Symmetry and Confinement

B. Caudy & JG, arXiv:0712.0999
B. Lucini & JG, in progress

Jeff Greensite

Quarks and Hadrons in Strong QCD

St. Goar, Germany

What is Confinement?

Juliet:

"What's in a name? That which we call a rose
By any other name would smell as sweet."

Romeo and Juliet (II, ii, 1-2)

What are people trying to prove, in order to “prove” confinement?
And what do they *mean* by that word?

1. linear static quark potential, rising to infinity ← most order parameters

2. colorless asymptotic particle states ← common terminology

These are not quite the same thing, which raises some semantic issues:

What are people trying to prove, in order to "prove" confinement?
And what do they *mean* by that word?

1. linear static quark potential, rising to infinity ← most order parameters

2. colorless asymptotic particle states ← common terminology

These are not quite the same thing, which raises some semantic issues:

against #1 - in *real* QCD, with quarks, the static potential rises and then levels off, due to string breaking.

so is real QCD not confining?

What are people trying to prove, in order to “prove” confinement?
And what do they *mean* by that word?

1. linear static quark potential, rising to infinity	←	most order parameters
2. colorless asymptotic particle states	←	common terminology

These are not quite the same thing, which raises some semantic issues:

against #1 - in *real* QCD, with quarks, the static potential rises and then levels off, due to string breaking.

so is real QCD not confining?

against #2 - asymptotic particle states are also colorless in a Higgs theory, where there is no linear potential at all.

so are Higgs theories confining?

The Fradkin-Shenker-Osterwalder-Seiler Theorem

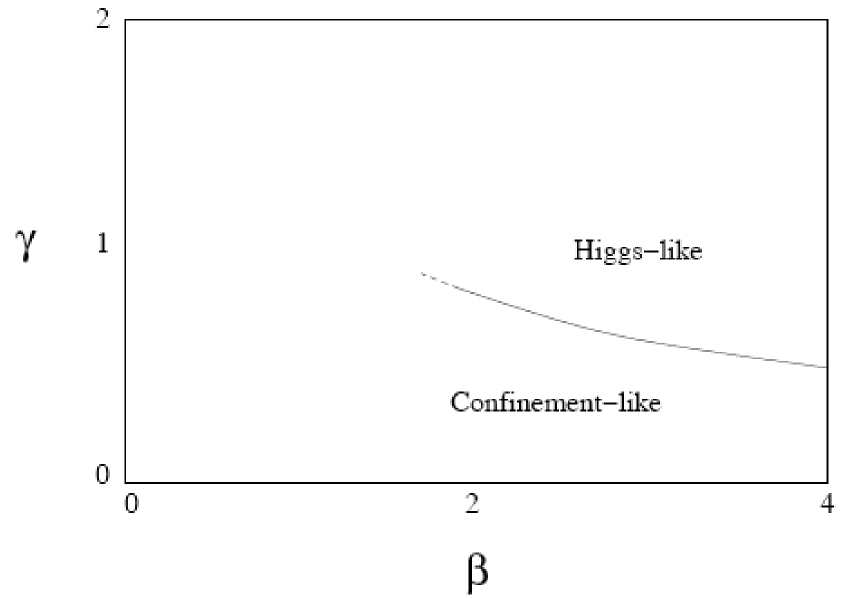
Consider an SU(2) gauge-Higgs theory with lattice action

$$S = \beta \sum_{plaq} \frac{1}{2}\mathrm{Tr}[UUU^\dagger U^\dagger] + \gamma \sum_{x,\mu} \frac{1}{2}\mathrm{Tr}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})]$$

It has a phase diagram something like this:

(Campos, 1997)

The theorem says that there is no complete separation between the Higgs-like and the confinement-like regions.



More precisely: between a point

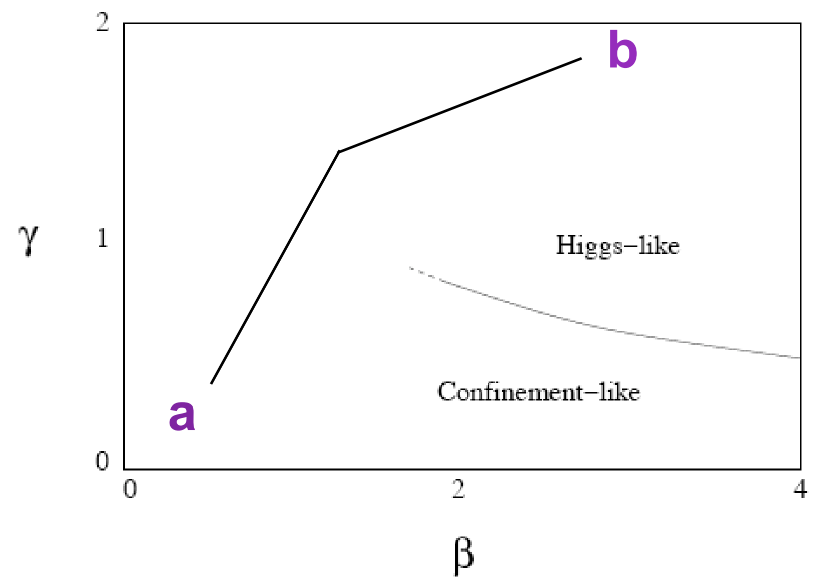
"**a**" deep in the confinement-like regime $(\beta, \gamma \ll 1)$, and a point

"**b**" deep in the Higgs regime $(\beta, \gamma \gg 1)$,

there is a path from **a** to **b** such that all Green's functions of all local, gauge-invariant operators

$$\langle A(x_1)B(x_2)C(x_3)...\rangle$$

vary analytically along the path.



This rules out an abrupt transition from a colorless to a color-charged spectrum.

Creation operators for three colorless vector mesons:

$$\vec{\phi}^\dagger(x)U_\mu(x)\vec{\phi}(x+\mu) \quad , \quad Re\left[\vec{\phi}^\dagger(x)U_\mu(x)\vec{\varphi}(x+\mu)\right] \quad , \quad Im\left[\vec{\phi}^\dagger(x)U_\mu(x)\vec{\varphi}(x+\mu)\right]$$

where

$$\vec{\varphi}(x) = \sigma_2\vec{\phi}(x)$$

Higgs-like region: "W-bosons"

Confinement-like region: Mesons (with scalar constituents)

So “color confinement” in the asymptotic spectrum doesn’t distinguish between Higgs and “real QCD”-like dynamics.

Question

Can the confinement phase of a gauge theory be regarded as the symmetric (or broken) realization of a gauge symmetry?

This idea appears in a number of popular approaches, in particular:

i) **Dual superconductivity**

ii) **the Kugo-Ojima criterion**

iii) **Coulomb-gauge confinement**

Naively, symmetry breaking of gauge invariance violates

Elitzur's Theorem:

Local gauge symmetries do not break spontaneously. In the absence of gauge fixing, $\langle\varphi\rangle = 0$ regardless of the shape of the Higgs potential.

However, although *local* symmetries can't break spontaneously, it is still possible to break a *global* subgroup of the local symmetry.

Subgroups of this kind are typically what remains of the local symmetry after a gauge choice.

Landau gauge

Remnant symmetries are homogenous gauge transformations

$$g(x) = g$$

There are also inhomogenous transformations

$$\begin{aligned} g(x) &= \exp[i\Lambda^a(\epsilon;x)\frac{1}{2}\sigma_a] \qquad \textit{(Hata, 1983)} \\ \Lambda^a(\epsilon;x) &= \epsilon^a_\mu x^\mu - \mathbf{g}\frac{1}{\partial^2}(A_\mu \times \epsilon_\mu)^a + O(\mathbf{g}^2) \end{aligned}$$

analogous to abelian transformations $A_\mu \rightarrow A_\mu + \partial_\mu \phi$
with

$$\phi(x) = c + \epsilon_\mu x^\mu$$

$\langle\varphi\rangle \neq 0$ ⇨ *remnant gauge symmetry breaking.*

Coulomb gauge

In addition to $g(x) = g$ there is a much larger, time-dependent remnant symmetry

$$g(\mathbf{x}, t) = g(t)$$

Define

$$L(\mathbf{x}, T) = P \exp\left[i \int_0^T dt A_0(\mathbf{x}, t) \right]$$

Then $\mathrm{Tr}[L(\mathbf{x}, T)] = \mathrm{Tr}[g L(\mathbf{x}, T) g^\dagger]$ for constant transformations, but

$$\mathrm{Tr}[L(\mathbf{x}, T)] \neq \mathrm{Tr}[g(0) L(\mathbf{x}, T) g^\dagger(T)]$$

$\langle \mathbf{Tr[L]} \rangle$ in Coulomb gauge probes the breaking of a remnant gauge symmetry which is ***different*** from that probed by $\langle \varphi \rangle$ in Landau gauge. It is insensitive to the breaking of the homogenous g(x)=g symmetry.

In principle, $\langle \mathbf{Tr[L]} \rangle$ and $\langle \boldsymbol{\varphi} \rangle$ could show transitions (0 → non-zero) at different places in the space of couplings.

Do different global subgroups of the gauge group break in different places in the phase diagram?

If they do, then there is an ambiguity is the phrase "spontaneously broken gauge symmetry". Precision requires specifying *which* symmetry is actually broken.

With this in mind, we revisit three confinement criteria which are based on the symmetric or broken realization of a gauge symmetry.

I. The Kugo-Ojima Criterion

Kugo and Ojima introduce a function $u^{ab}(p^2)$ defined by

$$u^{ab}(p^2)\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)=$$
$$\int d^4x\ e^{ip(x-y)}\langle 0|T[D_\mu c^a(x) g(A_\nu\times\overline{c})^b(y)|0\rangle$$

where $c^a(x)$ is the ghost field in a covariant gauge. They then show that the expectation value of charge vanishes in any physical state

$$\langle \text{phys}\ |Q^a|\text{phys}\rangle = 0$$ *(confinement?)*

providing the following conditions are satisfied:

1. Remnant symmetry under *g(x) = g* is unbroken

2. The criterion $u^{ab}(0) = -\delta^{ab}$ is satisfied.

It turns out that (2) implies that the spatially inhomogenous remnant symmetry in Landau gauge is also unbroken (Hata, Kugo).

Therefore, the Kugo-Ojima scenario requires that the entire remnant gauge symmetry in Landau gauge is unbroken, i.e. $\langle \varphi \rangle = 0$.

II. The Coulomb Criterion

Marinari, Parisi, Paciello, Taglienti (1993)
Olejnik, Zwanziger, JG (2004)

The idea is to show that

❶ The Coulomb energy of an isolated color charge is infinite;

❷ The color Coulomb potential is confining.

