The lattice gluon propagator in numerical stochastic perturbation theory

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work in collaboration with F. Di Renzo (Parma), C. Torrero (Regensburg)

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Introduction I

Perturbative results with increasing precision (more loops) are useful and needed:
Relate observables measured in lattice QCD to their physical counterpart via renormalisation

Separate non-perturbative effects in observables assumed to show confinement properties
Example: Extract gluon condensate from plaquette or from Creutz ratios of Wilson loops
Gluon and ghost propagators belong to these observables

Lattice perturbation theory (LPT) in diagrammatic approach much more involved compared to continuum PT of QCD
Only a limited number of two-loop results available in LPT
Remember: Standard PT derived from path integral quantisation

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Alternative:

Use Langevin equation as basis of stochastic quantisation
(Parisi and Wu, 1981)
Non-perturbative application:
Langevin simulations of lattice QCD (Batrouni et al., 1985)

Apply Langevin dynamics for weak coupling expansion
of lattice QCD

Powerful numerical approach for higher order calculations:
Numerical stochastic perturbation theory (NSPT)
(Di Renzo et al.,1994 ff.)

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New application:

Higher-loop contributions to gluon propagator in Landau gauge (Ilgenfritz, Perlt, Schiller, PoS (LATTICE2007))

We combined efforts with Di Renzo and Torrero

apeNEXT at Milano, storage possible of large lattices, one copy for all orders !

Our work at Leipzig and Berlin supported by DFG under contract FOR 365 (Forschergruppe Gitter-Hadronen-Phänomenologie).

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Langevin equation for lattice QCD

Use Euclidean lattice Langevin equation with "time" t

$$\frac{\partial}{\partial t} U_{x,\mu}(t;\eta) = \mathrm{i}\left(\nabla_{x,\mu} S_G[U] - \eta_{x,\mu}(t)\right)\; U_{x,\mu}(t;\eta)$$

$\eta = \eta^a T^a$ random field with Gaussian distribution
$\nabla_{x,\mu}$ left Lie derivative on the group

For $t \to \infty$ link gauge fields U are distributed according to the measure $\exp(-S_G[U])$

Discretise $t = n\epsilon$
Get solution at next time step $n+1$ in the Euler scheme

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We use the Wilson plaquette gauge action S_G

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Perturbative Langevin equations I

Use that solution for perturbative expansion:
Rescale $\varepsilon = \beta\epsilon$ and expand gauge fields U (and the "force" F)

$$U_{x,\mu}(n;\eta) \to 1 + \sum_{l>0} \beta^{-l/2} U^{(l)}_{x,\mu}(n;\eta)$$

This transforms the Langevin equation into a system of equations

$$\begin{aligned} U^{(1)}(n+1) &= U^{(1)}(n) - F^{(1)}(n) \\ U^{(2)}(n+1) &= U^{(2)}(n) - F^{(2)}(n) \\ &\quad + \frac{1}{2}(F^{(1)}(n))^2 - F^{(1)}(n)U^{(1)}(n) \\ &\cdots \end{aligned}$$

Random noise field η enters only in $F^{(1)}$. Higher orders become stochastic via noise propagation through lower order fields

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Perturbative Langevin equations II

Work also with field variables living in the algebra $A = \log U$
Expand

$$A_{x+\hat{\mu}/2,\mu}(t;\eta) \to \sum_{l>0} \beta^{-l/2} A^{(l)}_{x+\hat{\mu}/2,\mu}(t;\eta)$$

$$\begin{aligned} A^{(1)} &= U^{(1)} \\ A^{(2)} &= U^{(2)} - \frac{1}{2}(U^{(1)})^2 \\ &\dots \end{aligned}$$

Contributions of some order to an observable

$$\langle \mathcal{O} \rangle \to \sum_{l \geq 0} \beta^{-l/2} \langle \mathcal{O}^{(l)} \rangle$$

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Lattice gluon propagator

Continuum gluon propagator $D^{ab}_{\mu\nu} = \delta^{ab} D_{\mu\nu}$

$$D_{\mu\nu}(q) = \left(\delta_{\mu\nu} - \frac{q_\mu\, q_\nu}{q^2}\right) D(q^2) + \frac{q_\mu\, q_\nu}{q^2} \frac{F(q^2)}{q^2}$$

