From gluons to glue-lumps and hybrids adventures in Coulomb gauge

- Coulomb gauge and the “decouple vs conformal” conundrum

- Some phenomenology of heavy QQ systems (with glue)

Special thanks to :

Peng Guo

Hrayar Matavosyan

Patrick Bowman

Adam Szczepaniak
IU

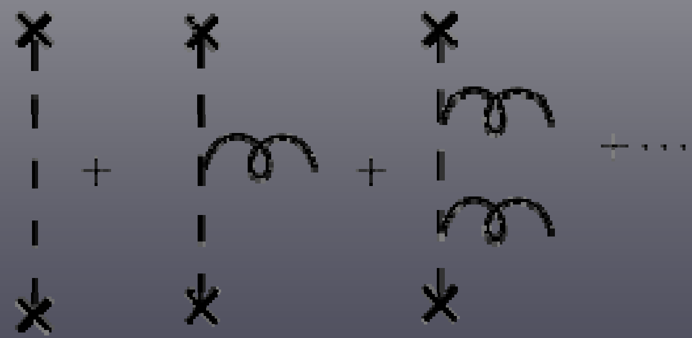
Hamiltonian Coulomb gauge QCD

$$H = H_D + H_{YM} + H_C$$

$$H_C = \int d\mathbf{x} d\mathbf{y} \rho^a(\mathbf{x}) K[\mathbf{x}, \mathbf{y}; \mathbf{A}]_{ab} \rho^b(\mathbf{y})$$

$$\boxed{K = \frac{1}{2} \frac{g}{\nabla \cdot D} (-\nabla^2) \frac{g}{\nabla \cdot D}}$$

$$K \to -\frac{g^2}{\nabla^2} = \frac{\alpha}{|\mathbf{x} - \mathbf{y}|}$$



$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

$|\Psi_0\rangle$ ground state (vacuum)

$|\Psi_1\rangle$ 1st excited state (Goldstone modes)

$|\Psi_{2,3,\ldots}\rangle$ higher excitations (bound states and continuum,excited state)

Gribov-Zwanziger $H = \sum_i E_i a_i^\dagger a_i + V$ $E_i = \sqrt{k^2 + m_i^2}$ $m_i \sim 1\ \mathrm{GeV}$

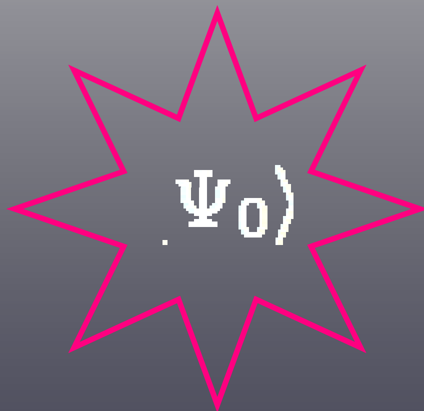
$|\rho^+\rangle \sim u^\dagger \bar{d}^\dagger |\Psi_0\rangle$ $G\rangle \sim g^\dagger g^\dagger |\Psi_0\rangle$ $H\rangle \sim q^\dagger g^\dagger \bar{q}^\dagger |\Psi_0\rangle$

other things of interest

$\langle\Psi_0|\mathbf{A}^a(\mathbf{k})\mathbf{A}^b(\mathbf{q})|\Psi_0\rangle = \delta^3(\mathbf{k}-\mathbf{q})\frac{\delta_{ab}}{2\omega(k)} \propto$ gluon "propagator"

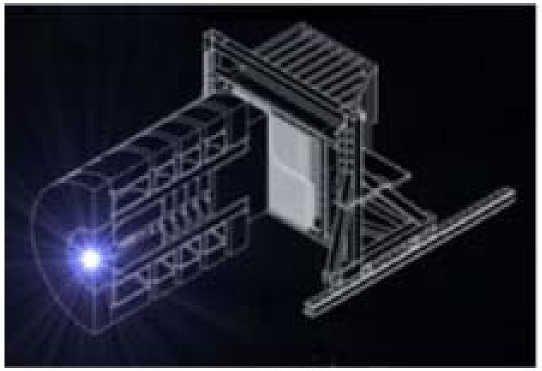
$\langle\Psi_0|\frac{g}{\nabla\cdot D}\Psi_0\rangle \propto$ ghost "propagator"

$\langle\Psi_0|\frac{g}{\nabla\cdot D}(\nabla^2)\frac{g}{\nabla\cdot D}|\Psi_0\rangle \propto$ Coulomb potential

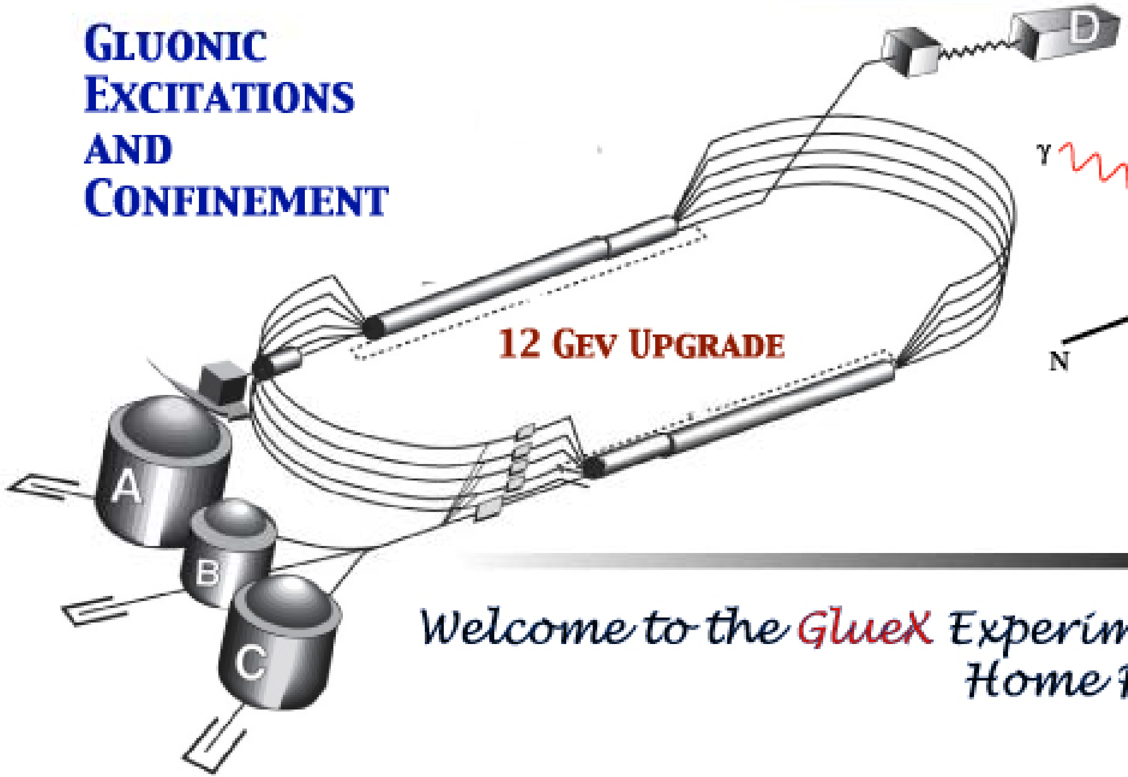


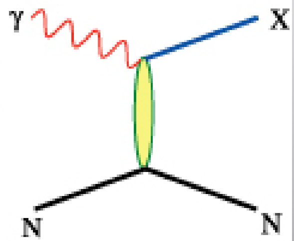
GLUE X CITATIONS PERIMENT

Hall D at Jefferson Lab



GLUONIC EXCITATIONS AND CONFINEMENT





Welcome to the GlueX Experiment Home Page

100 Physicists (including 16 theorists) from 6 countries including 10 states + D. of C.

- Australia
- Canada
- Poland
- Russia
- Scotland
- USA

Connecticut, D.C., Florida, Indiana, New York, New Mexico, North Carolina, Ohio, Pennsylvania, Tennessee, Virginia

- $$\langle A|\Psi\rangle = \Psi[A] \sim e^{-\frac{1}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3} |\mathbf{A}^a(\mathbf{k})|^2 k} \qquad (k >> \Lambda_{QCD})$$

  asymptotic freedom

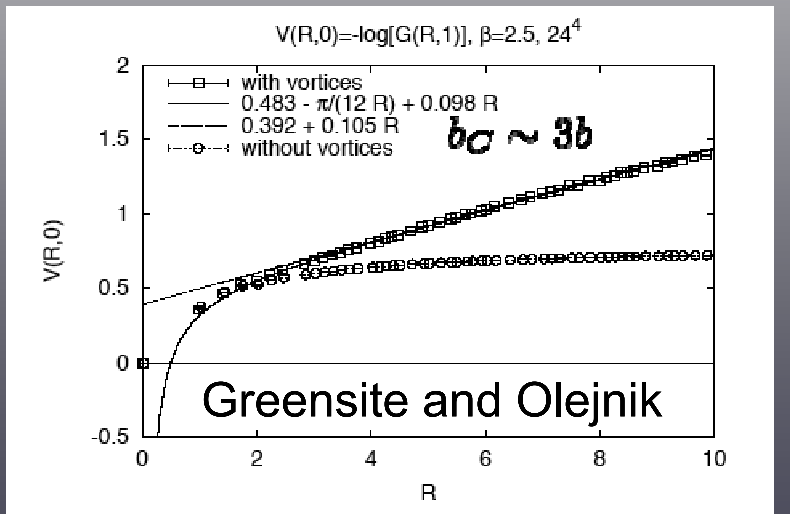
- $$\int d^3\mathbf{x}\,\psi^\dagger(\mathbf{x})\frac{\tau^a}{2}\gamma_5\psi(\mathbf{x})\Big| |\Psi\rangle \neq 0$$ χ-symmetry breaking

- $$\Psi[A] \sim e^{-\mu^4 G}$$ Zwanziger, Zwanziger Cuccieri

- $\Psi[A]$ **is large for A near a solitary wave solution**

  confinement, $U_A(1)$ breaking

- $$\Psi[A] \sim e^{-F\frac{1}{\sqrt{-D^2+m^2}}F}$$ Greensite



YM sector

- $$\langle A\,\Psi\rangle = \Psi[A] \sim e^{-\frac{1}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3} |\mathbf{A}^a(\mathbf{k})|^2 \omega(k)}$$

Reinhardt, AS, Zwanziger

$$\langle A|\rangle \sim e^{-\frac{1}{2}\int \frac{d^3\mathbf{k}}{(2\pi)^3} |\mathbf{A}^a(\mathbf{k})|^2 \omega(k)}$$

$$\int dx e^{i\mathbf{k}\mathbf{x}} \langle \frac{g}{-\vec{\nabla}\cdot\vec{D}} \rangle_{a,x;b,0} = \delta_{ab} \frac{d(k)}{k^2}$$

ghost propagator

$$\int dx e^{i\mathbf{k}\mathbf{x}} \langle \mathbf{A}^a(\mathbf{x})\mathbf{A}^b(0)\rangle = \frac{\delta_T(\mathbf{k})}{2\omega(k)}$$

gluon propagator

$$\frac{\partial}{\partial\omega}\langle H\rangle = 0$$

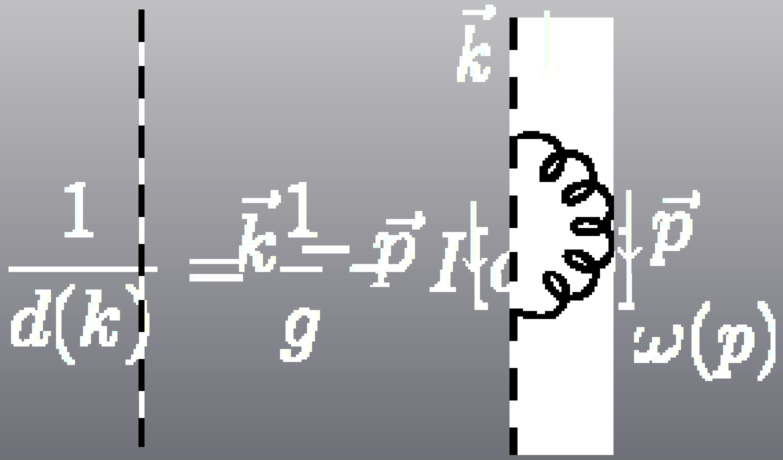
gap equation

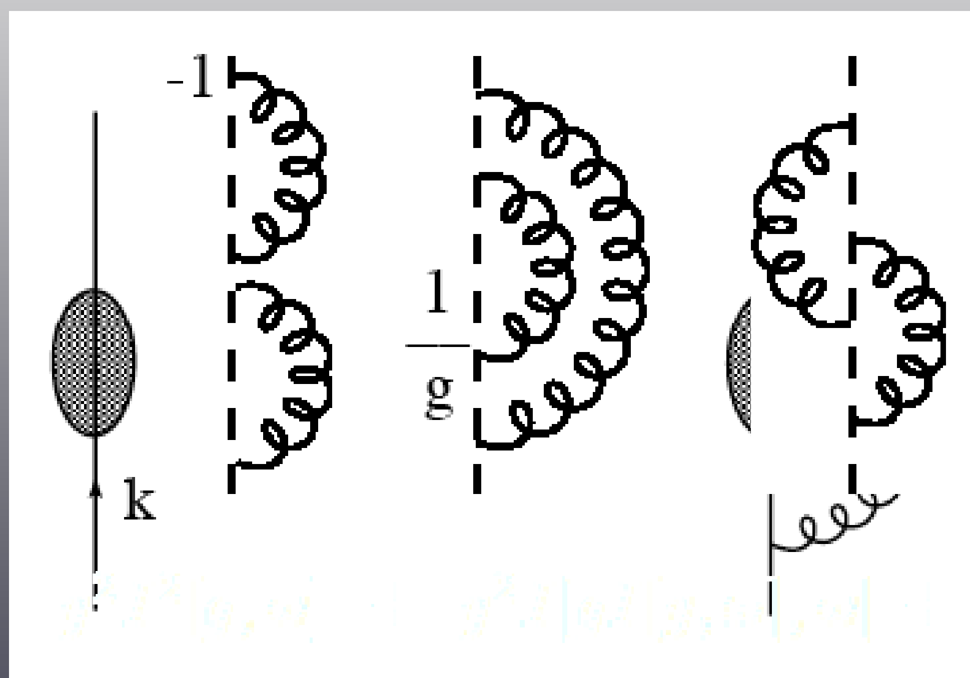
$$\int dx e^{i\mathbf{k}\mathbf{x}} \langle \frac{g}{-\vec{\nabla}\cdot\vec{D}} (-\vec{\nabla}^2) \frac{g}{-\vec{\nabla}\cdot\vec{D}} \rangle_{a,x;b,0} = \delta_{ab} \frac{d^2(k) f(k)}{k^2}$$

