Infrared Propagators in MAG on the Lattice*

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* Feynman gauge: Coming soon... (with A. Cucchieri, Axel Maas and Elton Santos)

IR propagators and Confinement

- Despite being gauge-dependent, gluon and ghost propagators are powerful tools in the (non-perturbative) investigation of the infra-red (IR) limit of QCD and of the mechanism of confinement.
- In MAG, the confinement scenario is based on the concepts of Abelian dominance and dual superconductivity.
  (Gribov-Zwanziger scenario may be valid for non-Abelian directions in gauge-configuration space.)
- IR behavior of propagators may be modified by the presence of condensates of mass dimension two (Capri et al., 2008).
- For the pure SU(2) case in MAG on the lattice we study: the IR gluon and ghost propagators, the ghost condensate and the smallest eigenvalue of the Faddeev-Popov matrix.
  Preliminary results: T. M., A. Cucchieri, A. Mihara, AIP Conf. Proc. 2007.

Propagators in MAG

From Capri et al., arXiV:0801.0566[hep-th] we expect

- Transverse off-diagonal gluon propagator of Yukawa type

$$D^{aa}(p^2) = \frac{1}{p^2 + m^2}$$

- Diagonal gluon propagator of Gribov-Stingl type

$$D^{33}(p^2) = \frac{p^2 + \mu^2}{p^4 + \mu^2 p^2 + 4\gamma^4}$$

- Symmetric (off-diagonal) ghost propagator

$$G^{aa}(p^2) = \frac{p^2 + \mu^2}{p^4 + 2\mu^2 p^2 + \mu^4 + v^4}$$

- Antisymmetric (off-diagonal) ghost propagator

$$G^{ab}(p^2) = \frac{v^2}{p^4 + 2\mu^2 p^2 + \mu^4 + v^4}\,\epsilon^{ab}$$

MAG on the lattice

On the lattice, for the $SU(2)$ case, the MAG is obtained by minimizing the functional

$$S = -\frac{1}{2dV} \sum_{x,\mu} Tr \left[\sigma_3 U_\mu(x) \sigma_3 U_\mu^\dagger(x) \right]$$

In any stationary point of S one has the conditions

$$\begin{aligned} \sum_\mu \left[U_\mu^-(x) A_\mu^\pm(x) - U_\mu^+(x - e_\mu) A_\mu^\pm(x - e_\mu) \right] &= 0 \\ \sum_\mu \left[U_\mu^+(x) A_\mu^\pm(x) - U_\mu^-(x - e_\mu) A_\mu^\pm(x - e_\mu) \right] &= 0 \end{aligned}$$

where $U_\mu^\pm(x) = U_\mu^0(x) \pm i\, U_\mu^3(x)$ and $A_\mu^\pm(x) = U_\mu^1(x) \pm i\, U_\mu^2(x)$. Here, we follow the notation $U_\mu(x) = U_\mu^0(x) \mathbb{1} + i\, \vec{\sigma} \cdot \vec{U}_\mu(x)$. We also fix the residual $U(1)$ degrees of freedom to Landau gauge.

MAG on the lattice (II)

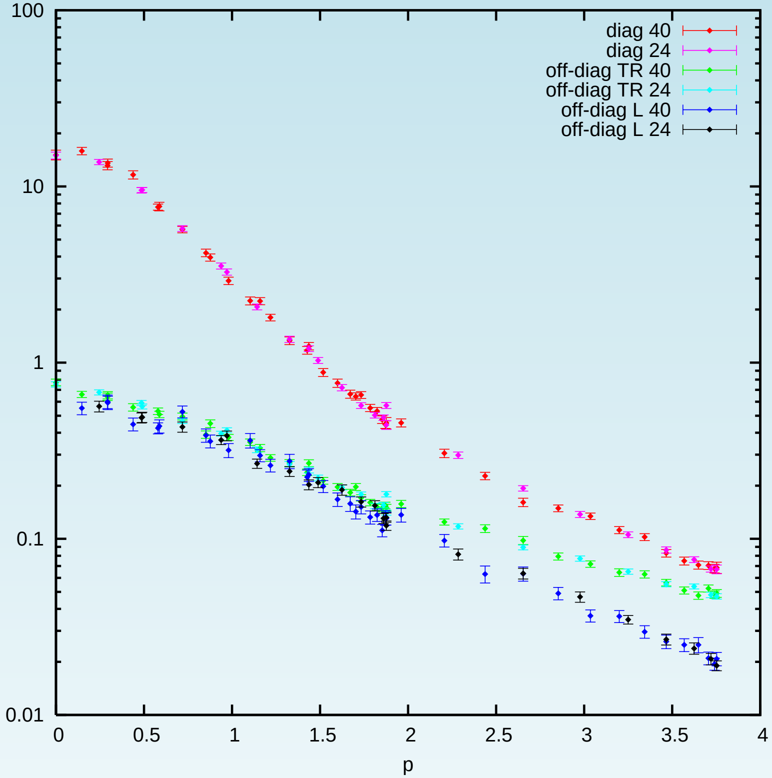
At any local minimum one also has that the Faddeev-Popov matrix

