Color-superconductivity from a Dyson-Schwinger perspective

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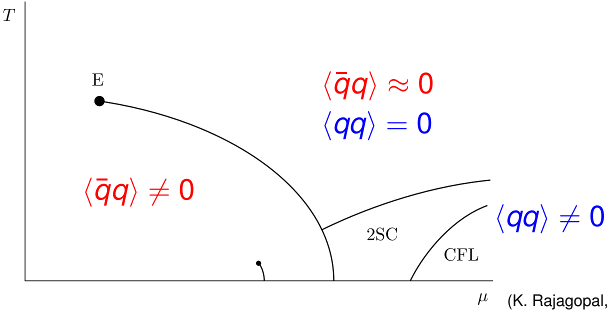
Quarks and Hadrons in strong QCD
March 2008, St. Goar

introduction and motivation

schematic phase diagram of QCD:



(K. Rajagopal, 1999)

- color-superconductivity by $\langle qq \rangle \neq 0$
- here: 2- and 3-flavor pairing
- phenomenology determined by strange-quark mass m_s

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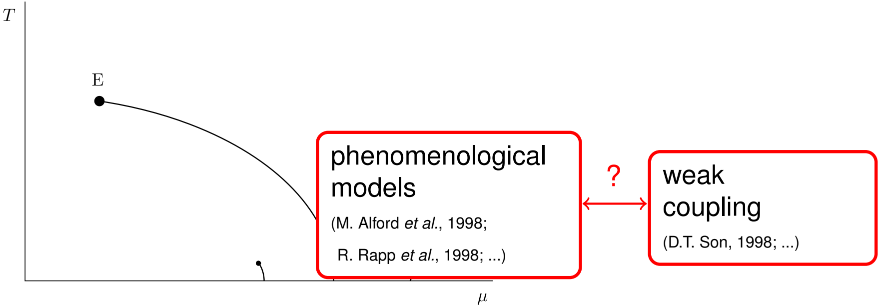

phenomenological models

(M. Alford *et al.*, 1998;

R. Rapp *et al.*, 1998; ...)

weak coupling

(D.T. Son, 1998; ...)

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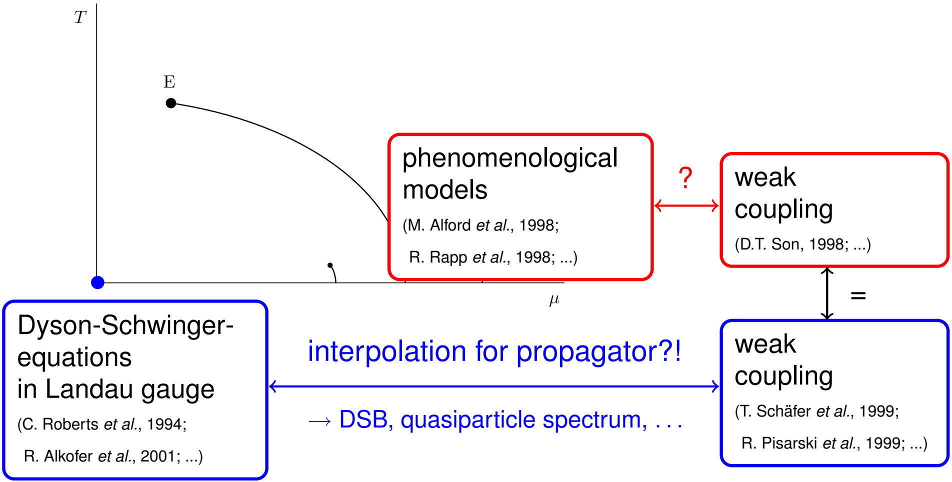

phenomenological models
(M. Alford *et al.*, 1998;
R. Rapp *et al.*, 1998; ...)

weak coupling
(D.T. Son, 1998; ...)

weak coupling
(T. Schäfer *et al.*, 1999;
R. Pisarski *et al.*, 1999; ...)

Dyson-Schwinger-equations
in Landau gauge
(C. Roberts *et al.*, 1994;
R. Alkofer *et al.*, 2001; ...)

interpolation for propagator?!

