Gribov confinement scenario 30 years later: status and perspectives

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Abstract

The infrared behavior of gluon and ghost propagators should be closely related to confinement in Yang-Mills theories. A nonperturbative study of these propagators from first principles is possible in lattice simulations, but one must consider significantly large lattice sizes in order to approach the infrared limit. We present data obtained for pure SU(2) gauge theory in Landau gauge, using the largest lattice sizes to date. We propose constraints based on the properties of the propagators as a way to gain control over the extrapolation of our data to the infinite-volume limit.

IR gluon propagator and confinement

- Green's functions carry all information of a QFT's physical and mathematical structure.
- Gluon propagator (two-point function) as the most basic quantity of QCD.
- Confinement given by behavior at large distances (small momenta) $\Rightarrow$ nonperturbative study of IR gluon propagator.

Landau gluon propagator

$$
\begin{aligned}
D^{ab}_{\mu\nu}(p) &= \frac{1}{V} \sum_{x,y} e^{-2i\pi k\cdot(x-y)} \langle A^a_\mu(x)\, A^b_\nu(y) \rangle \\
&= \delta^{ab} \left(g_{\mu\nu} - \frac{p_\mu\, p_\nu}{p^2} \right) D(p^2)
\end{aligned}
$$

IR gluon propagator and confinement (II)

- Gribov-Zwanziger confinement scenario (1978–) in Landau gauge predicts a gluon propagator $D(p^2)$ suppressed in the IR limit.
- In particular, $D(0) = 0$ implying that reflection positivity is maximally violated.
- This result may be viewed as an indication of gluon confinement.
- On large lattice volumes the gluon propagator decreases in the limit $p \to 0$, but $D(0) > 0$.

Can one find $D(0) = 0$ in lattice simulations? Yes in 2d (A. Maas) using lattices up to $(42.7fm)^2$. What about 4d and 3d?

Confining gluons

From the Wilson loop

$$W \equiv \langle Tr\, \mathcal{P} \exp\left[i\, g_0 \oint dx_\mu\, A_\mu(x) \right] \rangle$$

we can find the static QCD potential

$$V(r) = \lim_{t \to \infty} \left[-\frac{1}{t} \log\left(W_{t,r} \right) \right] .$$

If $W_{t,r} \sim \exp\left(-\sigma\, r\, t\right)$ (area law) then

$$V(r) \sim \sigma\, r\, .$$

One can prove (Seiler, 1978) that $V(r) \leq \sigma^{'}\, r$ and that (Zwanziger, 2003) there is no confinement without Coulomb confinement.

Confining gluons (II)

At the lowest order

$$W \;=\; 1 \,-\, \frac{g_0^2}{2} \, \mathrm{Tr} \oint dx_\mu \oint dy_\nu \, \langle \, A_\mu(x) \, A_\nu(y) \, \rangle \,+\, \ldots$$

Of course, the Wilson loop is a gauge-independent quantity, while the gluon propagator depends on the gauge.

Ax exact result (West, 1982):

$$W \;\leq\; \exp \left[- \frac{g_0^2}{2} \, \delta_{ab} \oint dx_\mu \oint dy_\nu \, D^{ab}_{\mu\nu}(x-y) \right] .$$

If in some gauge and for small momenta $D(k) \sim k^{-4}$, then $D(r) \sim 1$ and we obtain an area law.

IR ghost propagator and confinement

Ghost fields are introduced as one evaluates functional integrals by the Faddeev-Popov method, which restricts the space of configurations through a gauge-fixing condition. The ghosts are unphysical particles, since they correspond to anti-commuting fields with spin zero.

On the lattice, the (minimal) Landau gauge is imposed as a minimization problem and the ghost propagator is given by

$$G(p) \;=\; \frac{1}{N_c^2 - 1} \sum_{x,\,y,\,a} \frac{e^{-2\pi i\, k\cdot(x-y)}}{V} \,\langle\, \mathcal{M}^{-1}(a,x;a,y)\,\rangle\;,$$

where the Faddeev-Popov matrix $\mathcal{M}$ is obtained from the second variation of the minimizing functional.

IR ghost propagator and confinement (II)

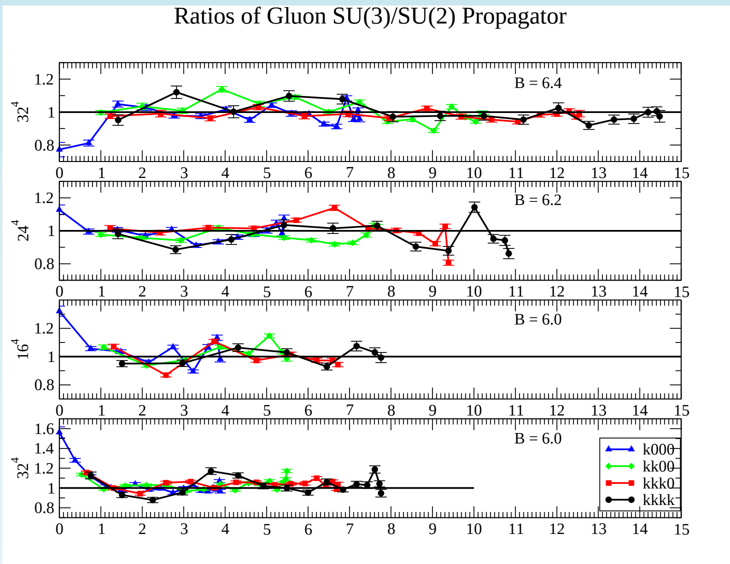
Gribov-Zwanziger confinement scenario: infinite volume favors configurations on the first Gribov horizon, where λ_{min} of $\mathcal{M}$ goes to zero. In turn, $G(p)$ should be IR enhanced, introducing long-range effects, related to the color-confinement mechanism. Large lattice sizes are needed to observe the predicted behavior.

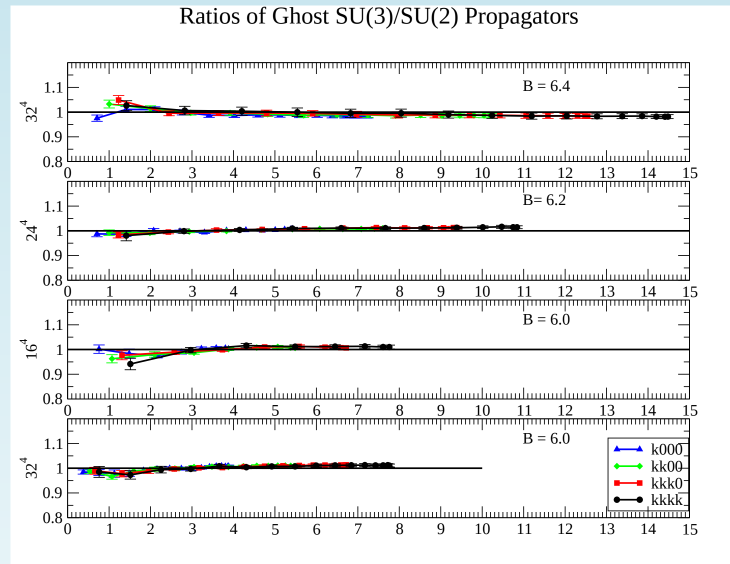
- Studies (with small lattices) in Landau and Coulomb gauge showed enhancement of $G(p)$.
- In MAG one finds an IR-finite $G(p)$.
- New results in Landau gauge on very large lattices seem to show no enhancement in the 3d and 4d cases.
- Enhancement is seen (A. Maas) in 2d.

Do we see an IR-enhanced Landau-gauge $G(p)$ in 3d and 4d?

$SU(2)$ vs. $SU(3)$

C., Mendes, Oliveira and Silva (2007)





Ratio $SU(3)/SU(2)$ for the Landau-gauge gluon propagator.

Ratio $SU(3)/SU(2)$ for the Landau-gauge ghost propagator.