$$\sum_{abxy} \gamma_a(x) M^{ab}(x,y) \gamma_b(y) \;=\; \sum_{\mu a b x} \gamma_a(x)\gamma_b(x)\delta_{ab}[V_\mu(x) + V_\mu(x-e_\mu)]$$
$$+\,2\,\gamma_a(x)\,\gamma_b(x-e_\mu)\{\delta_{ab}[1-2(U^0_\mu(x))^2]$$
$$-\,2\,[\epsilon_{ab}U^0_\mu(x)U^3_\mu(x) + \sum_{cd}\epsilon_{ad}\epsilon_{bc}U^d_\mu(x)U^c_\mu(x)]\}\,,$$

is positive-definite. Here the color indices take values $1,2$ and $V_\mu(x) = (U^0_\mu(x))^2 + (U^3_\mu(x))^2 - (U^1_\mu(x))^2 - (U^2_\mu(x))^2$. Notice that (as in Landau gauge) this matrix is symmetric under the simultaneous exchange of color and space-time indices. Using the relation $U_\mu(x) = e^{[-iag_0 A_\mu(x)]}$ one finds (in the formal continuum limit $a \to 0$) the standard continuum results for the stationary conditions above and for the matrix $M^{ab}(x,y)$.

The gluon propagators

3 gluon propagators: transverse diagonal, transverse off-diagonal and longitudinal off-diagonal, as functions of the momentum p.



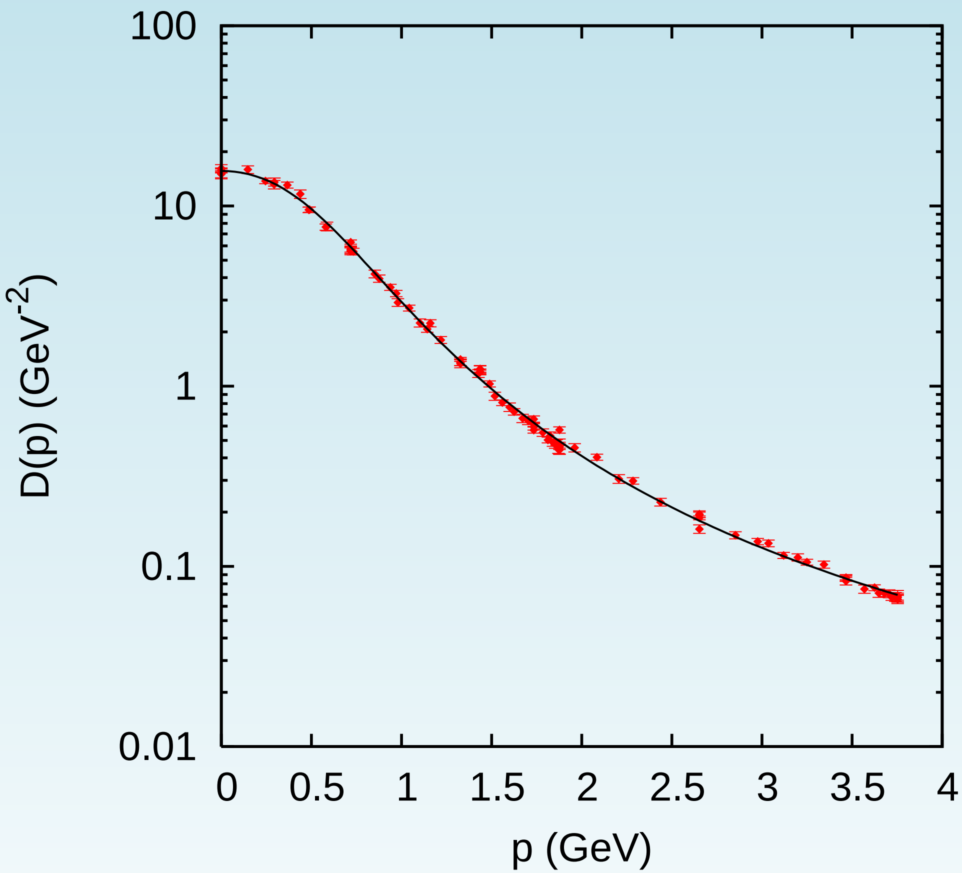
$D(p^2)$ as a function of p (both in physical units) for $V = 24^4$, 40^4 and $\beta = 2.2$. Red/pink points represent the (transverse) diagonal propagator, green/cyan the transverse off-diagonal propagator and blue/black the longitudinal off-diagonal propagator.

Results in agreement with the study by Bornyakov et al. (2003): we see a clear suppression of the off-diagonal propagators compared to the diagonal (transverse) one, supporting Abelian dominance.

Gluon fits (I)

Fit of all data (all values of V, $\beta = 2.2$) for $D(p^2)$ (transverse) diagonal.