It turns out that both of these are implied by unbroken remnant gauge symmetry

$$g(\mathbf{x},t) = g(t)$$

which means that

$$\left\langle \mathrm{Tr}\left[L(\mathbf{x},T)\right]\right\rangle = 0$$

Isolated Charge

$$\Psi_q^a = q^a(x)\Psi_0$$

propagation in time

$$\begin{aligned} G(T) &= \langle \Psi_q^a | e^{-(H-E_0)T} | \Psi_q^a \rangle \\ &\propto \left\langle \mathrm{Tr}\left[L(\mathbf{x},T)\right]\right\rangle \end{aligned}$$

infinite energy if ***G(T)=0*** , which implies $\langle \mathbf{Tr}[L] \rangle = 0$

Color-Coulomb Potential

$$V_{coul}(R) = -\lim_{T\to 0} \frac{d}{dT} \log\left[\mathrm{Tr}[L(\mathbf{x},T)L^\dagger(\mathbf{y},T)]\right]$$

$V_{coul}(R)$ goes flat at $G(R) \to \infty$ (no confinement) if $\langle \mathbf{Tr}[L] \rangle \neq 0$

So both conditions require unbroken remnant gauge symmetry.

Remnant Symmetry Breaking in the Gauge-Higgs Model

Landau Gauge

Coulomb Gauge

order parameters

$$\widetilde{\phi} = \frac{1}{V}\sum_x \phi(x)$$

$$\widetilde{U}(t) = \frac{1}{V_3}\sum_{\mathbf{x}} U_0(\mathbf{x},t)$$

$$Q_L = \left\langle \frac{1}{2}\mathrm{Tr}[\widetilde{\phi}\widetilde{\phi}^\dagger] \right\rangle$$

$$Q_C = \frac{1}{L_t}\sum_{t=1}^{L_t} \left\langle \frac{1}{2}\mathrm{Tr}[\widetilde{U}(t)\widetilde{U}^\dagger(t)] \right\rangle$$

symmetric phase

$$Q_L \propto \frac{1}{V}$$

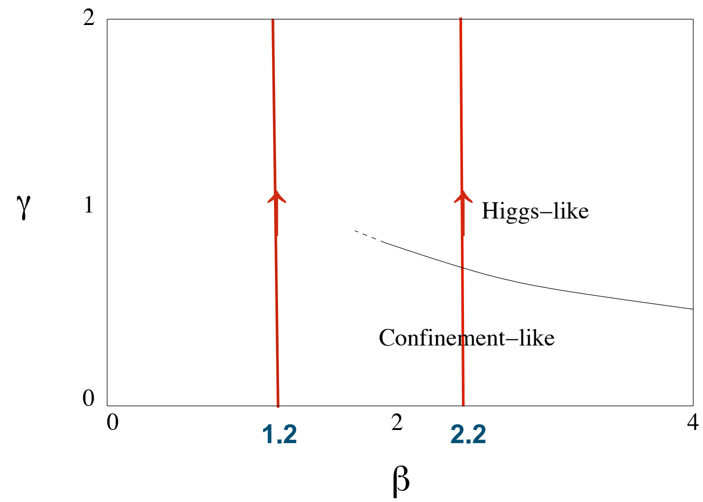
$$Q_C \propto \frac{1}{V_3}$$

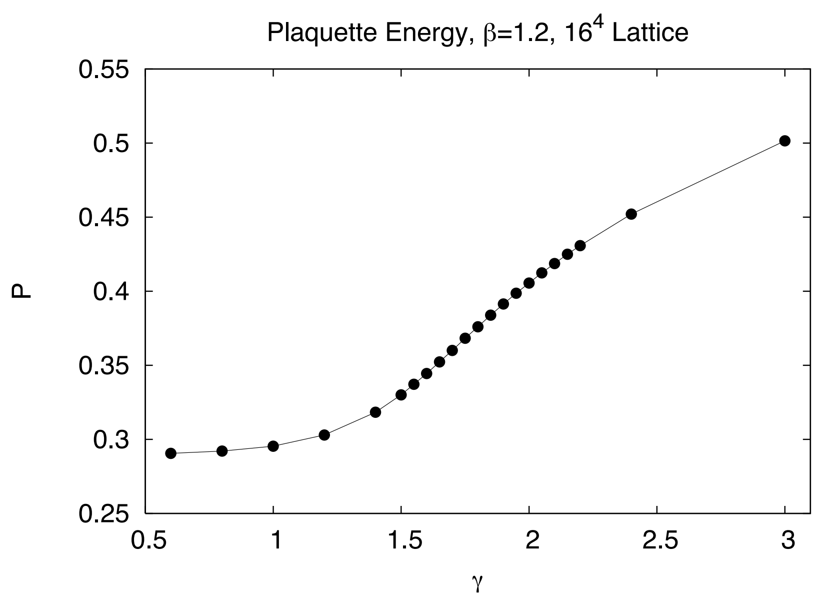
broken phase

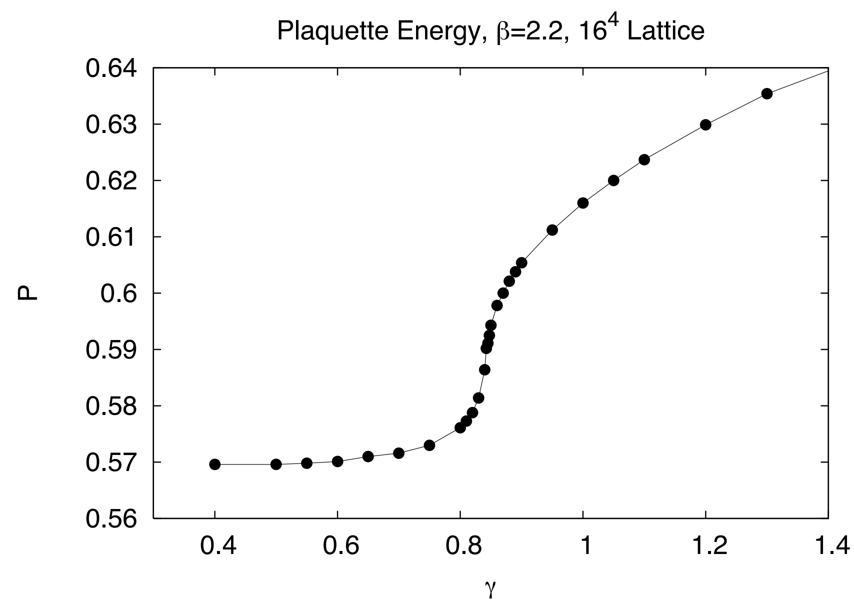
$$\lim_{V\to\infty} Q_L > 0$$

$$\lim_{V\to\infty} Q_C > 0$$

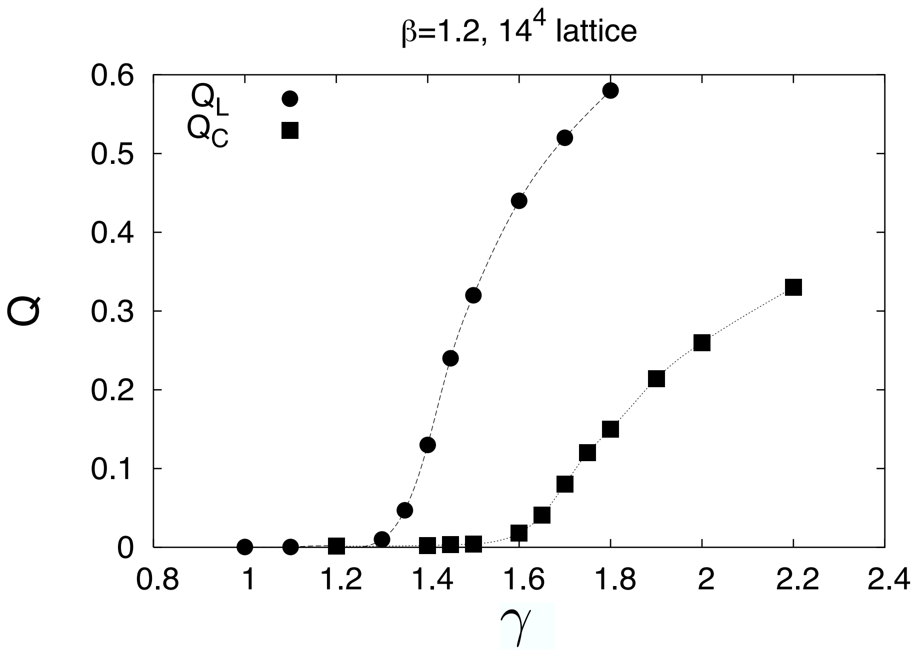
We see a thermodynamic transition (or sharp crossover) for $\beta > 2.0$







For $\beta > 2.0$, the Landau and Coulomb gauge transitions happen at about the same γ . For $\beta < 2.0$, these transitions happen at different γ .

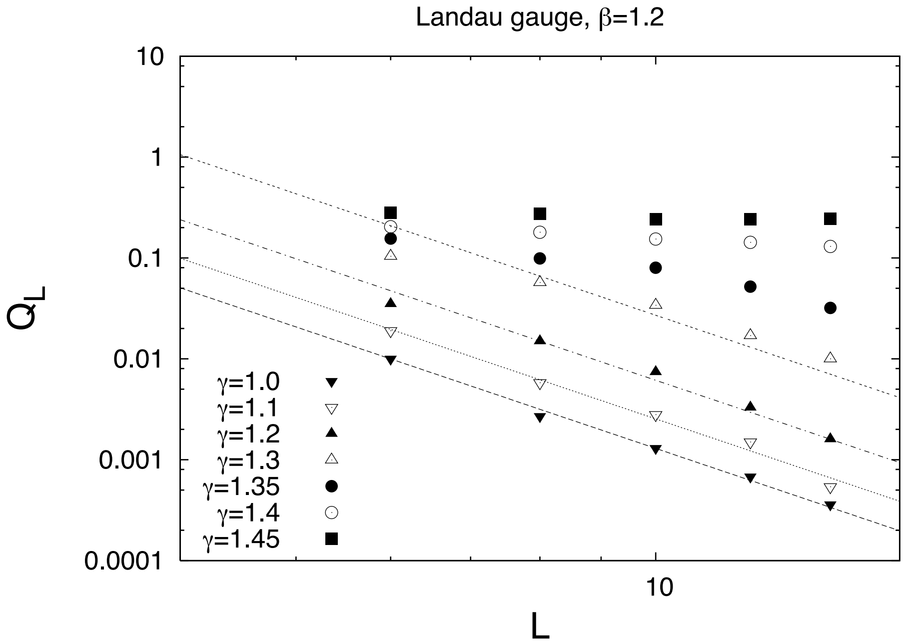


We need to pinpoint the location of the transitions...

