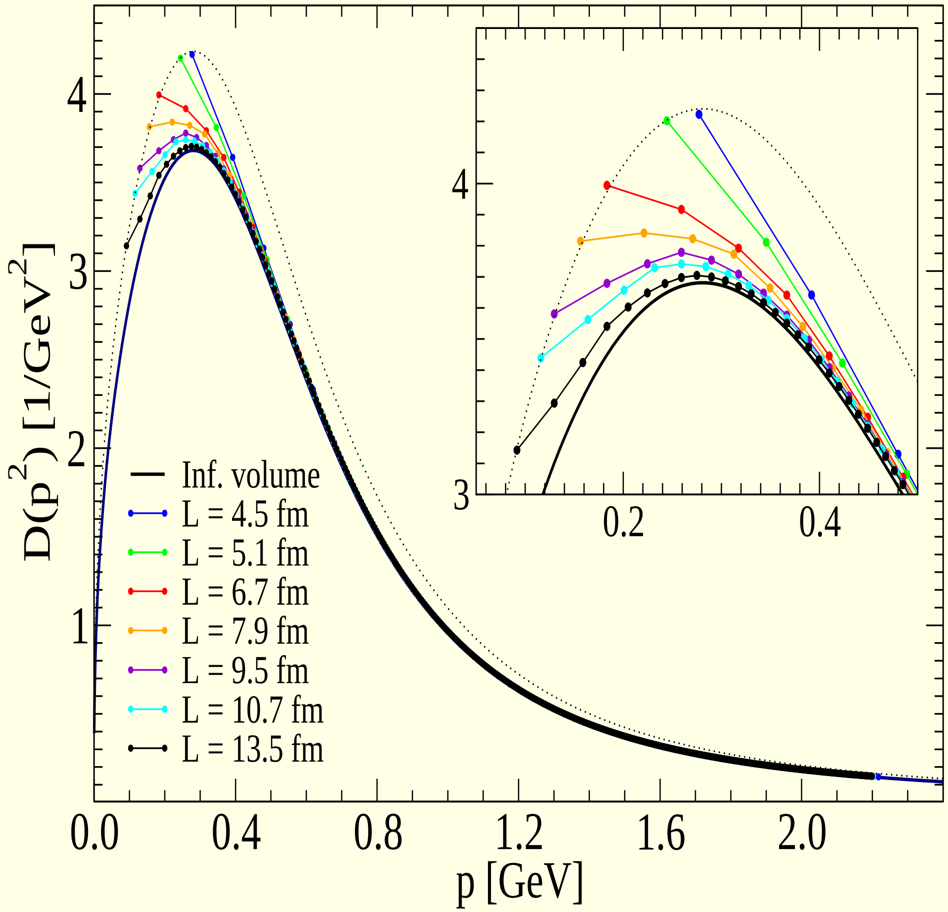

$\Longrightarrow$ For $L \geq 16\ a(\beta = 5.8) \simeq 24\ a(\beta = 6.0) \simeq 2.2$ fm

finite-size effect hard to resolve ?

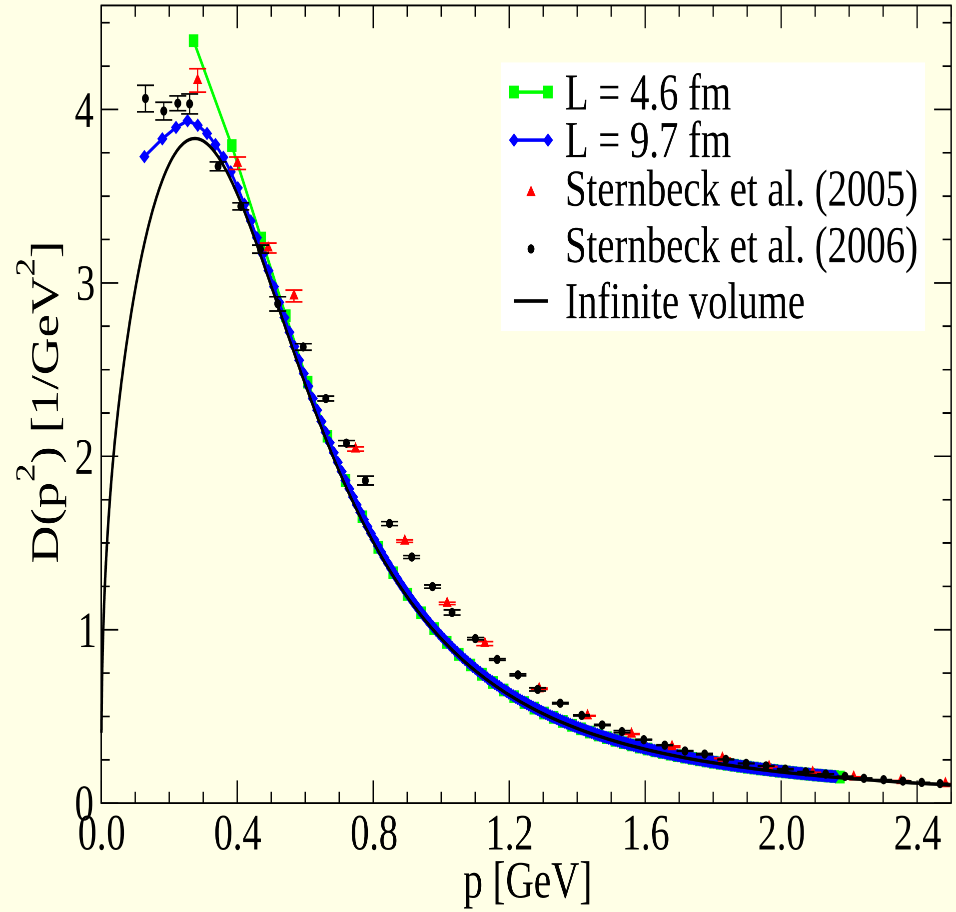
# DSE results for gluon propagator: infinite volume vs. torus