We find that the diagonal gluon propagator is best fitted by the form



$$D(p) = \frac{1 + d\,p^2}{a + b\,p^2 + c\,p^4},$$

with

$$a = 0.064(2)\,GeV^2\,,$$

$$b = 0.125(9)\,,$$

$$c = 0.197(9)\,GeV^{-2}\,,$$

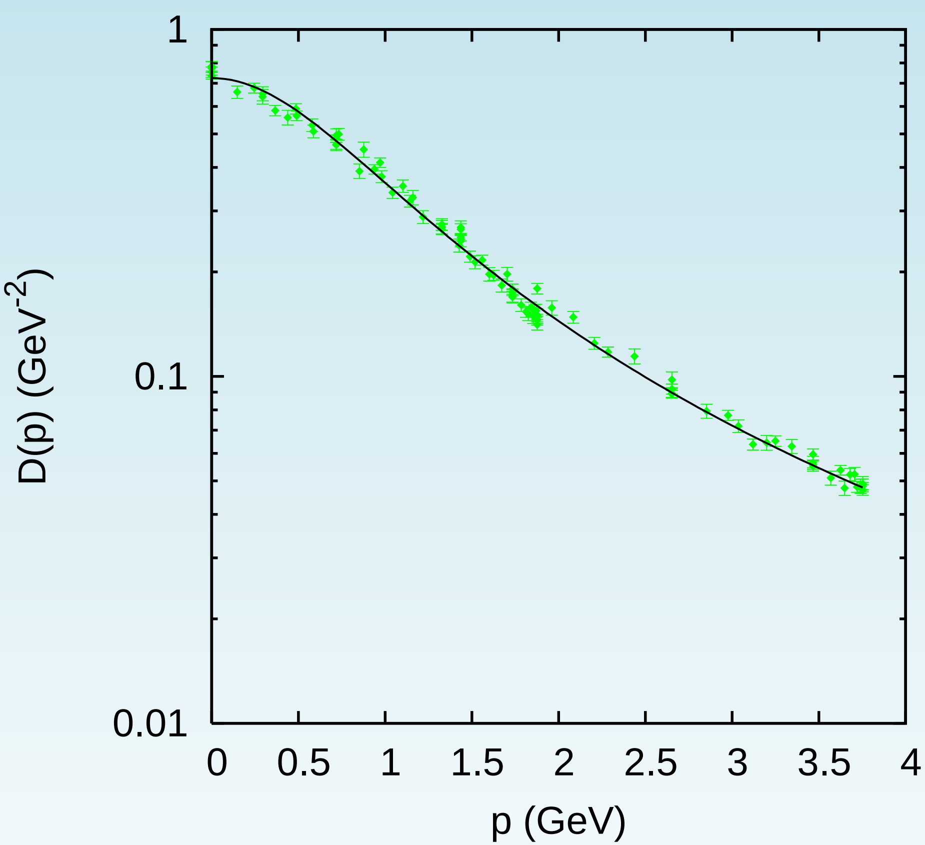
$$d = 0.13(1)\,GeV^{-2}\,.$$

Mass $m = \sqrt{a/b} \approx 0.72\,GeV$ from Stingl-Gribov fit.

Gluon fits (II)

Fit for $D(p^2)$ transverse off-diagonal.

The transverse off-diagonal gluon propagator is best fitted by



$$D(p) = \frac{1}{a + b\,p^2},$$

with

$$a = 1.38(2)\,GeV^2\,,$$

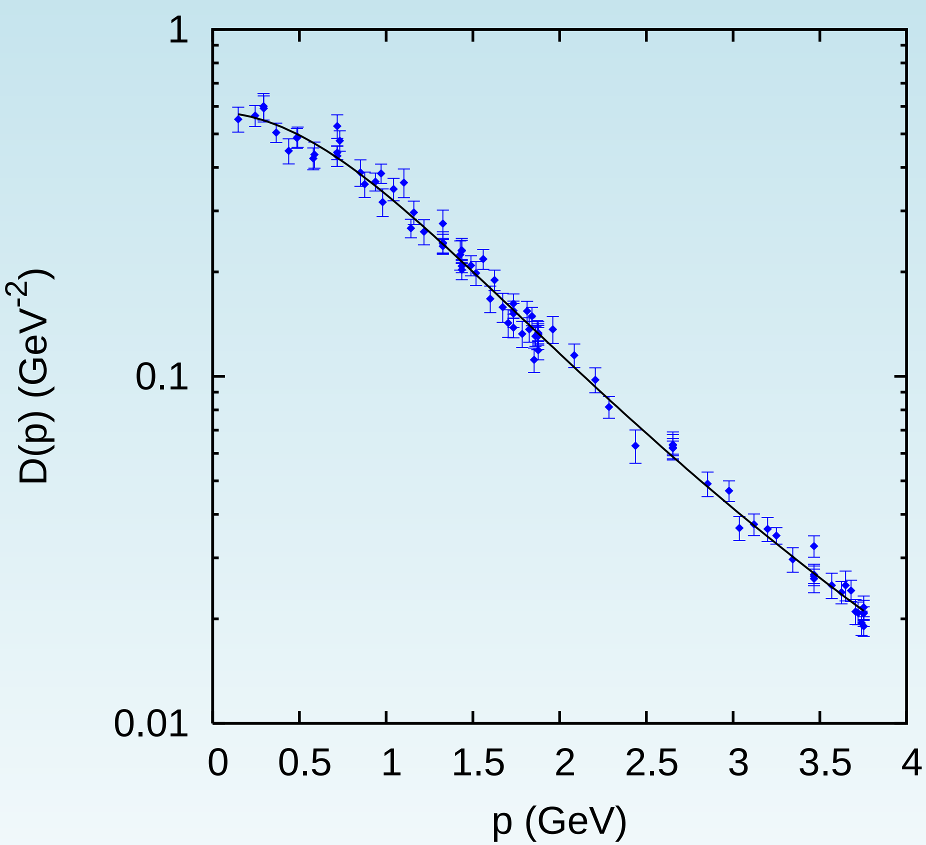
$$b = 1.39(1)\,.$$

Mass $m = \sqrt{a/b} \approx 0.97\,GeV$ from Yukawa fit.