Plotting Q vs lattice extension L shows that indeed

$$Q_L \propto \frac{1}{V} \qquad \text{and} \qquad Q_C \propto \frac{1}{V_3}$$

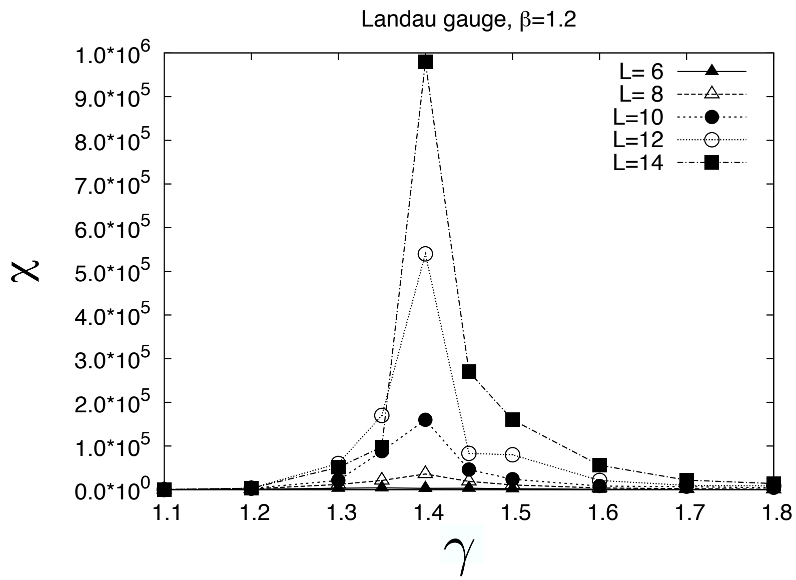
below the transition, and Q ≈ constant above....

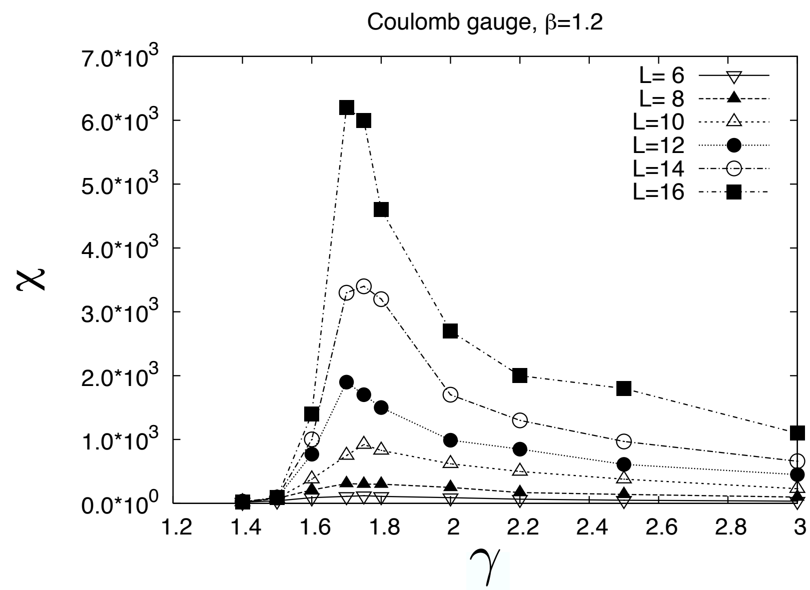


but a better method is to look for peaks in the susceptibilities.

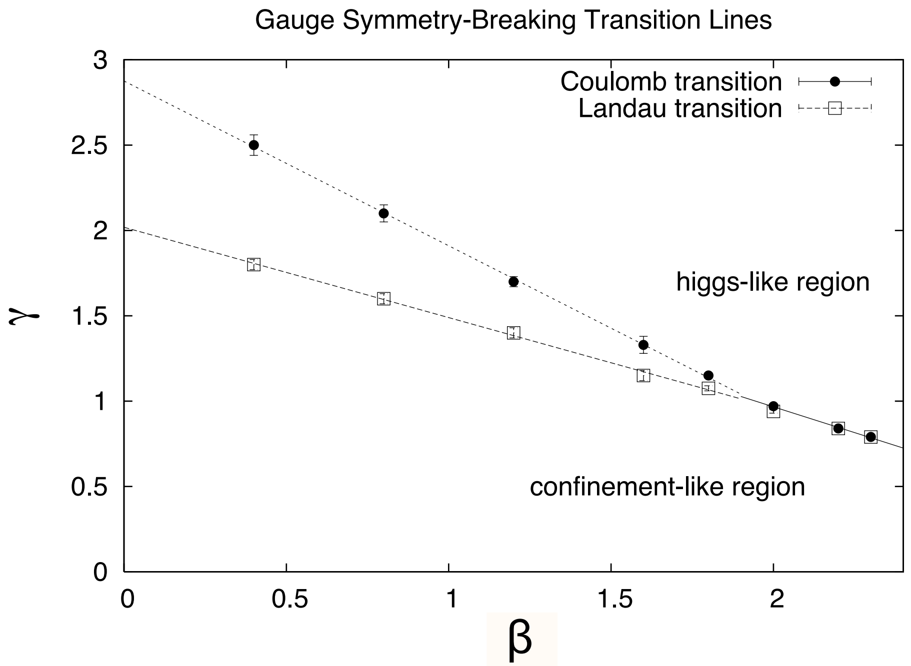
Let $Q = \langle \widetilde{Q} \rangle$ and define susceptibilities

$$\chi_L = V^2 \Big(\langle \widetilde{Q}_L^2 \rangle - Q_L^2 \Big)$$
$$\chi_C = V_3^2 \Big(\langle \widetilde{Q}_C^2 \rangle - Q_C^2 \Big)$$





The final result:



Coulomb and Landau remnant gauge symmetries break in different places, and they break in the absence of any discontinuity in the spectrum, or in the Green's functions.

Order parameters for confinement?? Unlikely!

III. Dual Superconductivity

Mandelstam and 't Hooft, mid-1970's

In compact U(1) gauge theories there is a conserved magnetic current

$$j^M_\mu = \partial^\nu \widetilde{F}_{\mu\nu}$$

associated with a dual U(1) gauge symmetry.

Spontaneous breaking of the dual gauge symmetry leads to confinement via a dual Meissner effect.

How to detect spontaneous breaking of a dual (global) gauge symmetry?

Pisa Proposal

Di Giacomo, Paffuti, D'Elia, Lucini, del Debbio...

The order parameter for dual symmetry breaking is a monopole creation operator, denoted μ, which doesn't commute with magnetic charge.

The monopole operator inserts a monopole field centered at a given point x

$$\mu(\mathbf{x})|A_i\rangle = |A_i + A_i^M\rangle$$

accomplished by

$$\mu(\mathbf{x}) = \exp\left[i \int d^3y \; A_i^M(y) E_i(y)\right]$$

(In a non-abelian theory, an abelian subgroup is picked out by abelian projection.)

In practice one computes

$$\rho = \frac{\partial}{\partial \beta} \log\langle\mu\rangle = \langle S\rangle_S - \langle S_M\rangle_{S_M}$$

A large negative peak in ρ at some $\beta=\beta_c$, growing with lattice volume, is the sign that $\langle\mu\rangle = 0$, and dual superconductivity disappears, for $\beta>\beta_c$.

In case after case, a symmetry restoration transition

$$\rho \to -\infty \;\;,\;\; \langle\mu\rangle \to 0$$

occurs at the deconfinement temperature.

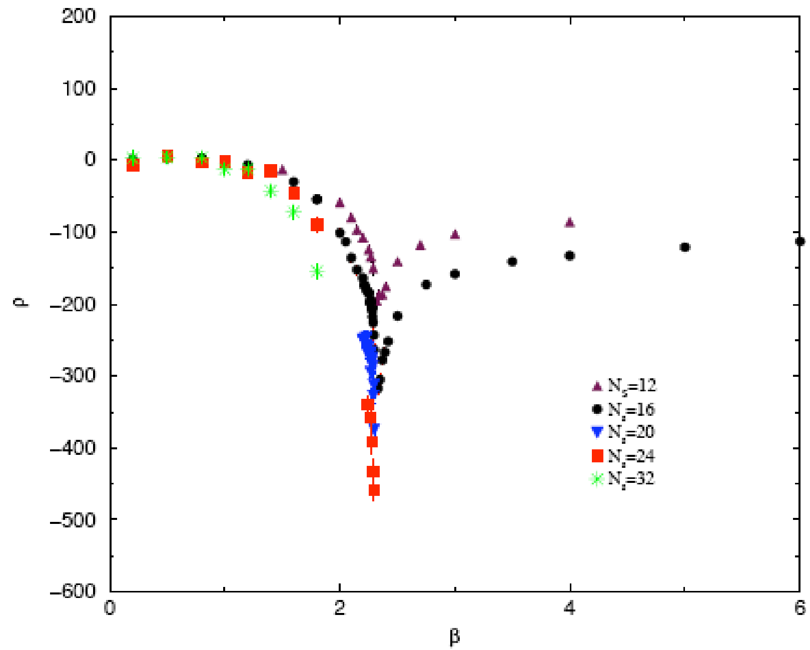


FIG. 3. ρ as a function of β for different spatial sizes at fixed $N_t = 4$. Plaquette projection.

Pure SU(2), N_T=4

Di Giacomo et al. (1999)

But what about the behavior of ρ near other types of transitions; e.g. in the gauge-Higgs model, at zero temperature?

Dual Gauge Symmetry Transitions

(Lucini & JG)

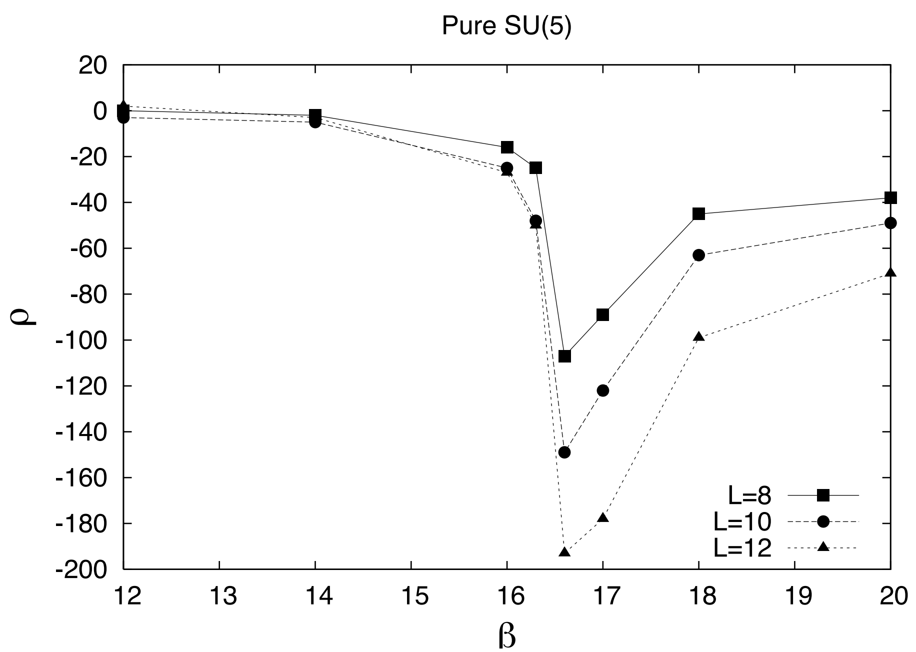
There is strong evidence of $\mu \to 0$ (dual symmetry restoration) transitions in the *absence* of any transition from a confining to a non-confining phase, and *even in the absence of any change of phase whatever.*

We find such $\mu \to 0$ transitions, at zero temperature, in

1. SU(5) gauge theory
2. mixed fundamental-adjoint SU(2) gauge theory
3. pure SU(2) (Wilson action)
4. gauge-Higgs theory
5. G(2) gauge theory (Cossu et al.)