Fischer, Maas, Pawlowski, von Smekal, hep-ph/0701051

DSE gluon propagator

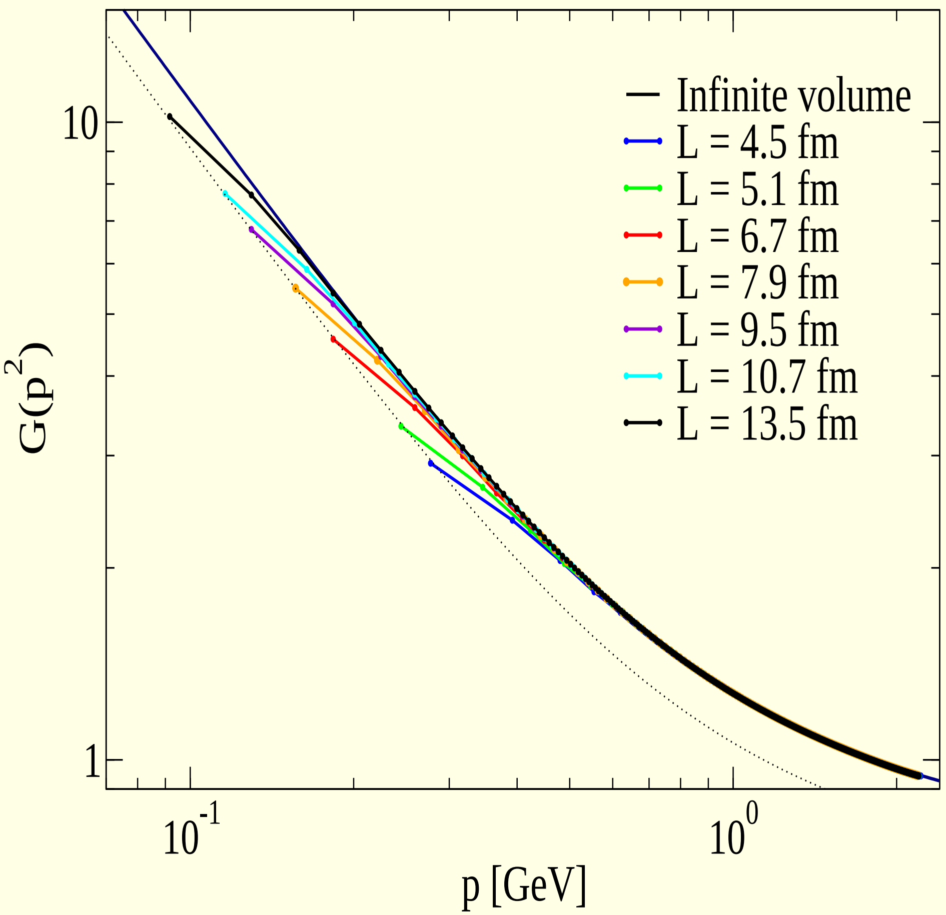



comparison with lattice

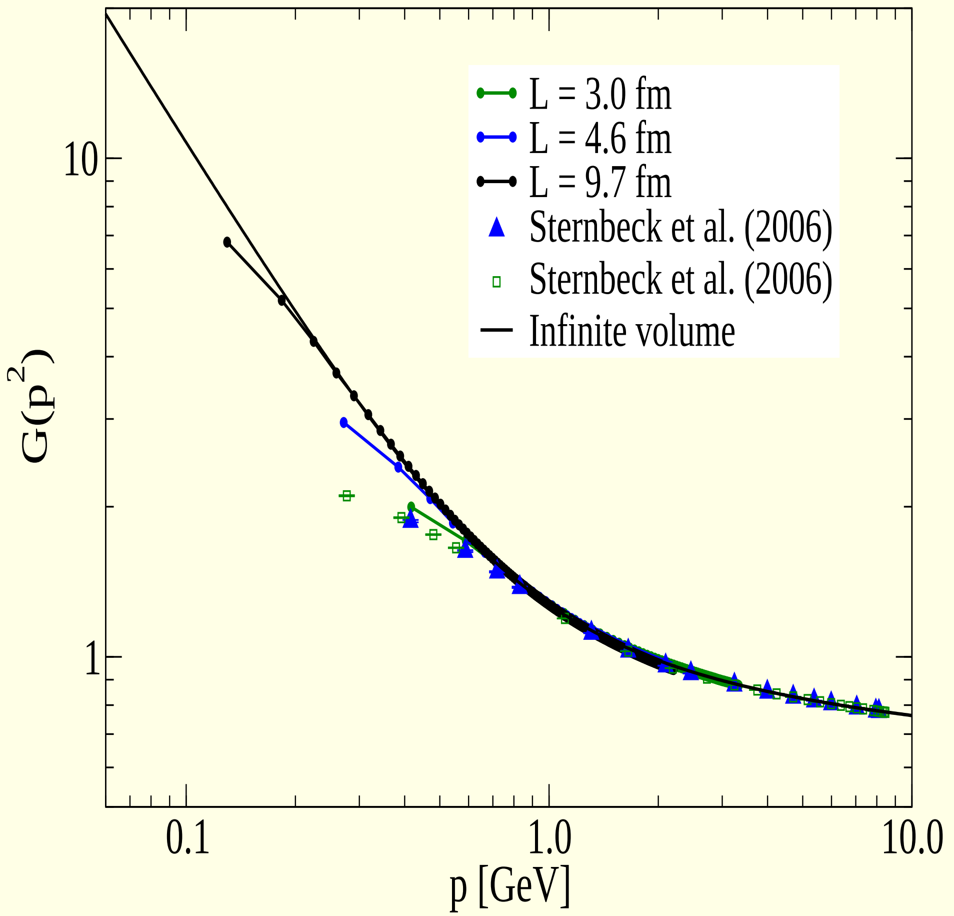


# DSE results for ghost dressing function: infinite volume vs. torus

DSE ghost dressing fct.




comparison with lattice



# 3. The running coupling

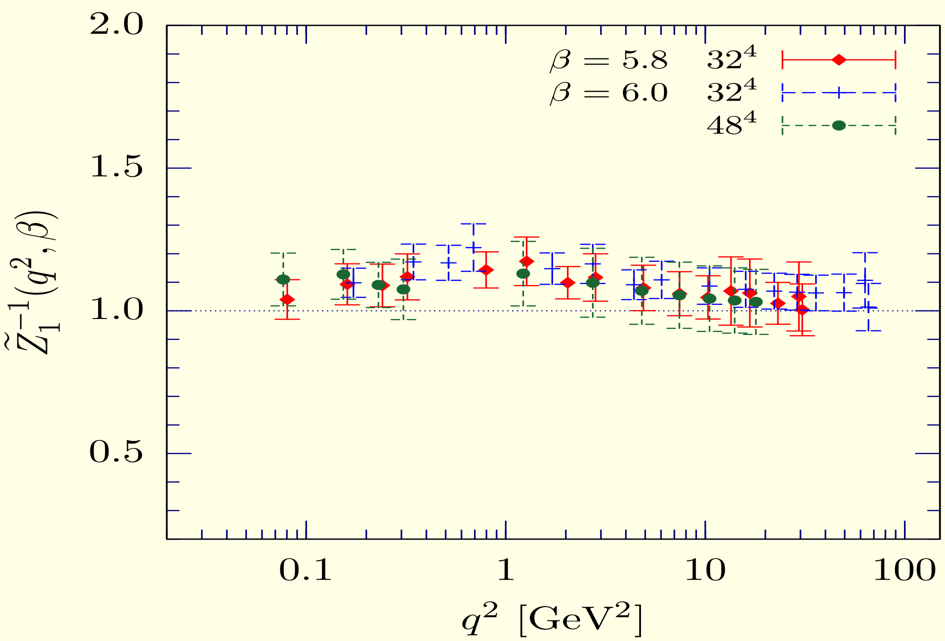