Gluon fits (III)

Fit for $D(p^2)$ longitudinal off-diagonal.

The longitudinal off-diagonal gluon propagator is best fitted by



$$D(p) = \frac{1}{a + b\,p^2 + c\,p^4},$$

with

$$a = 1.73(4)\,GeV^2\,,$$

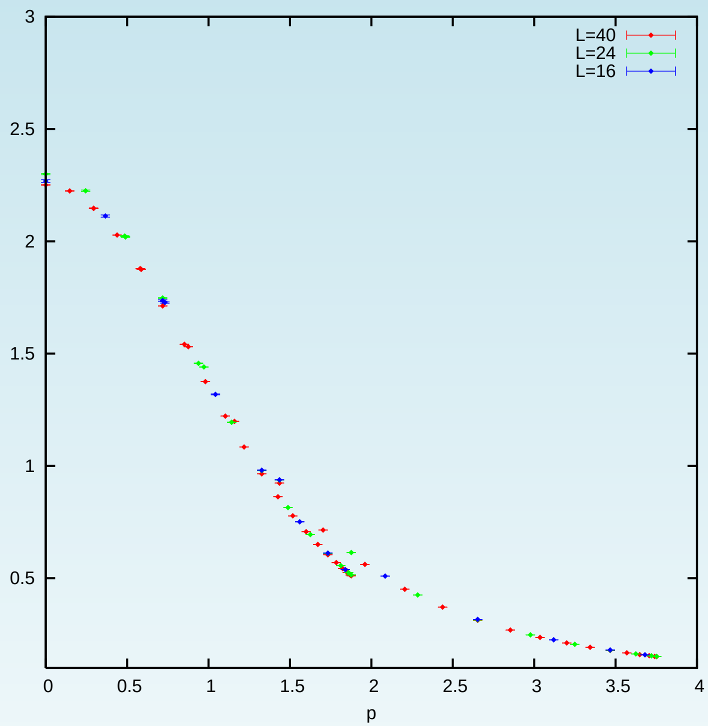
$$b = 1.11(4)\,,$$

$$c = 0.152(6)\,GeV^{-2}\,,$$

Mass $m = \sqrt{a/b} \approx 1.25\,GeV$ from Yukawa fit.

The ghost propagator

We also consider the ghost propagator $G(p^2)$ as a function of the momentum p.

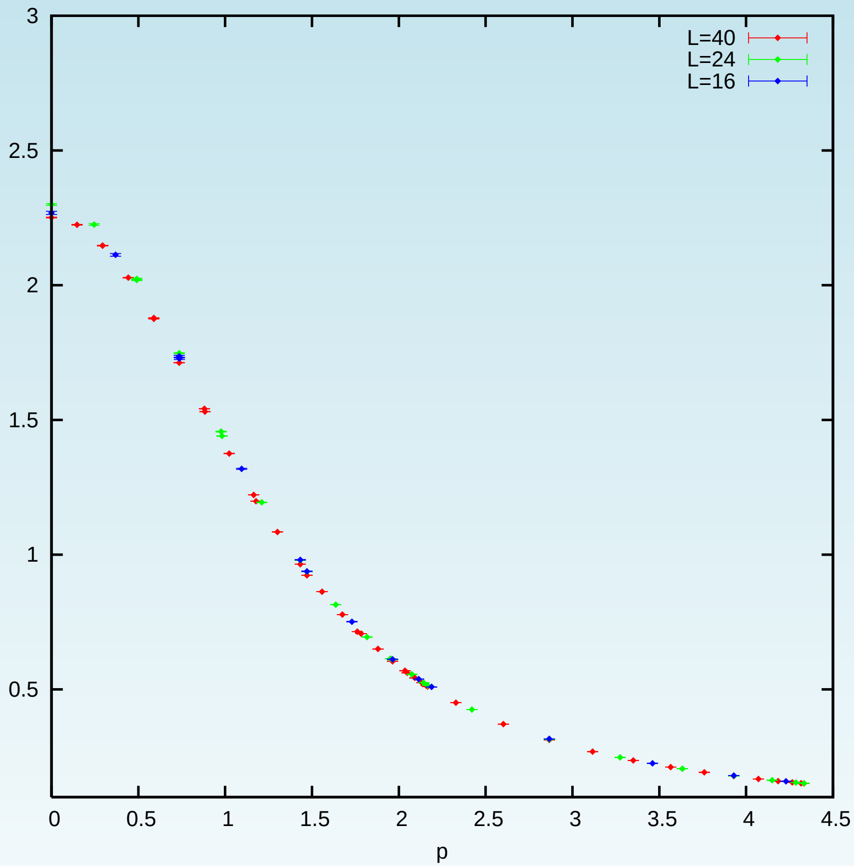


Plot of $G(p^2)$ as a function of p (both in physical units) for lattice volumes $V = 16^4$, 24^4, 40^4 and $\beta = 2.2$.

Note that in this case we can evaluate the ghost propagator at zero momentum. The data show little volume dependence at small p.