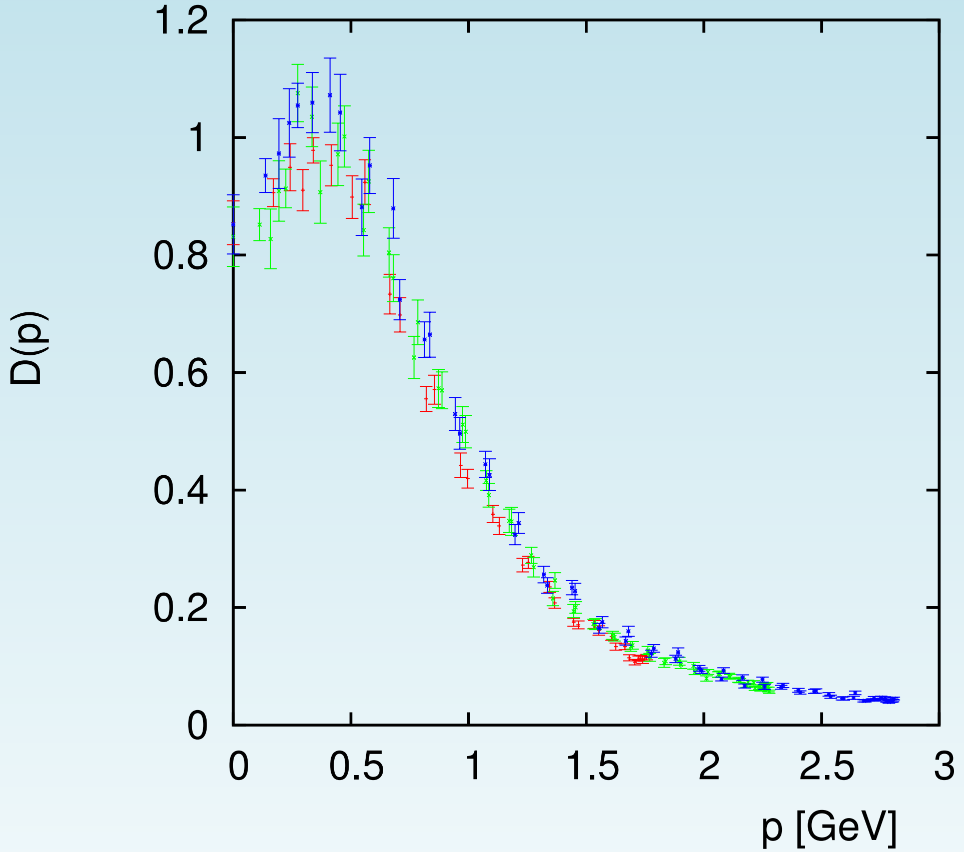
Results

for the Gluon Propagator

Gluon Propagator: status in 2000

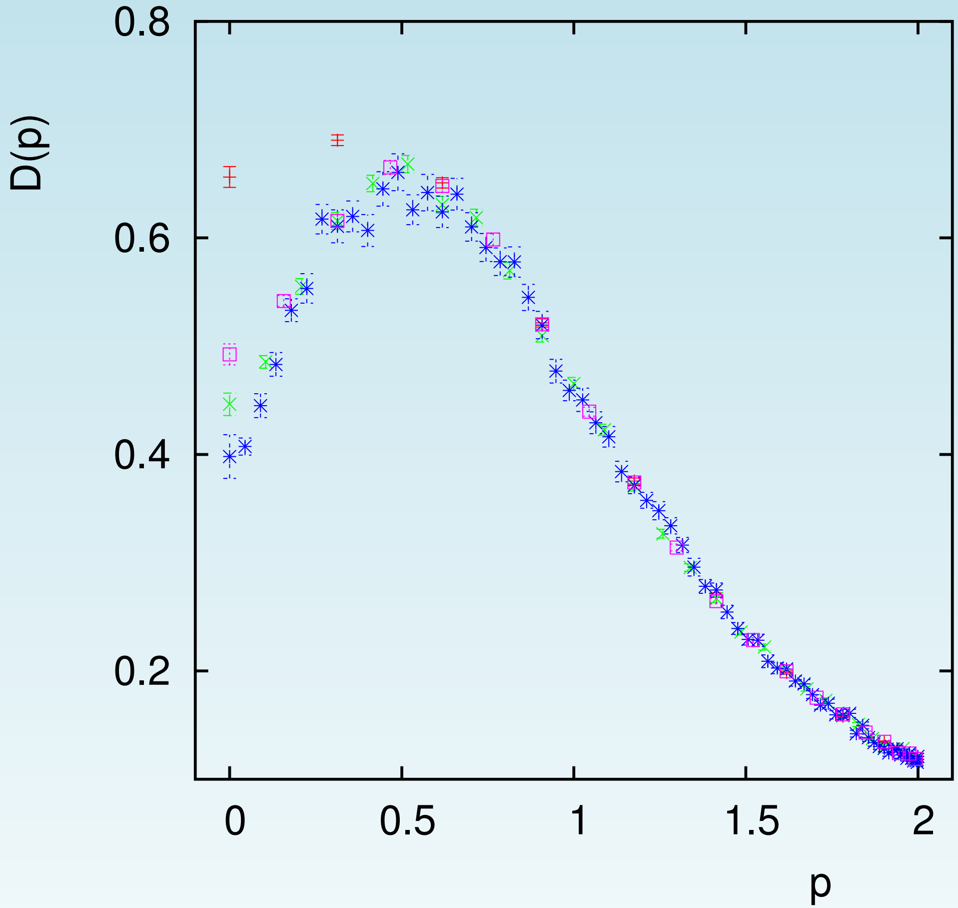
- Gribov noise for the gluon propagator is of the order of magnitude of the numerical accuracy (Heller et al., 1995; C., 1997).
- There are no finite-size effects at large momenta (study of the ultraviolet behavior: Williams et al., 1999; Becirevic et al., 1999).
- The gluon propagator is less singular than $p^{2-d}(k)$ in the infrared limit (C., 1999; Williams et al., 1999 & 2000).
- The gluon propagator decreases as the momentum goes to zero (C., 1997 & 1999; Nakajima and Furui, 1999).
- $D(0)$ decreases as the volume increases, but an extrapolation to infinite volume has never been attempted.

Infinite-volume limit in 3d (I)



Gluon propagator as a function of the lattice momentum p for $\beta = 3.4$ and 32^3 (+), $\beta = 4.2$ and 64^3 ($\times$), $\beta = 5.0$ and 64^3 ($*$) (C., 1999). About 100 days using a 0.5 Gflops workstation.

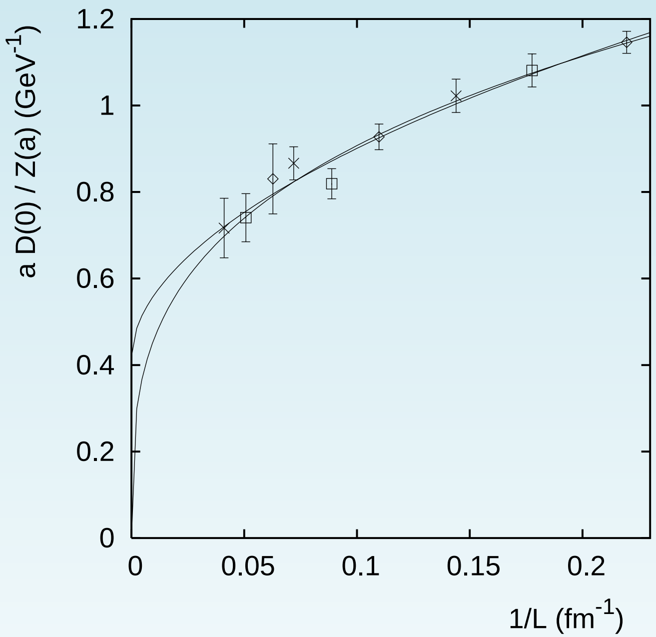
Infinite-volume limit in 3d (II)



Gluon propagator as a function of the lattice momentum p for lattice volumes $V = 20^3$, 40^3, 60^3 and 140^3 at $\beta = 3.0$ (C., Mendes and Taurines, 2003). About 100 days using a 13 Gflops PC cluster.

Old results in 3d

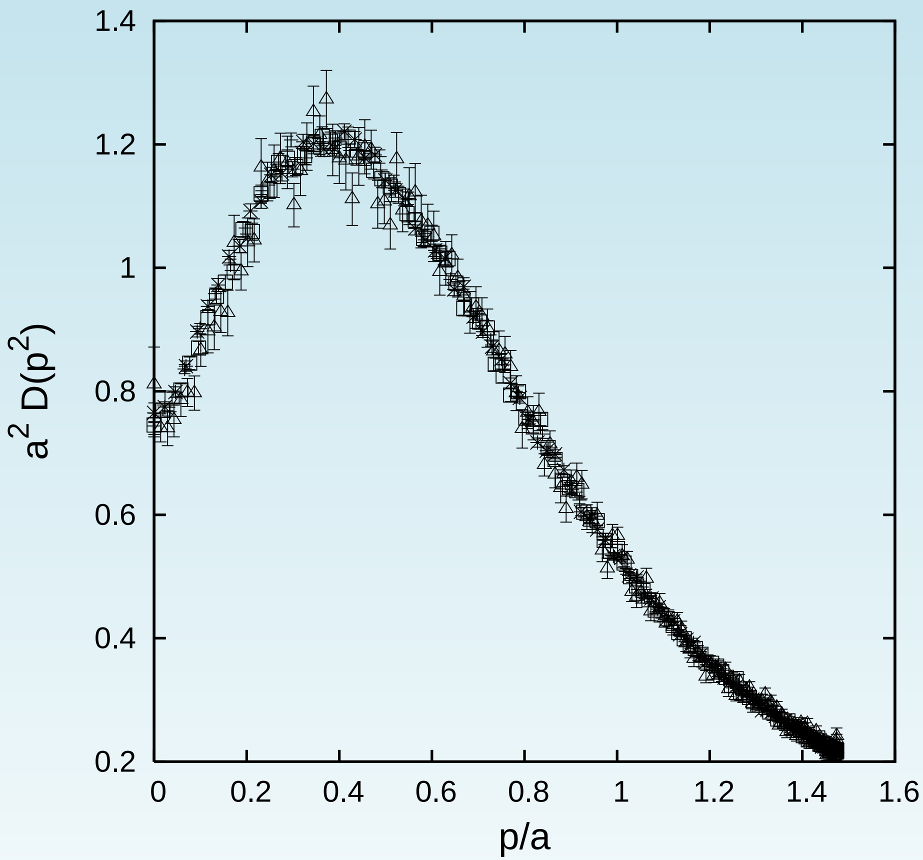
The gluon propagator using lattice volumes up to 140^3 and β values $4.2, 5.0, 6.0 \longrightarrow$ physical lattice sides almost as large as 25 fm.



Plot of the rescaled gluon propagator at zero momentum as a function of the inverse lattice side for $\beta = 4.2\,(\times),\ 5.0\,(\Box),\ 6.0\,(\Diamond)$. We also show the fit of the data using the Ansatz $d + b/L^c$ both with $d = 0$ and $d \neq 0$.