$F(q^2) = 0$ in Landau gauge $\partial_\mu A_\mu(x) = 0$

Lattice gluon propagator

$$D^{ab}_{\mu\nu}(\hat{q}) = \left\langle \widetilde{A}^a_\mu(k) \widetilde{A}^b_\nu(-k) \right\rangle = \delta^{ab} D_{\mu\nu}(\hat{q})$$

$\widetilde{A}^a_\mu(k)$ – Fourier transform of $A^a_{x+\hat{\mu}/2,\mu}$

$$\hat{q}_\mu(k_\mu) = \frac{2}{a} \sin\left(\frac{\pi k_\mu}{L_\mu}\right) = \frac{2}{a} \sin\left(\frac{a q_\mu}{2}\right),\ k_\mu \in \left(-\frac{L_\mu}{2}, \frac{L_\mu}{2}\right]$$

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Perturbative gluon propagator

Lattice gluon propagator in NSPT of loop order n (even powers in l)

$$\delta^{ab} D^{(n)}_{\mu\nu}(\hat{q}) = \left\langle \sum_{i=1}^{2n+1} \left[\widetilde{A}^{a,(i)}_{\mu}(k)\, \widetilde{A}^{b,(2n+2-i)}_{\nu}(-k) \right] \right\rangle$$

Tree level $D^{(0)}_{\mu\nu}$ arises from quantum fluctuations of gauge fields with $i = 1$

Inspired by continuum form we consider

$$\sum_{\mu=1}^{4} D^{(n)}_{\mu\mu}(\hat{q}) \equiv 3D^{(n)}(\hat{q})\,, \quad \sum_{\mu,\nu=1}^{4} \hat{q}_\mu D^{(n)}_{\mu\nu}(\hat{q})\, \hat{q}_\nu \xrightarrow{\text{Landau gauge}} 0$$

We present dressing functions $Z^{(n)}$ in two forms :

$$\hat{Z}^{(n)}(\hat{q}) = \hat{q}^2 D^{(n)}(\hat{q})\,, \quad Z^{(n)}(aq) = (aq)^2 D^{(n)}(\hat{q})$$

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Implementation of NSPT : three limits to be taken

• Limit $\varepsilon \to 0$

Solve coupled system of equation for $U^{(l)}$'s
Time sequence of gauge fields to all chosen orders
Measure perturbatively constructed observables
Different step sizes: $\varepsilon =$ 0.07,..., 0.01
up to 60000 Langevin steps for smallest ε

• Limit $V \to \infty$

Extract infinite volume loop results
Periodic lattices: $L = 6, 8, 10, 12$ (and 16) with orders of propagator: $n_{\max} = 4$ (1) (gauge field orders 10 (4))

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Compare with analytic results of standard LPT
Predict new precise numerical results in higher loops

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Landau gauge fixing

Perform Landau gauge fixing and measure gluon propagator (after each 20th Langevin step)

Condition for perturbative Landau gauge

$$\sum_{\mu} \partial_{\mu}^{L} A_{x,\mu}^{(l)} = 0 \,, \quad \partial_{\mu}^{L} A_{x,\mu}^{(l)} \equiv A_{x+\hat{\mu}/2,\mu}^{(l)} - A_{x-\hat{\mu}/2,\mu}^{(l)}$$

The Landau gauge reached by iterative gauge transformations chosen as perturbative variant of Fourier acceleration (Davies et al, 1987)

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Raw data examples for $\hat{Z}^{(n)}(\hat{q})$

Measured gluon dressing function averaged over equivalent 4-tuples of lattice momenta (k_1, k_2, k_3, k_4)
Contributions $\propto \beta^{-l/2}$ for odd l have to vanish !

Raw data examples for $\hat{Z}^{(n)}(\hat{q})$

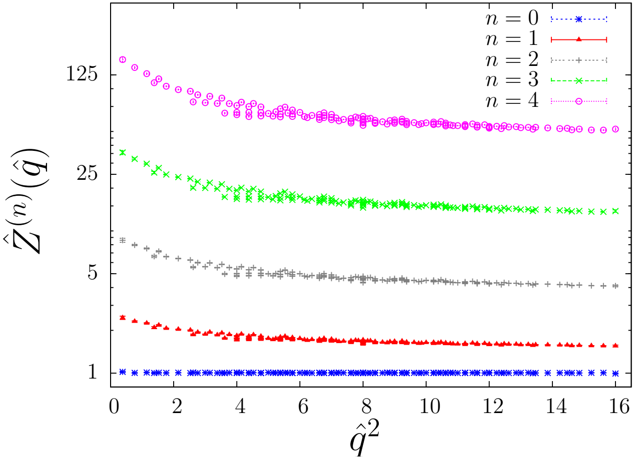

Figure: $\hat{Z}^{(n)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 10$ and $\varepsilon = 0.01$. Separate loop contributions.