The ghost propagator (II)

Using an improved definition of the momentum p (inspired by perturbation theory in Landau gauge)

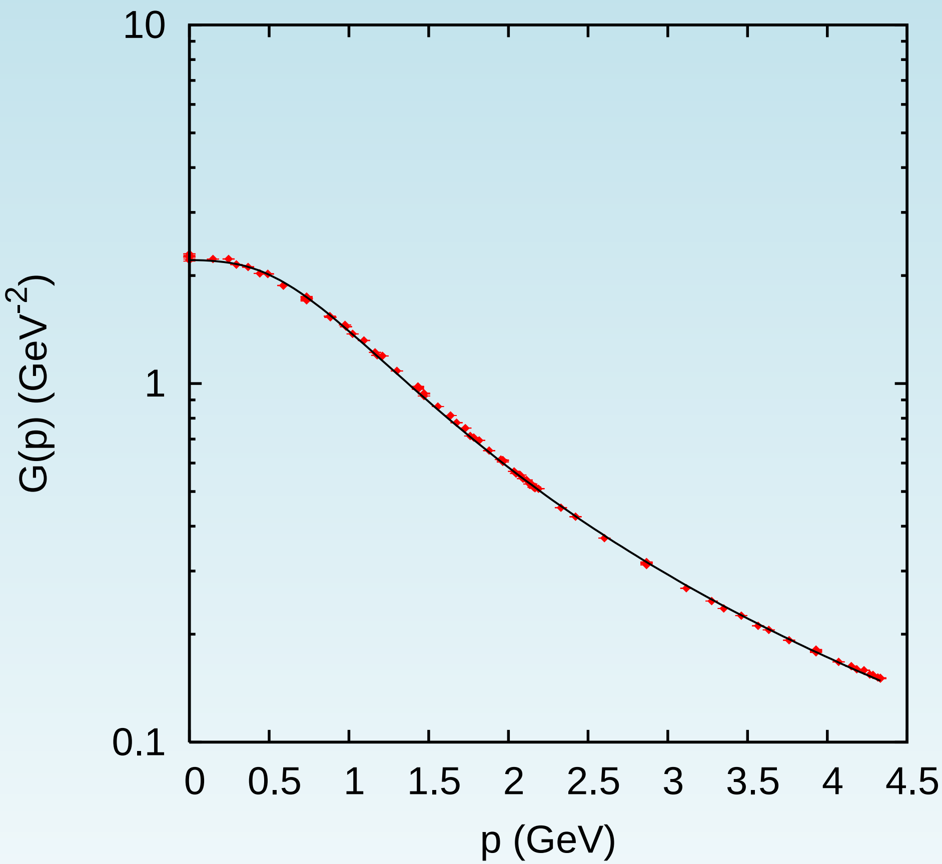


Plot of $G(p^2)$ as a function of "improved" p (both in physical units) for lattice volumes $V = 16^4$, 24^4, 40^4 and $\beta = 2.2$.

Ghost propagator is finite in the IR limit.

Ghost fit

Fit of all data (at $\beta = 2.2$) for $G(p^2)$ as a function of improved p.



$$G(p^2) = \frac{1 + d\,p^2}{a + b\,p^2 + c\,p^4},$$

with

$$a = 0.45(1)\,GeV^2\,,$$

$$b = 1.1(3)\,,$$

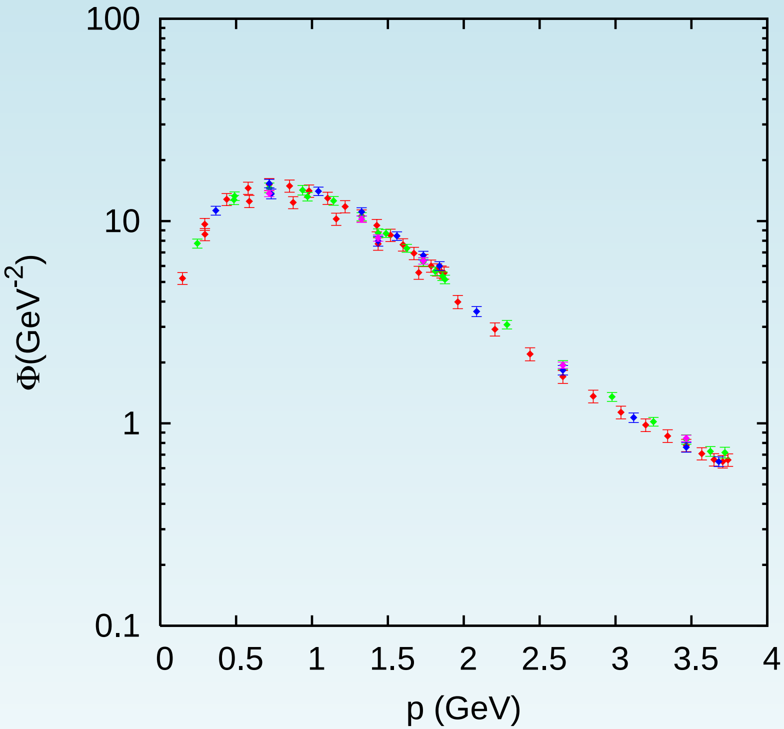
$$c = 0.73(30)\,GeV^{-2}\,,$$

$$d = 2.1(9)\,GeV^{-2}\,.$$

Mass $m = \sqrt{a/b} \approx 0.6\,GeV$ from Stingl-Gribov fit.

The ghost condensate

Following the analysis done in Landau gauge, we consider the quantity $\langle\,|\,\epsilon_{ab}G^{ab}(p^2)/2\,|\,\rangle$ rescaled by $L^2/\cos\left(\pi\,\widetilde{p}_\mu\,a/L\right)$, as a function of the momentum p for all lattice volumes and β values considered.