from ghost-ghost-gluon vertex: $\alpha_s(q^2) = \frac{g_0^2}{4\pi} Z(q^2) (J(q^2))^2$

assuming $Z_1(q^2) = 1, \quad q > 1\text{GeV}$

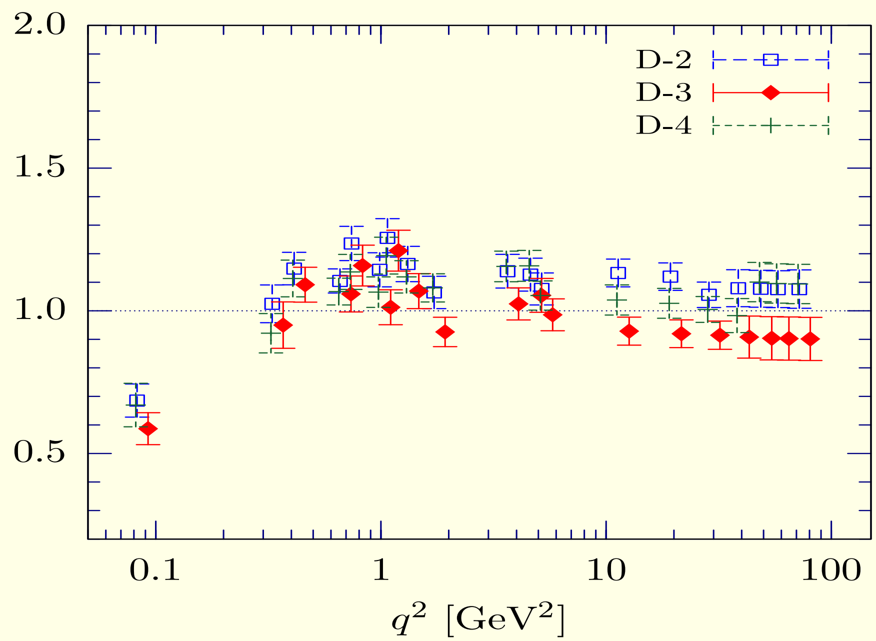
[perturbation theory: Taylor ('71) / lattice: Cucchieri et al. ('04)]

The vertex renormalization function $Z_1$, gluon momentum $k = 0$.