Coulomb form factor

Summation of all planar diagrams
can be expressed in
terms of two Dyson equation

$$\int dx e^{i\mathbf{k}\mathbf{x}} \langle \frac{g}{-\vec{\nabla}\cdot\vec{D}} \rangle_{a,x;b,0} = \delta_{ab} \frac{d(k)}{k^2}$$

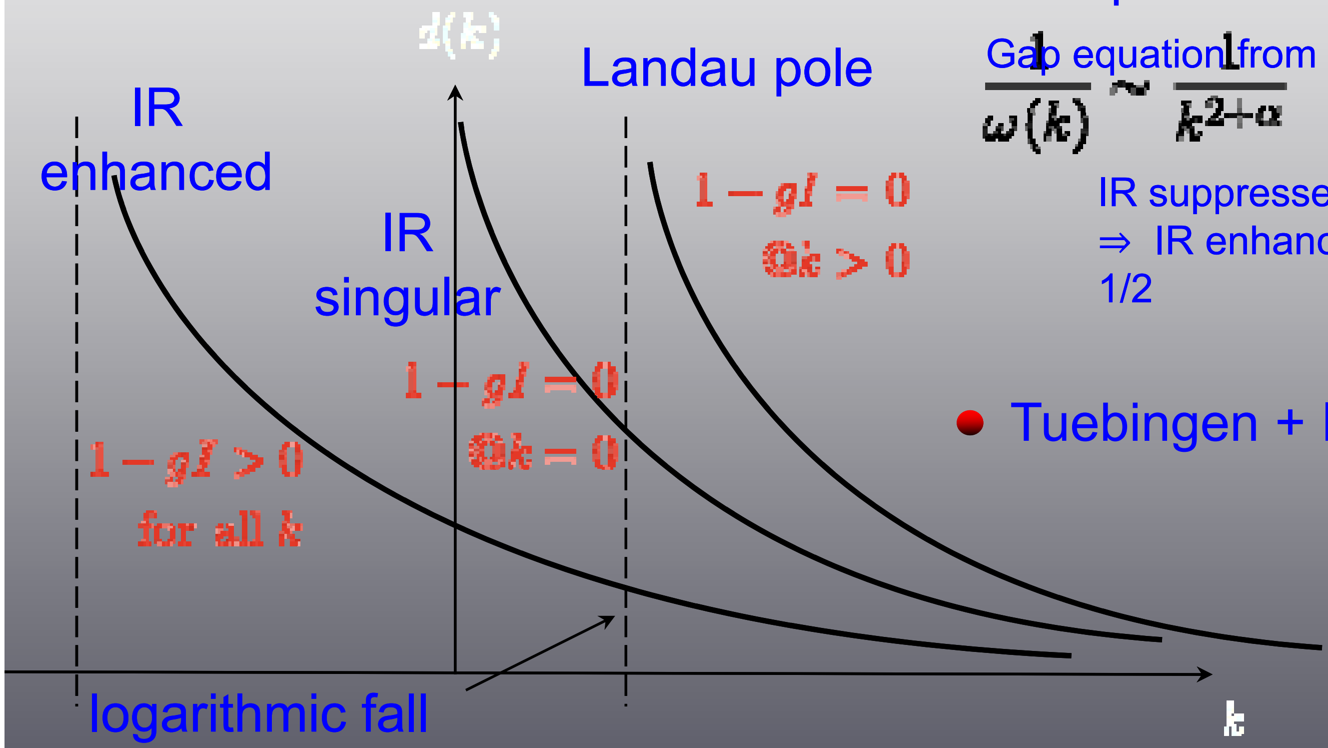




$$d = g[\ 1 \ + \ gI[g,\omega] \ + \ \cdots$$

$$d(k) = \frac{g(\Lambda)}{1 - g(\Lambda) I_\Lambda[d,\omega]}$$

Three scenarios (depending on g(Λ) and ω)



- Zwanziger:

$$\int_{A\in\Omega} \Rightarrow H \to H + \mu^4 G \qquad G \sim \int \frac{A^2}{\nabla D}$$

wave function ansatz diagonalizes the quadratic part (without self energy)

$$k \to \omega = \sqrt{k^2 + \mu^4 \frac{d(k)}{k^2}}$$

gluon mass

$$d(k) \sim \frac{1}{k^{1/3}} \qquad V(R) \sim R^{5/3}$$

- Szczepaniak: ω(0) = const
- Reinhardt et al. IR analysis

subcritical solutions d(0) = finite

Gap equation from min < ω|H| ω>

$$\frac{1}{\omega(k)} \sim \frac{1}{k^{2+\alpha}}$$

approximate analytical solution

$$d(k) \sim \frac{1}{k^\kappa} \qquad \alpha + 2\kappa = -1$$

IR suppressed gluon propagator α < -2
⇒ IR enhanced ghost propagator κ > 1/2

- Tuebingen + IU: Only IR finite solutions exist for the full set of coupled Dyson equations for ghost, gluon, and Coulomb form factors : Need to match "pre-factors" not only powers

$$\int_{A\in\Omega} DA\, \mathcal{J} \frac{\Psi^2[A]}{\nabla \cdot D[A]}$$

$$d(k) = \frac{g(\Lambda)}{1 \quad g(\Lambda) I_\Lambda[d,\omega]}$$

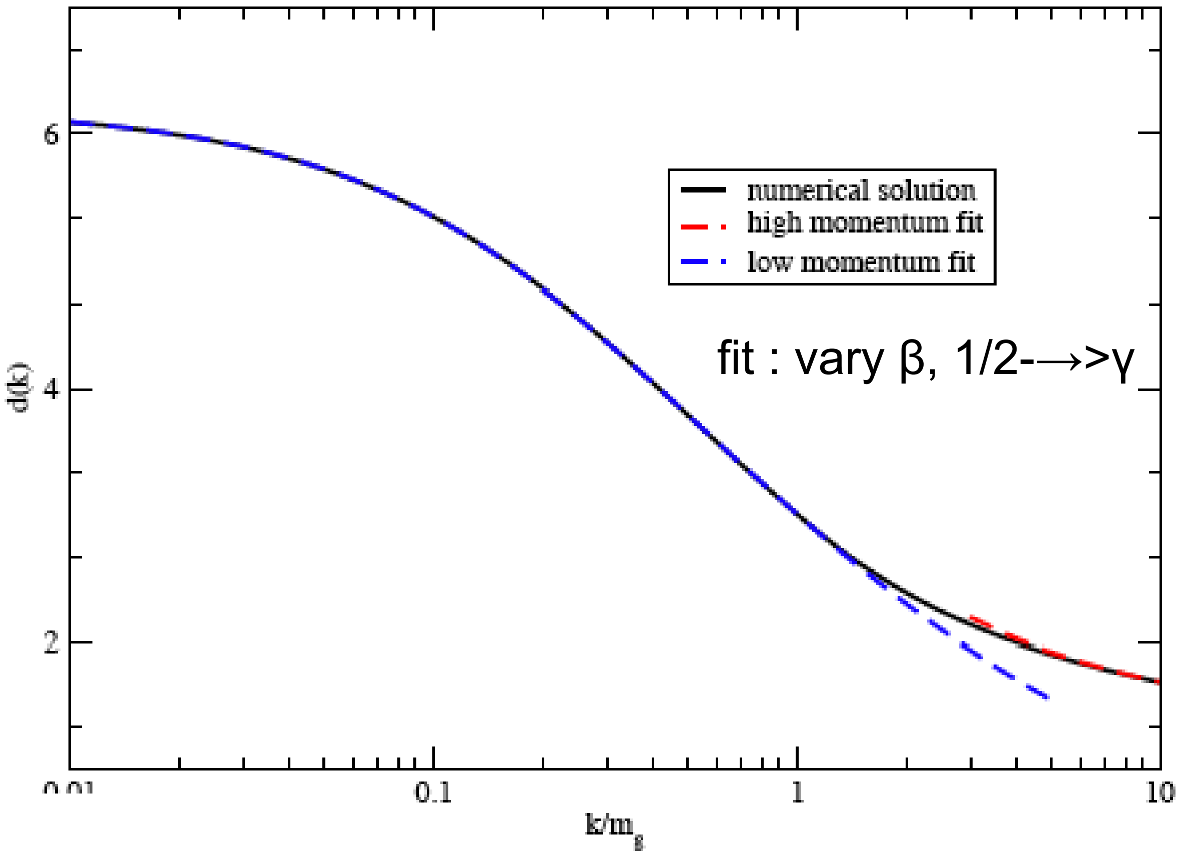
low momentum approximation

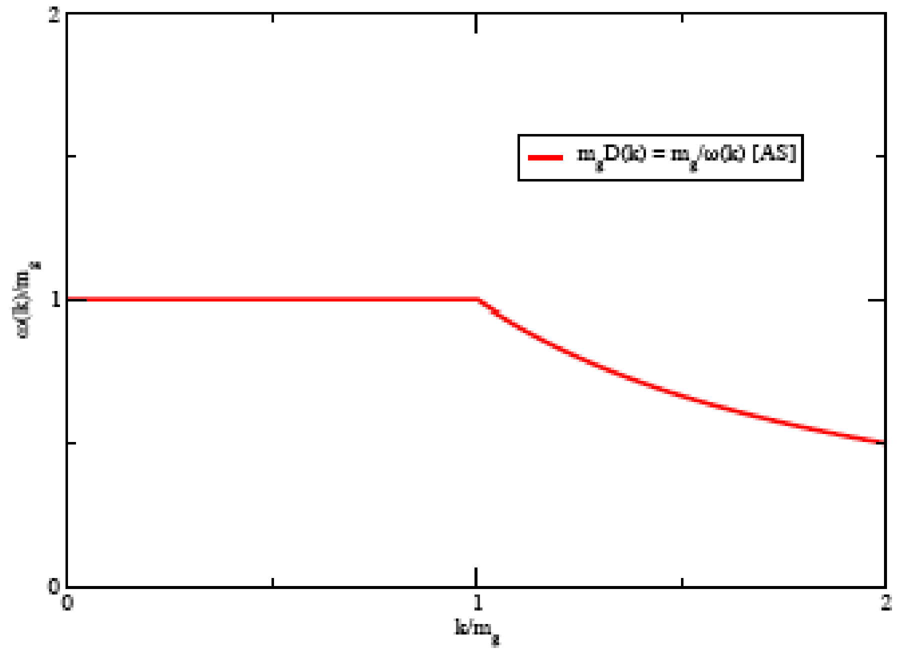
$$d(k) = \frac{g(\Lambda)}{\left[1+\beta^2 g^2(\Lambda)\left(\frac{k}{m_g} - \frac{\Lambda}{m_g}\right)\right]^{1/2}}$$

$$\beta = \frac{1}{\pi}\sqrt{\frac{40}{48}} \sim 0.27$$

critical coupling

$$g(\Lambda) = \frac{1}{\beta}\left(\frac{m_g}{\Lambda}\right)^{1/2} \sim \pi\left(\frac{m_g}{\Lambda}\right)^{1/2}$$





