Quark and gluon propagators in dense 2-colour matter

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Outline

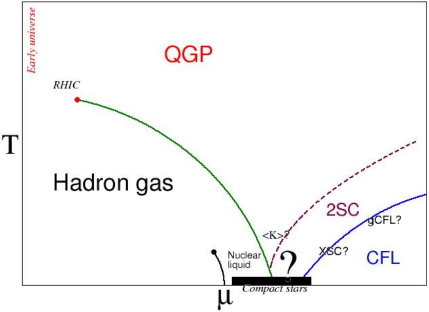
Background
- Global symmetries of QC_2D
- QC_2D vs QCD

Formalism
- Tensor structures
- Lattice formulation

Results
- Bulk thermodynamics
- Gluon propagator results
- Quark propagator results

Summary

Background



- A plethora of phases at high μ, low T
- Based on models and perturbation theory
- Details depend on diquark gaps and strange quark mass

Diquark condensation

- One-gluon exchange is attractive
- Energetically favourable to pair two quarks on opposite sides of Fermi surface $\Longrightarrow$ BCS instability
- In QCD this breaks the gauge symmetry $\Longrightarrow$ colour superconductivity
- Ground state at ultra-high densities has
$$SU(3)_L \otimes SU(3)_R \otimes SU(3)_c \to SU(3)_{L+R+c}$$
- Mismatch between strange and light quark Fermi momenta at intermediate densities $\Longrightarrow$ less symmetric pairing

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So standard Monte Carlo importance sampling can not be used!
We can still gain some insight:

- Approach from high(er) temperature
- Effective theories where problem is absent or reduced: HDET, NJL,. . .
- QCD-like theories without a sign problem
- QC_2D studies by Hands&Morrison; Muroya, Nakamura, Nonaka; Kogut&Sinclair; Allés, d'Elia, Lombardo; Chandrasekharan&Jiang,. . .

Global symmetries of QC_2D

Quarks and antiquarks are in the same representation

Anti-unitary symmetry: $\mathcal{K}\mathcal{M}\mathcal{K}^{-1} = \mathcal{M}^*$ with $\mathcal{K} \equiv C\gamma_5\tau_2$

$$\begin{aligned}\mathcal{L} &= \overline{\psi}(\gamma_\nu D_\nu - \mu\gamma_0 + m)\psi \\ &= i\Psi^\dagger\sigma_\nu(D_\nu - \mu B_\nu)\Psi + \frac{1}{2}m\Psi^T\sigma_2\tau_2\hat{M}\Psi\end{aligned}$$

$$\Psi = \begin{pmatrix} \psi_L \\ \sigma_2\tau_2\psi_R^* \end{pmatrix}, \quad B_\nu = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\delta_{\nu 0}, \quad \hat{M} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$m = \mu = 0$: global $SU(2N_f)$ symmetry

Chiral symmetry breaking

Chiral condensate

$$\overline{\psi}\psi = -\frac{1}{2}\Psi^T\sigma_2\tau_2\hat{M}\Psi + h.c.$$

$\langle\overline{\psi}\psi\rangle \neq 0$ breaks $SU(2N_f) \longrightarrow Sp(2N_f)$

$$\Rightarrow N_f(2N_f - 1) - 1\text{Goldstone modes}$$

$N_f = 2$: 5 modes

$\overline{\psi}\vec{\sigma}\gamma_5\psi$ pion $\psi^T\epsilon\tau_2 C\gamma_5\psi$, $\overline{\psi}\epsilon\tau_2 C\gamma_5\overline{\psi}^T$ scalar diquark

Note: Staggered fermions have different symmetry breaking pattern!

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Diquarks are colour singlets in QC_2D

$\rightarrow$ superfluidity rather than colour superconductivity

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Condensation of tightly bound diquarks (Goldstone baryons)

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Bardeen–Cooper–Schrieffer:

Pairing of quarks near the Fermi surface

$$\langle\psi\psi\rangle \propto \Delta\mu^2$$

QC_2D vs QCD

What we cannot learn

- Chiral dynamics
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What we might learn

- Gluodynamics
- Checks on model studies
- Qualitative features of deconfinement
- Generic features of gauge theories at high densities
- Medium effects on quarks, gluons and non-Goldstone hadrons

Issues of interest

Gluodynamics — SU(2) and SU(3) very similar?