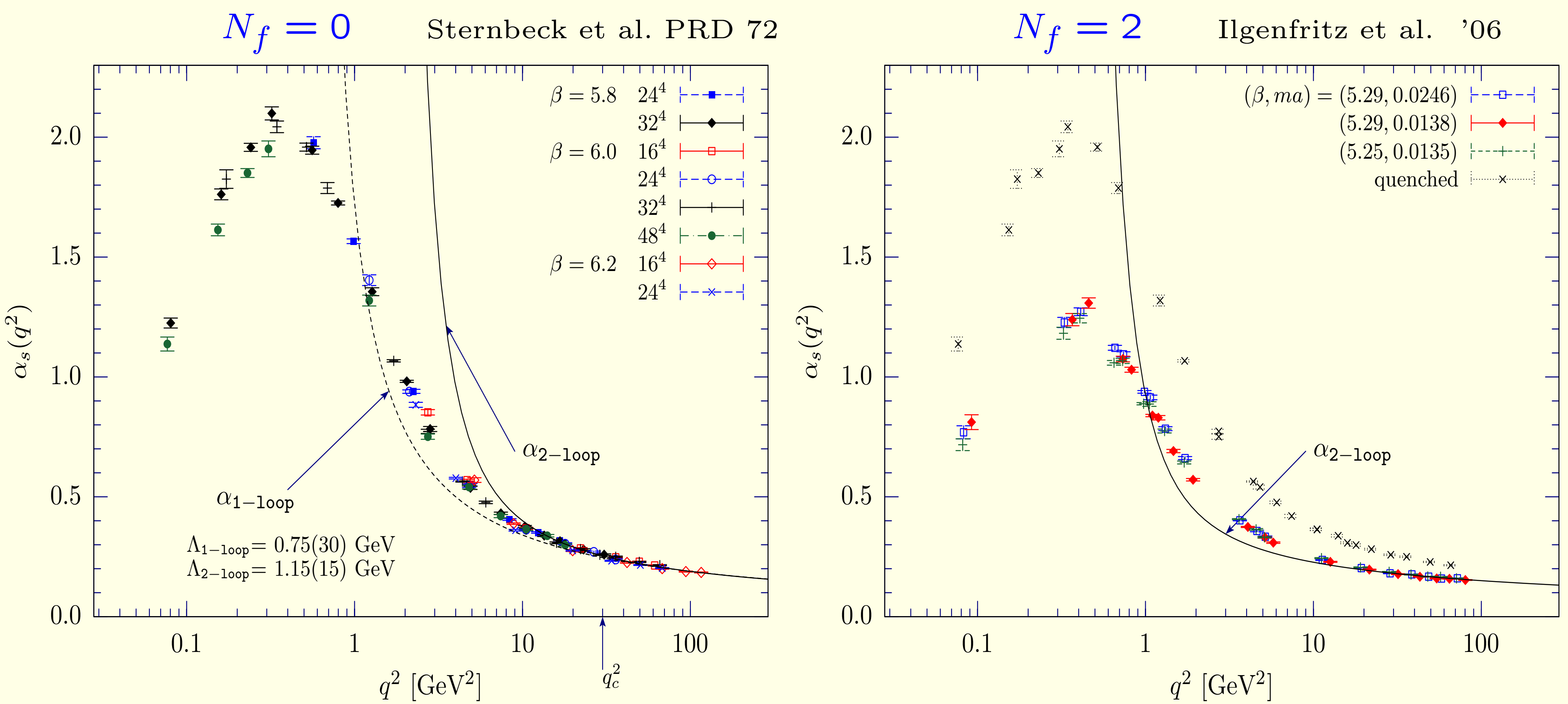
$N_f = 0$ Ilgenfritz et al. '05




$N_f = 2$ Sternbeck thesis '06

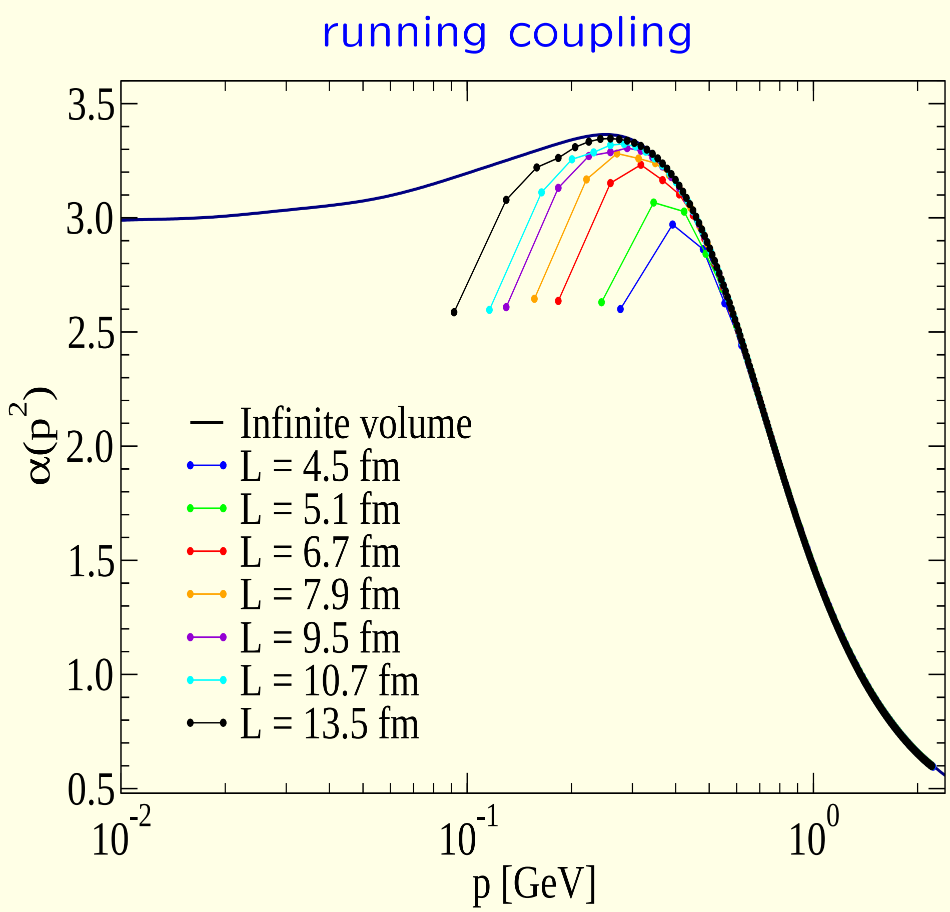


# Our result for the running coupling:




- Running coupling not monotonous, passes a maximum.
- Behavior agrees with other lattice studies, in particular for the three-gluon vertex.