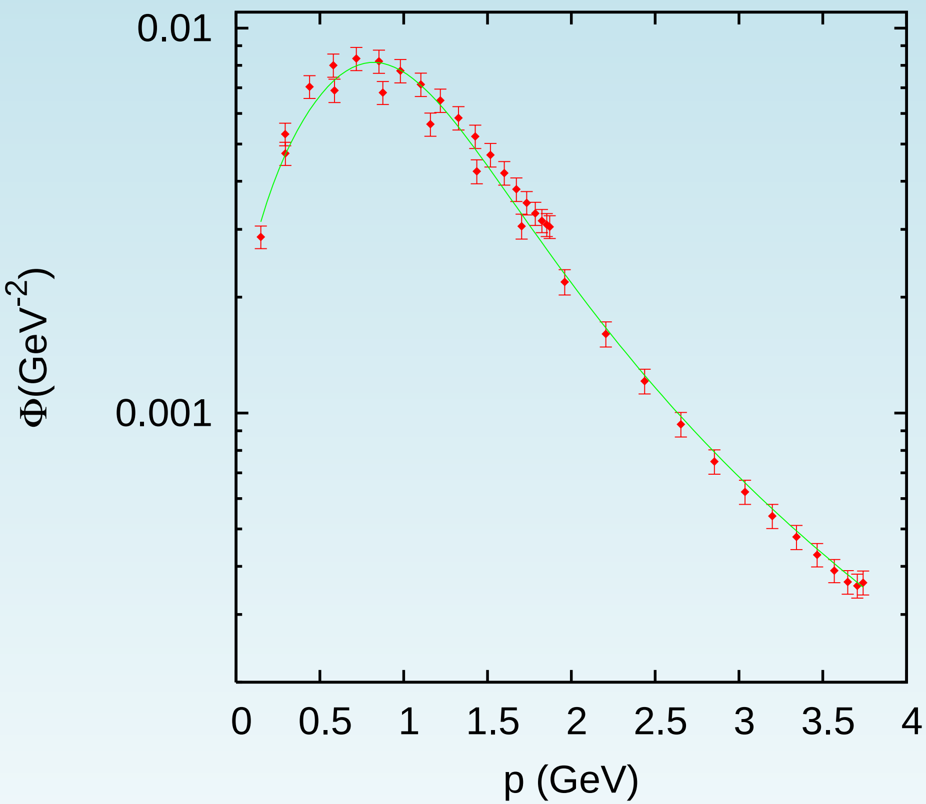
Plot of the quantity $\Phi(p^2)$ defined as $L^2/\cos\left(\pi\,\widetilde{p}_\mu\,a/L\right)\langle\,|\,\epsilon_{ab}G^{ab}(p^2)/2\,|\,\rangle$ as a function of p (both in physical units) for lattice volumes $V = 8^4$, 16^4, 24^4, 40^4 and $\beta = 2.2$.

The data show nice scaling for all cases considered.

Is there a ghost condensate?

Ghost condensate fit

Fit of data at $V = 40^4$ and $\beta = 2.2$ for $\Phi(p^2)$ as a function of p.



$$\Phi(p) = \frac{a + b\,p/L^2}{p^4 + v^2},$$

$$a = 0.0026(7)\,GeV^2\,,$$

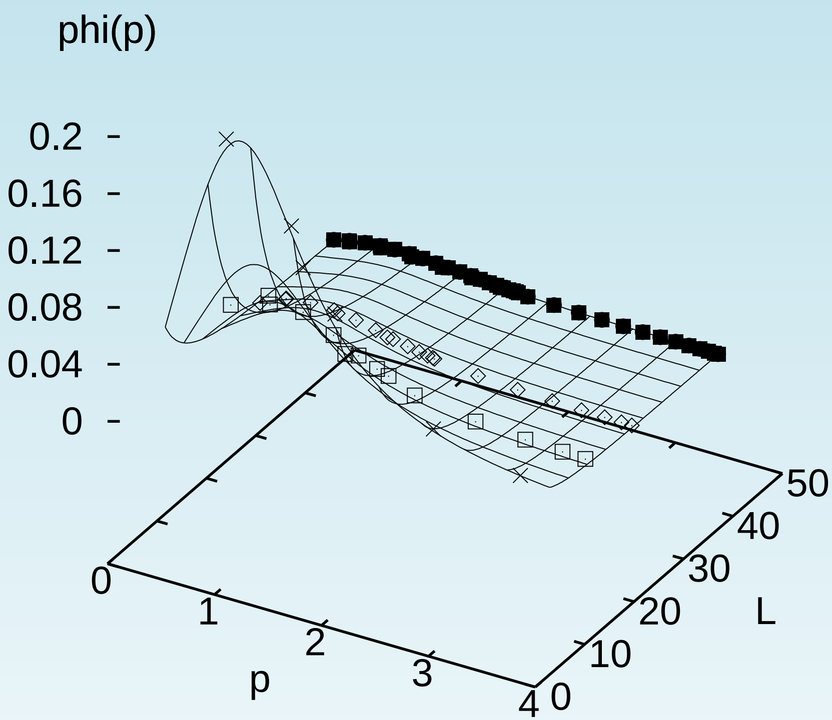
$$b = 32.6(7)\,GeV^{-1},$$

$$v^2 = 1.7(1)\,GeV^4\,.$$

Ghost condensate $v \approx 1.3\,GeV^2$ seems to be huge(!) but is $a \neq 0$??