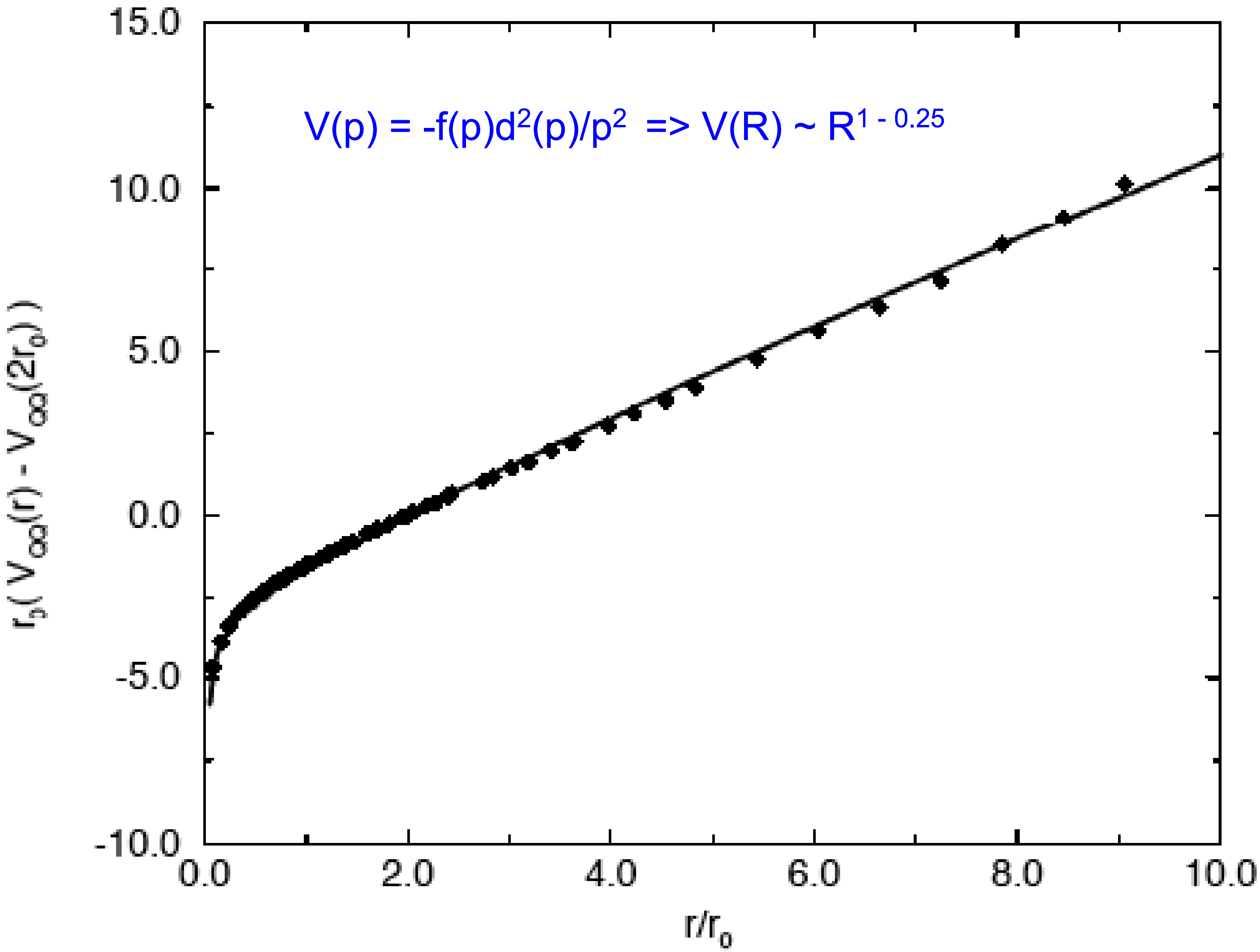
$$d(k) = \frac{g(\Lambda)}{\left[1+\beta^2 g^2(\Lambda)\ln(k/\Lambda)\right]^{1/2}}$$

$$\beta = \frac{1}{\pi} \sim 0.32$$

high momentum approximation (from "angular approximation to DS -> exact solution)

$$-\langle\omega|K[r,0,\mathbf{A}]_{ab}|\omega\rangle = V(r)\delta_{ab}$$

Szczepaniak, Swanson



Dyson-Schwinger approximation

- Rainbow-ladder (i.e. no vertex corrections)
- What is the role of the Gribov horizon (when $\omega(0)$ = finite, which is the only solution of the coupled set of DS. equations this is controlled by $g(\Lambda)$)

Use lattice to check ! Lattice "Lite"

Lattice "Lite"

- Discretize the spatial dimensions and use Monte-Carlo method to perform the functional integral using model wavefunction.

$$k_i = \frac{2\pi n_i}{aL_i} \quad |n_i \quad \in \left(-\frac{L_i}{2}, \frac{L_i}{2}\right], i \in \{1,2,3\},$$

$$\int^{FMR} \mathcal{D}A \to \sum_{A_i}$$

- Calculate expectation value of observables are

$$\langle \mathcal{O} \rangle = \frac{\langle \Psi | \mathcal{O} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

$$\Psi[A]|^2 = \exp\left\{ -\frac{1}{V} \sum_k \sum_{i=1}^{N_d} \sum_{c=1}^{N_c^2-1} A_i^c(k) A_i^c(-k) \frac{\omega(k)}{g^2} \right\}$$

$$A_i^a(-k) = A_i^{a*}(k)$$

- Setting $\mathcal{J} = 1$,

Gluon Propagator

Discrete form:

$$G(k) = \frac{1}{V}\frac{1}{N_d - 1}\frac{1}{N_c^2 - 1}\langle A_i^a(k) A_i^a(-k)\rangle$$

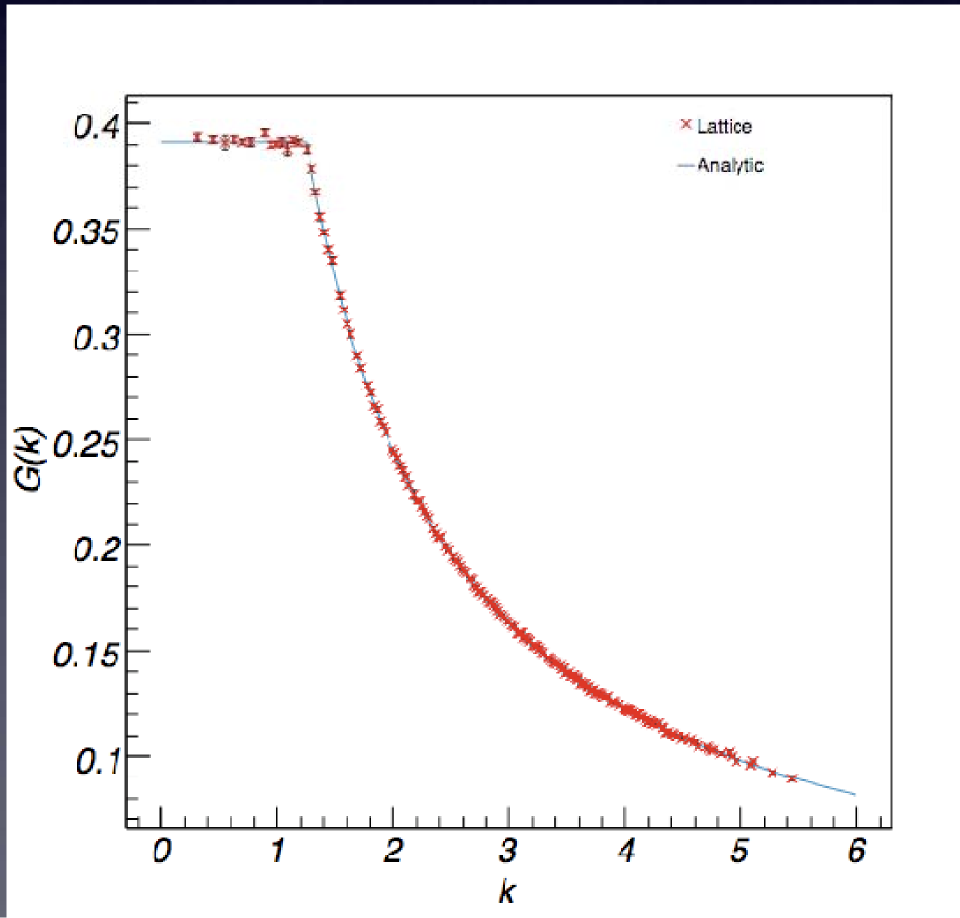
Analytic:

$$G(k) = \frac{1}{2\omega(k)}$$

Coulomb Gauge Condition:

$$\sum_{i=1}^{3} A_i(x) - A_i(x - \hat{i}) = 0$$

$$\sum_{i=1}^{3} (1 - \cos(k) + i\sin(k))A_i(k) = 0$$



Faddeev-Popov Operator

- Discrete form:

$$\begin{aligned}(M\phi)^a(x) &= \sum_{i=1}^{N_d} \delta^{ab}\left(\phi^b(x+\hat{i}) + \phi^b(x-\hat{i}) - 2\phi(x)\right) \\ &\quad - f^{abc}\left(\phi^b(x+\hat{i})A_i^c(x) - \phi^b(x-\hat{i})A_i^c(x-\hat{i})\right)\end{aligned}$$

real and symmetric.

- Check for positivity of lowest eigenvalues.

- Periodic boundary conditions produce trivial zero-modes for M, thus formally non-invertible.

$$M^{ab}(x,y)\phi^b(y) = \delta^{ab}\left(\delta(x) - \frac{1}{V}\right)$$

$$\begin{aligned}\sum_x e^{-i\mathbf{k}\cdot\mathbf{x}}\langle\phi^a(x)\rangle &= \sum_x e^{-i\mathbf{k}\cdot\mathbf{x}}\langle(M^{-1})^{aa}_{x0}\rangle - \frac{1}{V}\sum_{x,y} e^{-i\mathbf{k}\cdot\mathbf{x}}\langle(M^{-1})^{aa}_{xy}\rangle \\ &= \sum_x e^{-i\mathbf{k}\cdot\mathbf{x}} D(x) - \frac{1}{V}\sum_{x,y} e^{-i\mathbf{k}\cdot\mathbf{x}} D(x-y) = D(p) - \delta(p)\sum_x D(x)\end{aligned}$$

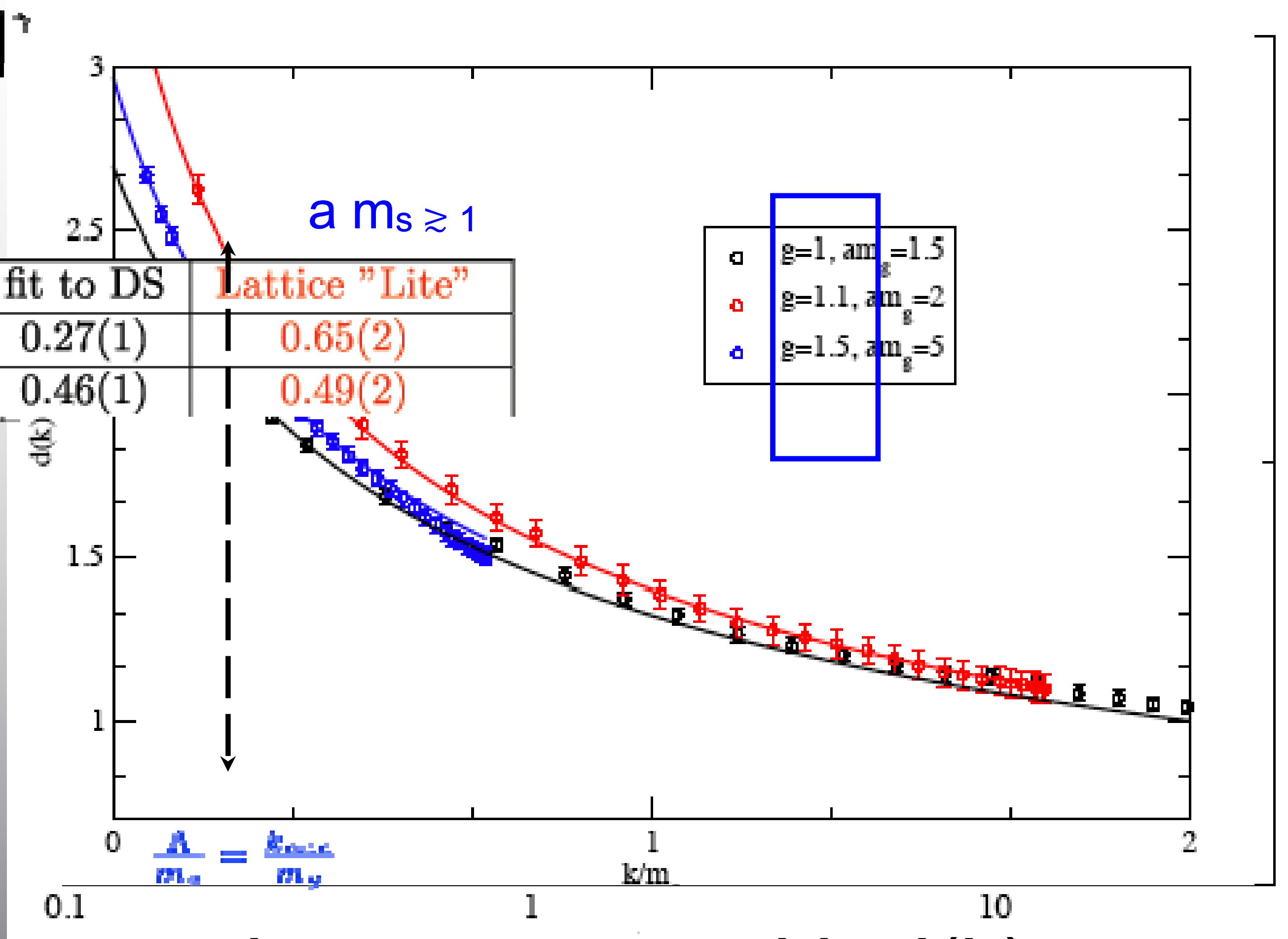
Low momentum

High momentum

$$d(k) = \frac{g(k;\Lambda)}{\left[1+\beta g^2 \ldots \ln\left(\frac{k}{m_g}/\Lambda\right) \ldots\right]^{\gamma}}$$

fit : β,γ

	approx. Dyson-Schwinger (fit)	fit to DS	Lattice "Lite"
β	0.27	0.27(1)	0.65(2)
γ	1/2	0.46(1)	0.49(2)



renormalized at fixed::

$g = g(k_{max}=\Lambda)$

$g = g(k_{min}=\Lambda)$

Summary:

$d(k)_{lattice}$ in very good agreement with $d(k)_{Dyson-Schwinger}$

$\beta_{lattice} > \beta_{Dyson-Schwinger}$: $g^c_{lattice} < g^c_{Dyson-Schwinger}$

Seems like lattice also prefers IR "decoupled" solutions : unable to find
g, am_g combination which is strictly critical

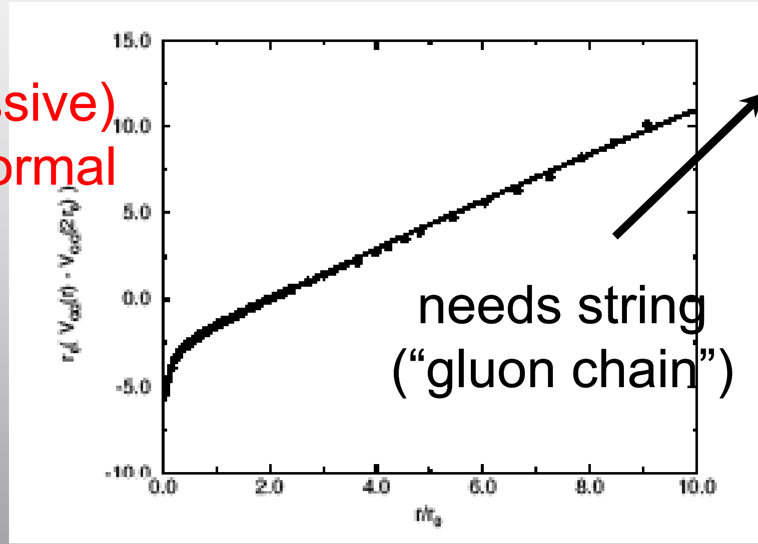
“To decouple or to not decouple this is the question”

From the point of view of phenomenology of confinement

- Temporal Wilson loop : static potential

$V_c(R) = b_c R^{1-\gamma}$ $(\gamma \gtrsim 0)$

for both Decoupled (massive) AND (critical) AND Conformal (critical)



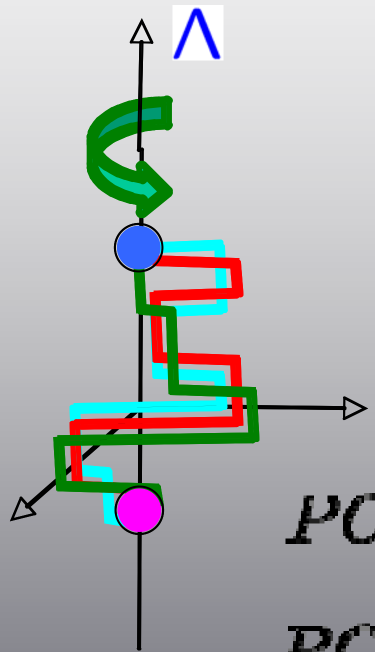
seems irrelevant for confinement from the point of view of temporal Wilson loop : something still missing in $|\Psi_0>$ anyway ! (“no Confinement with Coulomb confinement” should to be satisfied)

- Spacial Wilson loop :

Both Decoupled (massive) AND (critical) AND Conformal (critical) solutions lead to Perimer law !