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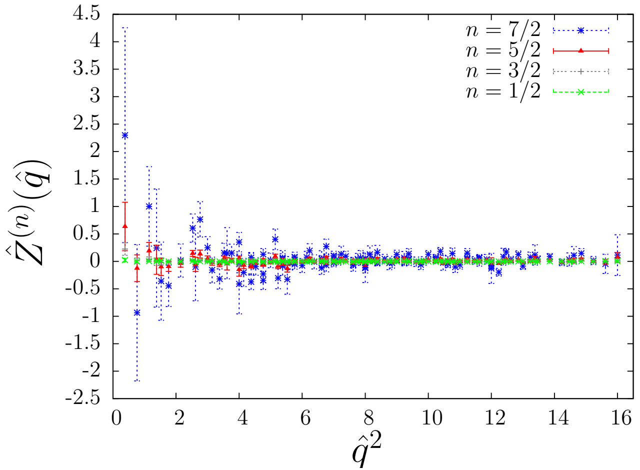

Figure: $\hat{Z}^{(n)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 10$ and $\varepsilon = 0.01$. Vanishing contributions.

Limit $\varepsilon \to 0$ for $\hat{Z}^{(n)}(\hat{q})$ – linear extrapolation

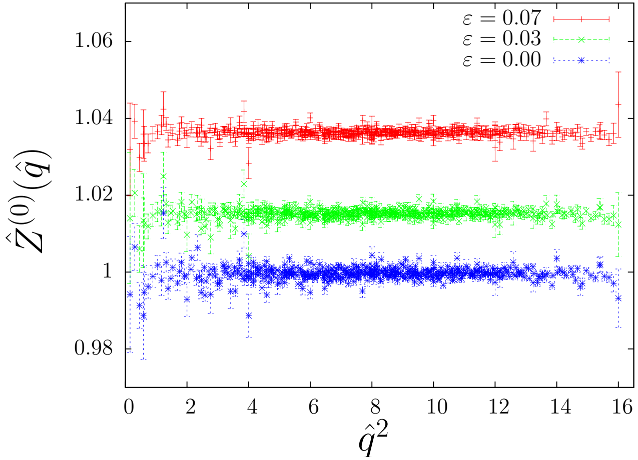

Figure: Tree level dressing function $\hat{Z}^{(0)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 16$.

Limit $\varepsilon \to 0$ for $\hat{Z}^{(n)}(\hat{q})$ – linear extrapolation

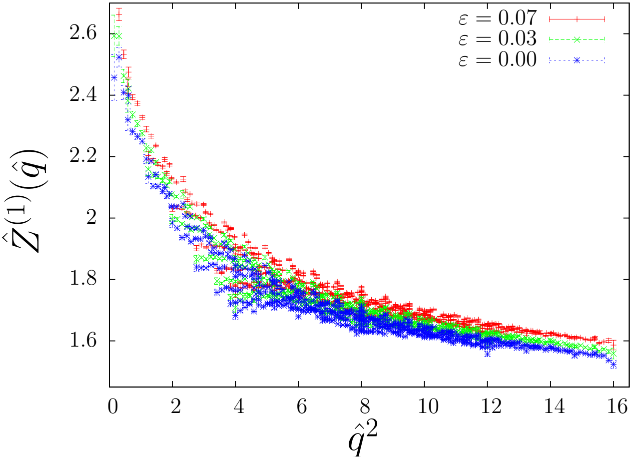

Figure: One-loop dressing function $\hat{Z}^{(1)}(\hat{q})$ vs. $\hat{q}^2$ at $L = 16$.