- Deconfinement at high density — effects on gluon propagator?
- Gap equation with effective or one-gluon interaction used to determine superconducting gap $\longrightarrow$ more realistic input?
- Static magnetic gluon: unscreened at all orders in perturbation theory!

Quark propagator

- Details of phase diagram depend critically on the effective quark mass in the medium.
- Dynamical quark masses $\longrightarrow$ effective strange quark mass?
- Direct determination of diquark gap?

Tensor structure in medium

The medium breaks Lorentz (Euclidean) symmetry to O(3)
$\Longrightarrow$ $1 \rightarrow 2$ scalar functions in gluon, $2 \rightarrow 4$ in quark:

$$D_{\mu\nu}(\vec{q}, q_t) = P^T_{\mu\nu} D_M(\vec{q}^2, q_t^2) + P^E_{\mu\nu} D_E(\vec{q}^2, q_t^2) + \xi \frac{q_\mu q_\nu}{q^4}$$

$$S^{-1}(\vec{p}, \tilde{\omega}) = i \vec{\not{p}} A(\vec{p}^2, \tilde{\omega}^2) + i\gamma_4 \tilde{\omega} C(\vec{p}^2, \tilde{\omega}^2) + B(\vec{p}^2, \tilde{\omega}^2) + i\gamma_4 \vec{\not{p}} D(\vec{p}^2, \tilde{\omega}^2)$$

$$S(\vec{p}, \tilde{\omega}) = i \vec{\not{p}} S_a + i\gamma_4 \tilde{\omega} S_c + S_b + i\gamma_4 \vec{\not{p}} S_d$$

where $\tilde{\omega} \equiv p_t - i\mu$.

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In general the form factors are complex!

Gor'kov formalism

Quarks and antiquarks are in the same representation.

Construct Gor'kov spinor $\Psi = \begin{pmatrix} \psi \\ \overline{\psi}^T \end{pmatrix}$

$$\implies \quad \langle \Psi(x)\bar{\Psi}(y)\rangle \equiv \mathcal{G}(x,y) = \begin{pmatrix} S_N & -S_A \\ \bar{S}_A & \bar{S}_N \end{pmatrix}$$

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S_A contains information about anomalous propagation

The corresponding self-energies are diquark gaps Δ (superfluid/superconducting)

Lattice formulation

We use Wilson fermions:

- Correct symmetry breaking pattern, Goldstone spectrum
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Diquark source J introduced to

- lift low-lying eigenmodes in the superfluid phase
- study diquark condensation without uncontrolled approximations

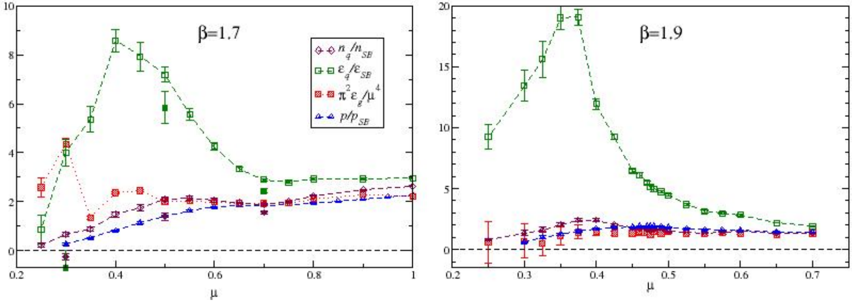
Simulation Parameters

We work on two lattices, 'coarse' and 'fine':

Name	β	κ	Volume	a	am_π	m_π/m_ρ
coarse	1.7	0.178	$8^3 \times 16$	0.26fm	0.79	0.80
fine	1.9	0.168	$12^3 \times 24$	0.20fm	0.65	0.80