- $\alpha_s \to 0$ for $q \to 0$ ? Strong volume dependence ?

# DSE results for the running coupling: infinite volume vs. torus




⟹ Extremely slow convergence to the infrared limit expected.

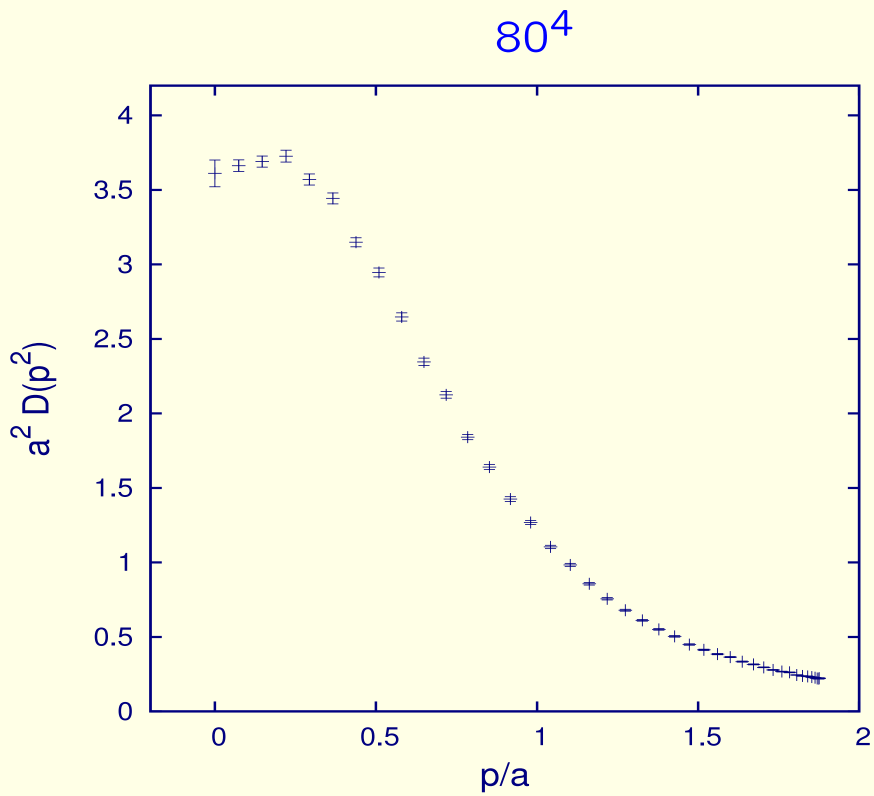
⟹ DSE on torus may provide extrapolation of lattice results.

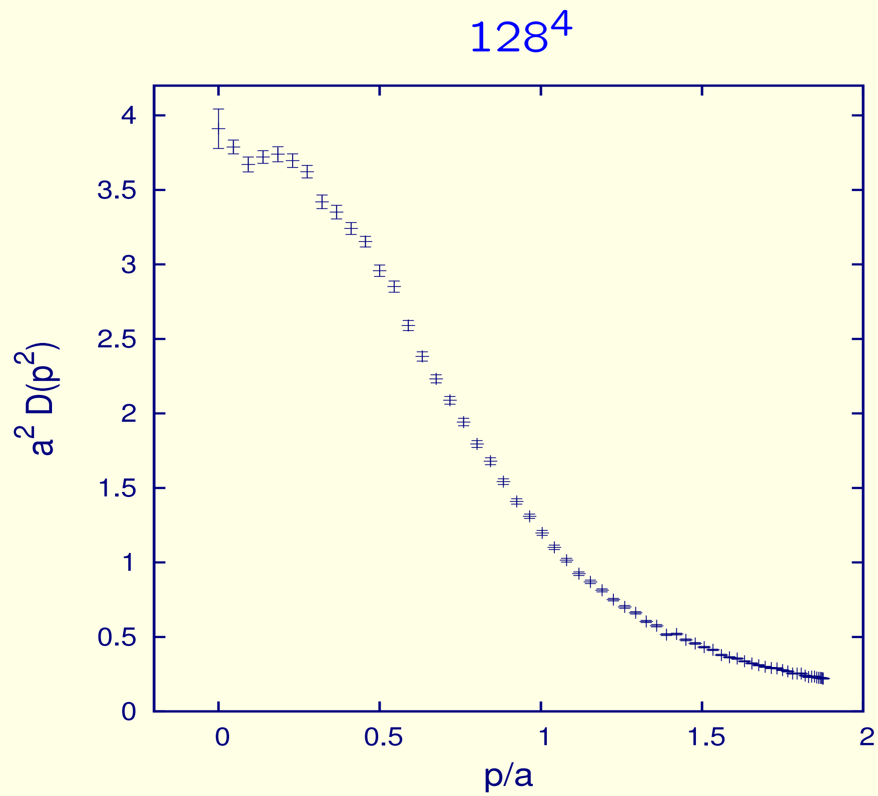
# 4. Gluon and ghost propagators: recent lattice results