Example 1 - SU(5)

Pure SU(5) gauge theory is known to have a first order transition around $\beta = 16.3$. And there, as it turns out, $\mu \to 0$. But the transition is *not* deconfining.

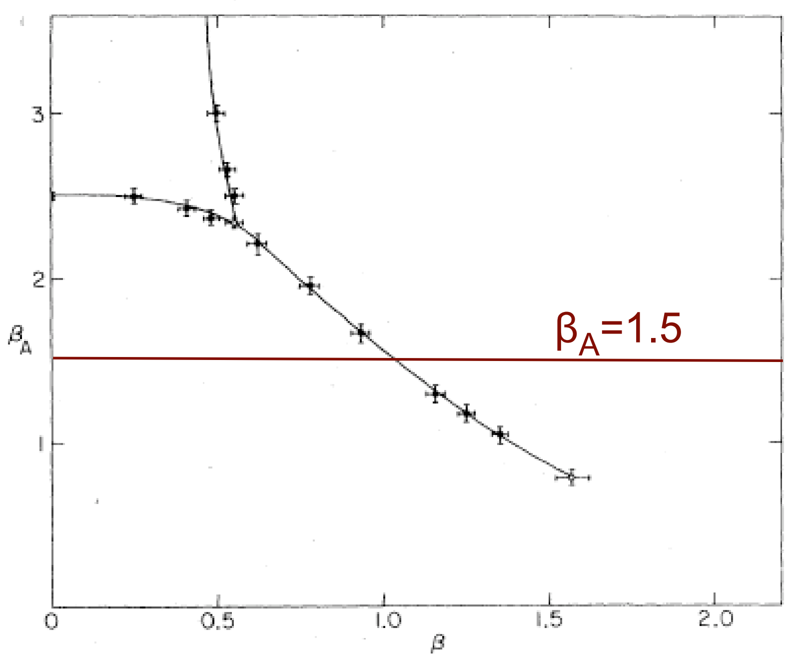


Example 2 - SU(2) mixed action

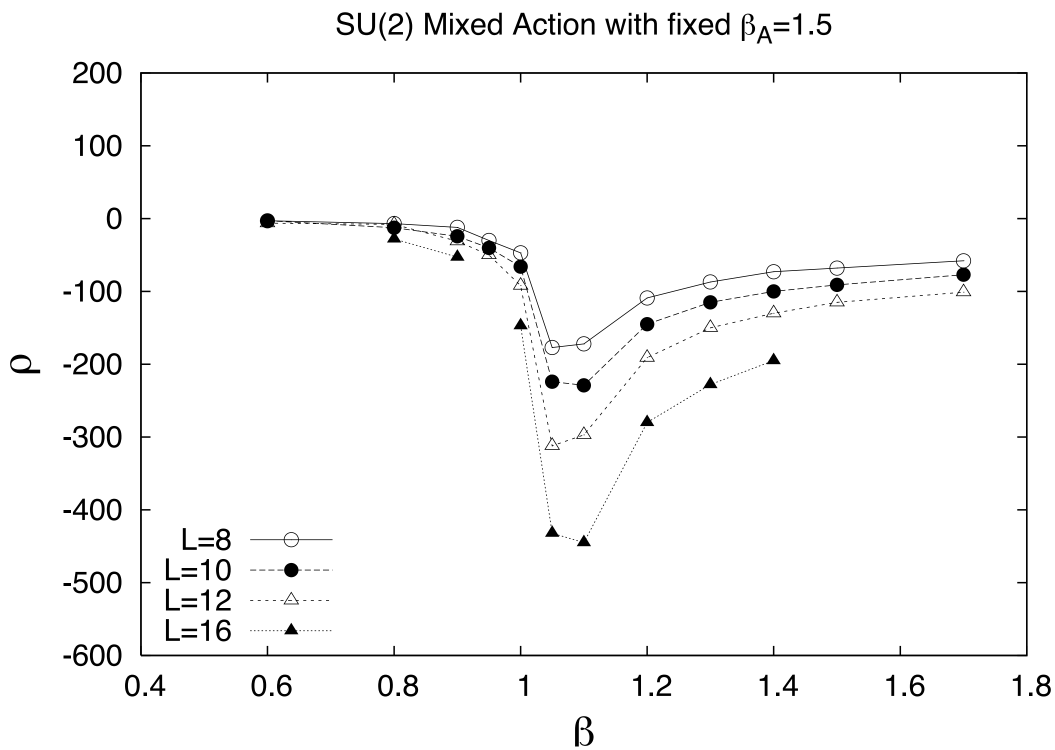
The mixed fundamental adjoint action is

$$S = \beta \sum \frac{1}{2}\mathrm{Tr}[U(P)] + \beta_A \sum \frac{1}{2}\mathrm{Tr}_A[U(P)]$$

and many years ago Creutz and Bhanot found the phase structure

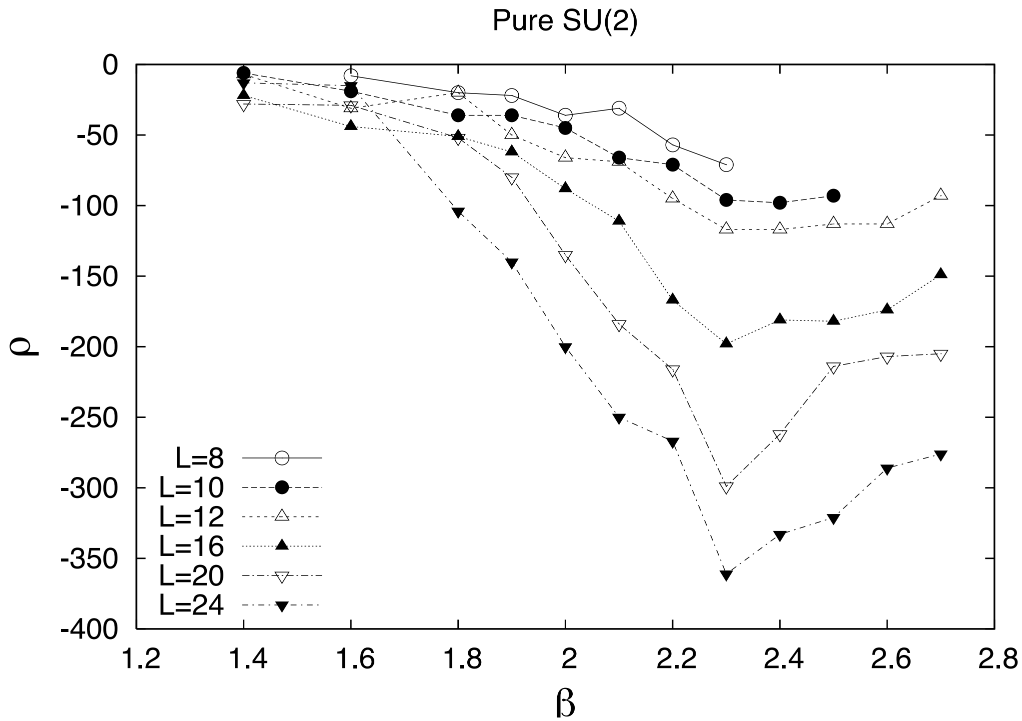


There is a $\mu \to 0$ transition along the (non-deconfining) Bhanot-Creutz transition line



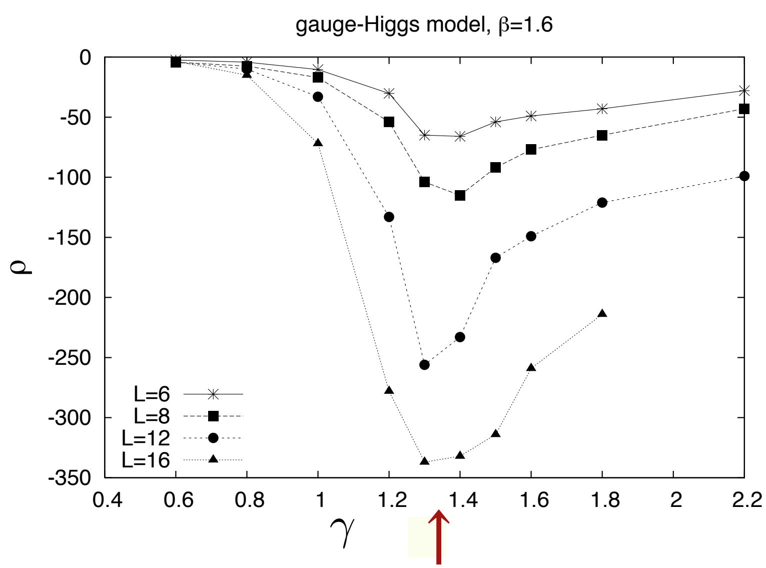
Example 3 - pure SU(2) Wilson action

There even appears to be a $\mu \rightarrow 0$ transition in pure (Wilson action) SU(2) LGT, although it requires rather large volumes to see the (much broader) peak at $\beta=2.3$.

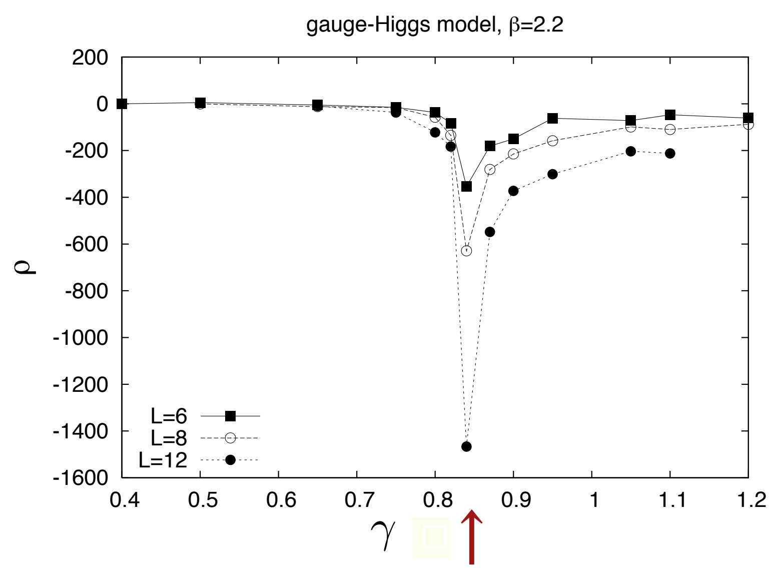


Example 4 - SU(2) gauge-Higgs action

We also find μ → 0 transitions in the gauge-Higgs model, where the Fradkin-Shenker theorem tells us that the phase diagram is connected.



β=1.6, γ=1.3
off the thermodynamic transition line



β=2.2, γ=0.84
on the thermodynamic transition line

Example 5 - G(2) gauge theory

Apart from us: the Pisa group themselves have found a $\mu \to 0$ transition, at zero temperature, in G(2) lattice gauge theory.