→ DSB, quasiparticle spectrum, . . .

outline

1. theoretical framework
2. warm-up: color-superconductivity in the chiral limit
3. color-flavor unlocking in neutral quark matter
4. back-reaction of Goldstone modes
5. conclusions

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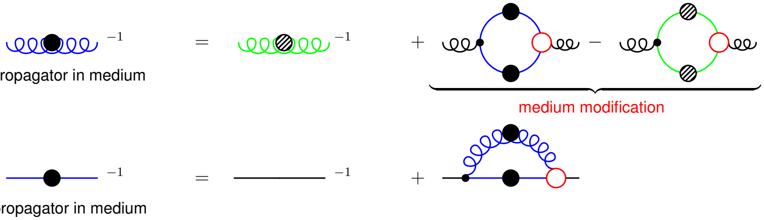
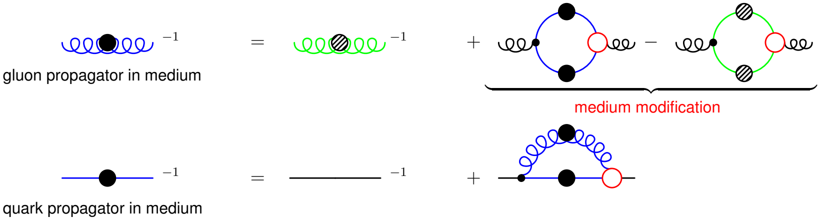
Dyson-Schwinger equation for quark propagator

truncated set of DSEs:



Dyson-Schwinger equation for quark propagator

truncated set of DSEs:



approximation: gluon and vertex separately

- medium modified interaction
  → reproduce weak-coupling
  → less sensitivity on coupling
- vertex adopted from vacuum investigations:
  → $\Gamma^a_\mu(p,q) \simeq i\, g\, \Gamma((p-q)^2)\, \gamma_\mu\, \frac{\lambda^a}{2}$
  (extended at the end of my talk)

truncated Dyson-Schwinger equation

$$\Rightarrow \quad S^{-1}(p) \quad = \quad Z_2 S_0^{-1}(p) + \frac{Z_2}{3\pi^3} \int d^4q \, \gamma_\mu S(q) \gamma_\nu \left(\frac{\alpha_s(k^2)}{k^2 + G(k)} P^T_{\mu\nu} + \frac{\alpha_s(k^2)}{k^2 + F(k)} P^L_{\mu\nu} \right)$$

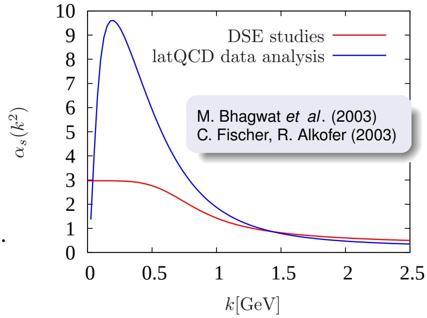
- similar to HDL
- screening and damping included through

$$m_g(k^2)^2 = \frac{N_f \, \mu^2 \, \alpha_s(k^2)}{\pi}$$

$$F(k) = 2 \, m_g(k^2)^2 + \dots$$

$$G(k) = \frac{\pi}{2} m_g(k^2)^2 \frac{k_4}{|\vec{k}|} + \dots$$

- strong running coupling $\alpha_s(k^2)$



M. Bhagwat *et al*. (2003)
C. Fischer, R. Alkofer (2003)

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Nambu-Gor'kov formalism

In Nambu-Gor'kov space with bispinors $\Psi = \begin{pmatrix} \psi \\ \psi^c = C\bar{\psi}^T \end{pmatrix}$, the quark DSE

$$\mathcal{S}^{-1} = Z_2 \mathcal{S}_0^{-1} + Z_{1F} \Sigma$$

includes the gap-equation

$$\Sigma = \begin{pmatrix} \Sigma^+ & \Phi^- \\ \Phi^+ & \Sigma^- \end{pmatrix} = -\int \frac{d^4q}{(2\pi)^4} \Gamma^{\mu}_{0\,a} \mathcal{S}(q) \Gamma^{\nu}_{b}(p,q) D^{ab}_{\mu\nu}(p-q).$$