Unrenormalized gluon propagator for $SU(2)$ on very large lattices:

Cucchieri, Mendes, contr. LATTICE '07, arXiv:0710.0412 [hep-lat]

$\beta = 2.20, \quad \Longrightarrow \quad (128a)^4 \simeq (27\ \text{fm})^4$



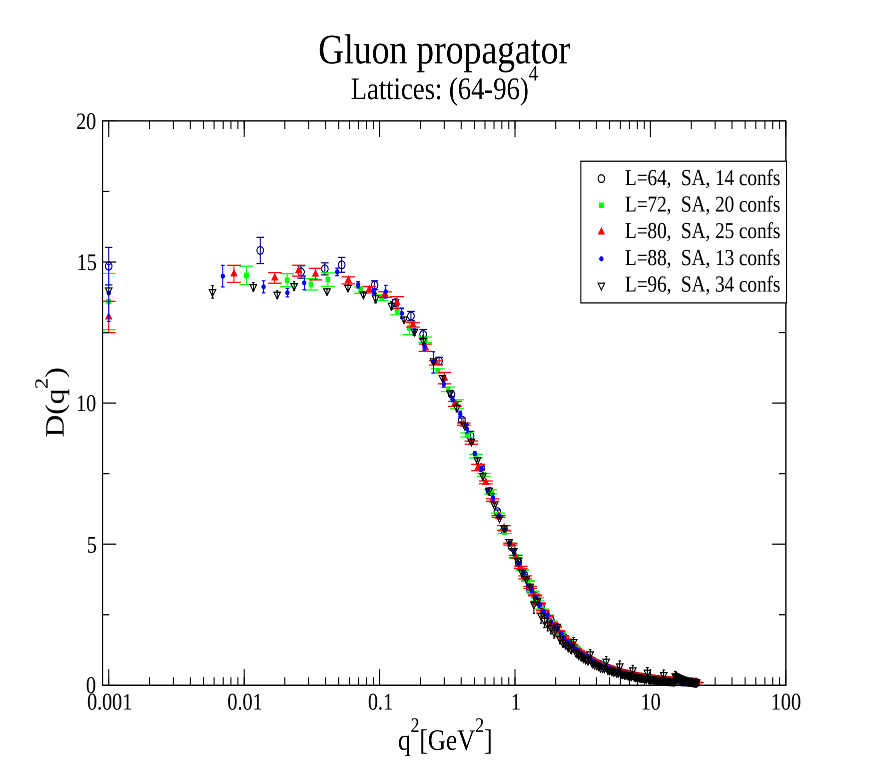

$\Longrightarrow \quad D(q^2) = Z(q^2)/q^2$ still no signal for vanishing $D \to 0$ for $q^2 \to 0$ ??.

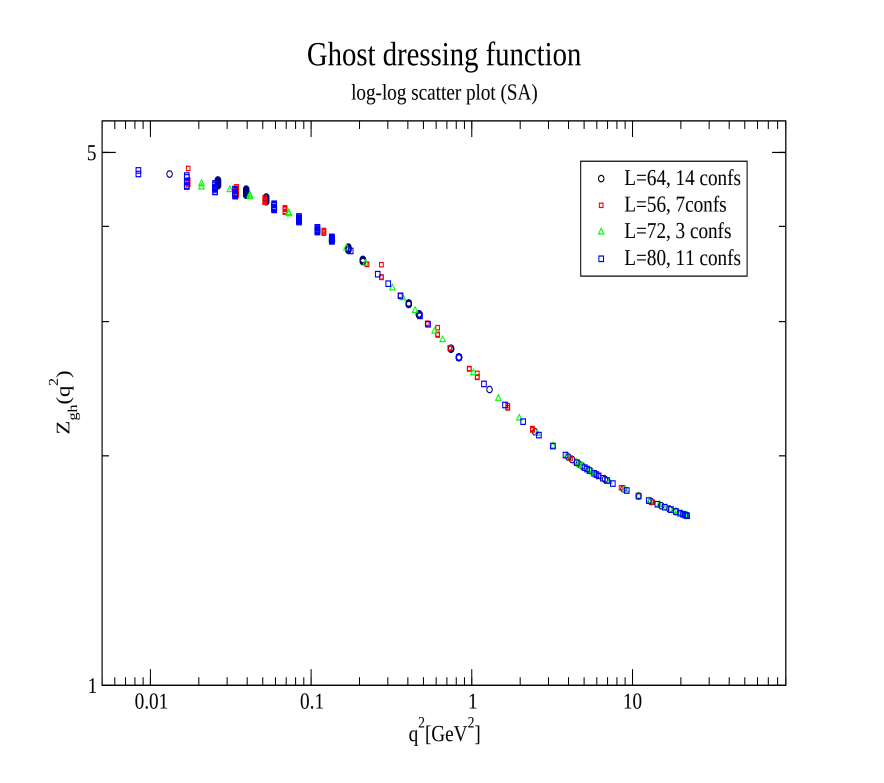
# Gluon and ghost propagators for $SU(3)$ on large lattices

Bogolubsky, Ilgenfritz, M.-P., Sternbeck, contr. LATTICE '07

$\beta = 5.70, \quad \Longrightarrow \quad (80a \; - \; 96a)^4 \simeq (13.2 \; - \; 15.8)^4 \text{ fm}^4.$

Simulated annealing used for gauge fixing.