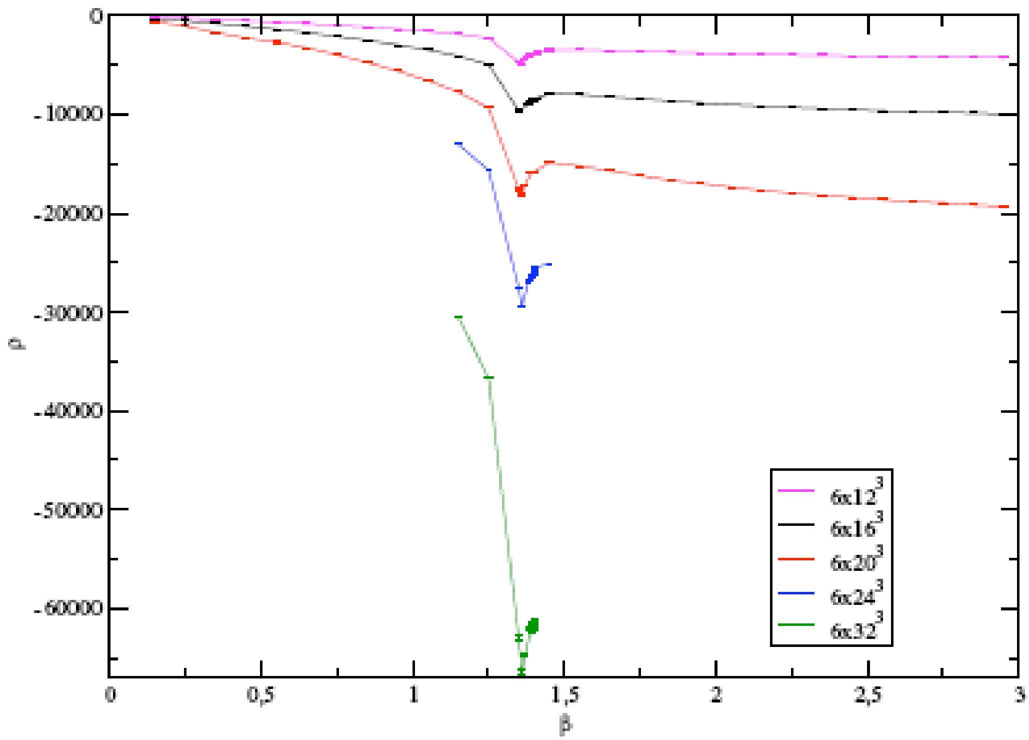


Figure 3: Results on the ρ parameter at different lattice volumes, strong dip occours at the bulk transition point.

Cossu, D'Elia, Di Giacomo, Lucini, Pica (2006)

Is it possible to find some other order parameter μ' such that

i) $\mu'(\beta) = \mu(\beta)$ at large $\beta \geq \beta_{large}$

ii) $\mu'(\beta)$ has no transition

?

Answer: No! Given (i), a transition $\mu \to 0$ *guarantees* a transition in μ'. Here's why:

$$\log[\mu(\beta_{large})] = \lim_{V\to\infty} \int_0^{\beta_{large}} d\beta \;\rho(\beta) + \log[\mu(0)]$$

If $\mu(\beta_{large})=0$ then, by (i), also $\mu'(\beta_{large})=0$.

All that can happen is that, since $\rho'(\beta) \neq \rho(\beta)$, the transition may happen at a different $\beta < \beta_{large}$, the peak may be broader, and the transition harder to see at small (or even fairly large) volumes.

to summarize

Global gauge symmetries associated with the

a) Kugo-Ojima
b) Coulomb confinement
c) Dual superconductor

scenarios are found to have transitions where there is no transition to/from a confinement phase, and even where there is no change of phase whatever.

in consequence

Global gauge symmetries do not seem to provide us with good order parameters for confinement.

So, what's in a name?

If "confinement" means:

color-singlet spectrum

then there is probably no meaningful distinction between the confined and Higgs phases, at least in terms of symmetries

But there is a difference in physics! *Flux tube formation, linear potential, Regge trajectories*....as opposed to a Yukawa potential.

If we focus on these, rather than on color neutrality, then it is better to say that confinement is the phase of

magnetic disorder

Then there *is* a relevant associated symmetry.

Magnetic Disorder

means: the existence of vacuum fluctuations strong enough to induce an area law falloff in Wilson loops at arbitrarily large scales.

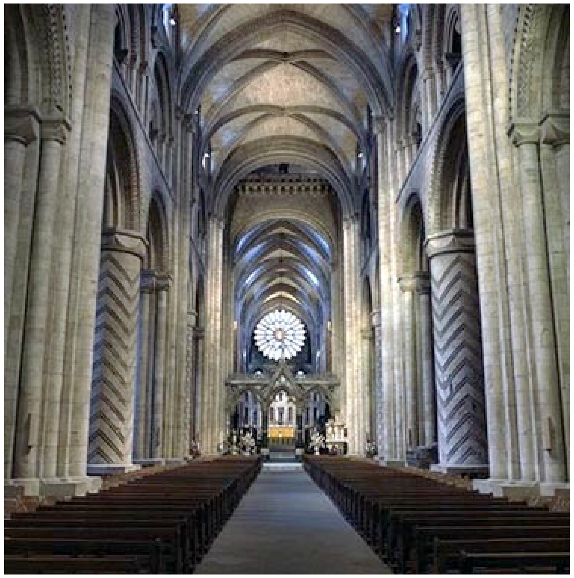
The vacuum of the gauge-Higgs theory satisfies this condition only as $\gamma \to 0$, and there is a symmetry which distinguishes $\gamma = 0$ from $\gamma > 0$:

Magnetic Disorder

means: the existence of vacuum fluctuations strong enough to induce an area law falloff in Wilson loops at arbitrarily large scales.

The vacuum of the gauge-Higgs theory satisfies this condition only as $\gamma \rightarrow 0$, and there is a symmetry which distinguishes $\gamma = 0$ from $\gamma > 0$:

Center Symmetry



When center symmetry is broken, either spontaneously

deconfinement
adjoint rep matter fields

or explicitly,

fundamental rep matter fields

or doesn't exist

G(2) gauge group

magnetic disorder is lost.

The traditional order parameters for confinement test center symmetry:

A. finite asymptotic string tension $\sigma > 0$ (implies linear potential)

$$W(C) = \left\langle P \exp[i \oint_C dx^\mu A_\mu] \right\rangle \sim \exp[-\sigma \text{Area}(C)]$$

B. vanishing Polyakov lines (isolated charge has infinite energy)

$$P(\vec{x}) = \left\langle P \exp[i \int_0^T dt \ A_0(\vec{x},t)] \right\rangle = 0$$

C. 't Hooft loop (center vortex creation operator)

$$B(C) \sim \exp[-\mu \text{Perimeter(C)}]$$

D. center vortex free energy:

if $\quad F_v = L_z L_t \exp[-\sigma' L_x L_y] \quad$ then $\quad \sigma \geq \sigma'$

None of these conditions are satisfied if global center symmetry is broken, either spontaneously or explicitly.

Question:

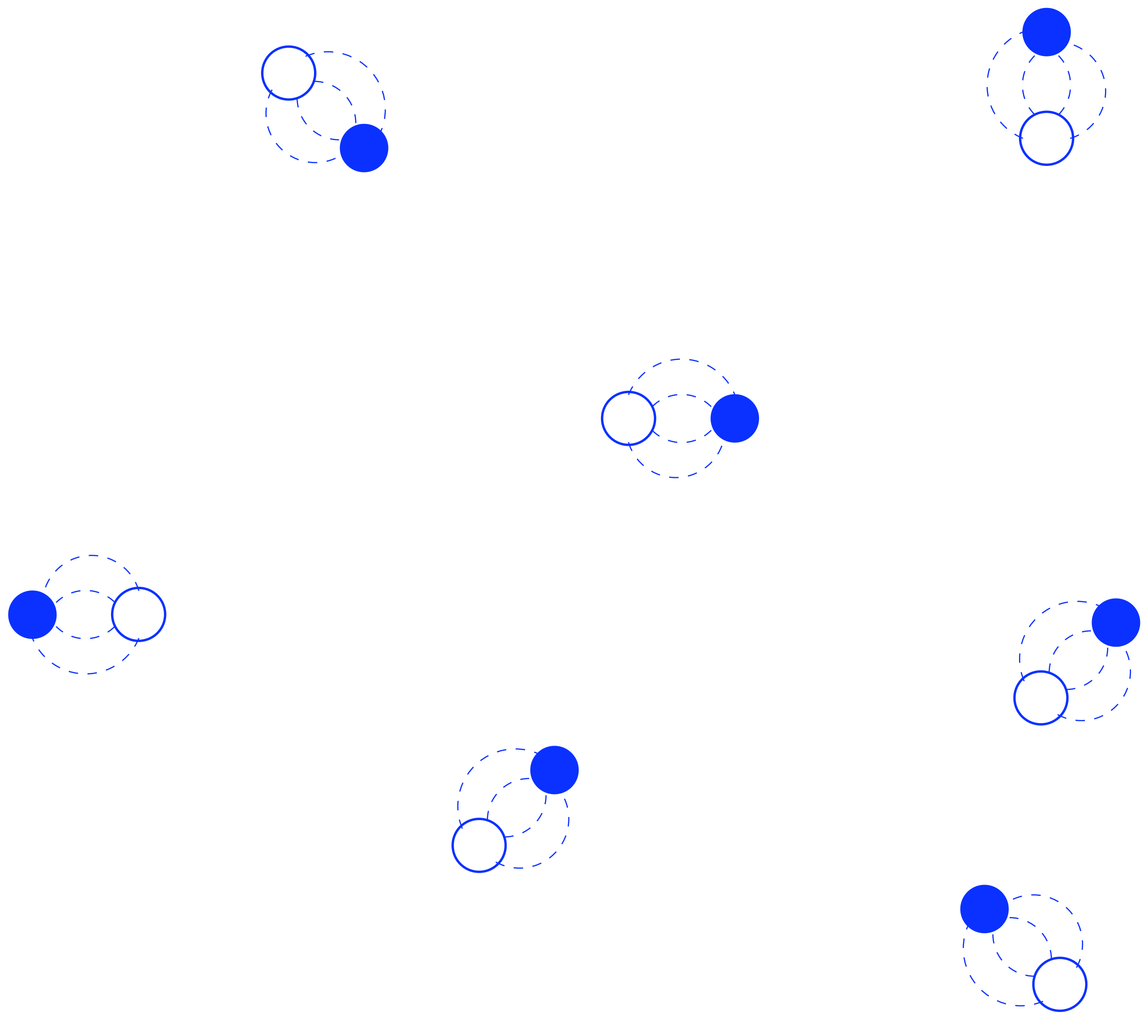
"If the center is so important, then what confines gluons?"

spectrum sense!