Limit $V \to \infty$ and momentum cuts for $Z^{(n)}(aq)$

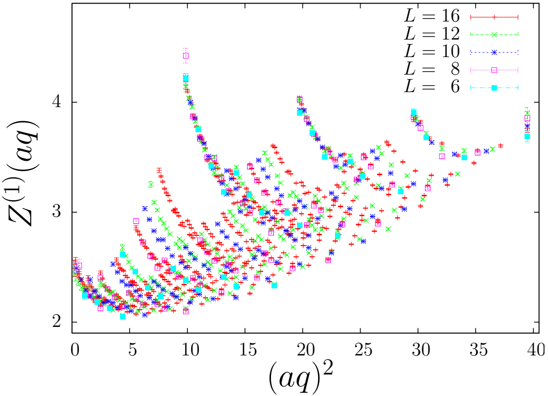

Figure: One-loop dressing function $Z^{(1)}(aq)$ vs. $(aq)^2$ at all volumes for *all* inequivalent 4-tuples.

Different branches for off-diagonal tuples due to hypercubic group

Limit $V \to \infty$ and momentum cuts for $Z^{(n)}(aq)$

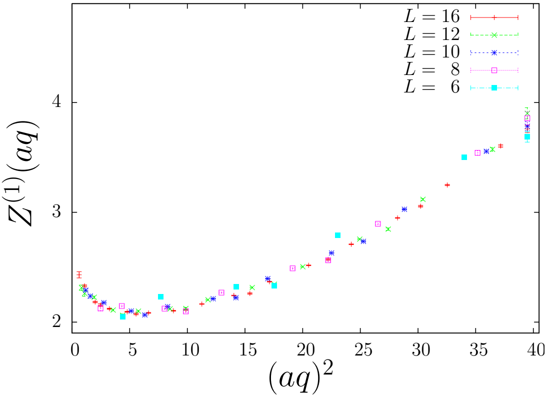

Figure: One-loop dressing function $Z^{(1)}(aq)$ vs. $(aq)^2$ at all volumes for 4-tuples $(k,k,k,k),(k \pm 1,k,k,k), k > 0$.

Universal $(aq)^2$ (or $\hat{q}^2$) dependence for larger V near diagonal momenta, behaviour is similar for all loop contributions.

$\hat{Z}(\hat{q}, n_{\max}) = \sum_{n=0}^{n_{\max}} \hat{Z}^{(n)}(\hat{q})/\beta^n$

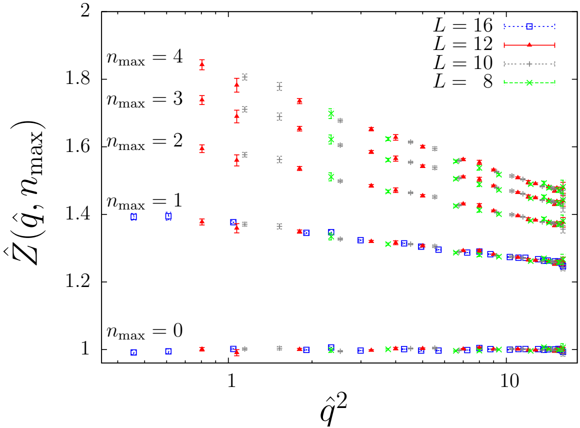

Figure: Summed dressing function near diagonal $\hat{Z}(\hat{q}, n_{\max})$ up to four loops (in steps of one loop) vs. $\hat{q}^2$ using $\beta = 6.0$ at $L = 8, 10, 12$ (up to only one loop for $L = 16$).

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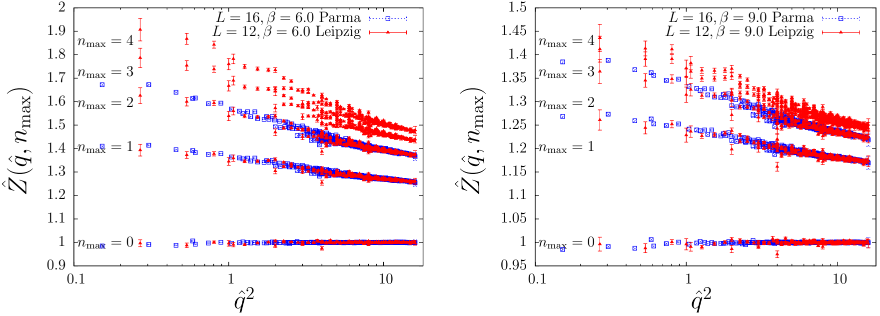

Figure: Summed dressing function, now including the Parma data for two loops at $L = 16$. Left: $\beta = 6.0$. Right: $\beta = 9.0$.