$\Psi_0[A]$ needs to feel the presence of center disorder !

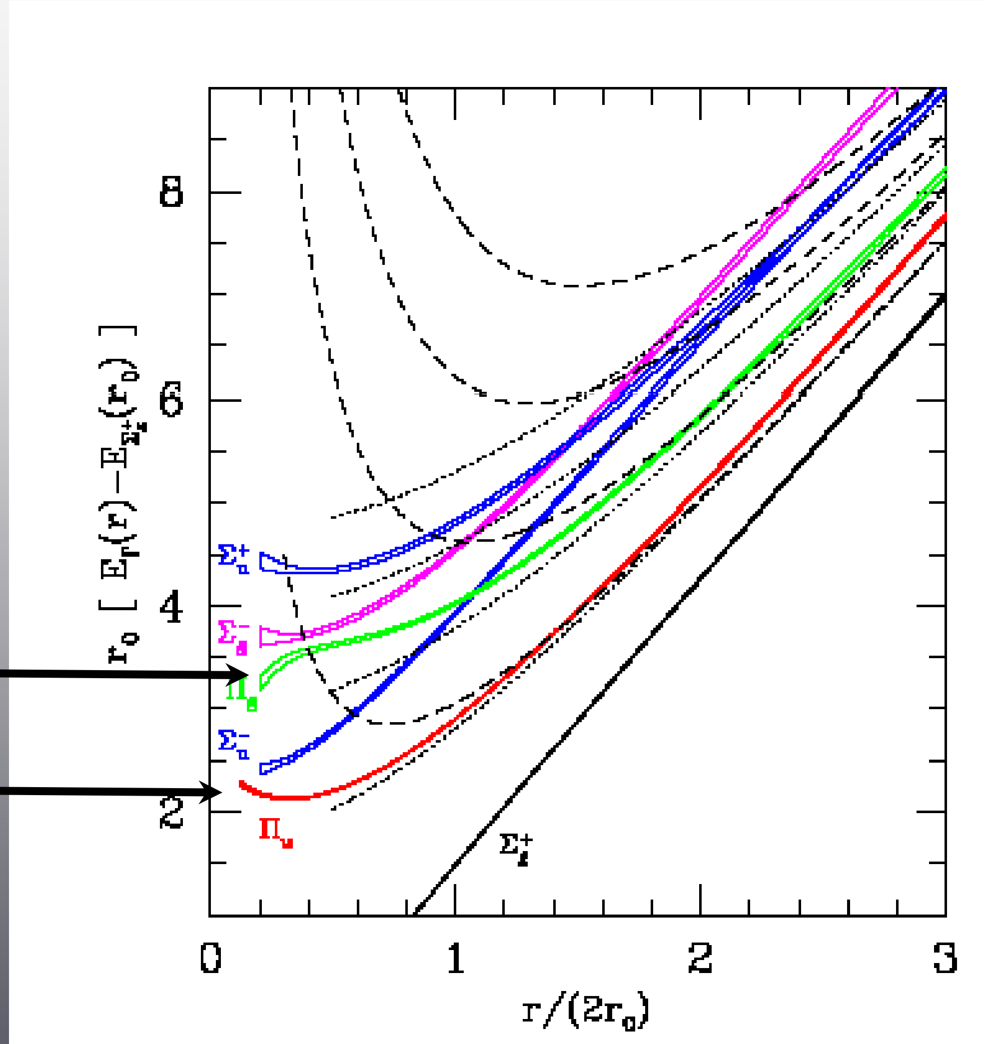
Gluonic excitations in presence of static sources



$$PC = +1$$

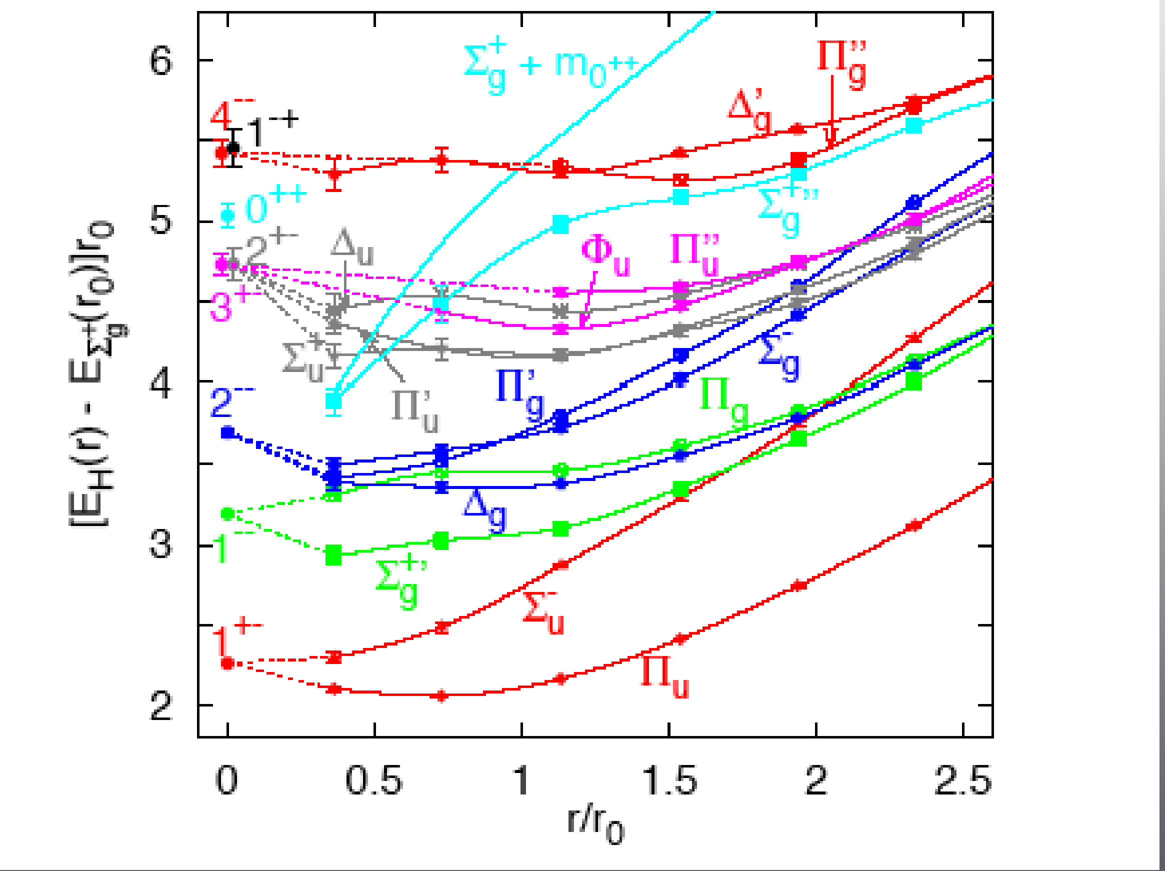
$$PC = -1$$

$$\Lambda^Y_{PC} = (\Sigma, \Pi, \Delta, \cdots)^+_{u,g}$$



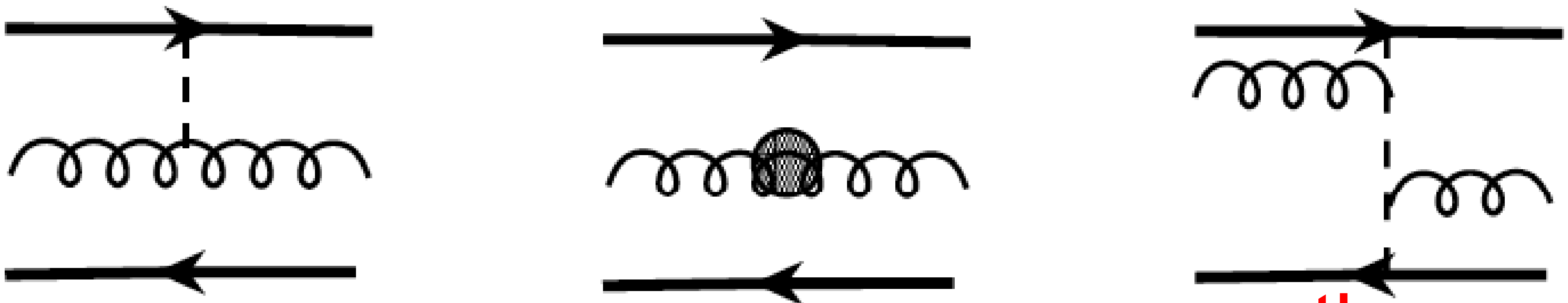
Glue-lumps : (“R=0” static hybrids)

(our V_c is good for small R: so lets do static quarks and not too many gluons)



$$E_G^{J^{P'}} = E_H(r) - E_{QQ} - V_C^8 + V_C^0$$

Single gluon hybrids (with static sources)



two-body potential

one-body potential

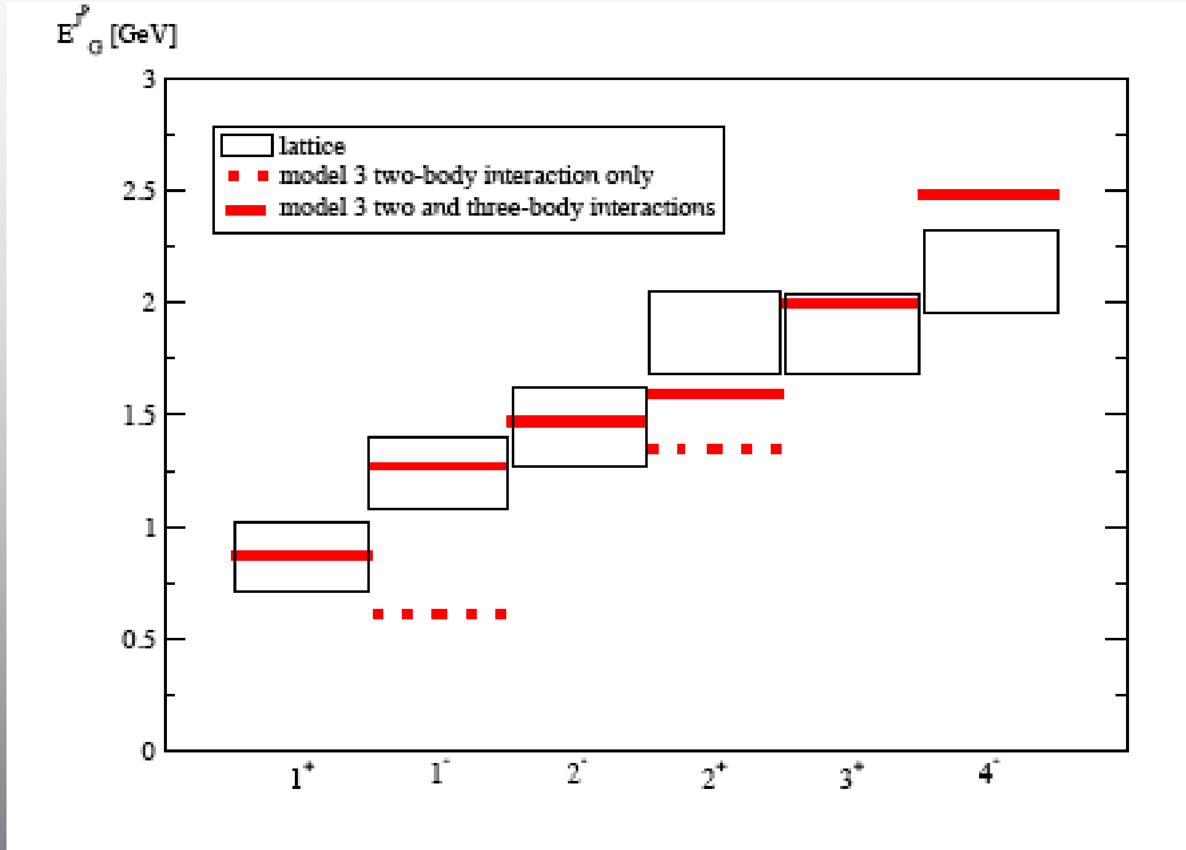
three-body potential

$$H|\Lambda_Y^{PC}\rangle = E(r)|\Lambda_Y^{PC}\rangle$$

one-body Schrodinger eq.