Answer:

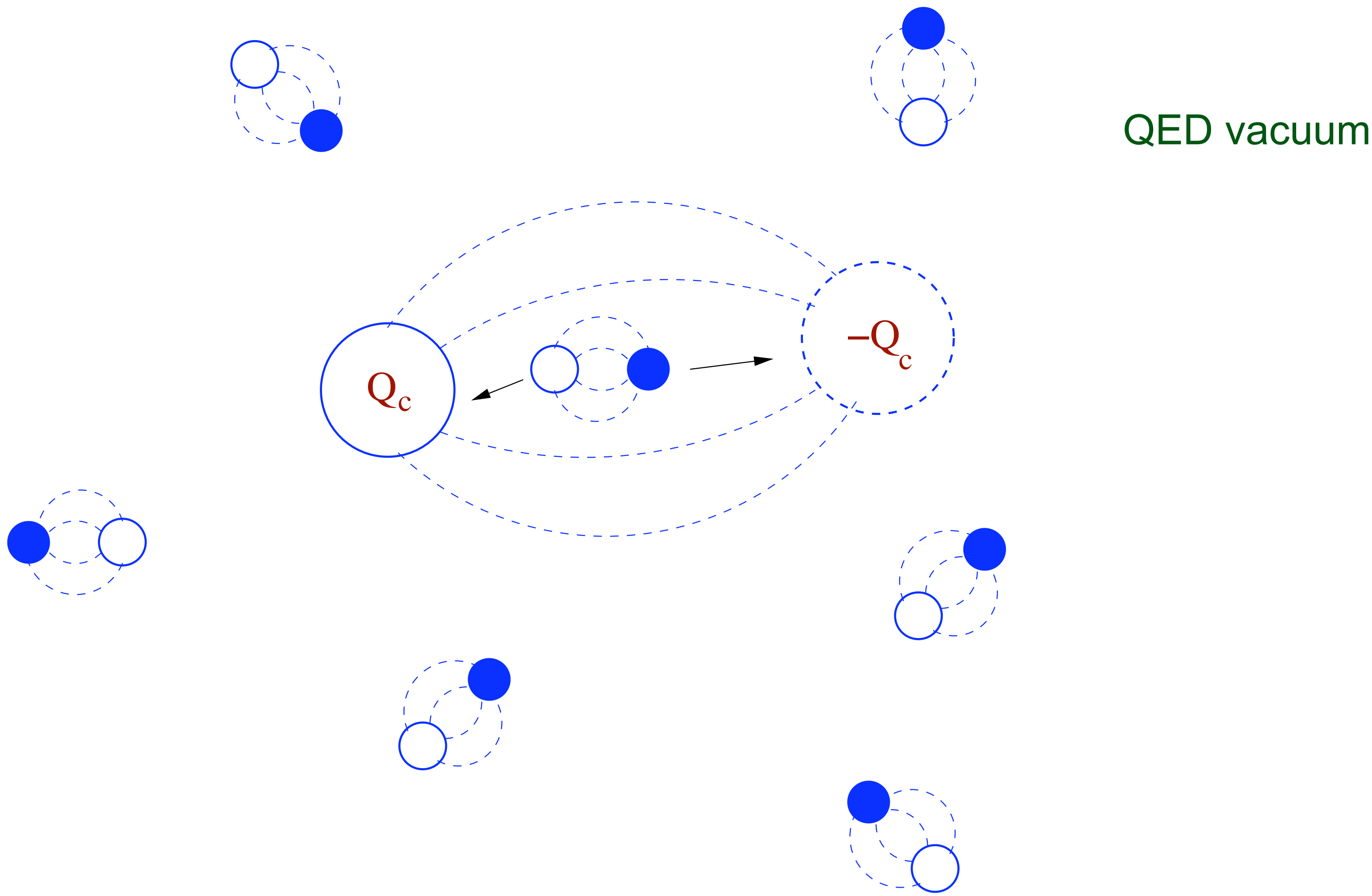
The same thing that "confines" large electric charge in QED.

In QED it is impossible to have an object of nuclear size having an electric charge much greater than $|Q_c| \approx 170$.

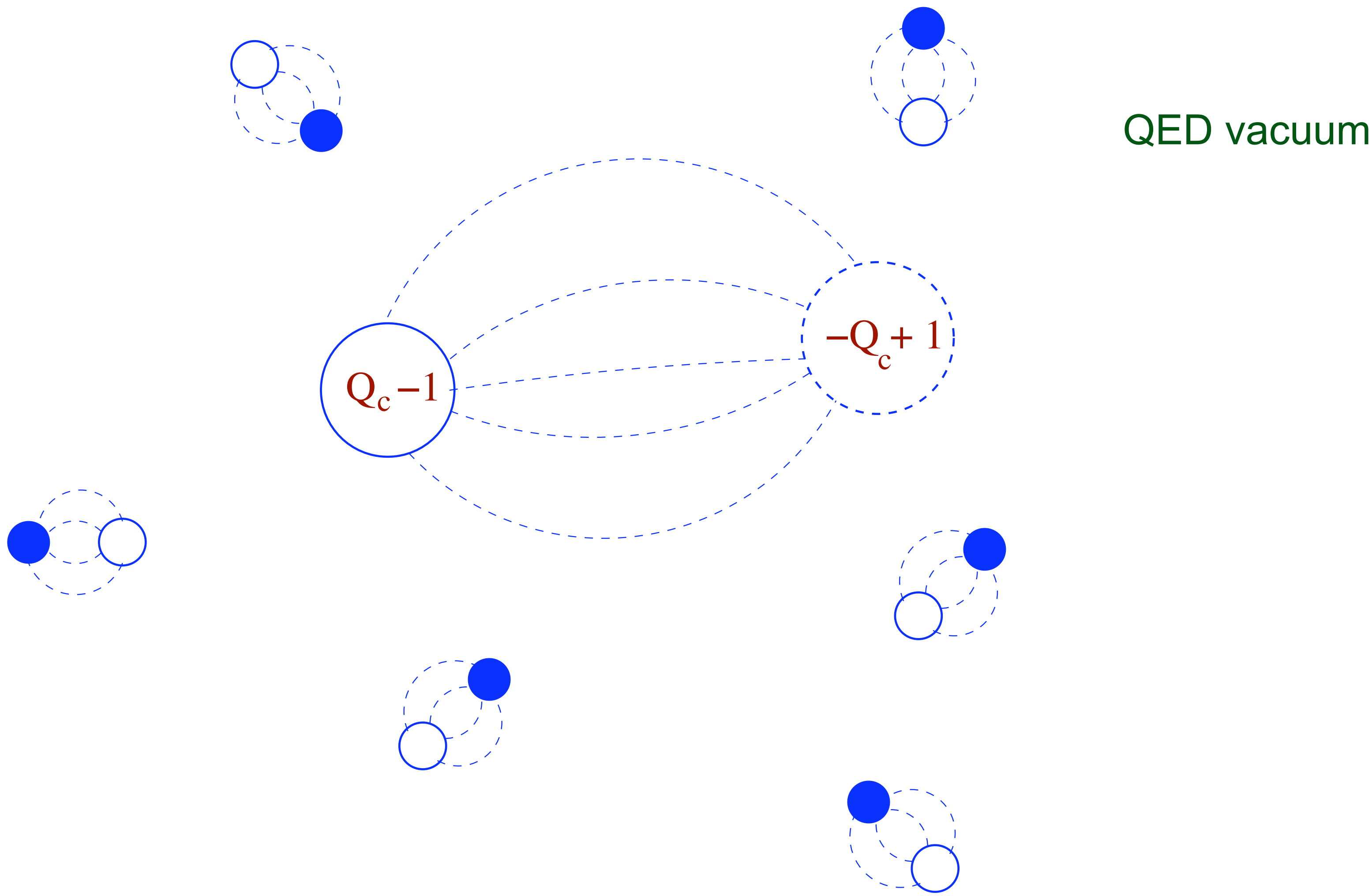


QED vacuum

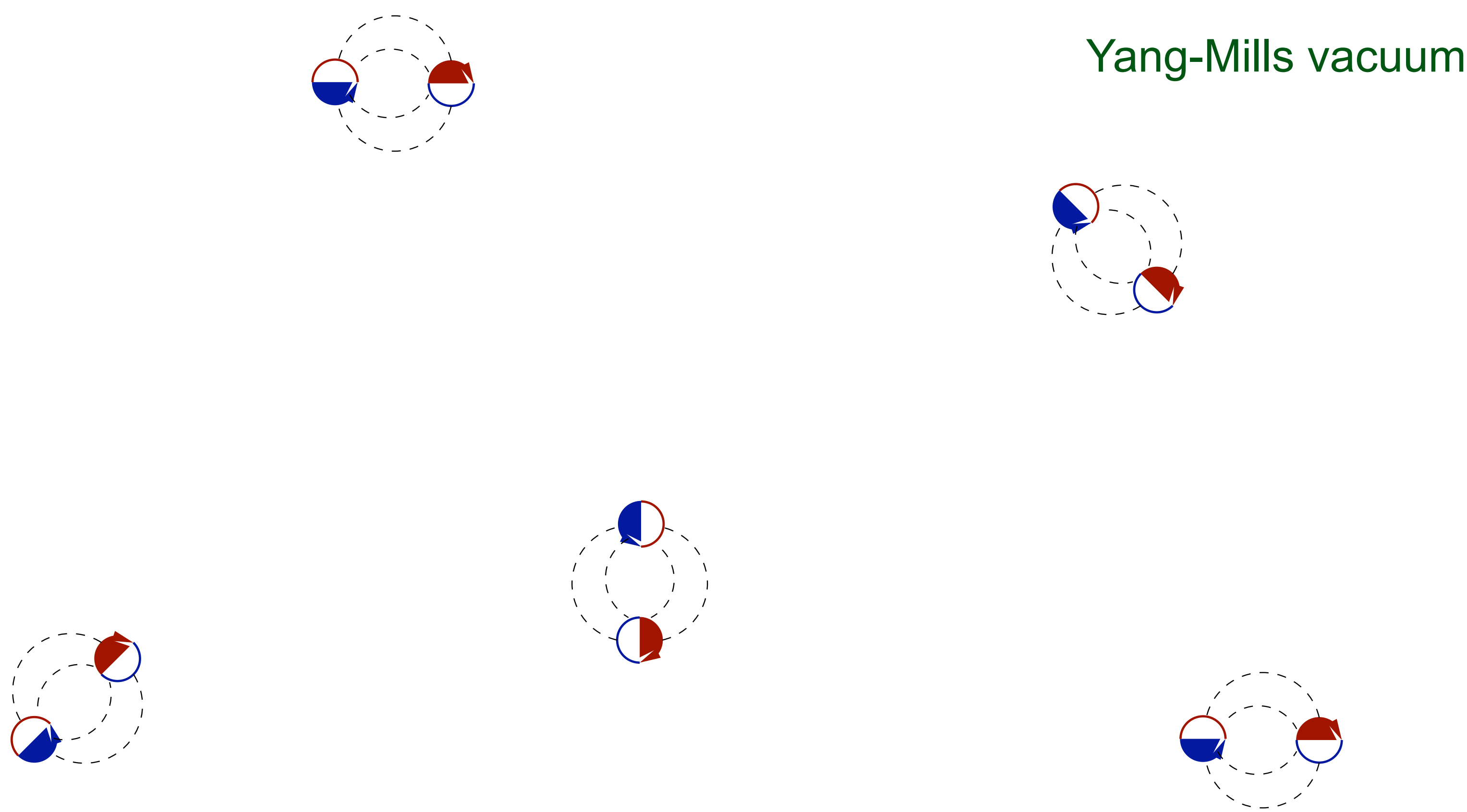
In QED it is impossible to have an object of nuclear size having an electric charge much greater than $|Q_c| \approx 170$.