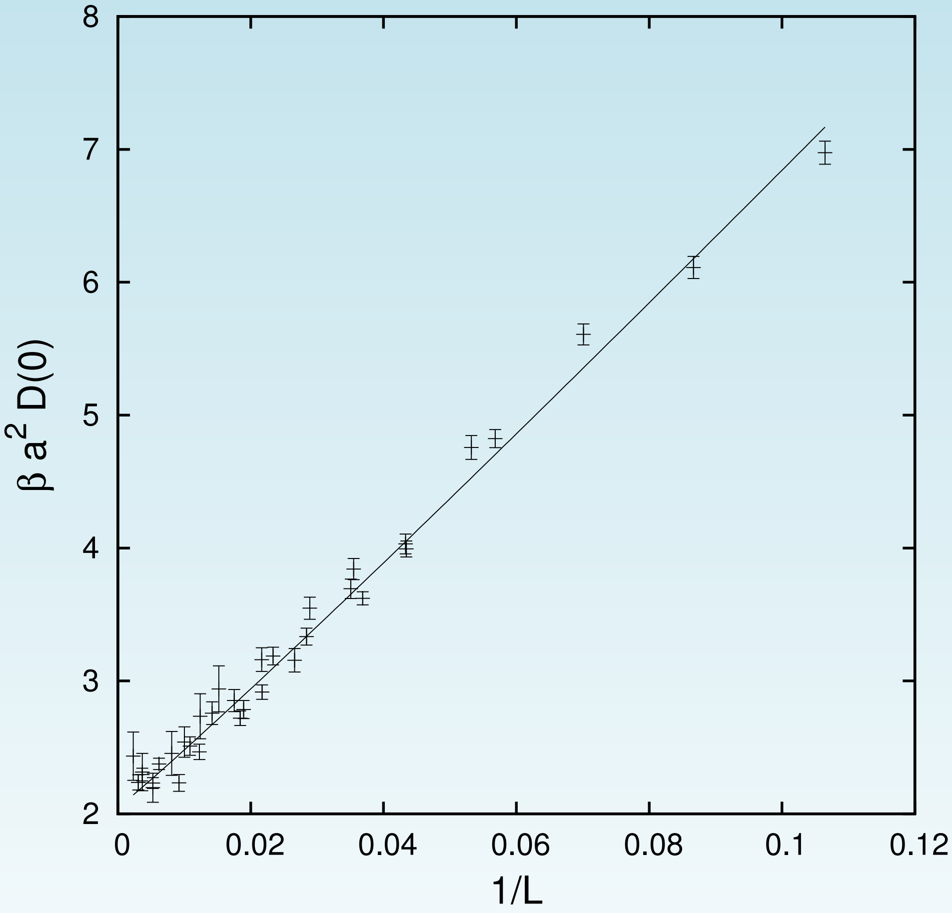
Can we go to even larger lattice volumes?

Infinite-volume limit in 3d (III)



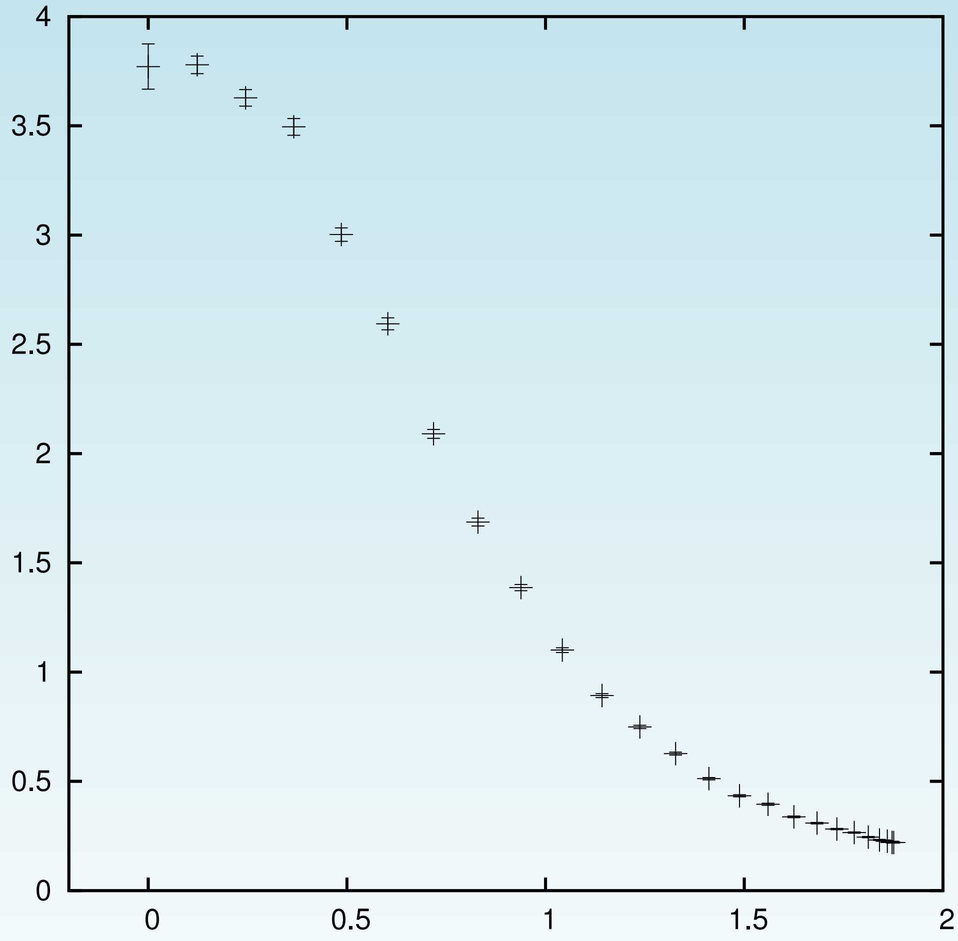
Gluon propagator as a function of the lattice momentum p including lattices of up to 320^3 in the scaling region. (C. and Mendes, 2007) About 5 days on a 4.5Tflops IBM supercomputer.

New data: infinite-volume limit in 3d



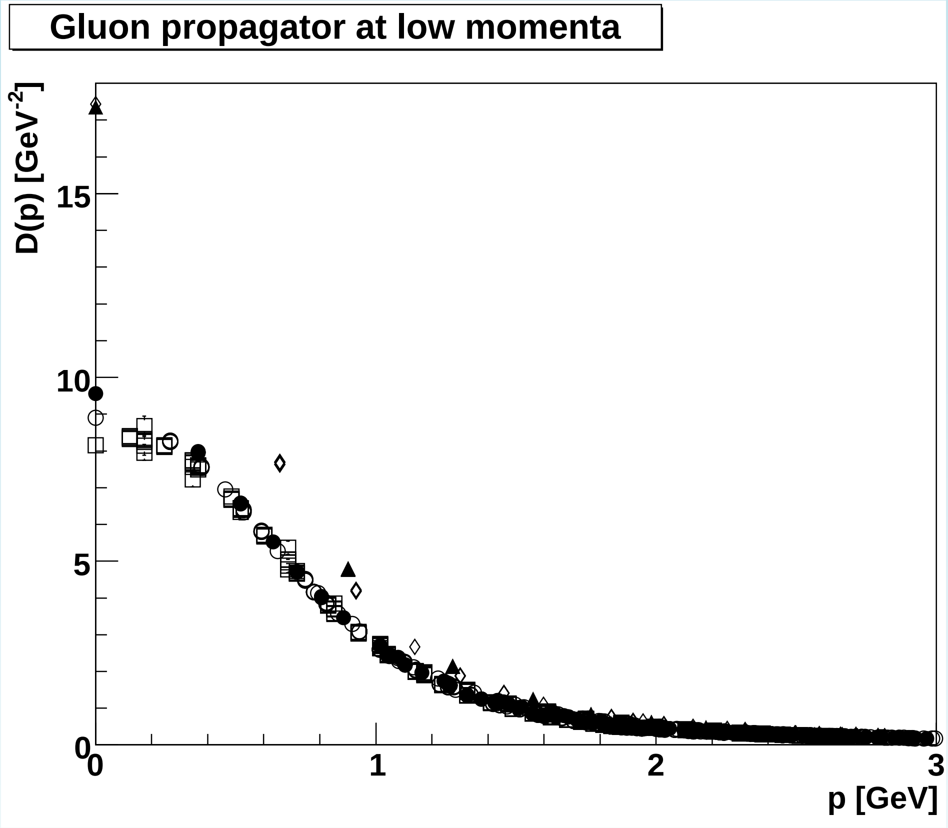
Gluon propagator at zero momentum as a function of the inverse lattice side $1/L$ (in fm^{-1}) and extrapolation to infinite volume. New data, up to 320^3 for $\beta = 3.0$.

Infinite-volume limit in 4d (I)



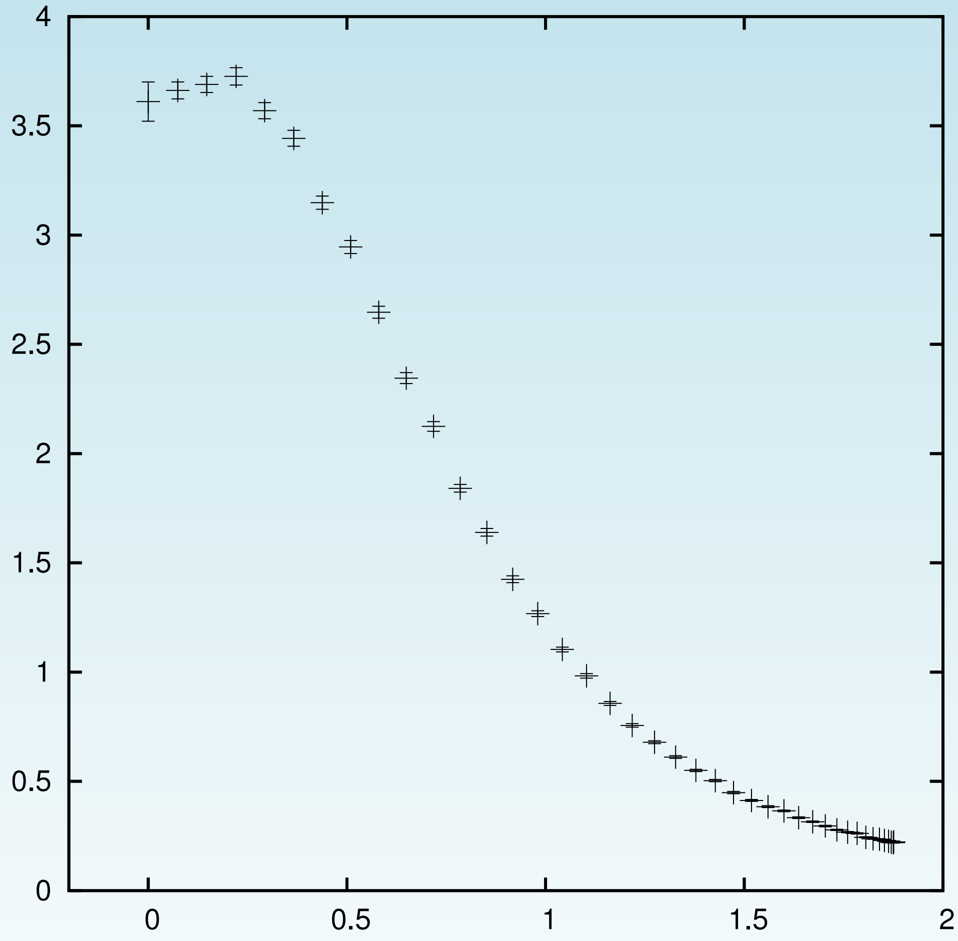
Gluon propagator as a function of the lattice momentum p for lattice volume $V = 48^4$ at $\beta = 2.2$.