$$|\Lambda_Y^{PC}\rangle = \int d\mathbf{k}|Q(r/2)\bar{Q}(\;\; r/2);\mathbf{k}\lambda\rangle\Psi_{\Lambda_Y^{PC}}(\mathbf{k},\lambda)$$

Glue-lumps : (“R=0” static hybrids)



$$E_G^{J^{PC}} = E_H(r) - E_{QQ} - V_C^8 + V_C^0$$

Real Heavy Hybrids via Foldy - Wouthuysen Hamiltonian

$$
\begin{aligned}
H_{FW} &= m\int d^3x\Psi^\dagger(\mathbf{x})\beta\Psi(\mathbf{x}) + \frac{1}{2}\int d^3x(\mathbf{E}_a^2 + \mathbf{B}_a^2 + \rho^a A_0^a) \\
&+ \frac{1}{2m}\int d^3x\Psi^\dagger(\mathbf{x})\beta\mathbf{D}^2\Psi(\mathbf{x}) - \frac{g}{2m}\int d^3x\Psi^\dagger(\mathbf{x})\beta\mathbf{\Sigma}\cdot\mathbf{B}^c(\mathbf{x})T^c\Psi(\mathbf{x}) \\
&- \frac{1}{16m^2}\int d^3x(\mathbf{J}_+^a\cdot\mathbf{J}_-^a - \nabla\cdot\mathbf{J}_+^a\frac{1}{\nabla^2}\nabla\cdot\mathbf{J}_-^a) \\
&- \frac{1}{16m^2}\int d^3x d^3y d^3z \nabla\cdot\mathbf{J}_+^a\mathcal{D}_{ab}\nabla^2\mathcal{D}_{bc}\nabla\cdot\mathbf{J}_-^c \\
&- \frac{1}{4m^2}\int d^3x\Psi^\dagger(\mathbf{x})\varepsilon_{ijk}\Sigma_j(E_i^a - \nabla_i A_0^a)(igT^a\nabla_k + \frac{g^2}{2}\{T^a,T^b\}A_k^b)\Psi(\mathbf{x}) \\
&- \frac{1}{4m^2}\int d^3x\Psi^\dagger(\mathbf{x})\frac{g^2}{2}f^{abc}T^b A_i^c\Psi(\mathbf{x})(E_i^a - \nabla_i A_0^a) \\
&- \frac{1}{8m^2}\int d^3x\Psi^\dagger(\mathbf{x})\varepsilon_{ijk}\Sigma_i(igT^a\nabla_j E_k^a)\Psi(\mathbf{x}) \\
&+ \frac{1}{8m^2}\int d^3x\Psi^\dagger(\mathbf{x})gT^a\Psi(\mathbf{x})\nabla^2 A_0^a + \mathcal{O}(\frac{1}{m^3})
\end{aligned}
$$

expected degeneracies

$$H_{FW} = m\int d^3x \Psi^\dagger(\mathbf{x})\beta\Psi(\mathbf{x}) + \frac{1}{2}\int d^3x(\mathbf{E}_a^2 + \mathbf{B}_a^2 + \rho^a A_0^a) + \frac{1}{2m}\int d^3x \Psi^\dagger(\mathbf{x})\beta\mathbf{D}^2\Psi(\mathbf{x})$$

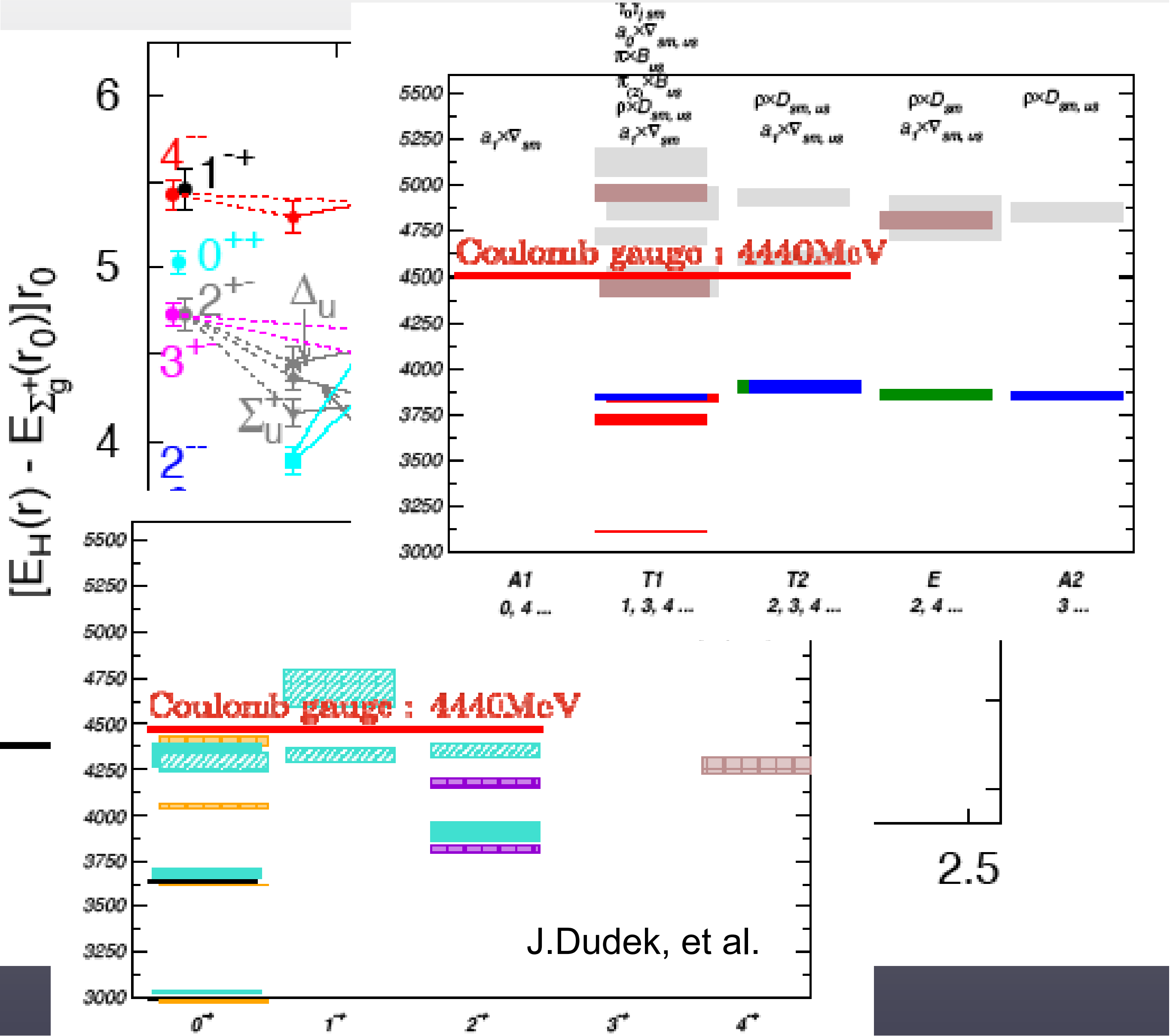
J^{PC} glue

J^{PC} $Q\bar{Q}$

$$1^{--} \times 0^{-+}_{S_{QQ}} = \boxed{1^{--}}$$

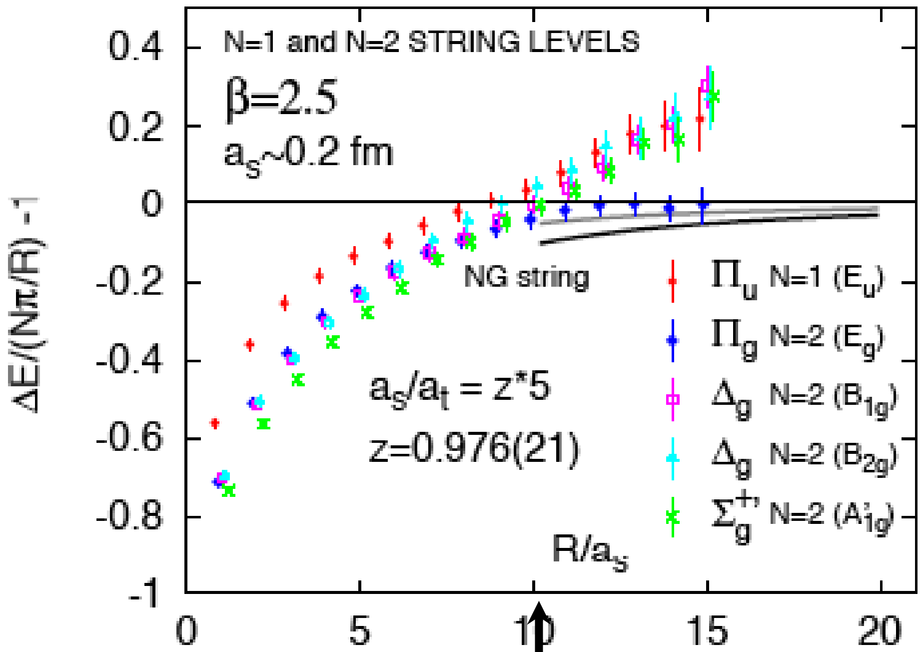
$$1^{+-} \times 1^{--}_{S_{QQ}=1} =$$

$$\boxed{0^{-+}, 1^{-+}, 2^{-+}}$$



where is the string limit ?

FIG. 1: Energy gaps ΔE above Σ_g^+ are shown in string units for quantum numbers in continuum and lattice notation. The Nambu-Goto string is discussed in the text.



Morningstar et al. 2 fm

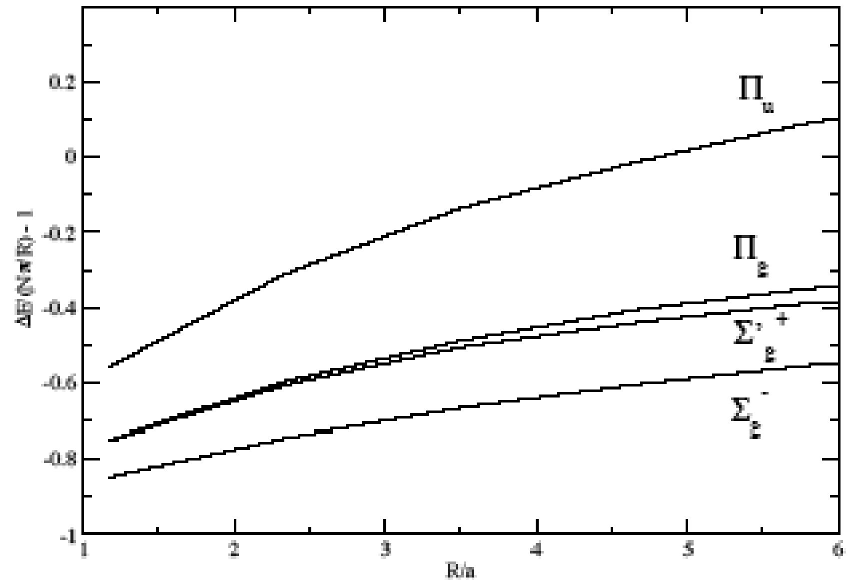
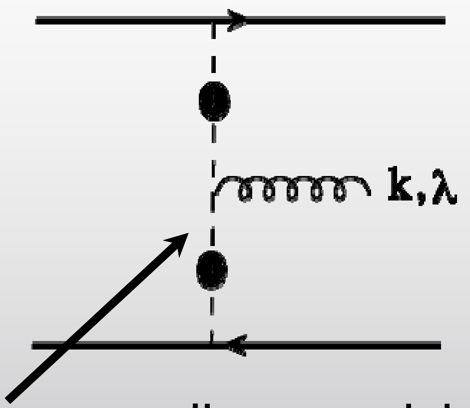


FIG. 8: Splittings between the excited energies of quasi-gluons and the ground state $Q\bar{Q}$ energy. The values N expected from the string model are, $N = 1$ for Π_u, $N = 2$ for Π_g and Σ_g' and $N = 4$ for Σ_g^- ($a = 0.2$ fm).