A_μ definition dependence of the dressing function

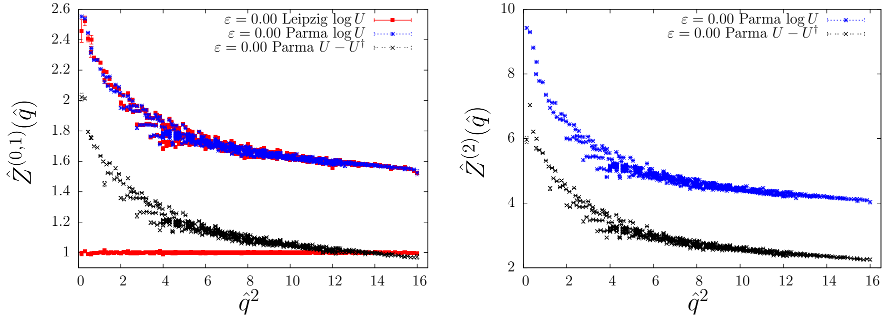

Figure: Left: tree-level and one-loop dressing function. Right: two-loop dressing function.

Influence of the definition of A_μ on the summed loops

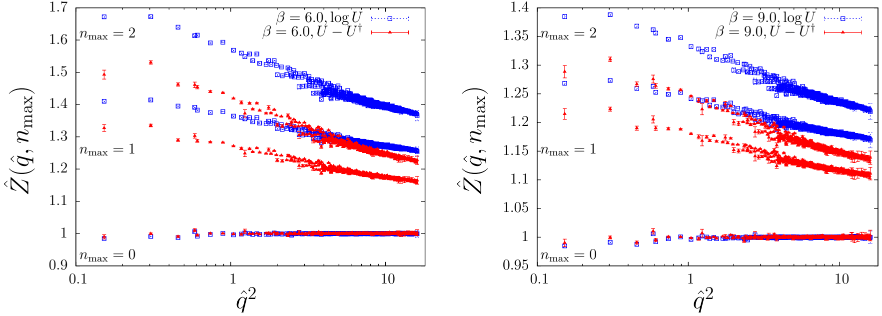

Figure: The summed loop contributions for a 16^4 lattice. Left: for $\beta = 6.0$. Right: for $\beta = 9.0$.

Extracting expansion coefficients I

Aim: find gluon wave function renormalisation constant Z_A in LPT
Use regularisation independent scheme (RI')

$$Z_A^{RI'}(a,\mu,\alpha^{RI'})\,\Pi_T(a,q,\alpha^{RI'})\Big|_{q^2=\mu^2} = 1$$

Dressing function $Z(a,q,\alpha)$=1/$\Pi_T(a,q,\alpha)$ – transverse part of the gluon polarisation tensor

$\longrightarrow$ at $\mu^2 = q^2$, in bare α_0

$$Z(a,q,\alpha_0) = Z_A(a,q,\alpha_0)$$

Expected form using $\beta = N_c/(8\pi^2\alpha_0)$:

$$Z(a,q,\beta) = 1 + \frac{1}{\beta}\left[D_{1,1}\,\log(aq)^2 + D_{1,0}\right] + \frac{1}{\beta^2}\left[D_{2,2}\,\log^2(aq)^2 + D_{2,1}\,\log(aq)^2 + D_{2,0}\right] + \cdots$$

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Extracting expansion coefficients II

Determine $D_{i,j}$ for $a \to 0$ and $V \to \infty$ from measured Z (Landau gauge, quenched approximation)
Minimize the coefficient number using results from renormalisation group in continuum QCD PT:

Outcome for lowest orders:
Leading log coefficients $D_{n,n}$ known
Non-leading log coefficients more complicated, e.g. $D_{2,1}(D_{1,0})$
$D_{1,0}$ calculated in lattice PT (Kawai et al.,1981)

$$Z^{2-\mathrm{loop}}(a,q,\beta) = 1 + \frac{1}{\beta}\left(-0.24697 \log(aq)^2 + 2.29368\right) + \frac{1}{\beta^2}\left(0.0821078 \log^2(aq)^2 - 1.48445 \log(aq)^2 + D_{2,0}\right)$$