Infinite-volume limit in 4d (I)



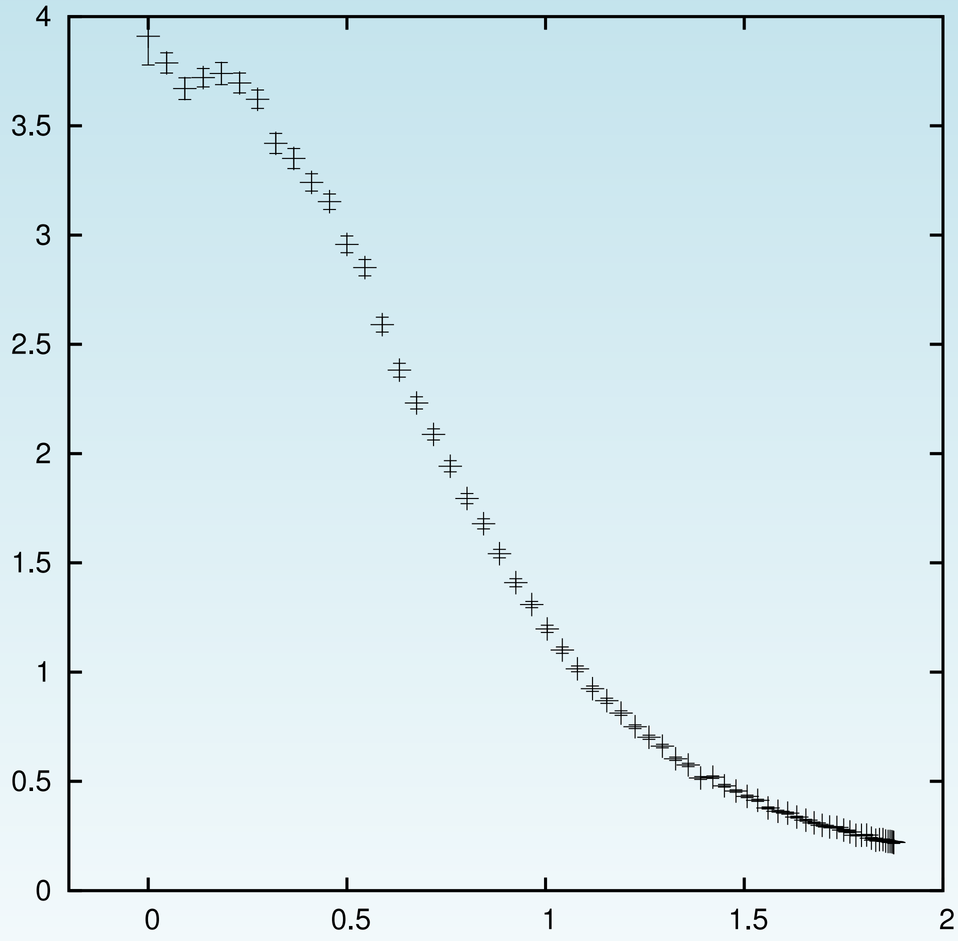
Gluon propagator as a function of the lattice momentum p for lattice volume $V = 48^4$ at $\beta = 2.2$ (new data C., Maas and Mendes, 2008).

Infinite-volume limit in 4d (III)



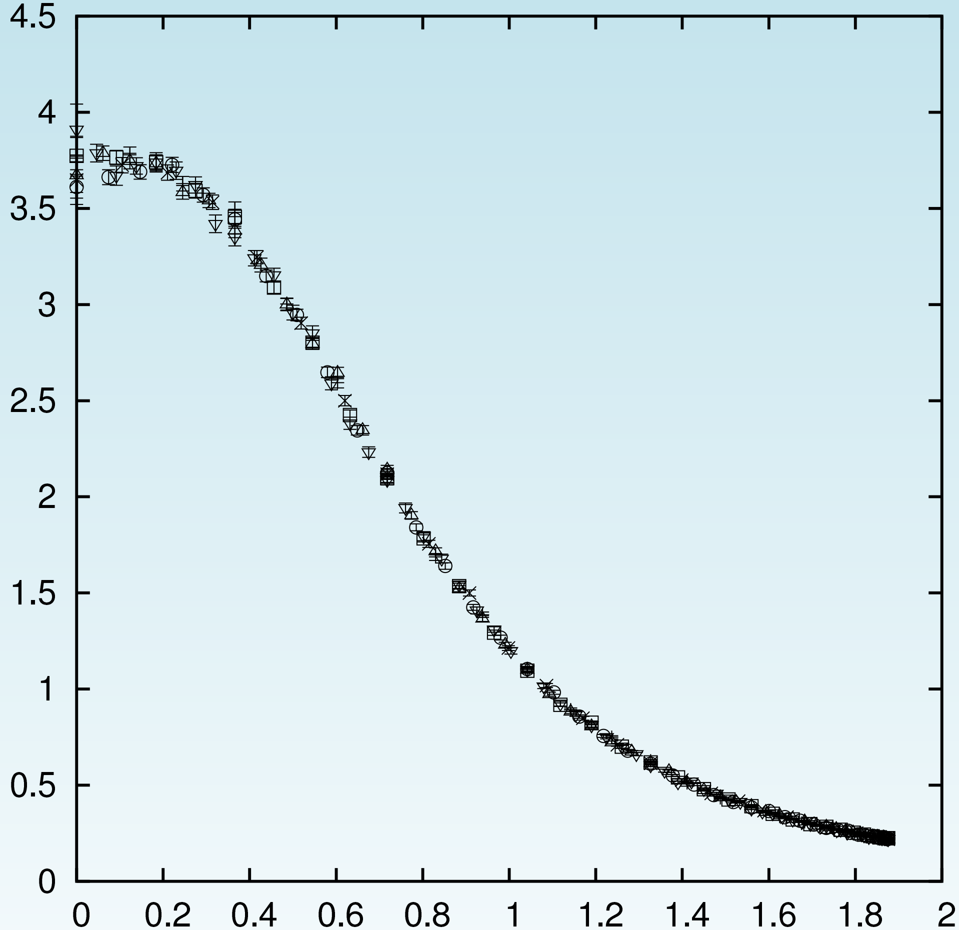
Gluon propagator as a function of the lattice momentum p for lattice volume $V = 80^4$ at $\beta = 2.2$.

Infinite-volume limit in 4d (IV)



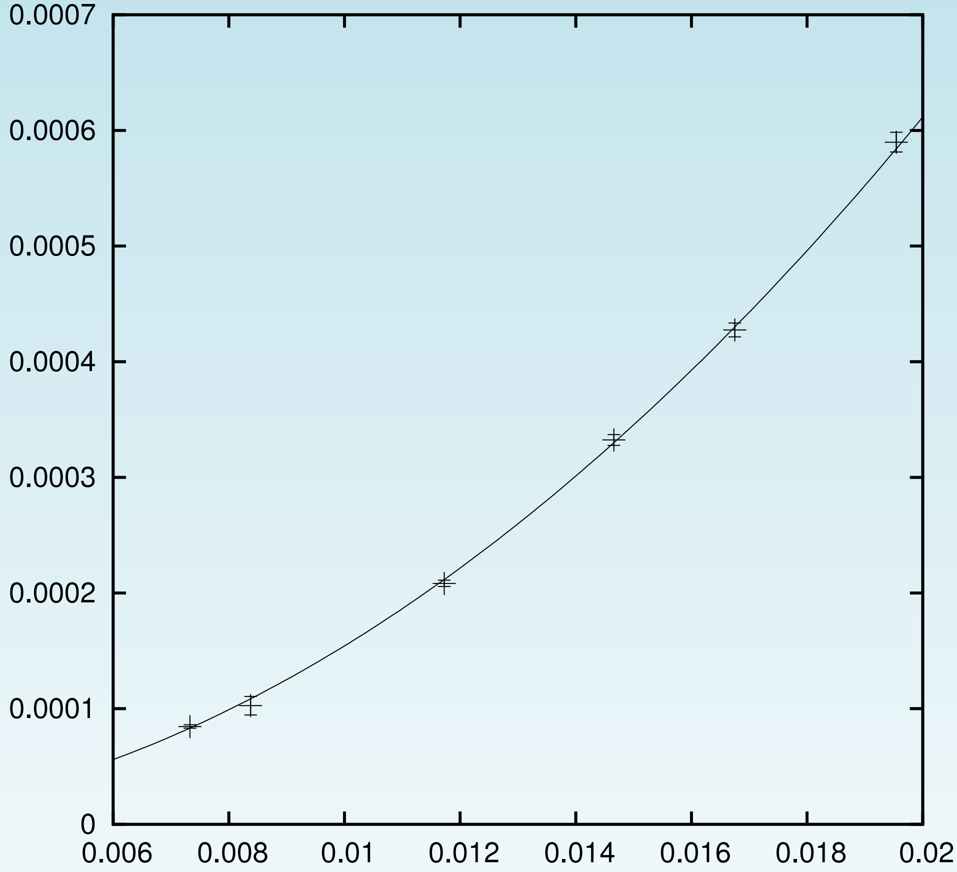
Gluon propagator as a function of the lattice momentum p for lattice volume $V = 128^4$ at $\beta = 2.2$.