$\Rightarrow$ room for diquark condensation $\leftrightarrow$ $\Phi^{\pm}$

Dirac and color-flavor structure in chiral limit

Dirac structure

T- and χ- symmetric, even-parity and color-flavor symmetric

(R. Pisarski, D. Rischke, 1999)

$$\begin{aligned}\Sigma_i &= \gamma_4 \left(\Sigma_i^+ \Lambda^+ + \Sigma_i^- \Lambda^-\right)\\ \phi_i &= \gamma_5 \left(\phi_i^+ \Lambda^+ + \phi_i^- \Lambda^-\right)\end{aligned}$$

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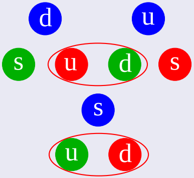
color-flavor structure of gap functions (similar for Σ^+)

2SC: $\Phi^+ = \phi_{2SC}\, \lambda_2 \otimes \tau_2$



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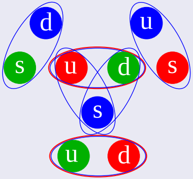
color-flavor structure of gap functions (similar for Σ^+)

2SC: $\Phi^+ = \phi_{2SC}\,\lambda_2 \otimes \tau_2$

CFL: $\Phi^+ = \underbrace{\phi_{\bar{3}} \sum_A \lambda_A \otimes \tau_A}_{\textit{attractive}} + \underbrace{\phi_6 \sum_S \lambda_S \otimes \tau_S}_{\textit{induced}}$

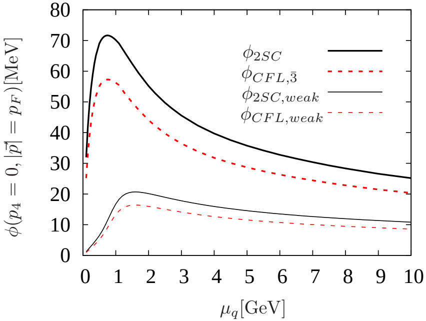


gap-functions on the Fermi surface

analytical result in weak coupling (Q. Wang, D. Rischke, 2001):

$$\phi^{+}_{weak} = 512\,\pi^4 \left(\frac{2}{N_f g^2}\right)^{\frac{5}{2}} e^{-\frac{\pi^2+4}{8}}\,\mu\, e^{-\frac{3\pi^2}{\sqrt{2}g}} \times \begin{cases} 1 & \mathrm{2SC} \\ 2^{-1/3} & \mathrm{CFL} \end{cases}$$

comparison:



(D.N., R. Alkofer, J. Wambach, 2006)

- large deviations from extrapolated result! $\phi^{+}_{2SC}(\sim 400\mathrm{MeV}) > 60\mathrm{MeV}!$
- similar results for stronger coupling

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neutrality and background fields

'bare' propagator in Dyson-Schwinger equation

$$S_0^{-1}(p) \quad = \quad -i\,\vec{p}\cdot\vec{\gamma} - i\,(p_4 + i\mu + \underbrace{gA_4}_{\frac{i}{2}\sum_a \mu_a\lambda_a})\gamma_4 + m - \mu_{el}Q$$

homogenous background fields → 'color chemical potentials'

(A. Gerhold, A. Rebhan, 2003; D. Dietrich, D. Rischke,2003)

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homogenous background fields → 'color chemical potentials'

(A. Gerhold, A. Rebhan, 2003; D. Dietrich, D. Rischke,2003)

- 'color neutrality' corresponds to equation of motion of gA_4:

$$\rho_a = \langle\psi^\dagger\frac{\lambda_a}{2}\psi\rangle = \int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\left(Z_2\,\gamma_4\frac{\lambda_a}{2}\,S^+(p)\right) = 0$$