$$E(r) = \boxed{Na} + br + \frac{N\pi}{r}$$

$$\frac{\Delta E(r)}{N\pi/r} - 1 = \frac{a}{\pi} r$$

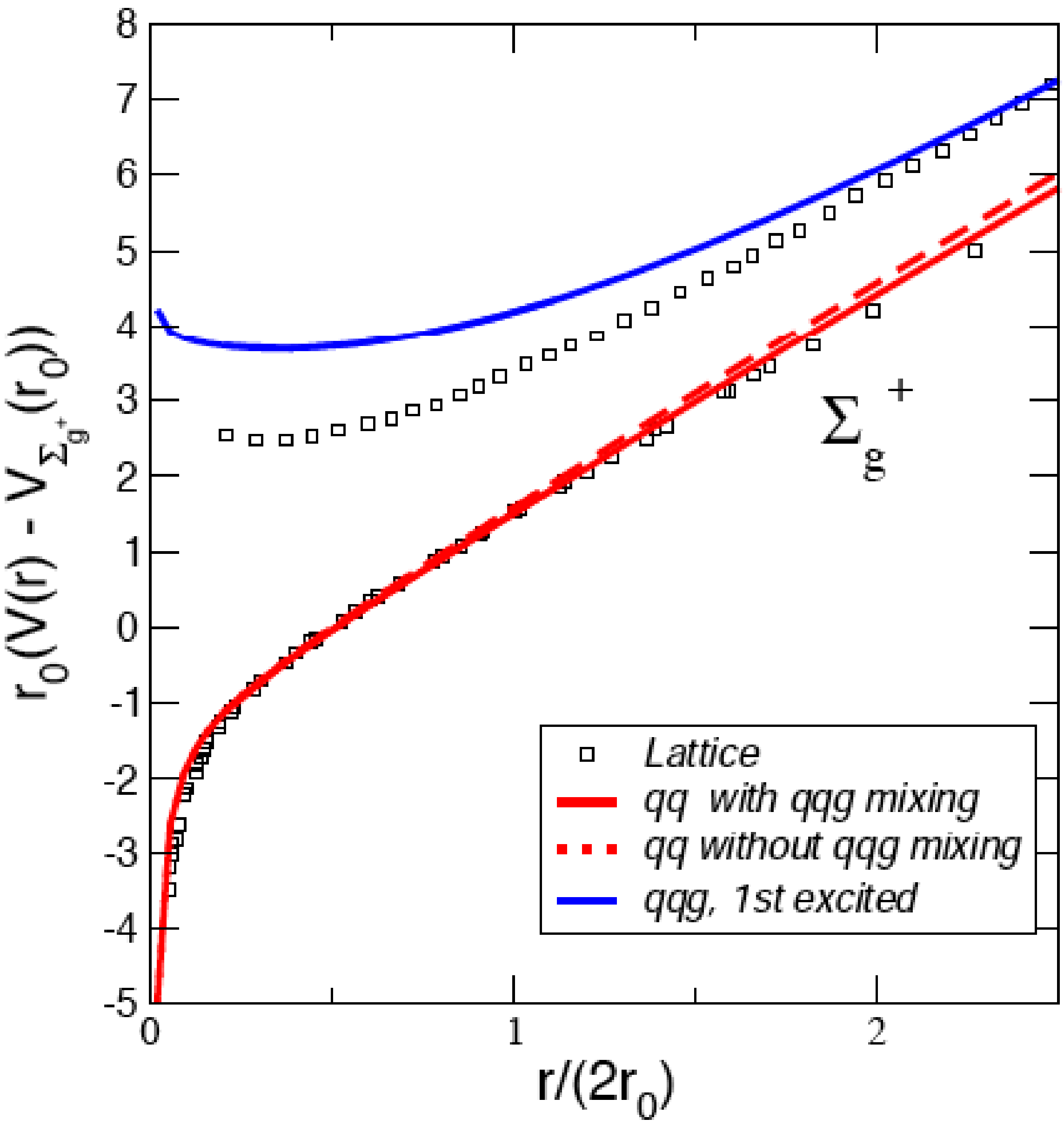
Szczepaniak, Krupinski



P-wave coupling : vanishes as

$R \to 0$

and $V_{mix}/R \to 0$

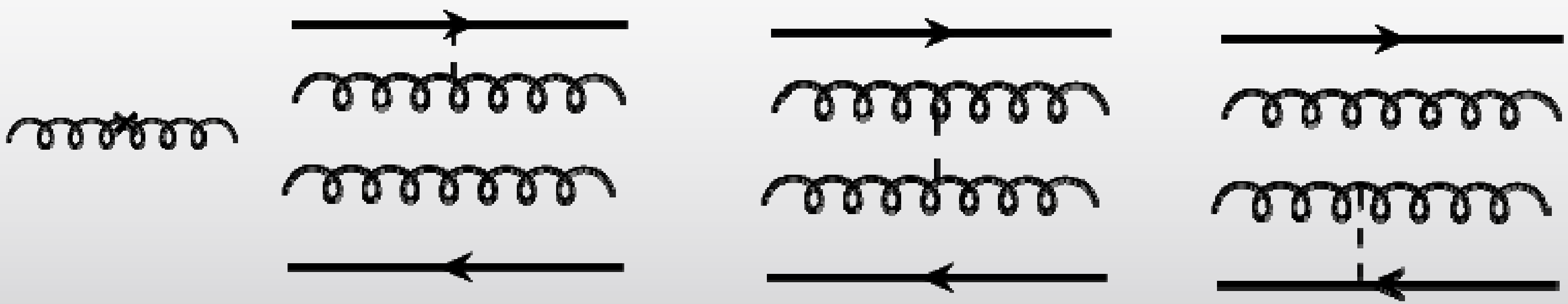


Szczepaniak,Krupinski

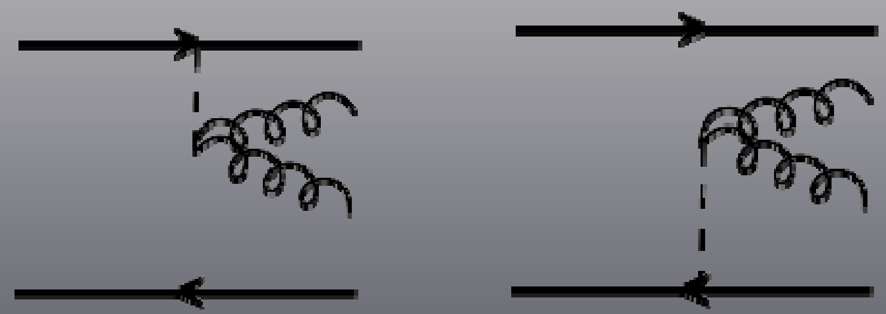
(lattice :Morningstar et al.)

Towards the gluons chain

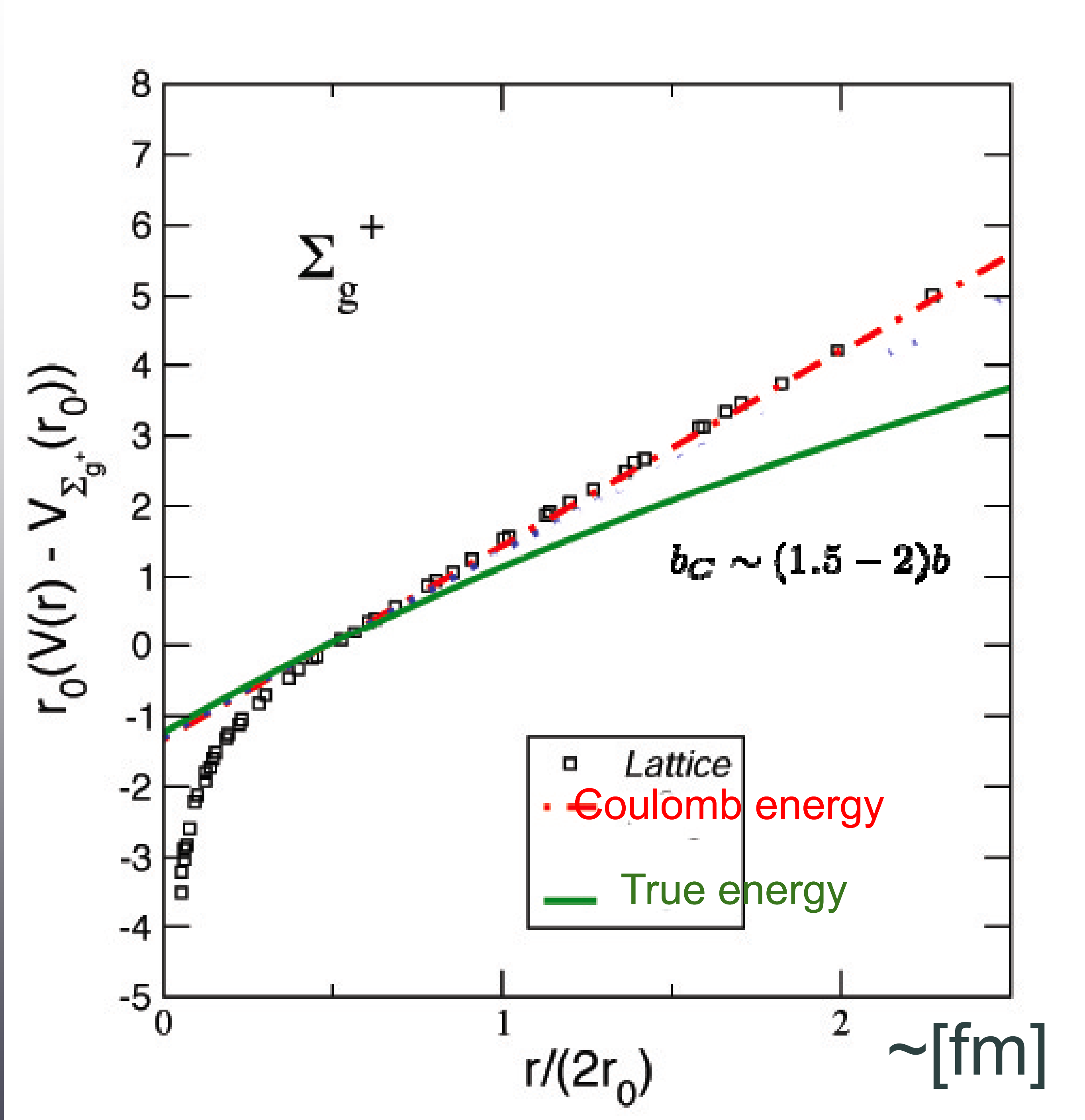
(Thorn, Greensite)



$$H_{n'n} = [n m_g + C_F b R]\delta_{n'n}$$



$$H_{n'n} = \gamma C_F R[\delta_{n',n+2} + \delta_{n'+2,n}]$$



with up to 40 gluons

Summary :

- Coulomb gauge perfect for QCD inspired hadron phenomenology
- Center vortices coming soon so stay tuned

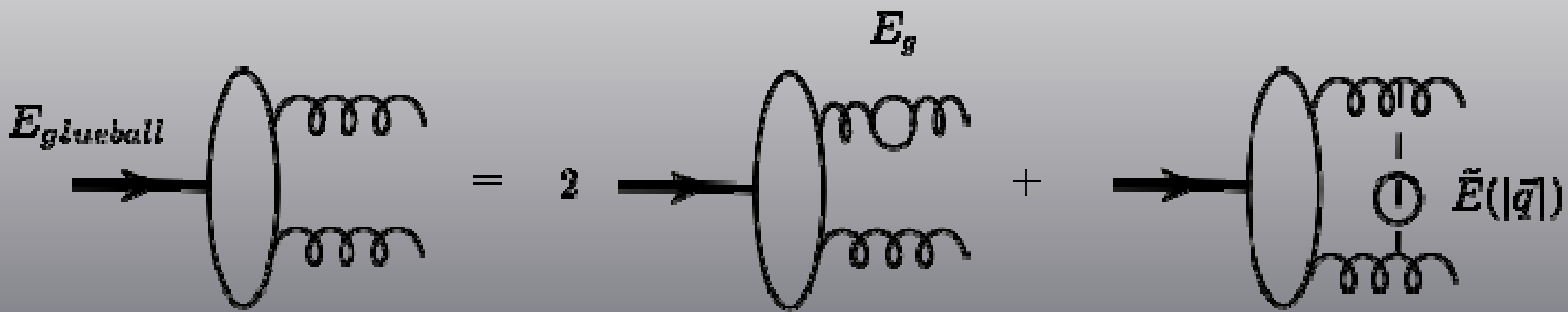
Quasi-gluons and glueballs

$\omega(|\vec{k}|)$ plays the role of average one-gluon kinetic energy in a color singlet state

$\omega(0) = 0.6\ \mathrm{GeV}$

Not to be confused with a one-gluon energy which is IR unstable

$$E_g \propto \int \frac{d\vec{k}}{(2\pi)^3} \tilde{E}(|\vec{k}|)$$

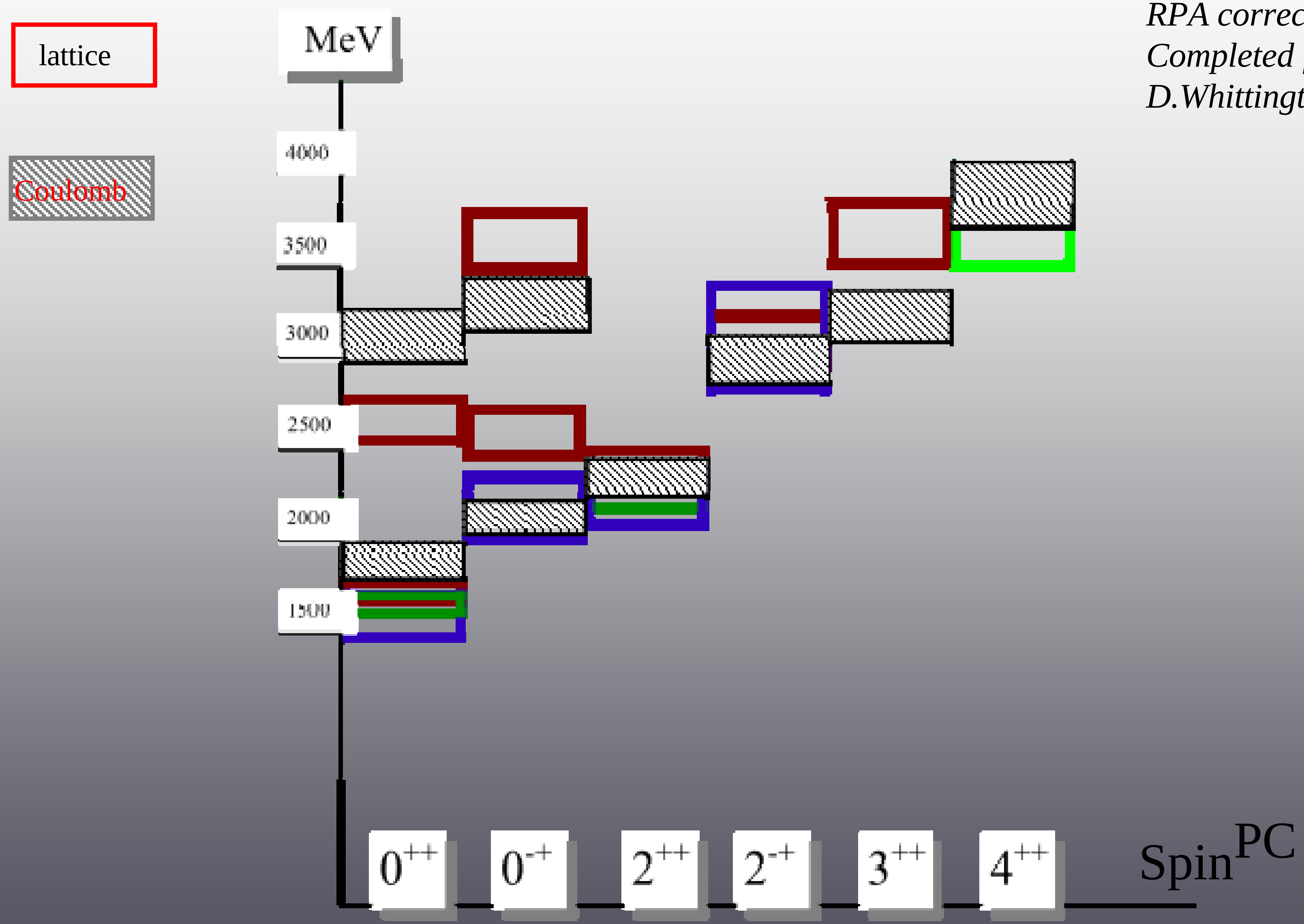


sum is IR finite (for color singlet)