Check $D_{1,0} = 2.29368$ and predict $D_{2,0}$

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One-loop $D_{1,0}$

Fit non-log contribution of $Z^{(1)}(aq)$ near diagonal

$$D_{1,0;L}(aq) = D_{1,0;L} + c_1(aq)^2 + c_2(aq)^4$$

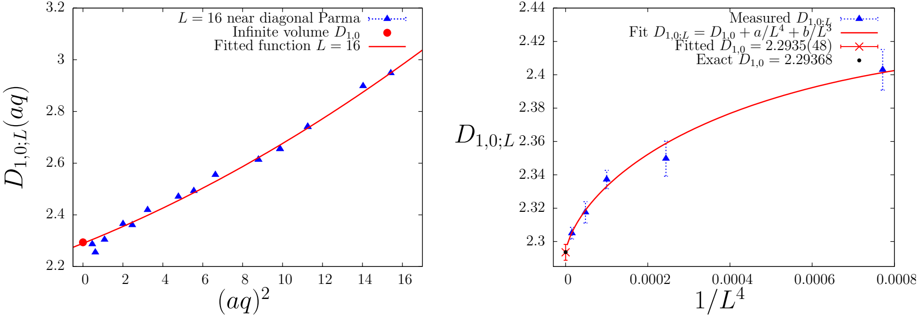

Figure: Left: Limit $a \to 0$: $D_{1,0;16} = 2.3050(35)$. Right: Limit $V \to \infty$:

$$D_{1,0;\infty} = 2.2935(48)$$

Nice agreement !

Two-loop $D_{2,0}$ (preliminary, up to 12^4 only)

Same fit ansatz for the non-log contribution

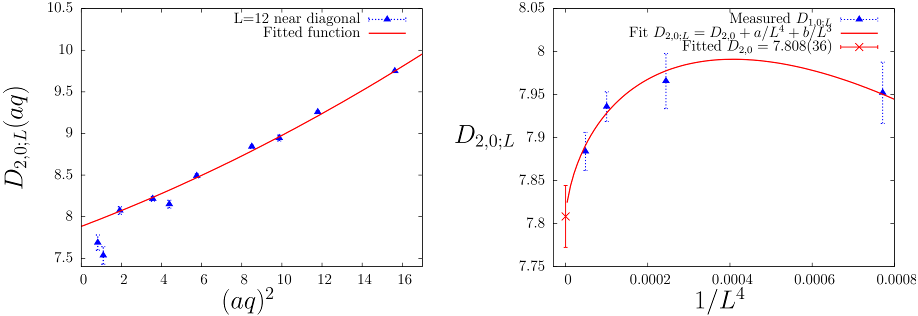

Figure: Left: Limit $a \to 0$: $D_{2,0;12} = 7.884(23)$. Right: Limit $V \to \infty$:

$$D_{2,0;\infty} = 7.808(36)$$

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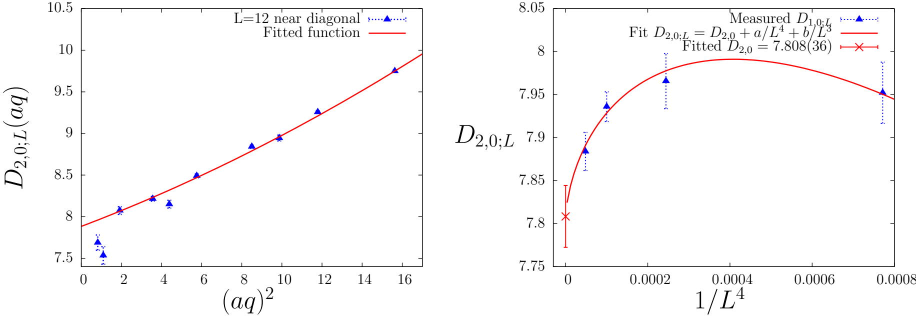