  (constrain to ρ_3 and ρ_8)
- electrical neutrality and β-equilibrium

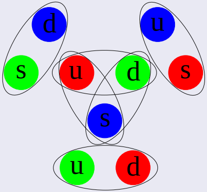
$$\rho_Q = \langle\psi^\dagger Q\psi\rangle = \frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \frac{1}{3}\rho_s - \rho_{el} = 0$$

$$\mu_d = \mu_s = \mu_u + \mu_{el}$$

color-flavor locking

color-flavor locking for three equal flavors (similar for Σ^+)

$$\Phi^+ = \underbrace{\phi_{\bar{3}} \sum_A \lambda_A \otimes \tau_A}_{attractive} + \underbrace{\phi_6 \sum_S \lambda_S \otimes \tau_S}_{induced}$$



$$\phi_i = (\gamma_4 \, \hat{p} \cdot \vec{\gamma} \, \phi_A + \gamma_4 \, \phi_B + \phi_C + \hat{p} \cdot \vec{\gamma} \, \phi_D) \, \gamma_5$$

CFL via symmetry pattern

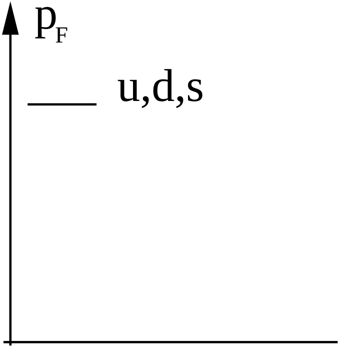
$$SU(3)_L \otimes SU(3)_R \otimes SU(3)_{color} \quad \rightarrow \quad SU(3)_{V+c}$$

generated by $\tau_a - \lambda_a^T$, $a = 1, \ldots, 8$

(D.N., J. Wambach, R. Alkofer, 2006)

unlocking under neutrality constraints

different stress on pairing pattern

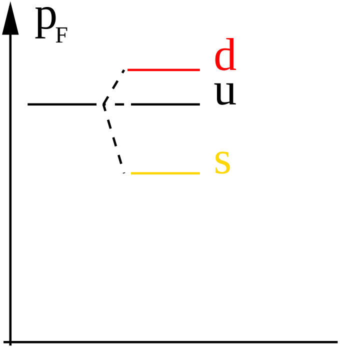


$$\Phi = \begin{array}{c} \left(\begin{array}{ccccccccc} \phi_1 & \phi_4 & \phi_6 & & & & & & \\ \phi_5 & \phi_2 & \phi_7 & & & & & & \\ \phi_8 & \phi_9 & \phi_3 & & & & & & \\ & & & & \phi_{10} & & & & \\ & & & \phi_{11} & & & & & \\ & & & & & & \phi_{12} & & \\ & & & & & \phi_{13} & & & \\ & & & & & & & & \phi_{14} \\ & & & & & & & \phi_{15} & \end{array} \right) \begin{array}{c} u \\ d \\ s \\ d \\ u \\ s \\ u \\ s \\ d \end{array} \\ \begin{array}{ccccccccc} u & d & s & d & u & s & u & s & d \end{array} \end{array}$$

CFL: residual $SU(3)_{V+c}$ symmetry

unlocking under neutrality constraints

different stress on pairing pattern



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\begin{pmatrix}
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\phi_5 & \phi_2 & \phi_7 & & & & & & \\
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 & & & & \phi_{10} & & & & \\
 & & & \phi_{11} & & & & & \\
 & & & & & & \phi_{12} & & \\
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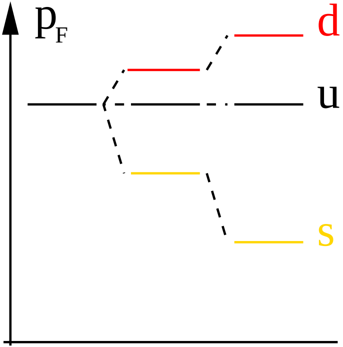
\end{pmatrix} & \begin{array}{c} u \\ d \\ s \\ d \\ u \\ s \\ u \\ s \\ d \end{array} \\
\begin{array}{ccccccccc} u & d & s & d & u & s & u & s & d \end{array} &
\end{array}$$