$\Longrightarrow$ Gluon propagator runs into plateau for $q^2 \to 0$.

$\Longrightarrow$ Ghost dressing fct. may become constant, too ?

$\Longrightarrow$ Size dependence acc. to the claim of L. von Smekal ?

# 5. Improved gauge fixing: new hope ?

Improved gauge fixing $\Longrightarrow$ getting closer to the FMR:
simulated annealing plus global $\mathbb{Z}(N)$ flips

- Simulated annealing (SA):
  Find $g$'s randomly with statistical weight:

$$W \propto \exp(\frac{F_U(g)}{T}) .$$

  Let "temperature" $T$ slowly decrease. Infinitely slow cooling ends at the global extremum. In practice SA clearly wins for large lattice sizes.

- $\mathbb{Z}(N)$ flips:
  Gauge functional $F_U(g)$ maximized by enlarging the gauge orbit with respect to $\mathbb{Z}(N)$ non-periodic gauge transformations:

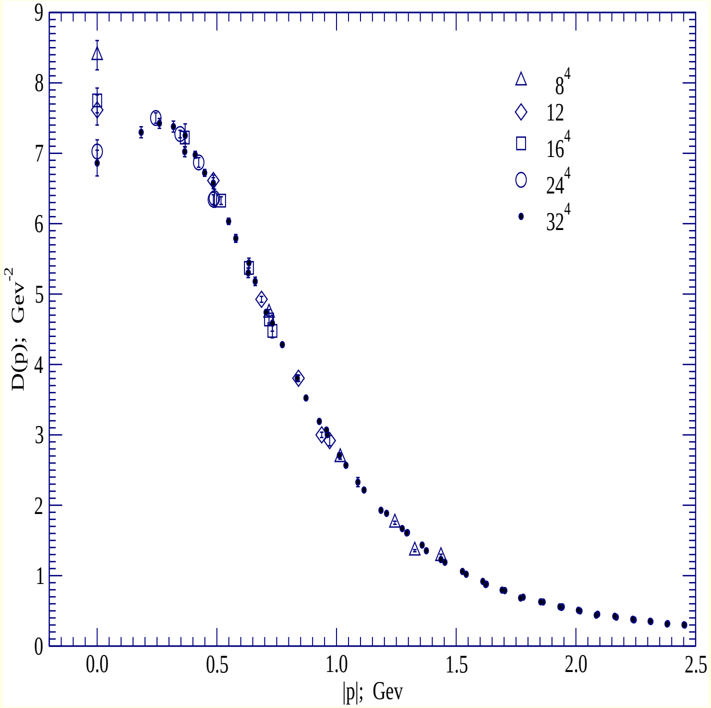
$$g(x + L\hat{\nu}) = z_\nu g(x) , \quad z_\nu \in \mathbb{Z}(N) .$$

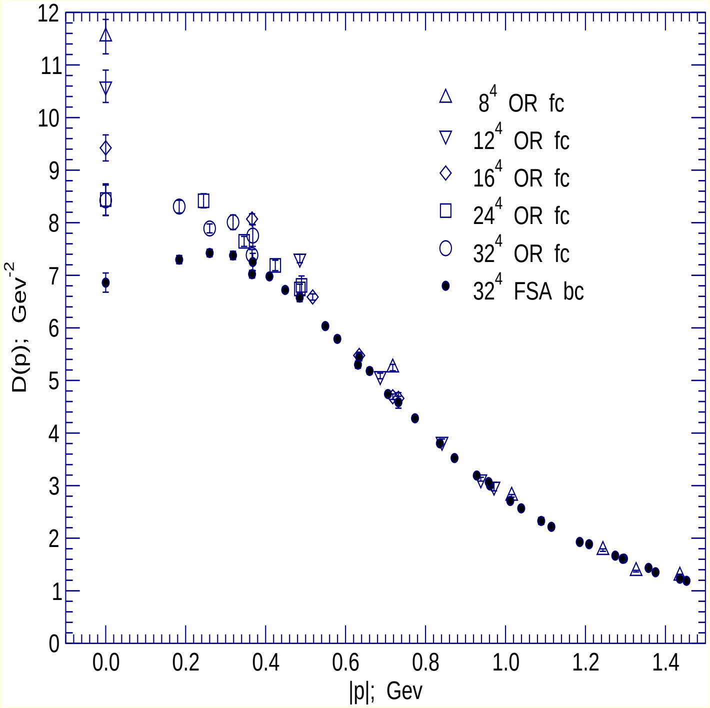
## $SU(2)$ results for the gluon propagator ($8^4, \ldots, 32^4$, $\beta = 2.2$):

Bogolubsky, Bornyakov, Burgio, Ilgenfritz, Mitrjushkin, M.-P.,

arXiv:0707.3611 [hep-lat], LATTICE '07

$\mathbb{Z}(2)$ flips + SA versus OR (cf. Cucchieri, Mendes




$\Rightarrow$ Gribov copies important for the gluon propagator, too!