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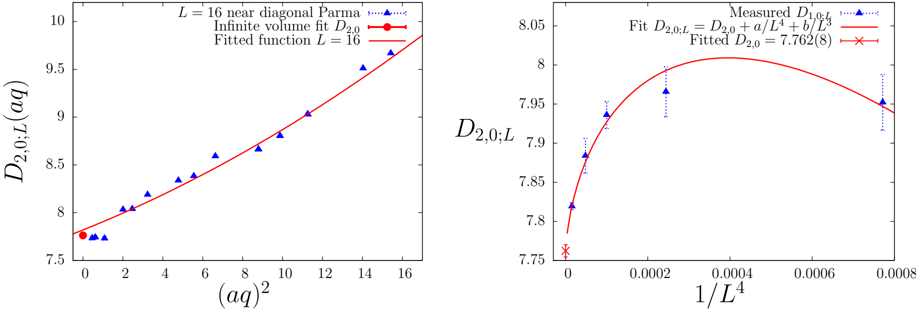

Figure: Left: Limit $a \to 0$: $D_{2,0;16} = 7.772(12)$. Right: Limit $V \to \infty$:

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Measurements in progress at larger volumes (Parma, 20^4) to confirm that prediction.

NSPT works where standard lattice PT is extremely difficult ! The two-loop result is not completely known !

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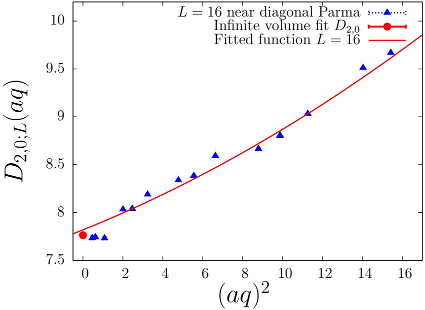

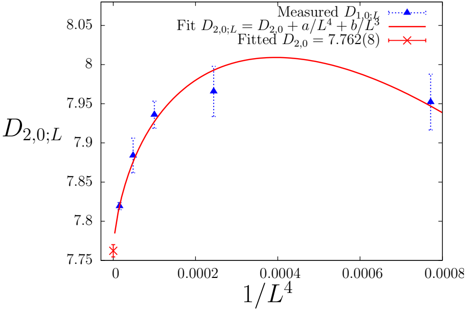

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Some formulae for the ghost

The perturbative Faddeev-Popov operator (see the book by Rothe) :
with the gluon field $\phi_\mu^a = i\, A_\mu^a$,
in adjoint representation $[\Phi_\mu(x)]_{bc} = [t^a]_{bc}\, \phi_\mu^a(x) = -i\, f_{abc}\, \phi_\mu^a(x)$
and in fundamental representation $[\phi_\mu(x)]_{ij} = [T^a]_{ij}\phi_\mu^a(x)$,
one can write the covariant derivative as

$$D_\mu[\phi] = \left(1 + \frac{i}{2}\Phi_\mu(x) - \frac{1}{12}(\Phi_\mu(x))^2 + \ldots\right)\, \partial_\mu^R + i\, \Phi_\mu(x)\, .$$

Then the Faddeev-Popov operator becomes (with left and right derivatives):

$$M_{xy}^{ab} = [\partial_\mu^L\, D_\mu]_{xy}^{ab}\, .$$

Since the operator M can be expanded,

$$M = M^{(0)} + \sum_{l>0} \beta^{-l/2}\, M^{(l)}\, ,$$

Some formulae for the ghost

the ghost propagator can be expanded, too,

$$M^{-1} = [M^{-1}]^{(0)} + \sum_{l>0} \beta^{-(l/2)} \, [M^{-1}]^{(l)}$$

with all orders recursively defined without inversion
(except for $[M^{-1}]^{(0)} = [M^{(0)}]^{-1}$ (the free Laplacian)) :

$$[M^{-1}]^{(l)} = -[M^{-1}]^{(0)} \sum_{j=0}^{l-1} M^{(l-j)} \, [M^{-1}]^{(j)} \, .$$

With a plane wave source η and $\psi^{(l)} = [M^{-1}]^{(l)} \, \eta$
the ghost propagator in momentum space is

$$G^{(l)}(\hat{q}(k)) = \xi^* \cdot \psi^{(l)} \, .$$

Then the ghost dressing functions of order l are

$$J^{(l)}(aq) = (aq)^2 \; G^{(l)} \quad \text{and} \quad \hat{J}^{(l)}(\hat{q}) = \hat{q}^2 \; G^{(l)} \, .$$

Results of standard lattice perturbation theory

H. Kawai (checked by Mathematica):
The ghost self-energy in Landau gauge

$$\Pi^{ab}(q^2) = \delta_{ab} q^2 \left\{ 1 + \frac{1}{\beta} \left[-0.0854897 \; \log(a^2 q^2) + 0.525314 \right] \right\}$$