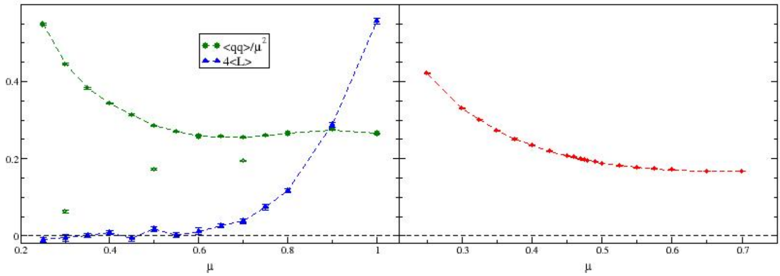
- Simulations performed with $j = J/\kappa = 0.04$ for $\mu = 0.3 - 1.0$
- 300–500 trajectories for each μ.
- Simulations with $j = 0.02, 0.06$ for $\mu = 0.3, 0.5, 0.7, 0.9$
  $\rightarrow$ enable extrapolation to $j = 0$.

Thermodynamics results



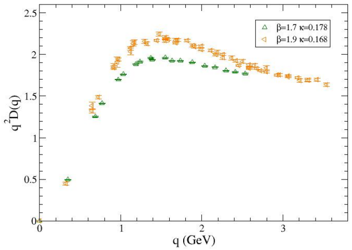
- Close to SB scaling for $\mu > \mu_d$
- $\varepsilon_q \sim 2\varepsilon_{SB} \rightarrow k_F > E_F \Longrightarrow$ binding energy?
- 30–40% of total energy from gluons!?

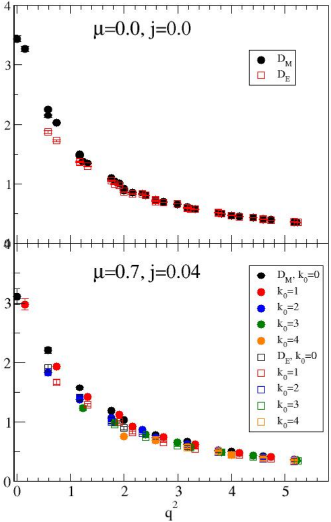
Phase transitions



- Deconfining transition on coarse lattice — goes away on fine lattice?
- BEC → BCS crossover becoming softer?

Gluon propagator results

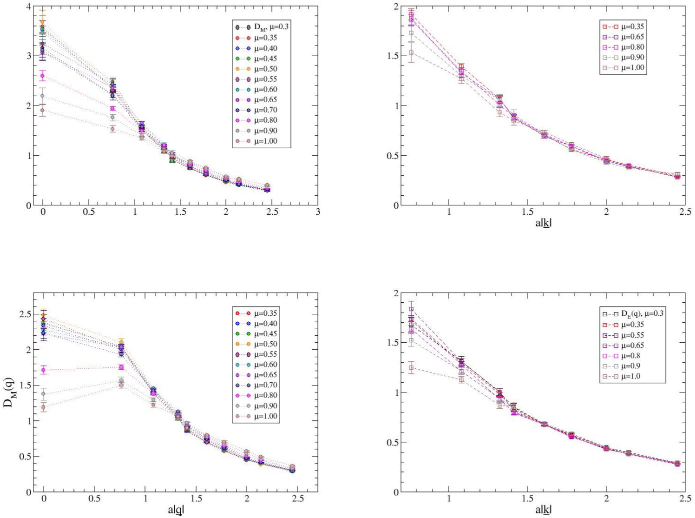




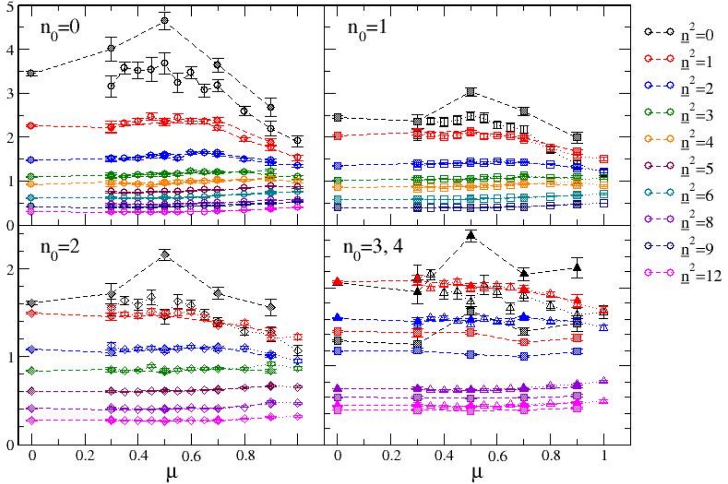
Some finite volume and lattice spacing effects at $\mu = 0$