Ghost condensate fit (II)

Fit of data at several V's and β's for $\Phi(p^2)$ as a function of p and L.



$$\Phi(p) = \frac{a + b\,p/L^2}{p^4 + v^2},$$

$$a = 0.0033(6)\,GeV^2\,,$$

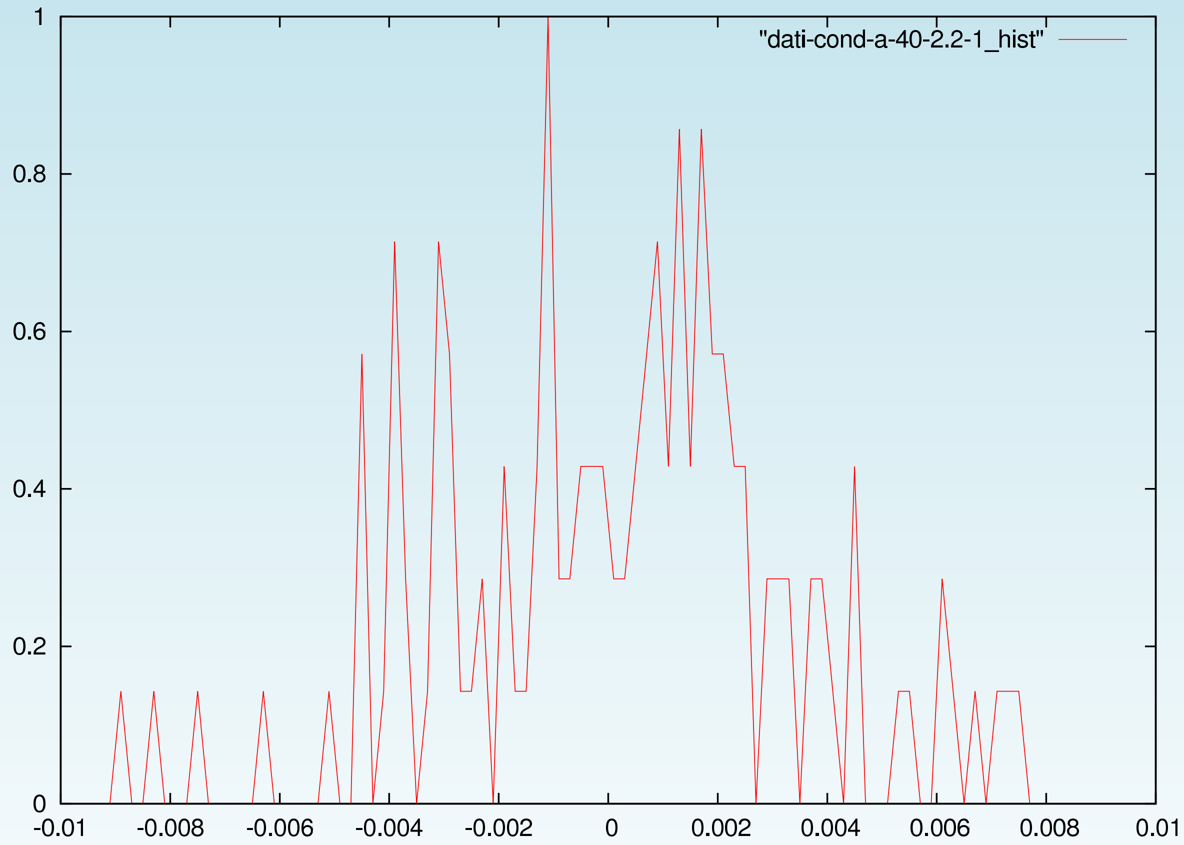
$$b = 35.8(5)\,GeV^{-1},$$

$$v^2 = 1.87(8)\,GeV^4\,.$$

Fit parameters seem to change little with the (physical) lattice volume.