$\Rightarrow$ Finite-size effects weaker when approaching the FMR $\Lambda$.

$\Rightarrow$ Shall we see a turn to $D(q^2 \to 0) = 0$ ?.

# Bounds for the gluon propagator

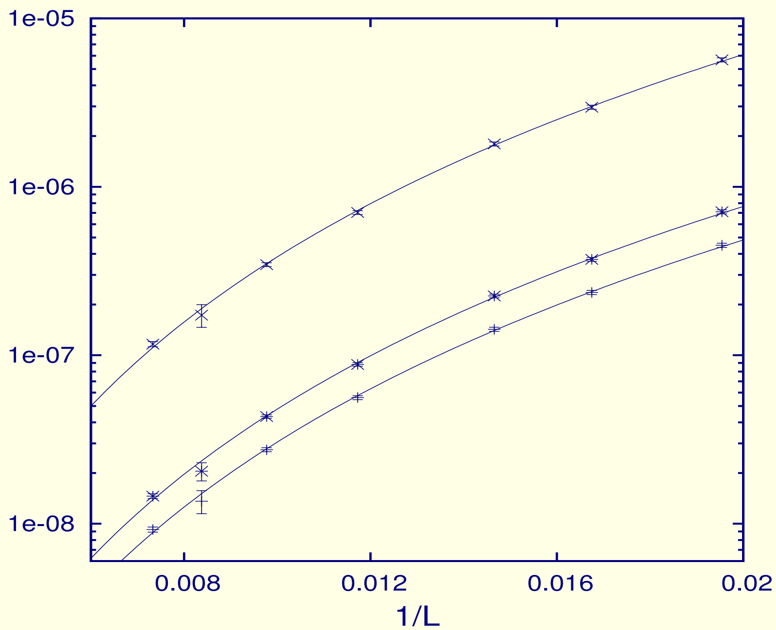
Cucchieri, Mendes, arXiv:0712.3517 [hep-lat]

zero momentum modes (ZMM):

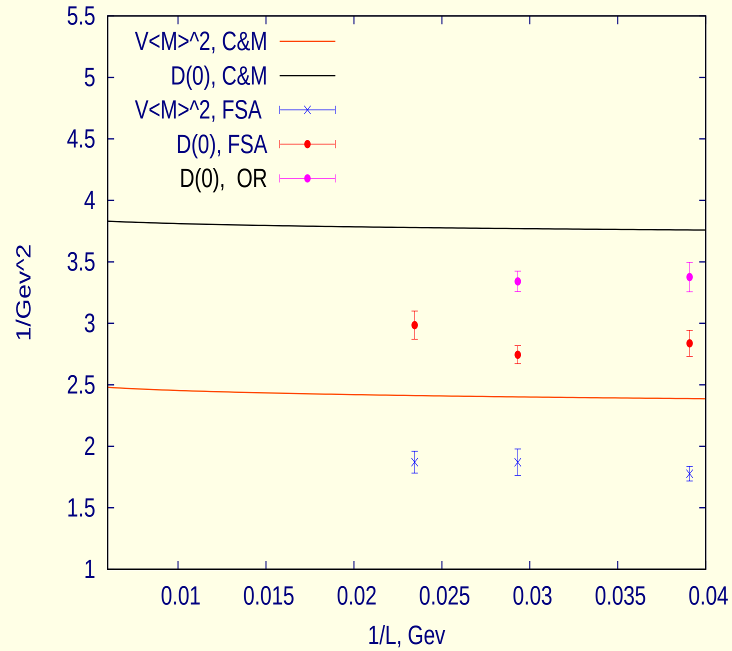
$$\tilde{A}^b_\mu(0) = (1/V)\sum_x A^b_\mu(x)\,, \quad M(0) = (1/12)\sum_{\mu,b} |\tilde{A}^b_\mu(0)|$$

$$\Longrightarrow \quad \text{Bounds:} \quad \langle M(0)\rangle^2 \leq D(0)/V \leq 12\langle M(0)^2\rangle$$

OR results Cucchieri, Mendes




$\mathbb{Z}(2)$ flips + SA versus OR




⇒ For $\mathbb{Z}(2)$ flips + SA: $D(0)$ as well as ZMM bounds are shifted downwards

⇒ Question remains: $D(0) \to 0$ for $V \to \infty$ ?.

# 6. Conclusion and outlook

- $SU(2)$ : For $d = 2$ on huge lattices power-like behavior has been reported. For $d = 3$ still not conclusive, but Gribov copies become important [talk by A. Maas].

- $SU(3)$ : Finite-size effects on the lattice - in particular for the ghost - look different than for DSE on a torus.
DSE truncation effect ?

- Comparison with confinement scenarios:
$D(q^2) \to 0$ and $J(q^2) \to \infty$ for $q^2 \to 0$ still possible ? Lattice data favor to go to a constant $\neq 0$, the latter to be "subtracted"? [talk by L. von Smekal]

- $SU(2)$ : Gribov copies produce finite-size effects, $\mathbb{Z}(N_c)$-flips important. Does this solve the puzzle ? We hope to give a more convincing answer soon.
- Debate on analytic results: please, continue !

Thank you for your attention.