Infinite-volume limit in 4d (V)



Gluon propagator as a function of the lattice momentum p for lattice volume up to $V = 128^4$ at $\beta = 2.2$.

Infinite-volume limit in 4d

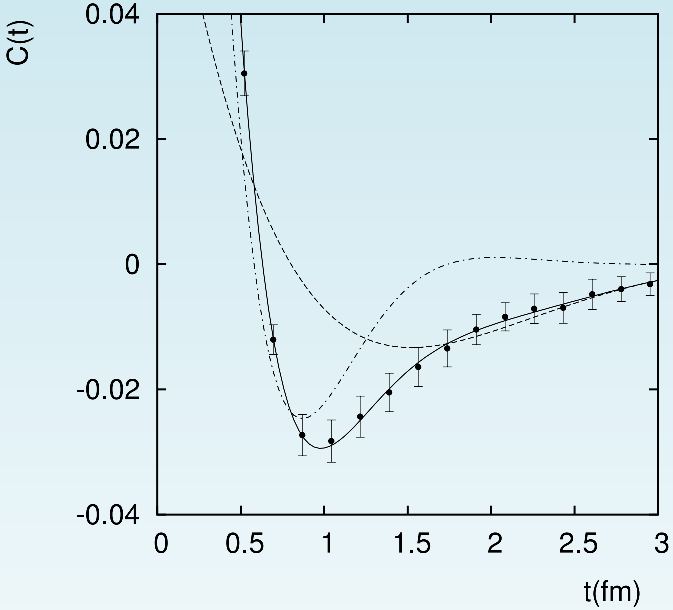


Average absolute value of the gluon field at zero momentum $|A^b_\mu(0)|$ (for $\beta = 2.2$) as a function of the inverse lattice side $1/L$ (in fm^{-1}) and extrapolation to infinite volume. Recall that $D(0) \propto V \sum_{\mu,b} |A^b_\mu(0)|^2$. We also show the fit of the data using the Ansatz b/L^c (with $c = 1.99 \pm 0.02$).

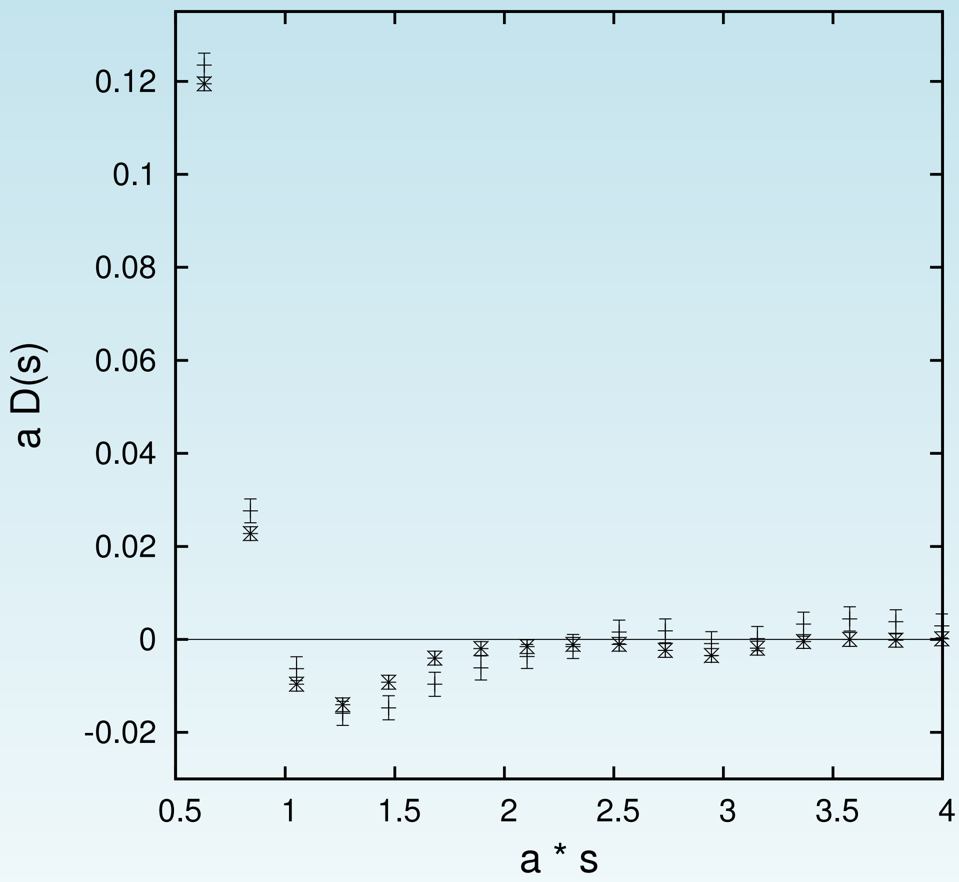
Violation of reflection positivity in 3d

- The transverse gluon propagator decreases in the IR limit for momenta smaller than p_{dec}, which corresponds to the mass scale λ in a Gribov-like propagator $p^2/(p^4 + \lambda^4)$. We can estimate $p_{dec} = 350^{+100}_{-50}$ MeV.

- Clear violation of reflection positivity: this is one of the manifestations of gluon confinement. In the scaling region, the data are well described by a sum of Gribov-like formulas, with a light-mass scale $M_1 \approx 0.74(1)\sqrt{\sigma} = 325(6)\,MeV$ and a second mass scale $M_2 \approx 1.69(1)\sqrt{\sigma} = 745(5)\,MeV$.



Violation of reflection positivity in 4d



Clear violation of reflection positivity for lattice volume $V = 128^4$ at $\beta = 2.2$.

Lower bound for $D(0)$

We can obtain a lower bound for the gluon propagator at zero momentum $D(0)$ by considering the quantity

$$M(0) \;=\; \frac{1}{d(N_c^2-1)} \sum_{b,\mu} \langle | A_\mu^b(0) | \rangle \; .$$

Consider the Cauchy-Bunyakovski-Schwarz inequality $|\vec{x}\cdot\vec{y}|^2 \leq \|\vec{x}\|^2 \|\vec{y}\|^2$, a vector $\vec{y}$ with all components equal to 1 and a vector $\vec{x}$ with components x_i, we find

$$\left(\frac{1}{m} \sum_{i=1}^{m} x_i \right)^2 \;\leq\; \frac{1}{m} \sum_{i=1}^{m} x_i^2 \; ,$$

where m is the number of components of the vectors $\vec{x}$ and $\vec{y}$.

Lower bound for $D(0)$ (II)

We can now apply this inequality first to the vector with $m = d(N_c^2 - 1)$ components $\langle |A_\mu^b(0)| \rangle$, where

$$A_\mu^b(0) \; = \; \frac{1}{V} \sum_x A_\mu^b(x)$$

is the gluon field at zero momentum. This yields

$$M(0)^2 \; \leq \; \frac{1}{d(N_c^2 - 1)} \sum_{b,\mu} \langle |A_\mu^b(0)| \rangle^2 \; .$$

Then, we can apply the same inequality to the Monte Carlo estimate of the average value

$$\langle |A_\mu^b(0)| \rangle \; = \; \frac{1}{n} \sum_c |A_{\mu,c}^b(0)| \; ,$$

where n is the number of configurations. In this case we obtain

$$\langle |A_\mu^b(0)| \rangle^2 \; \leq \; \langle |A_\mu^b(0)|^2 \rangle \; .$$

Lower bound for $D(0)$ (III)

Thus, by recalling that

$$D(0) \;=\; \frac{V}{d(N_c^2 - 1)} \sum_{b,\mu} \langle | A_\mu^b(0) |^2 \rangle \;,$$