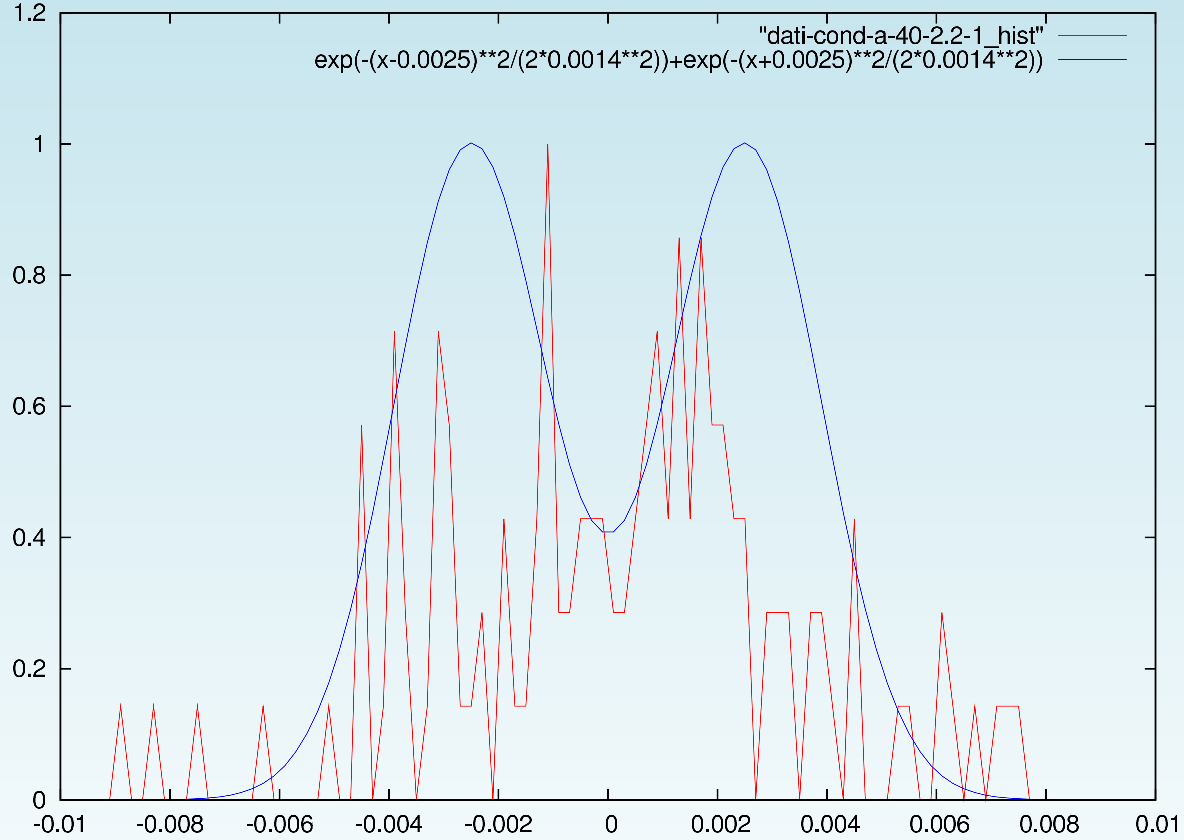
Distribution of $G^{ab}(p_{min})$ at $V = 40^4$

Histogram of data at $V = 40^4$ and $\beta = 2.2$ for G^{ab} at the smallest nonzero p.

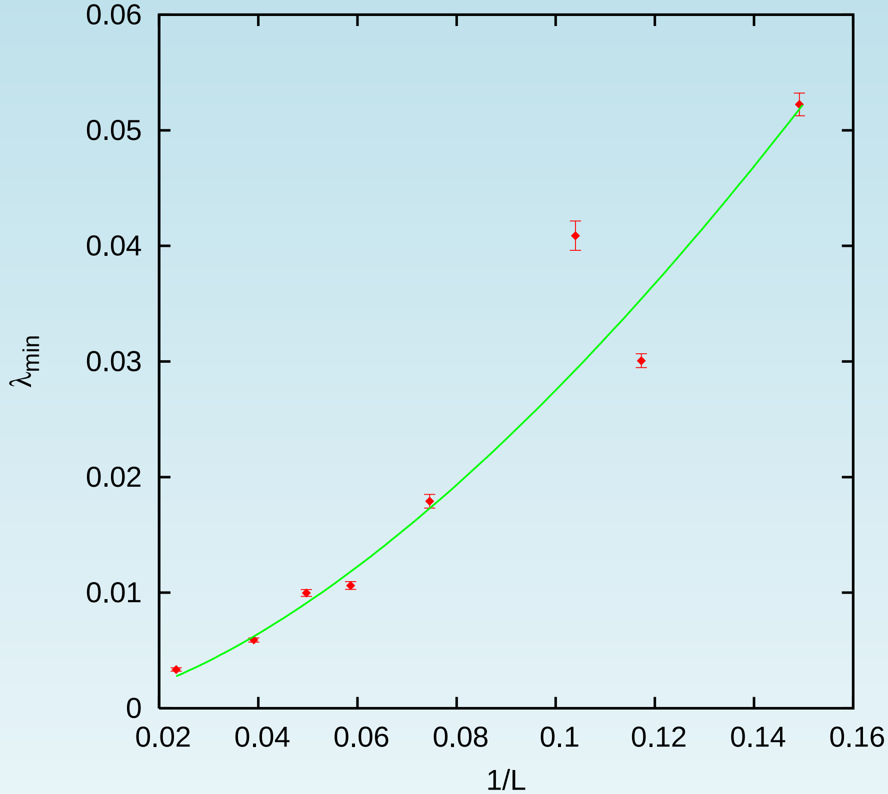


Distribution of $G^{ab}(p_{min})$ at $V = 40^4$ (II)

Histogram of data at $V = 40^4$ and $\beta = 2.2$ for G^{ab} at the smallest nonzero p. Gaussian or two-peak??



Smallest eigenvalue of the FP matrix



Plot of λ_{min} for several lattice volumes and values of β as a function of $1/L$, both in physical units.

Fit to $a\,(1/L)^b$ shows $b = 1.6(1)$, therefore vanishes more slowly than $(1/L)^2$ (Laplacian).

Conclusions

- Ongoing study of gluon and ghost propagators for the pure $SU(2)$ case in minimal MAG.
- Gluon propagator in agreement with the study by Bornyakov et al. (2003): suppression of the off-diagonal propagators compared to the diagonal (transverse) one, supporting Abelian dominance.
- Results for the ghost propagator show a finite value at zero momentum. This might be related to 1) the fact that the smallest nonzero eigenvalue of the Faddeev-Popov matrix vanishes more slowly than $1/L^2$ in the infinite-volume limit and 2) to the presence of dimension-two condensates.
- To confirm the presence of a ghost condensate we may need larger volumes.