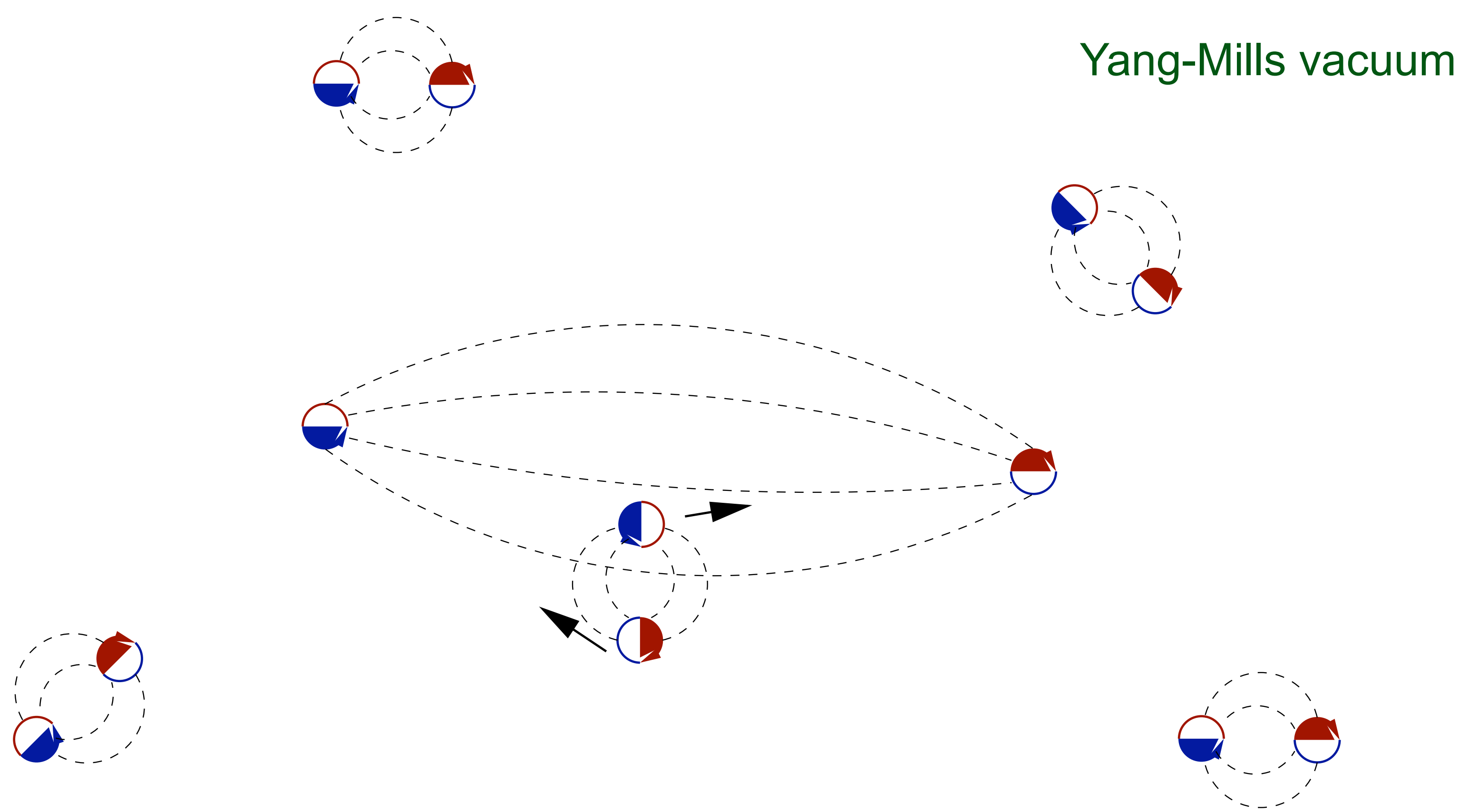
In QED it is impossible to have an object of nuclear size having an electric charge much greater than $|Q_c| \approx 170$.



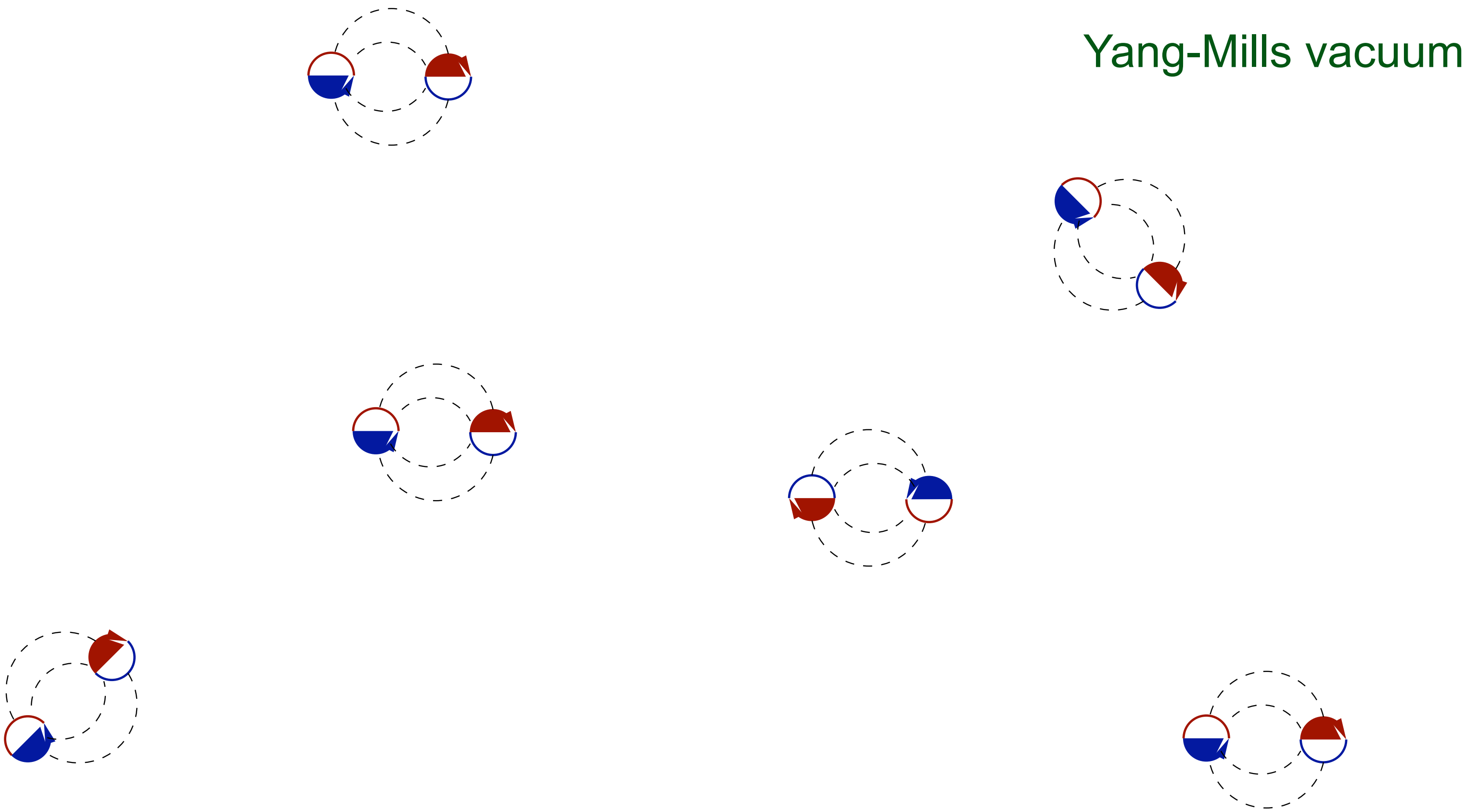
The same process goes on for adjoint charges in non-abelian theories, given sufficient charge separation



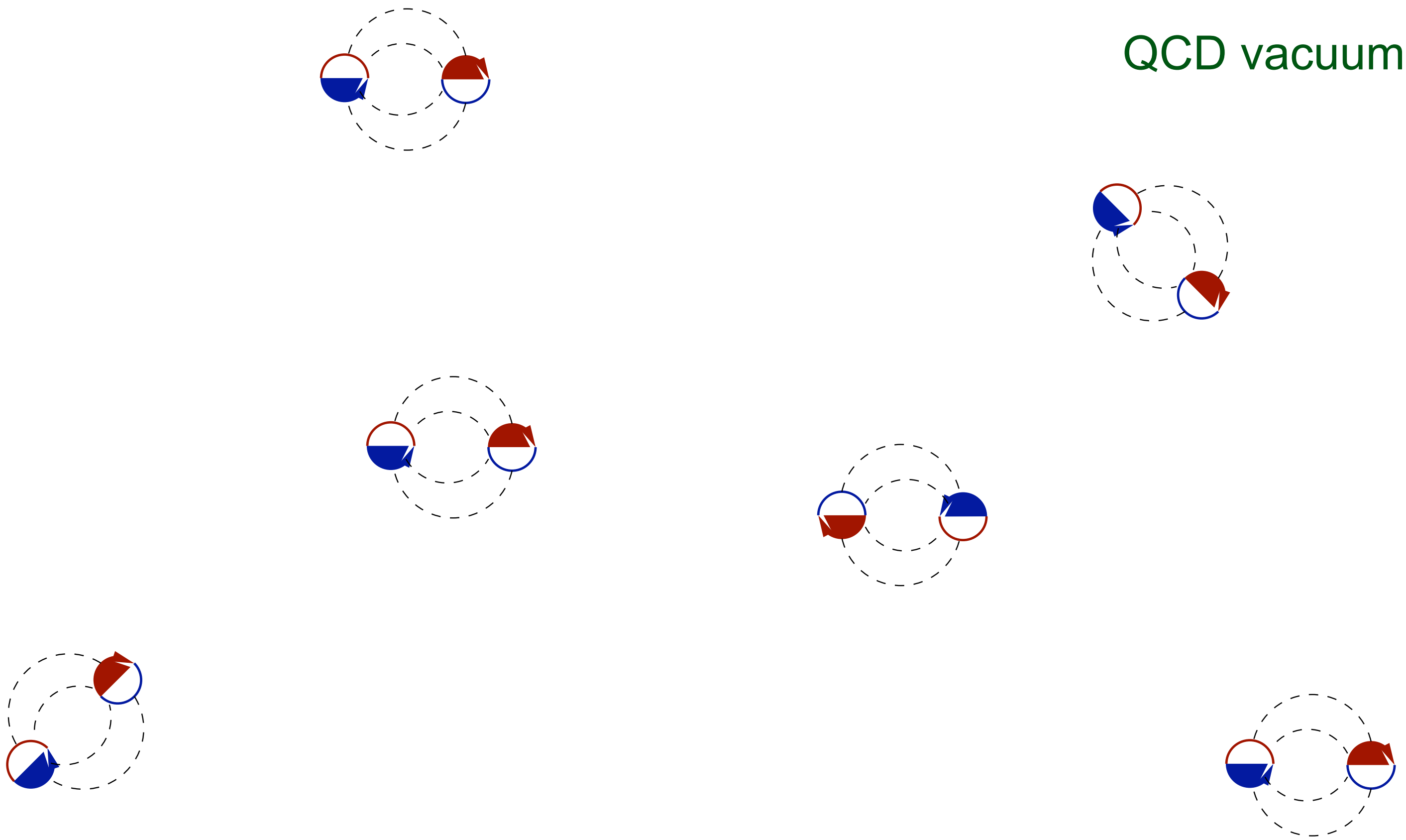
The same process goes on for adjoint charges in non-abelian theories, given sufficient charge separation



The same process goes on for adjoint charges in non-abelian theories, given sufficient charge separation



The same process goes on for adjoint charges, with sufficient separation



I prefer to call this “color screening”, rather than color “confinement”.