CFL: approximate $U(1)_{V+c} \otimes U_{V+c}(1)$ symmetry

unlocking under neutrality constraints

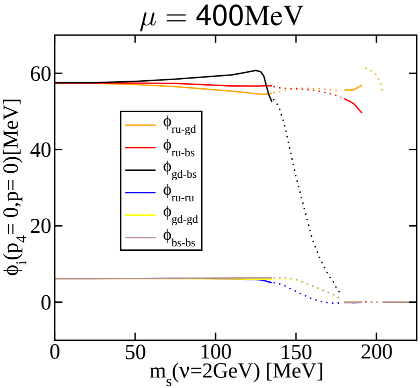
different stress on pairing pattern



$$\Phi = \begin{array}{cccccccccc} u & d & s & d & u & s & u & s & d & \\ \hline & \phi_4 & \phi_6 & & & & & & & u \\ \phi_5 & & & & & & & & & d \\ \phi_8 & & & & & & & & & s \\ & & & & \phi_{10} & & & & & d \\ & & & \phi_{11} & & & & & & u \\ & & & & & & & \phi_{12} & & s \\ & & & & & \phi_{13} & & & & u \\ & & & & & & & & & s \\ & & & & & & & & & d \end{array}$$

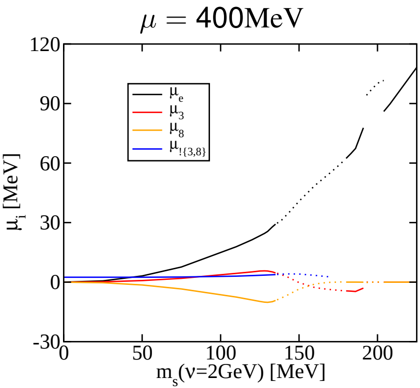
always up quarks involved in pairing → uSC phase
possible occurrence of 2SC for larger strange quark masses

gap functions



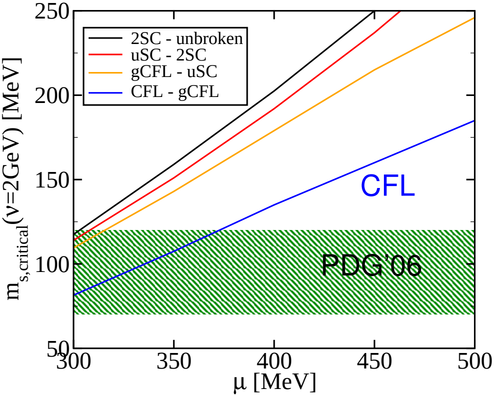
- CFL → gCFL → uSC → 2SC → unbroken

chemical potentials



- $\mu_{el} \neq 0$ in CFL phase!
  $\rightarrow$ long-range interaction? ’fully gapped’?
- μ_3 and μ_8 comparatively small

critical strange quark mass



(D.N., R. Alkofer, J. Wambach, 2008)

- light quark screen interaction also in strange quark sector
  → only small dynamical chiral symmetry breaking (different to NJL)!!!
  → other phases never favored for physical value of strange quark mass!?

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4. **back-reaction of Goldstone modes**
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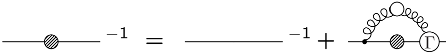
extension

quark self-energy



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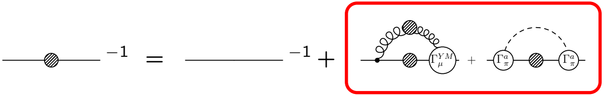


- in vacuum → 'pion cloud' (C. Fischer, D.N., J. Wambach, 2007)
- in CFL Phase (chiral limit):

$$SU_L(3)\otimes SU_R(3)\otimes SU_c(3)\otimes U_A(1)\otimes U_B(1)\longrightarrow \underbrace{SU_{c+V}(3)}_{\text{generated by } \tau_a-\lambda_a^T}$$

therefore we have **8**⊕**8**⊕**1**⊕**1** Goldstone modes (in Landau gauge)