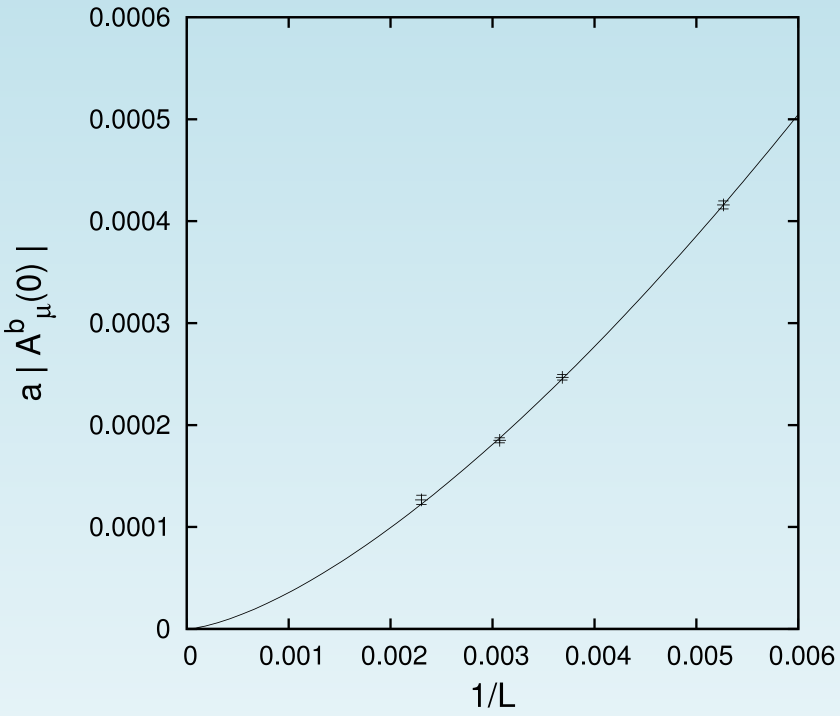
we find

$$\left[V^{1/2} M(0) \right]^2 \;\leq\; D(0) \;.$$

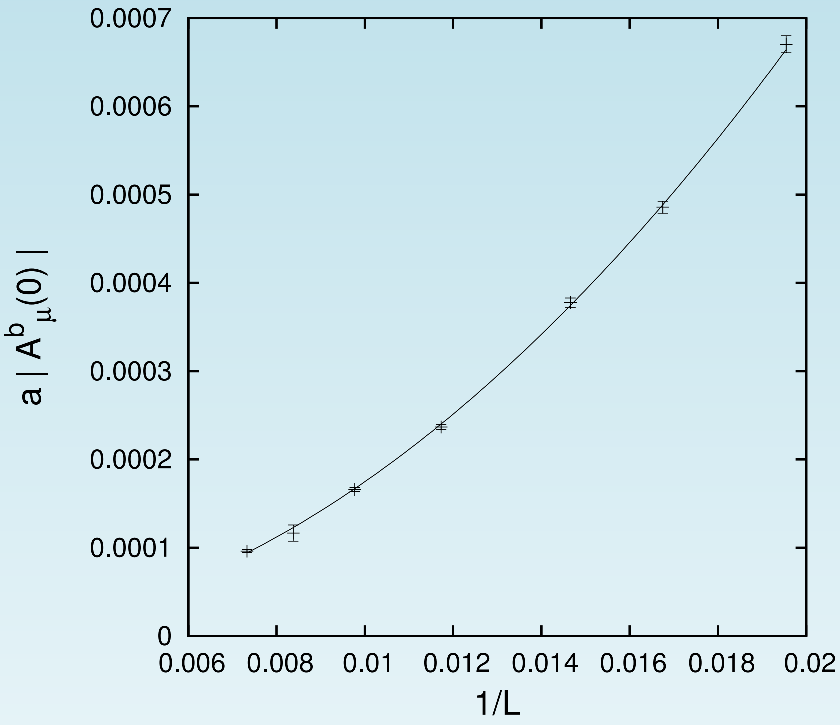
From our fits we obtain that $M(0)$ goes to zero exactly as $1/V^{1/2}$!

This gives $D(0) \geq 0.5(1)$ (GeV^{-2}) in 3d and $D(0) \geq 2.5(3)$ (GeV^{-2}) in 4d.

Lower bound for $D(0)$ (IV)



Fit of $M(0)$ using the Ansatz B/L^c, with $B = 1.0(1)$ (GeV^{-2}), $c = 1.48(3)$ and $\chi/d.o.f. = 1.00$ in 3d.



Fit of $M(0)$ using the Ansatz B/L^c, with $B = 1.7(1)$ (GeV^{-2}), $c = 1.99(2)$ and $\chi/d.o.f. = 0.91$ in 4d.

Upper bound for $D(0)$

We can now consider the inequality

$$\langle \sum_{\mu,b} |A_\mu^b(0)|^2 \rangle \;\leq\; \langle \Big\{ \sum_{\mu,b} |A_\mu^b(0)| \Big\}^2 \rangle \;.$$

This implies

$$D(0) \;\leq\; V d (N_c^2 - 1) \, \langle M(0)^2 \rangle \;.$$

Thus

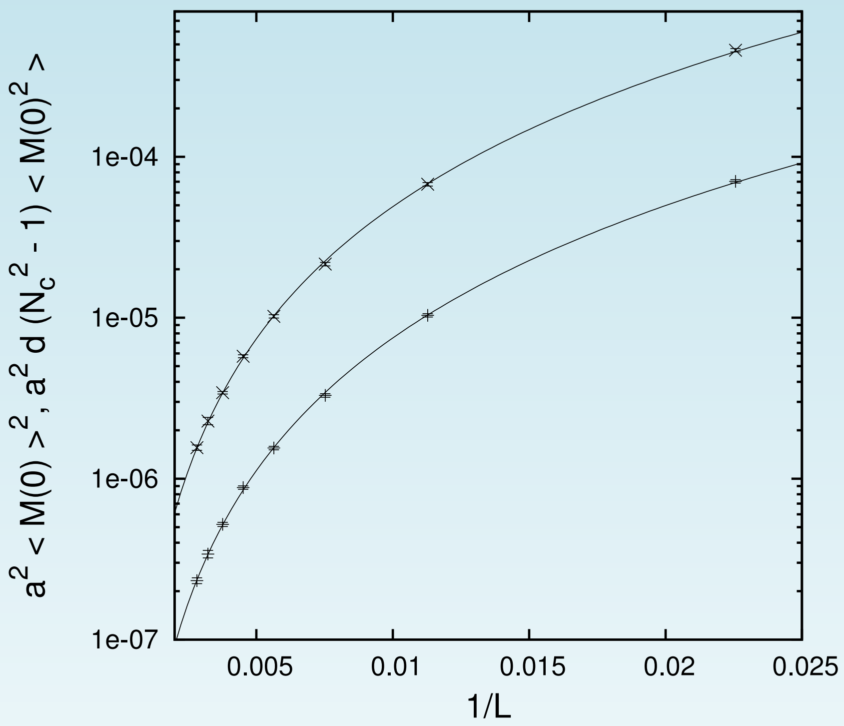
$$V \, \langle M(0) \rangle^2 \;\leq\; D(0) \;\leq\; V d (N_c^2 - 1) \, \langle M(0)^2 \rangle \;.$$

In summary, if $M(0)$ goes to zero as $V^{-\alpha}$ we find that

$$D(0) \to 0, \quad 0 < D(0) < +\infty \quad \text{or} \quad D(0) \to +\infty$$

respectively if α is larger than, equal to or smaller than 1/2.

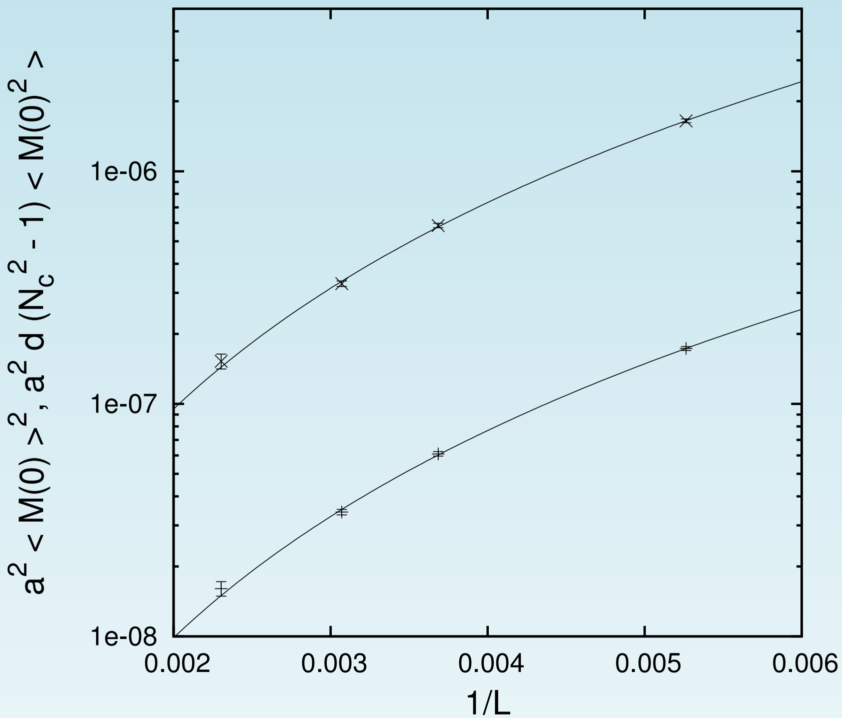
Upper and lower bounds for $D(0)$

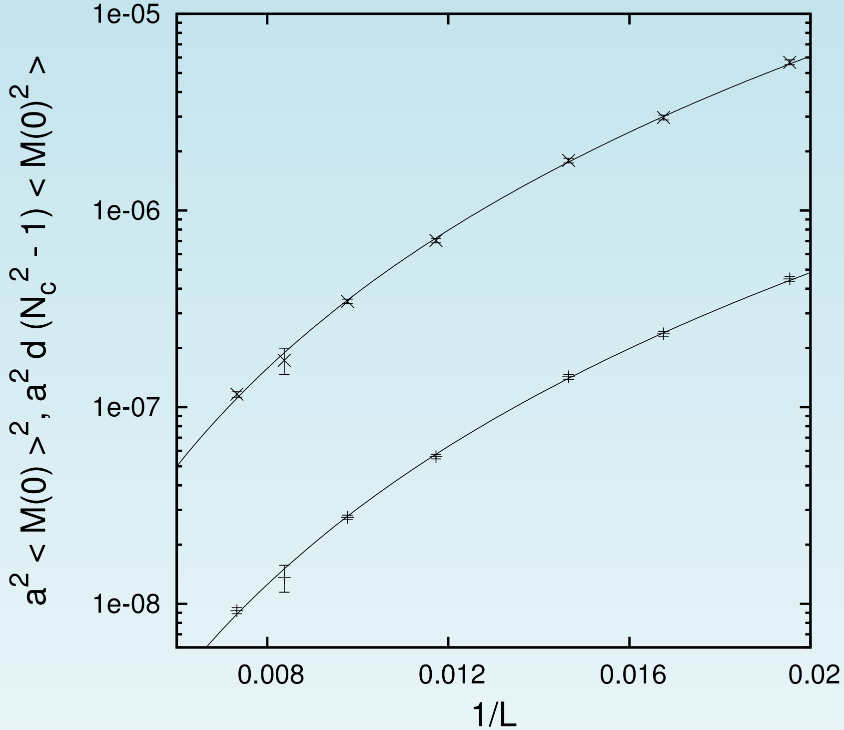


Two-dimensional case: B_l/L^c (for $a\langle M(0)\rangle$) and the Ansatz B_u/L^e (for $a^2\langle M(0)^2\rangle$), with $B_l = 1.48(6)$, $c = 1.367(8)$ and $\chi/d.o.f. = 1.00$ and $B_u = 2.3(2)$, $e = 2.72(1)$ and $\chi/d.o.f. = 1.02$.