Group Disorder and Center Disorder

K. Langfeld, S. Olejnik, H. Reinhardt, T. Tok, & J.G.   (2006)

Representation dependence of the string tension:

- Casimir scaling at intermediate distances;
- N-ality dependence asymptotically.

The latter is due to string-breaking by gluons and/or matter fields. No big mystery.

But this is a “particle” explanation...

What is the “field” explanation, for both Casimir *and* N-ality behavior, in terms of vacuum fluctuations which dominate the relevant functional integral?

Basic idea: In a surface slice, the vacuum is dominated by overlapping center domains on some scale R. Fluctuations within each domain (beyond some confinement scale r_c) are subject only to the weak constraint that the total magnetic flux adds up to a center element of the gauge group.

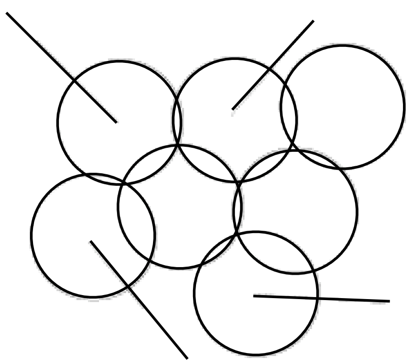


FIG. 1: A 2D slice of the D=4 Yang-Mills vacuum. Circular regions with (Dirac) lines correspond to $z = -1$ domains, circular regions without lines denote $z = +1$ domains.

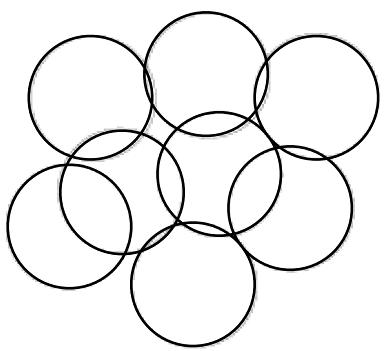
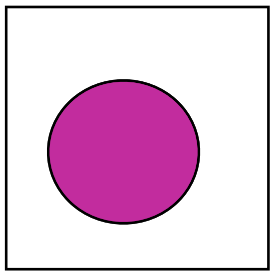


FIG. 2: A 2D slice of the D=4 vacuum of G(2) gauge theory. There is only one type of domain, corresponding to the single element of the center subgroup.

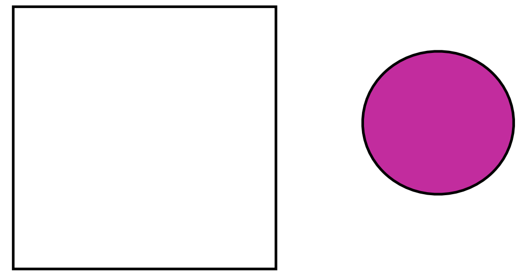
Fluctuations within a domain → Group disorder, Casimir scaling

Existence of domains → Center disorder, N-ality

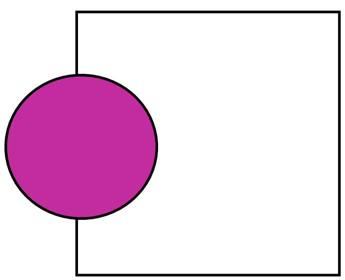
A simple model: center domain in the plane of a Wilson loop contributes a factor



$$z_n \in Z_N$$



$$z_0 = 1$$



$$\begin{aligned} z &= \bar{G}_r[\alpha^n] \\ &= \frac{1}{d_r}\chi_r\left[\exp[i\vec{\alpha}^n \cdot \vec{H}]\right] \end{aligned}$$

where χ_r is the group character, $\vec{H}$ the generators of the Cartan subalgebra, and the $\vec{\alpha}^n$ depend on the overlap of the domain with the interior of the loop.

$\bar{G}_r[\alpha]$ is proportional to the quadratic Casimir for small α, and goes to a center element (which may be $z_0 = 1$) for enclosed domains.

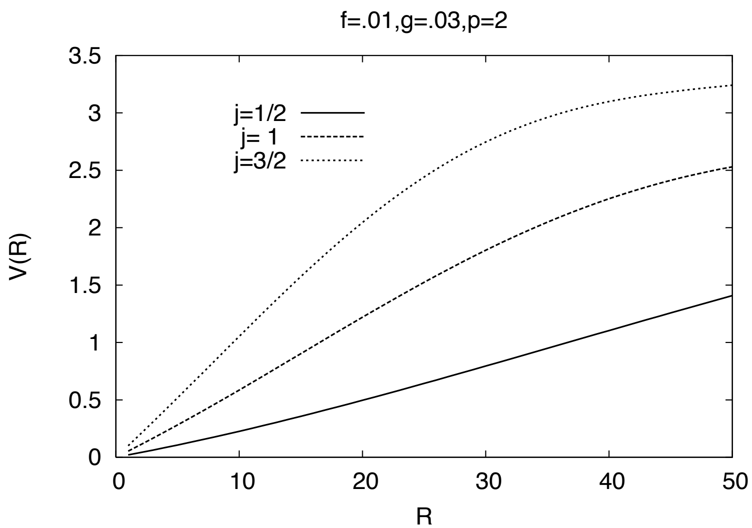
$\vec{\alpha} \cdot \vec{H}$ represents the average magnetic flux in the overlap region of loop and domain.

We suppose that fluctuations in different regions of each domain are correlated only by the constraint that they add up to center element. If A_D is the area of the domain, and A is the area contained in the loop, then for SU(2) we get

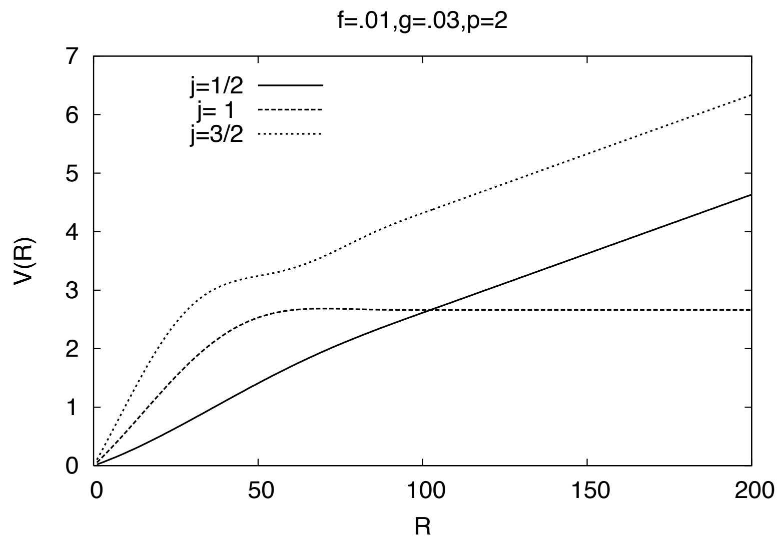
$$\begin{aligned} \left(\alpha^1(x)\right)^2 &= \text{const.}\left[\frac{A}{A_D} - \frac{A^2}{A_D^2}\right] + \left(2\pi\frac{A}{A_D}\right)^2 \\ \left(\alpha^0(x)\right)^2 &= \text{const.}\left[\frac{A}{A_D} - \frac{A^2}{A_D^2}\right] \end{aligned}$$

Difference between G(2) and SU(2): G(2) has only one type of center domain, only α^0 contributes, string tension is asymptotically zero.

For SU(2), the domain model gives results for the static potential like these:



Casimir scaling
(short distance)



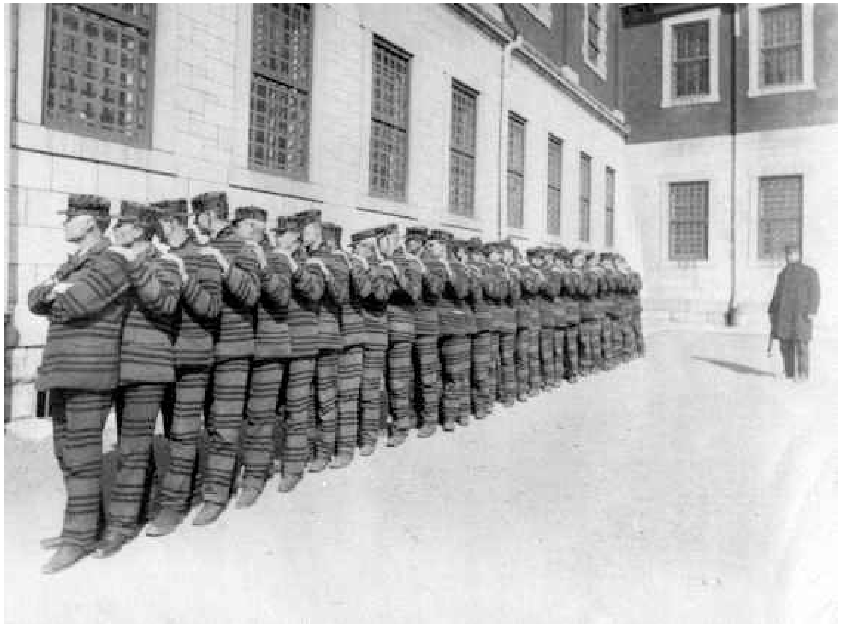
Color screening
(asymptotic)

Conclusions

If "confinement" means:

color-singlet spectrum

then there is probably no meaningful distinction between the confined and Higgs phases, at least in terms of global gauge symmetries



These symmetries show transitions in the wrong places!

But, if confinement means

magnetic disorder

Then the relevant symmetry is center symmetry.

A Question

to people who compute ghost/gluon propagators on the lattice, and who see, e.g., an infrared enhanced ghost propagator:

what happens to κ in the gauge-Higgs coupling plane?
is there a transition at the remnant-symmetry transition?