The one-loop lattice ghost dressing function for near-diagonal momenta has the form

$$\hat{J}(\hat{q}) = \gamma_G \; \log(\hat{q}^2) + \hat{J}_{1,0;L}(\hat{q}^2)$$

with $\gamma_G = -0.854897$ and a fit ansatz $\hat{J}_{1,0;L}(\hat{q}) = \hat{J}_{1,0;L} + c_1 \hat{q}^2 + c_2 \hat{q}^4$ to account for remaining artefacts near the diagonal

First test results on small lattices

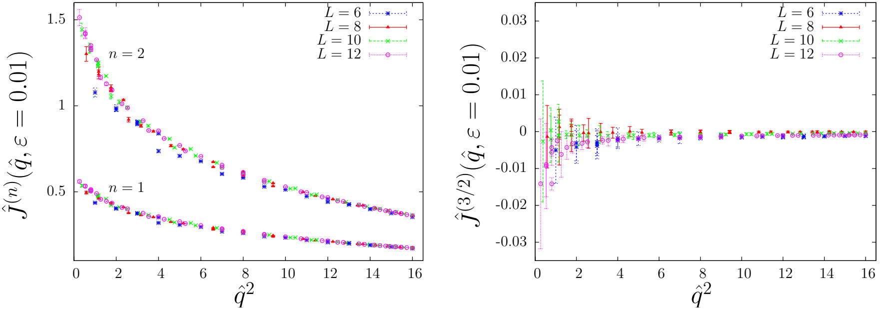

Figure: Raw ghost dressing function at one- and two-loop level (left) for all momentum components near the diagonal and the raw ghost dressing function at order $\beta^{-3/2}$ (right) for lattices with $L = 8$, 10 and 12 with $\varepsilon = 0.01$.

Extrapolation $\varepsilon \to 0$

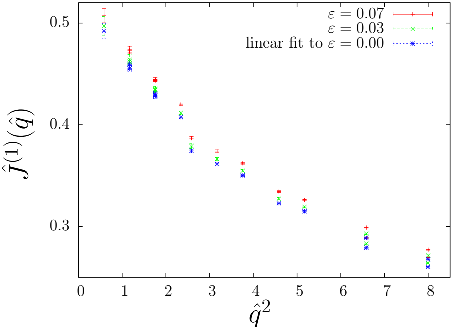

Figure: Linear extrapolation to zero Langevin time step in the momentum region $\hat{q}^2 < 8$ for a lattice of size 8^4.

Some fit results

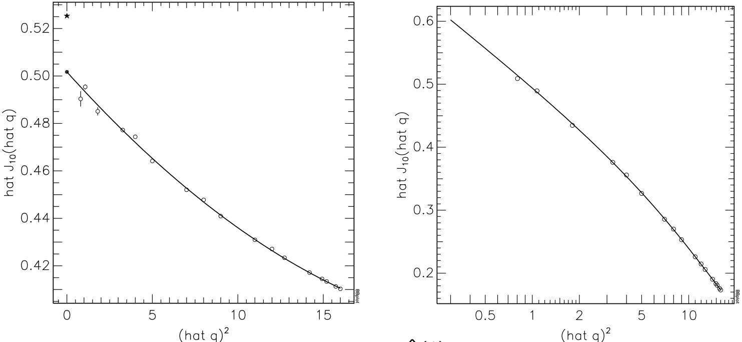

Figure: Fit of the one-loop expression for $\hat{J}^{(1)}(\hat{q})$. Left: fit of the non-logarithmig terms with given γ_G. Right: the full one-loop dressing function $\hat{J}^{(1)}(\hat{q})$, for $L = 12$ and $\varepsilon = 0.01$.

The result of the fit $\hat{J}_{1,0;L=12} = 0.5017(7)$
(with $c_1 = -0.00802(13)$ and $c_2 = 0.00014(1)$)
comes close to the expected result $\hat{J}_{1,0;\infty} = 0.525314$.

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- Very good agreement with one-loop standard LPT
- First quantitative prediction for two-loop contribution (prelim.)
  Large constants as result of lattice artefacts (tadpoles)
  Tadpole improvement needed
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