In-medium modifications, incl. violations of Lorentz symmetry, visible in magnetic gluon at $\mu = 0.7$

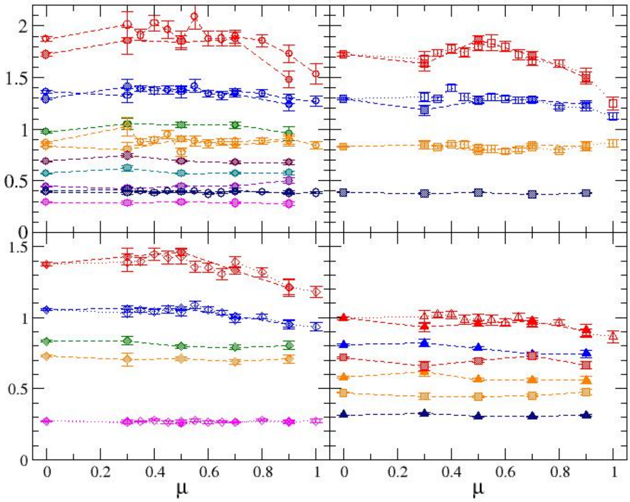
Coarse lattice results



Magnetic gluon (coarse lattice)

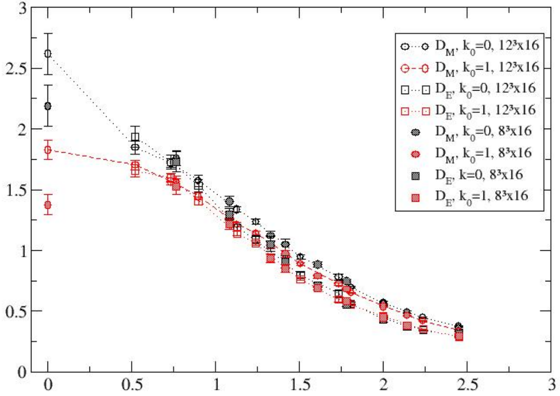


Electric gluon (coarse lattice)

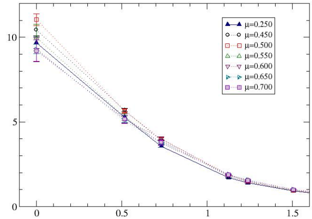


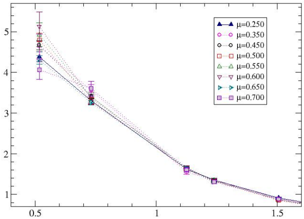
Volume dependence

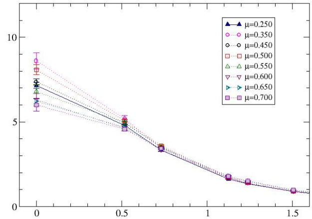
$[\mu = 0.9, j = 0.04]$

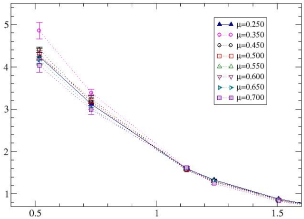


Fine lattice results









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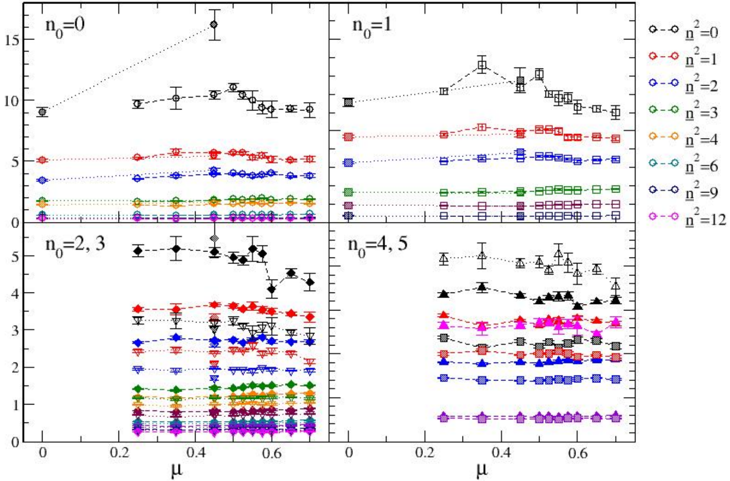