Glueball Spectrum

Szczepaniak,Swanson

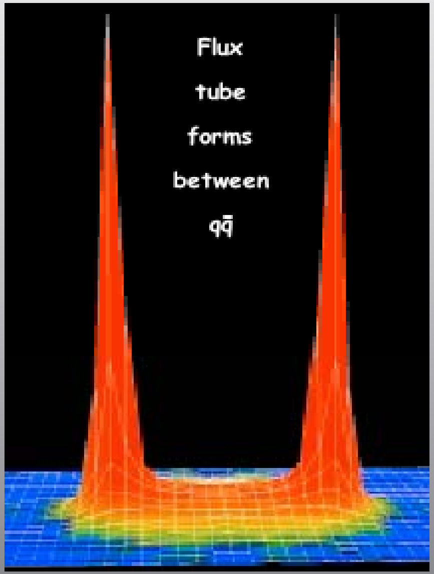
RPA corrections
Completed for 0^{++}
D.Whittington,APS



Gluon degrees of freedom
Alternatives to lattice

Bag Model

Flux tube model

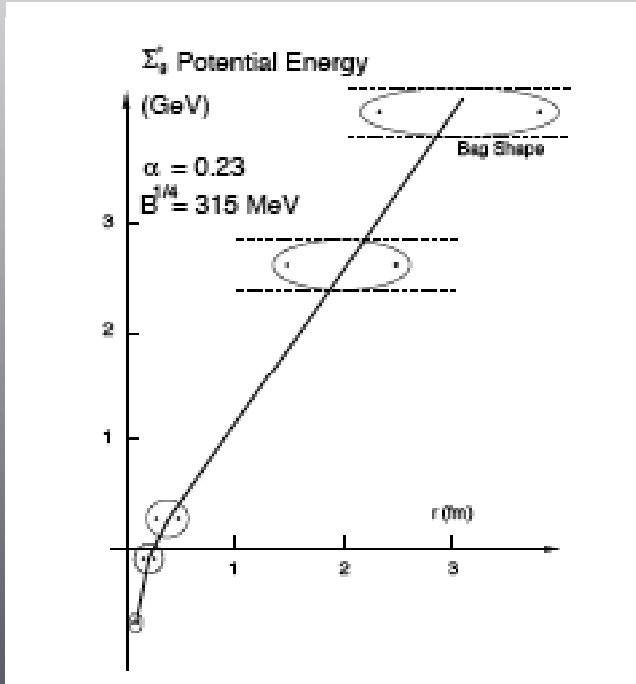


Quasi-
particles

gluons in the bag

$a = 1$ fm	$q\bar{q}(J^P = 1^-)$	g, $J^{PC} = 1^{+-}$	g, $J^{PC} = 1^{--}$
E	810 MeV	550 MeV TE	900 MeV TM

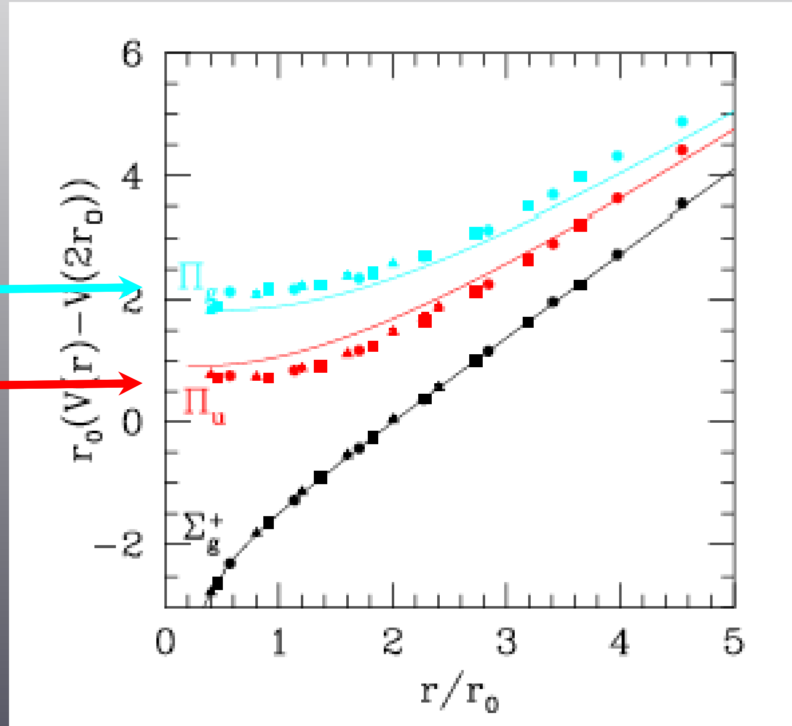
deformed bags



$P \times C = +$

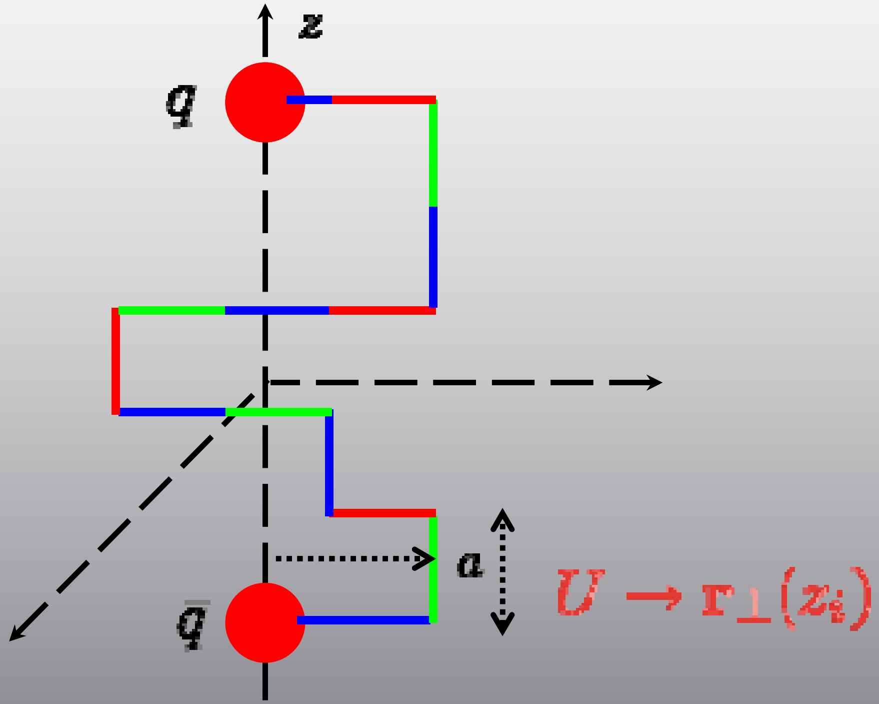
$P \times C = -$

Juge, Kuti, Morningstar

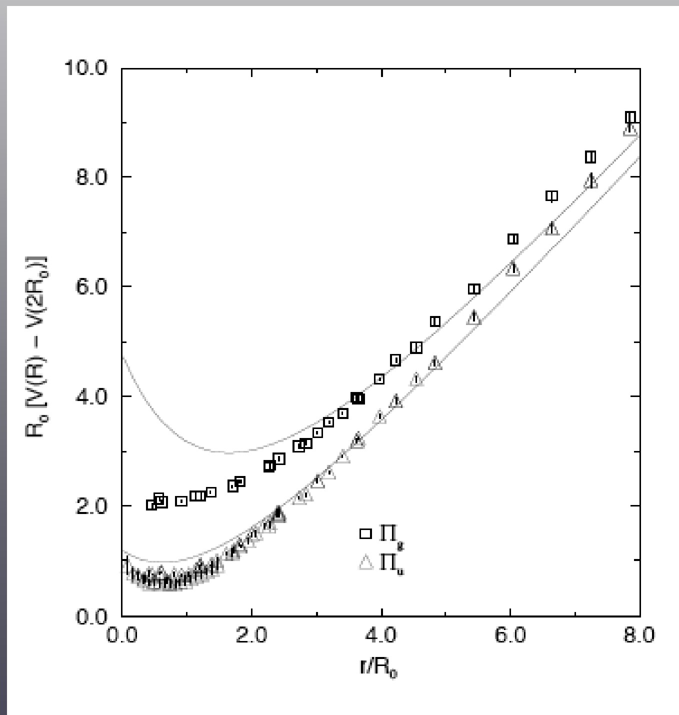


$$\mathbf{A}^a(\mathbf{x}) \to U_{ij}$$

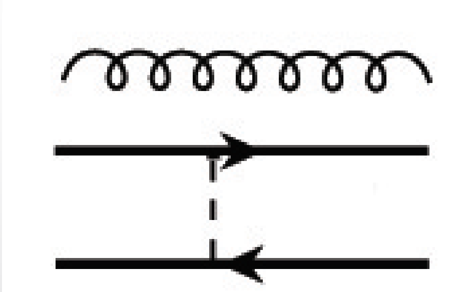
$$H = br_{q\bar{q}} - \frac{1}{2ba}\sum_i \frac{\partial^2}{\partial \mathbf{r}_i^2} + \frac{ba}{2}\sum_{i>j}(\mathbf{r}_i - \mathbf{r}_j)^2$$

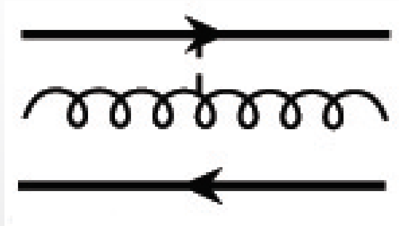


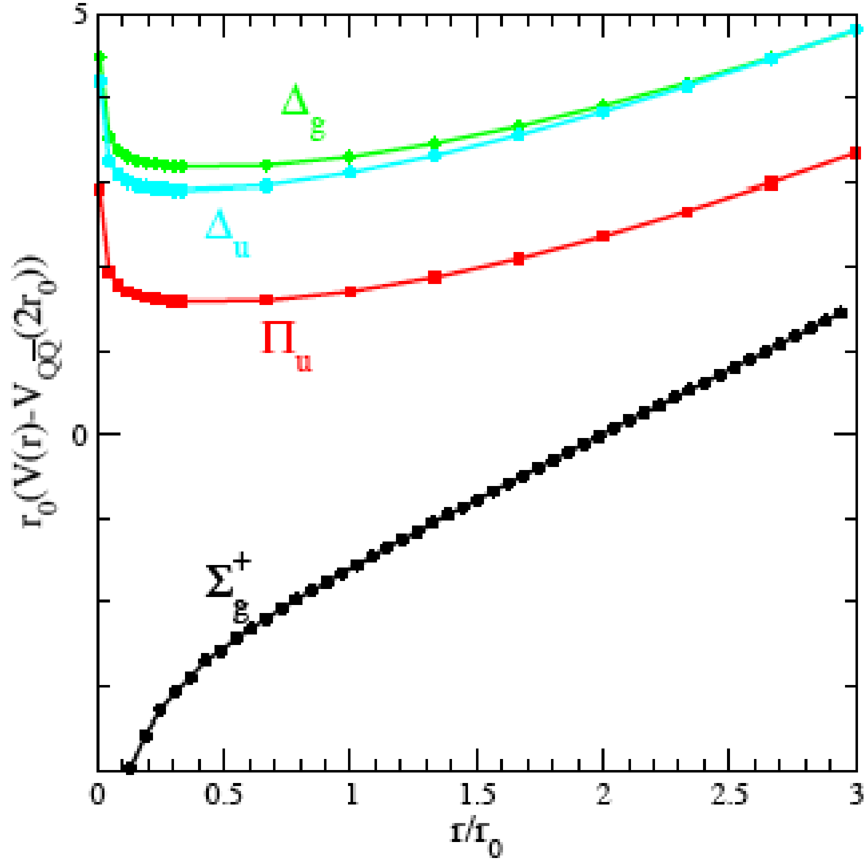
$$(P = -1) \times (C = +1) = -1$$



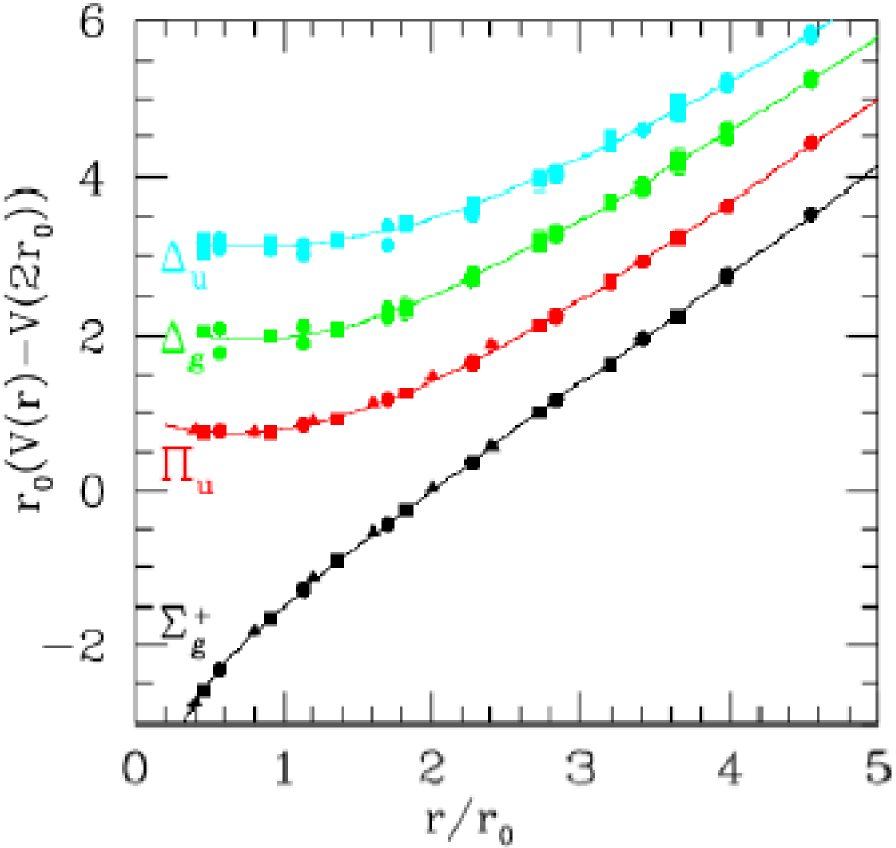
Excited states without 3-body interactions