Goldstone boson amplitudes

Ward identities for propagators (axial case)

$$P_\mu \Gamma^M_{5\mu}(k;P) = \mathcal{S}^{-1}(k+P/2) i\gamma_5 \mathcal{T}^M_5 + i\gamma_5 \mathcal{T}^M_5 \mathcal{S}^{-1}(k-P/2) \xrightarrow{P\to 0} f^M_5 \Gamma^M_5(k;0)$$

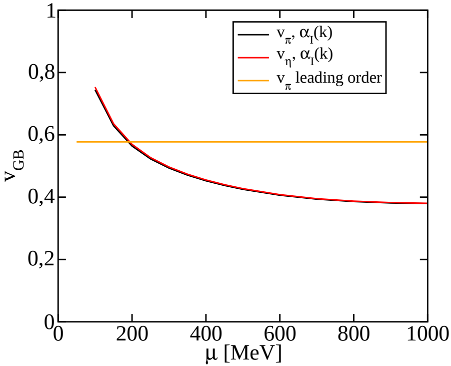
(Goldberger-Treiman)

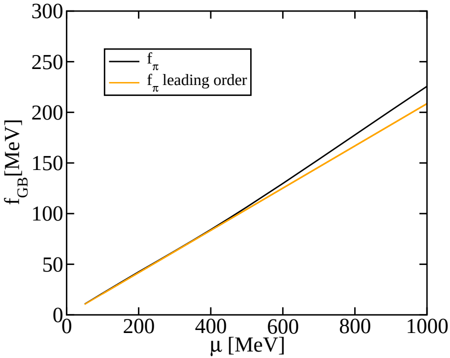
$$\Gamma^M_{5\mu}(k;P) = \frac{({v_M}^2\vec{P},P_4)_\mu}{P_4^2 + {v_M}^2\vec{P}2} f^M_5 \Gamma^M_5(k;P) + O(P_\mu)$$

decay constants / velocity

$$f^M_5({v_M}^2\vec{P},P_4)_\mu = \frac{Z_2}{2}\int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}\left[\mathcal{S}(q_+)\gamma_\mu\gamma_5\mathcal{T}_M\mathcal{S}(q_-)\bar{\Gamma}^M_5(q;-P)\right]$$

low-energy constants





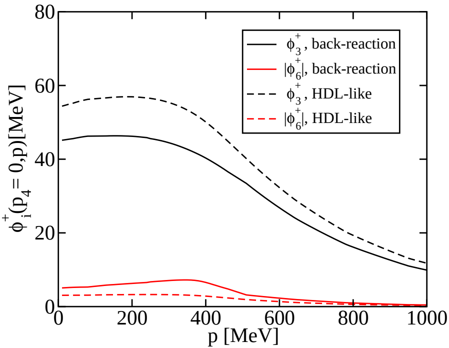
- 'leading order' results:

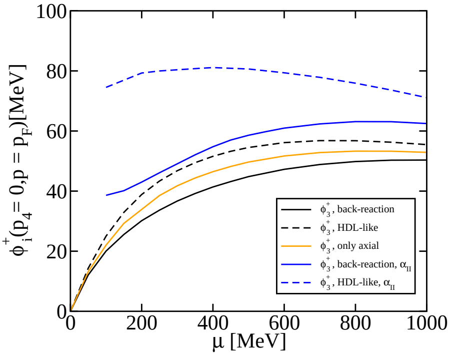
$$v_{GB} = \frac{v_F}{\sqrt{3}}$$

$$f_\pi^2 = \frac{21 - 8\ln 2}{36}\frac{p_F^2}{\pi^2}$$

no dependence on gap!

gap-functions





- back-reaction on gap-functions modest (at least for weaker coupling)
- reduced sensitivity on coupling
- no additional parameters

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conclusions

summary

- selfconsistent solution of DSE by approximating gluon propagator, incorporating medium effects
- huge deviations from extrapolated weak coupling results
- including finite masses and neutrality constraints
- CFL phase for physical strange quark mass at zero temperature!
- low-energy constants of Goldstone bosons in CFL phase
- modest back-reaction of Goldstone bosons