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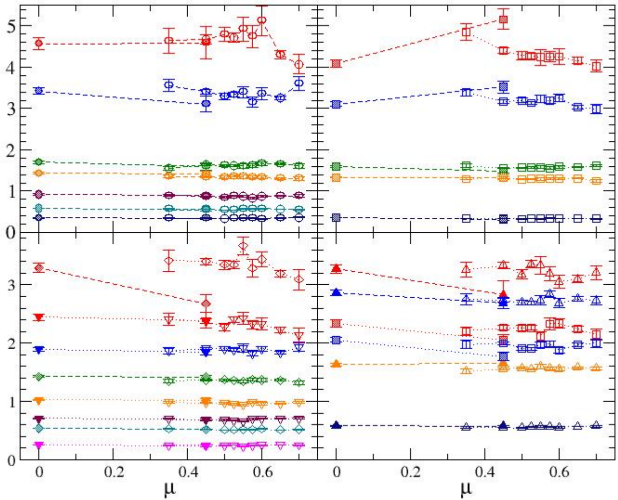
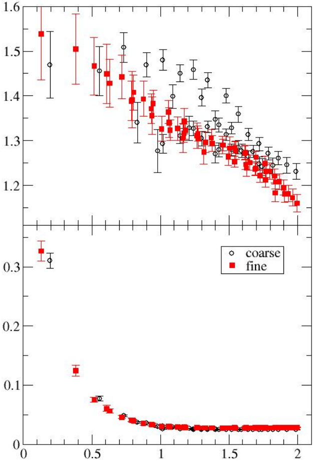


Quark propagator results

Quark propagator in vacuum

Raw data!

Large lattice artefacts on coarse lattice
Unusual momentum behaviour?



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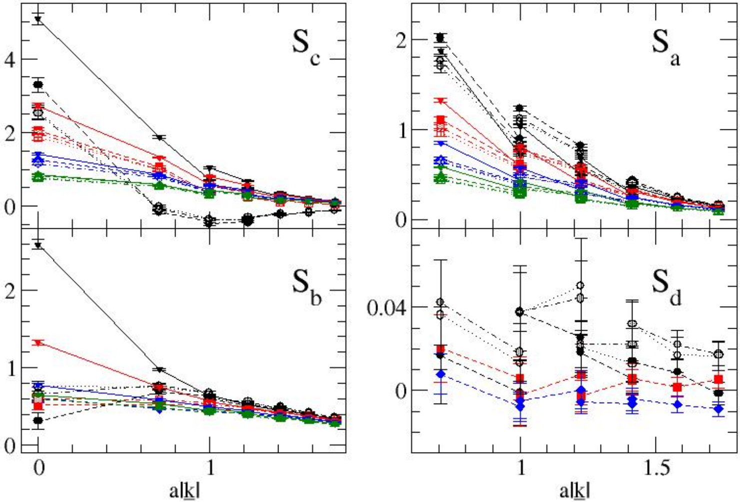
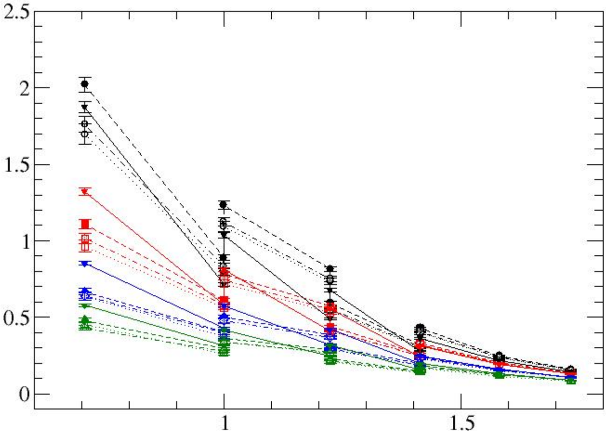
- Anomalous parts are complex
- All are consistent with zero??