Upper and lower bounds extrapolate to zero, implying $D(0) = 0$.

Upper and lower bounds for $D(0)$ (II)

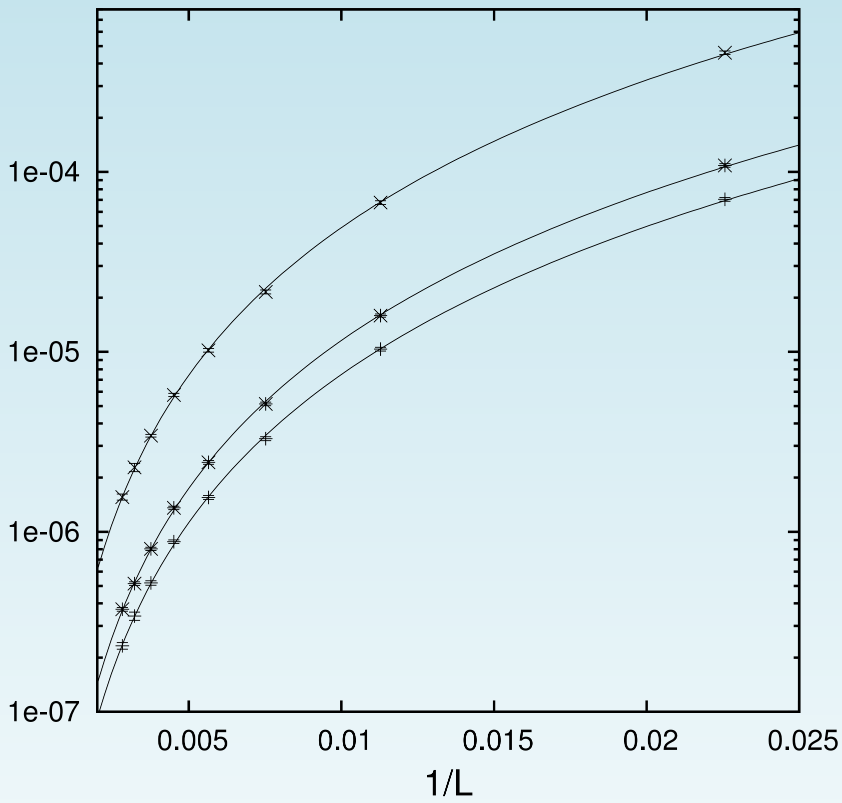




Similarly for 3d: $B_u = 1.0(3)$, $e = 2.95(5)$ and $\chi/d.o.f. = 0.95$.

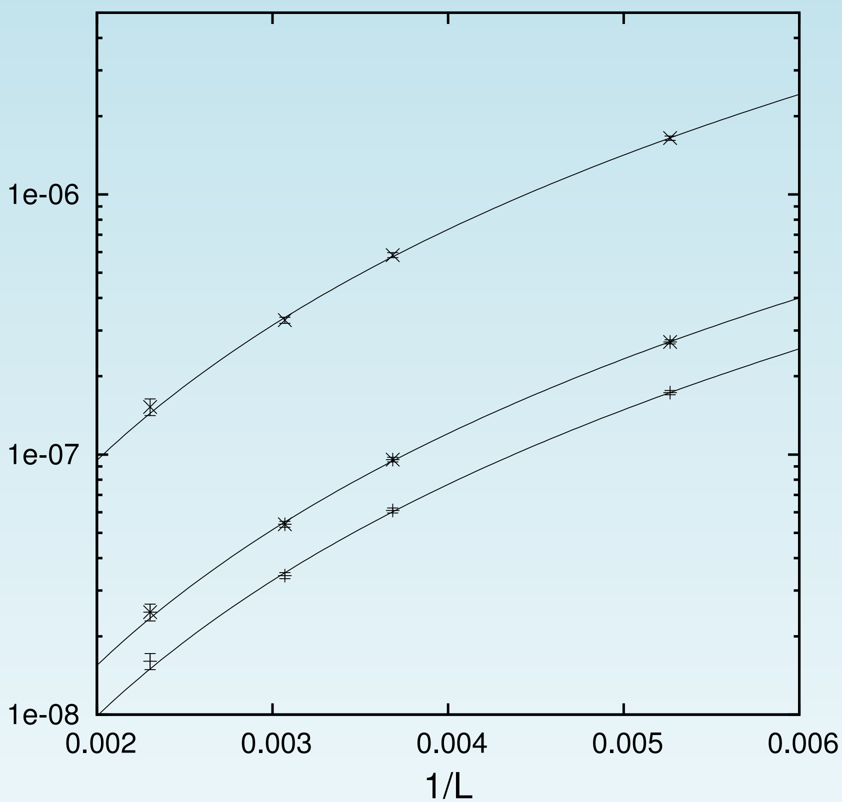
Similarly for 4d: $B_u = 3.1(5)$, $e = 3.99(4)$ and $\chi/d.o.f. = 0.96$.

Upper and lower bounds plus $D(0)/V$

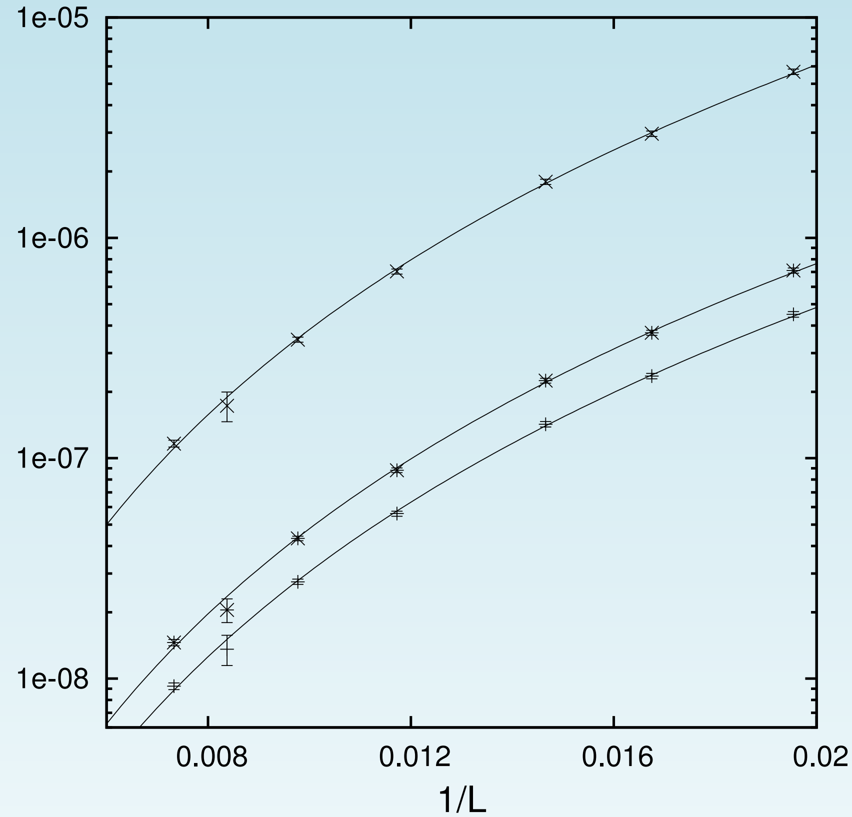


2d case

Upper and lower bounds plus $D(0)/V$ (II)



3d case



4d case

Gluon Propagator at Infinite Volume

- Gluon propagator in Landau gauge IR finite in 3d and 4d, as a consequence of "self-averaging" of a magnetization-like quantity [i.e. $M(0)$, without the absolute value].
- May think of $D(0)$ as a response function (susceptibility) of this observable ("magnetization"). In this case it is natural to expect $D(0) \sim const$ in the infinite-volume limit.
- 2d case is different, the magnetization is "over self-averaging", the susceptibility is zero.

Results

for the Ghost Propagator

Upper and Lower Bounds for $G(p)$

Consider eigenvectors $\psi_i(a,x)$ and associated eigenvalues λ_i of the FP matrix $\mathcal{M}(a,x;b,y)$. The ψ's form a complete orthonormal set

$$\sum_{i=1}^{(N_c^2-1)V} \psi_i(a,x)\,\psi_i(b,y)^* \;=\; \delta_{ab}\delta_{xy} \quad \text{and} \quad \sum_{a,x} \psi_i(a,x)\,\psi_j(a,x)^* \;=\; \delta_{ij} \;.$$

If we now write

$$\mathcal{M}^{-1}(a,x;b,y) \;=\; \sum_{i,\lambda_i\neq 0} \frac{1}{\lambda_i}\psi_i(a,x)\,\psi_i(b,y)^* \;,$$

we get for $G(p)$ the expression

$$G(p) \;=\; \frac{1}{N_c^2-1} \sum_{i,\lambda_i\neq 0} \frac{1}{\lambda_i} \sum_a \langle\, |\widetilde{\psi}_i(a,p)|^2 \,\rangle \;,$$

where

$$\widetilde{\psi}_i(a,p) \;=\; \frac{1}{\sqrt{V}} \sum_x \psi_i(a,x)\, e^{-2\pi i k\cdot x} \;.$$

Upper and Lower Bounds for $G(p)$ (II)

From the above expression we immediately get for $G(p)$ the lower bound

$$\frac{1}{N_c^2 - 1} \frac{1}{\lambda_{min}} \sum_a \langle \, | \widetilde{\psi}_{min}(a,p) |^2 \, \rangle \, \leq \, G(p)$$

and the upper bound

$$G(p) \, \leq \, \frac{1}{N_c^2 - 1} \frac{1}{\lambda_{min}} \sum_{i, \lambda_i \neq 0} \sum_a \langle \, | \widetilde{\psi}_i(a,p) \, |^2 \rangle \; .$$