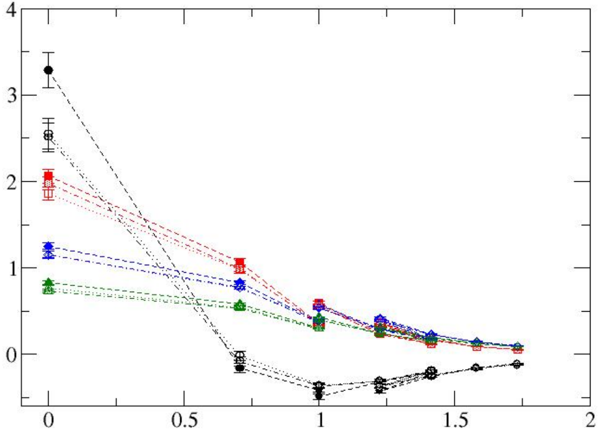
Quark propagator $\mu = 0.5$ (Preliminary!)



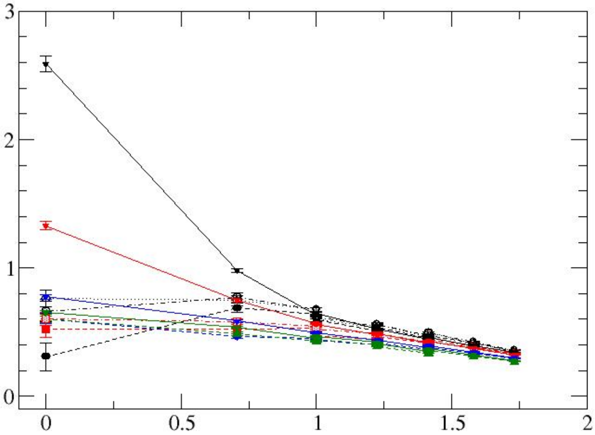
Quark propagator: spatial vector part



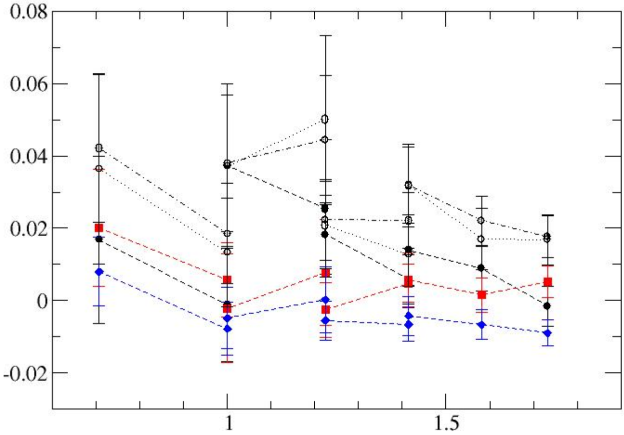
Quark propagator: temporal vector part



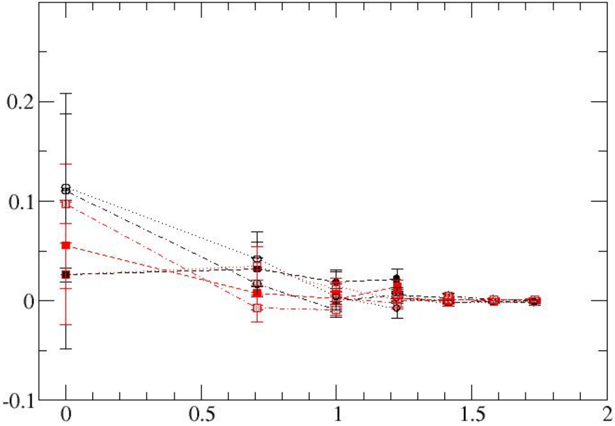
Quark propagator: scalar part



Quark propagator: tensor part



Anomalous propagation



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- Need to understand diquark source dependence
- Need to understand lattice artefacts