Now by adding and subtracting the contribution from the null eigenvalue and using the completeness relation, the upper bound may be rewritten as

$$G(p) \, \leq \, \frac{1}{\lambda_{min}} \left[1 \, - \, \frac{1}{N_c^2 - 1} \sum_{j, \lambda_j = 0} \sum_a \langle \, | \widetilde{\psi}_j(a,p) \, |^2 \rangle \right] \; .$$

Upper and Lower Bounds for $G(p)$ (III)

In Landau gauge the eigenvectors corresponding to null λ are constant modes. Thus for any nonzero p we have

$$\frac{1}{N_c^2 - 1} \frac{1}{\lambda_{min}} \sum_a \langle \, | \widetilde{\psi}_{min}(a,p) |^2 \, \rangle \, \leq \, G(p) \, \leq \, \frac{1}{\lambda_{min}} \, .$$

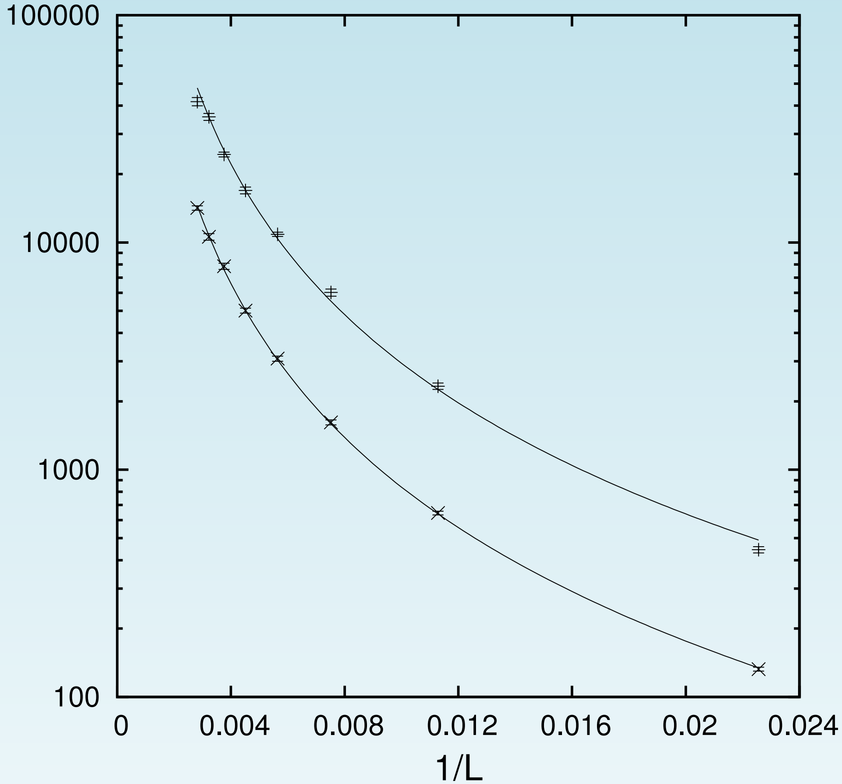
Now, assuming $\lambda_{min} \sim N^{-\alpha}$ and the power-law behavior $p^{-2-2\kappa}$ for the IR ghost propagator, we expect to have

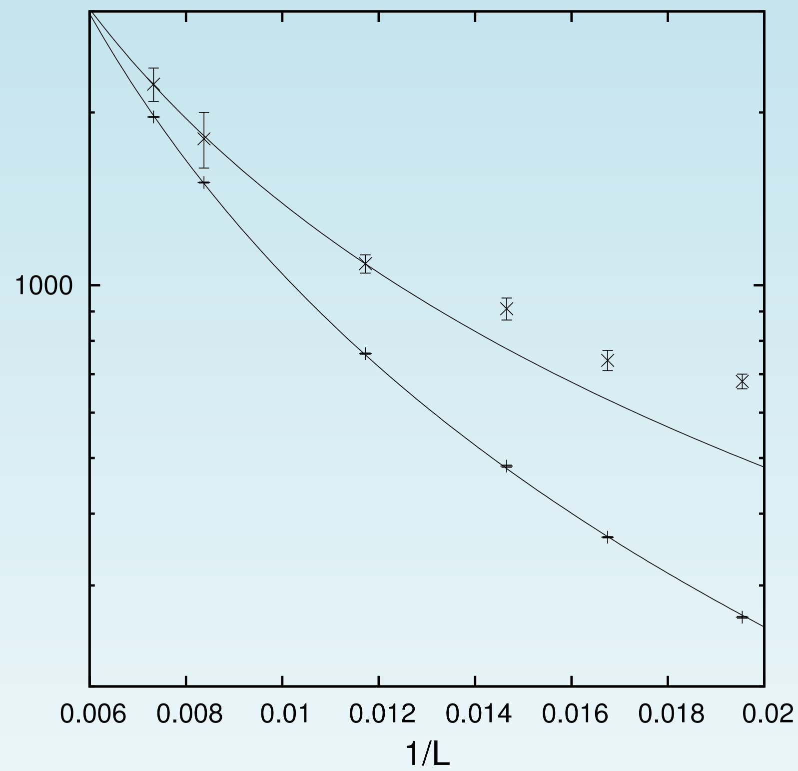
$$2 + 2\kappa \, \leq \, \alpha$$

and a necessary condition for IR enhancement of $G(p)$ is

$$\alpha \, > \, 2 \, .$$

Upper bound for $G(p_{min})$





For 2d: $2\kappa = 0.251(9)$, $\alpha = 2.20(4)$.

For 4d: $2\kappa = 0.043(8)$, $\alpha = 1.53(2)$.

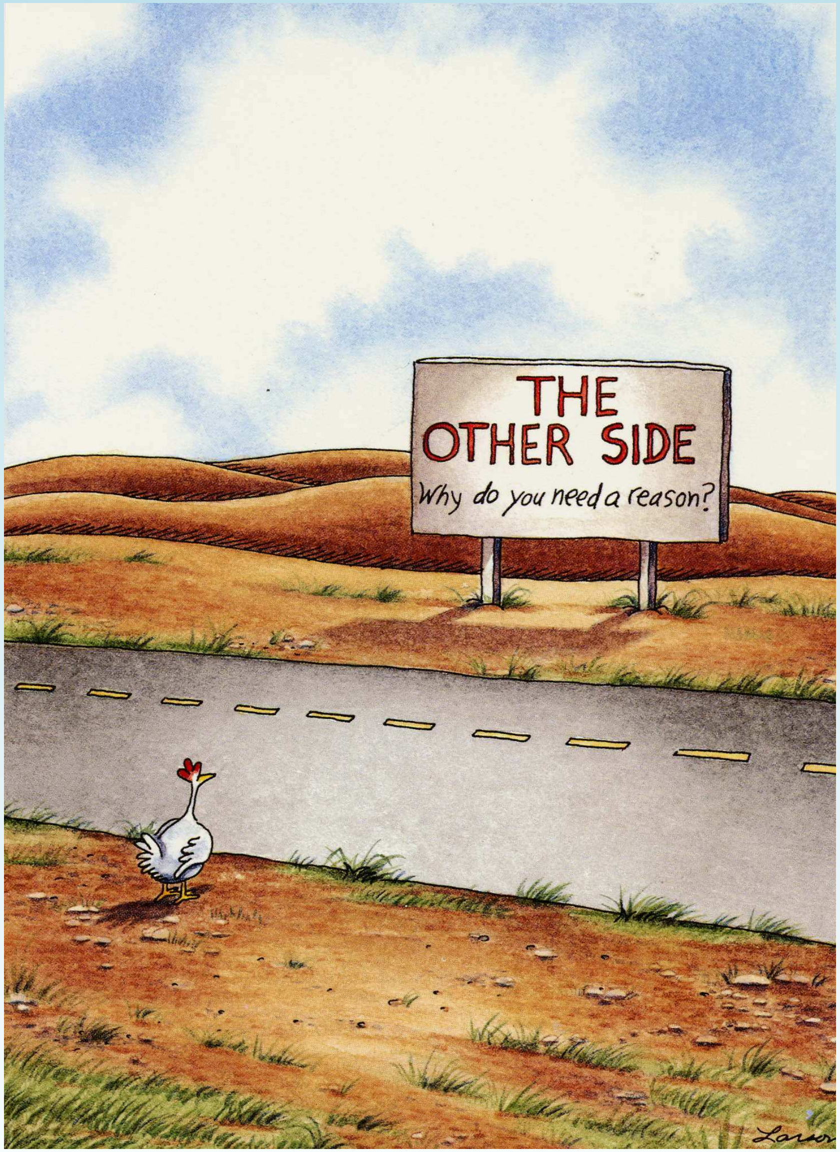
Ghost Propagator at Infinite Volume

- From present fits we have $\alpha > 2$ in 2d [implying IR enhancement of $G(p)$], but $\alpha < 2$ in 4d.
- On the other hand the expected relation $2 + 2\kappa \leq \alpha$ is not satisfied, although the upper bound is.
- In 4d the upper bound seems to saturate, so main contribution comes from λ_{min}.

Conclusions (our work)

- We are able to find simple properties of gluon and ghost propagators that constrain (by upper and lower bounds) their IR behavior.
- For the gluon case we define a magnetization-like quantity, while for the ghost case we relate the propagator to λ_{min} of the FP matrix.
- We propose the study of these quantities as a function of the lattice volume, to gain better control of the infinite-volume limit of IR propagators.

From Latticeland to the Continuum



Conclusions (this workshop)

- There are two solutions in the continuum (from functional methods): the conformal solution and the decoupling solution.
- In 3d and in 4d the lattice gives results in agreement with the decoupling one. $\longrightarrow$ Theoretical imputs from Sorella and collaborators.
- In 2d the lattice results agree with the conformal solution. $\longrightarrow$ The 2d case should be easy to understand.
- Gribov copies + discretization (Maas, Von Smekal) could give us the conformal solution also in 3d and in 4d. $\longrightarrow$ How do we explain two different continuum limits?