Swanson
Szczepaniak,Krupins
ki

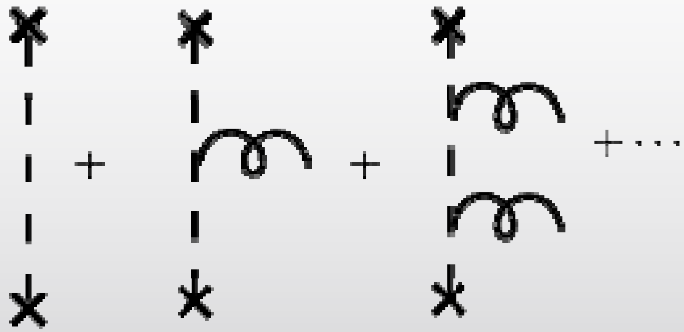


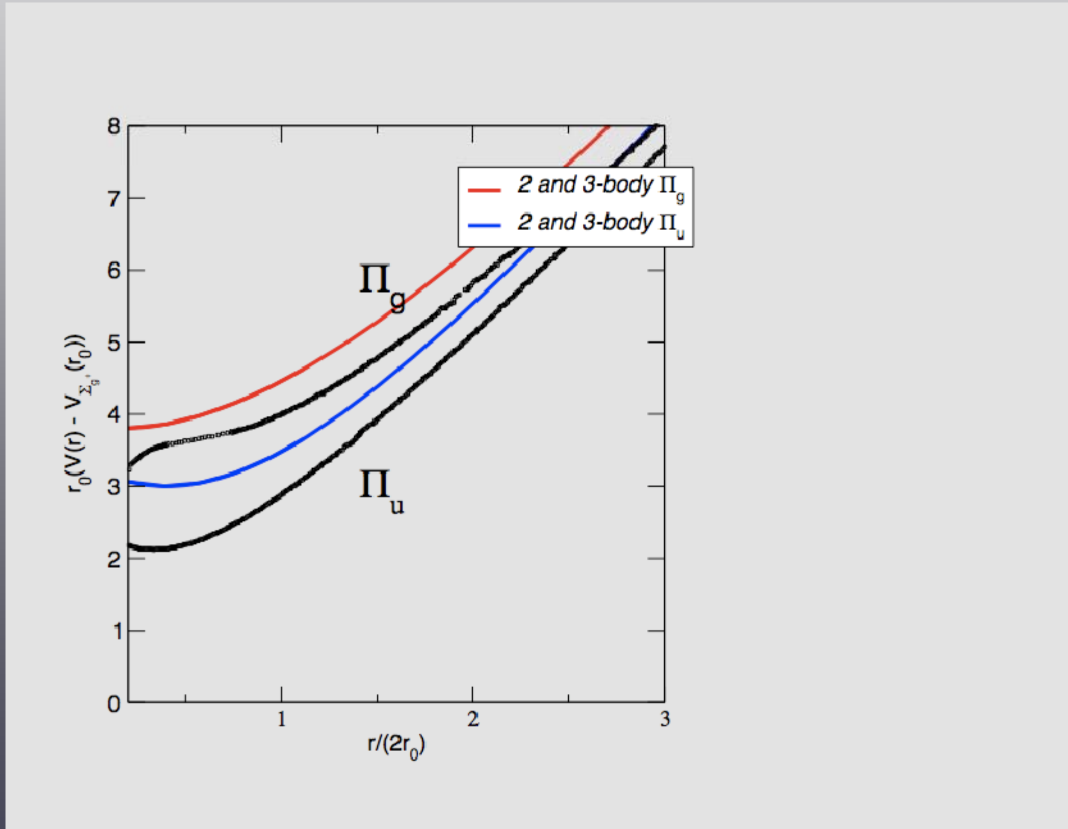
Lattice
(Morningstar et al.)

QED Coulomb Energy

$$V_O = \frac{1}{r}$$

QCD Coulomb Energy





In the quasi-particle representation generates a 3-body force

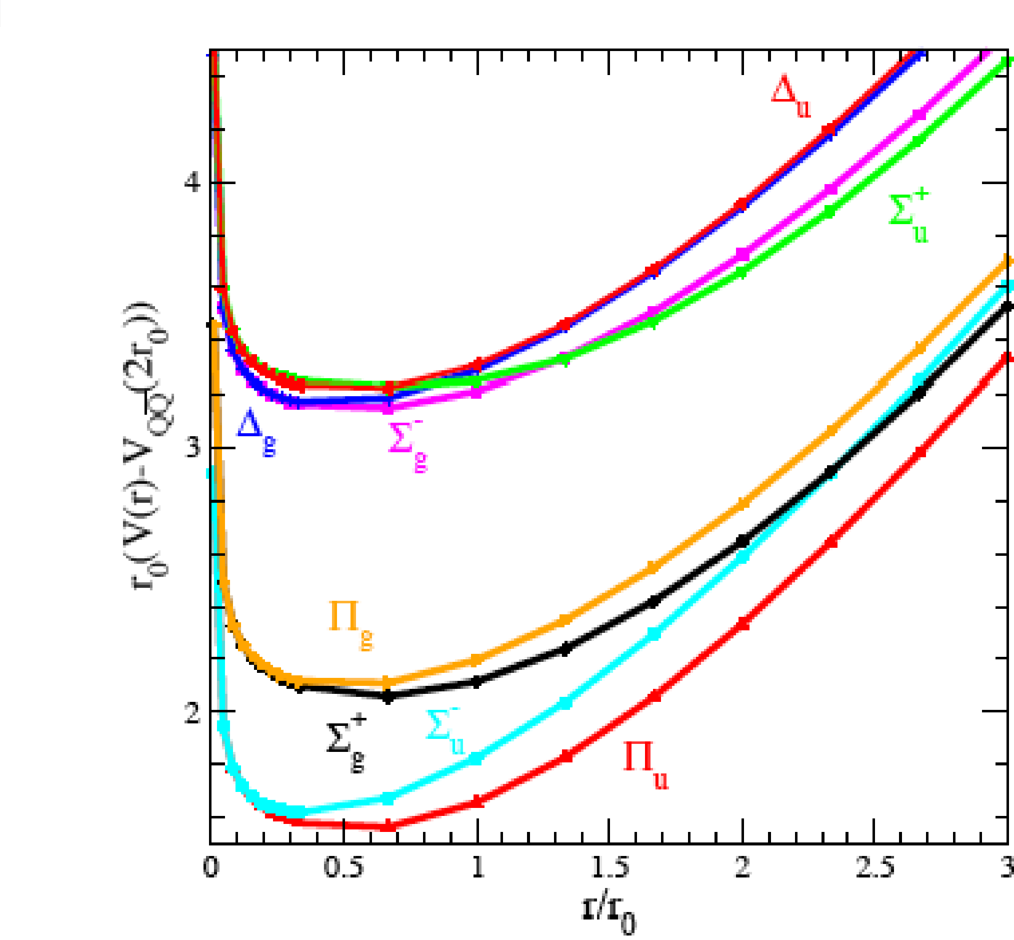
@ R =0

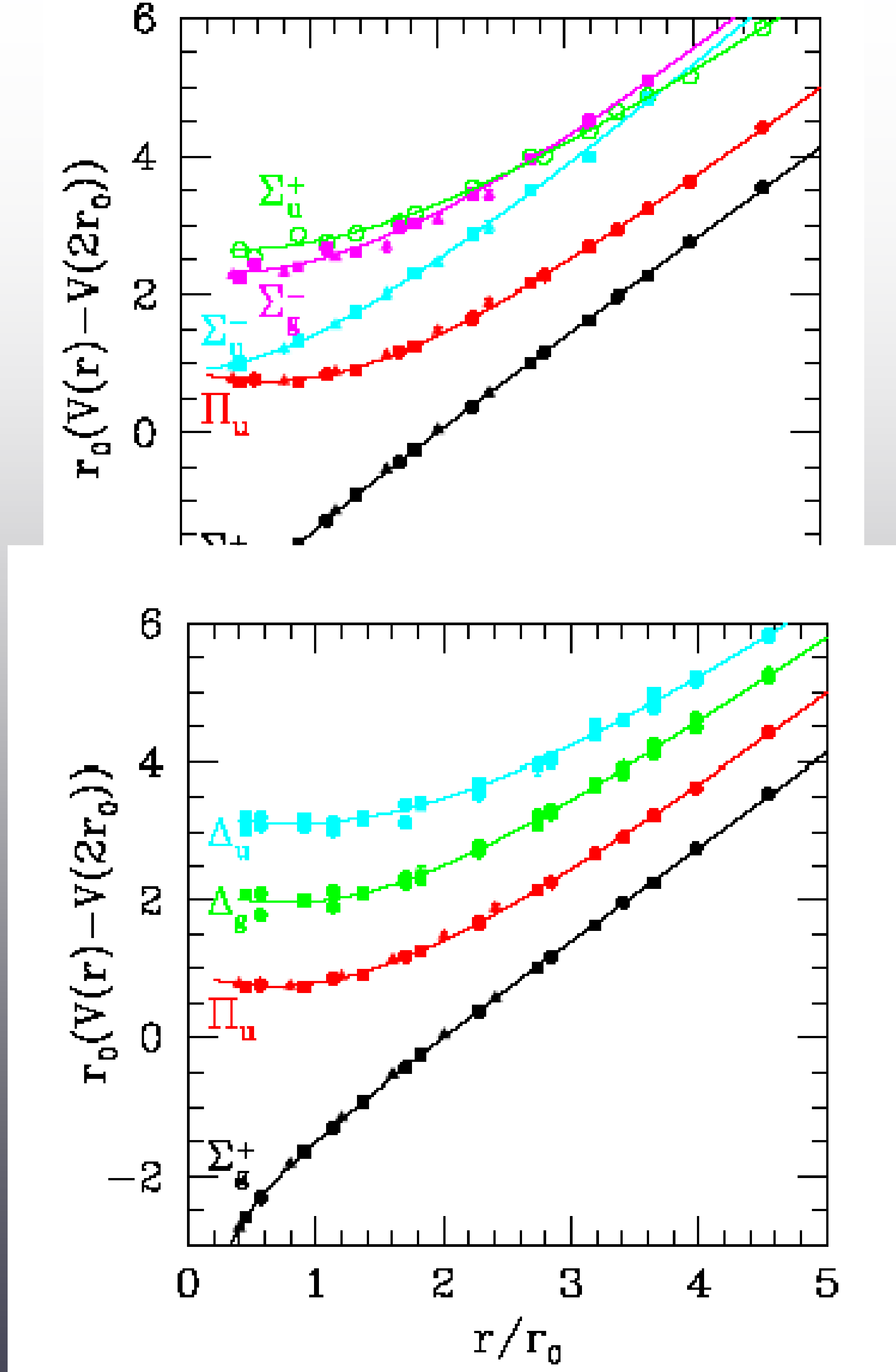
$$\delta\Pi_u = 0$$

$$\delta\Pi_g > 0$$

Krupinski,Szczepaniak

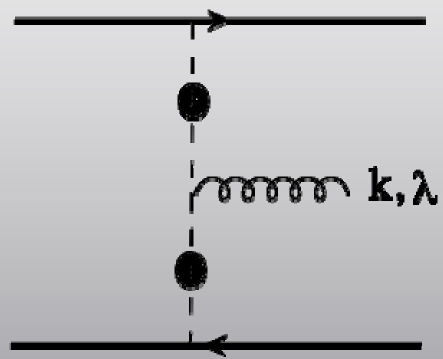
With 3-body interactions

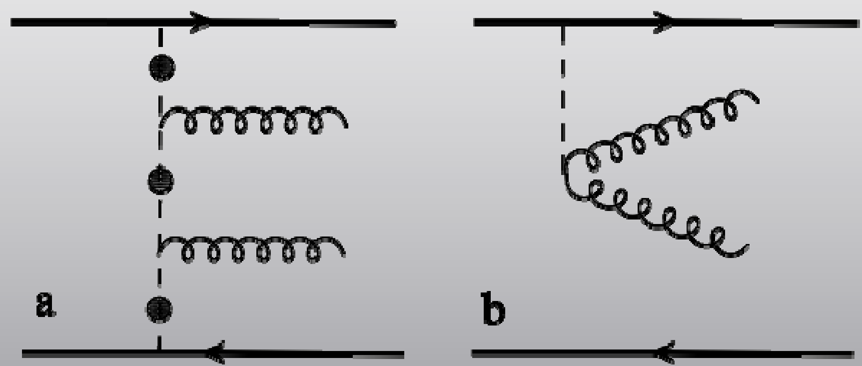




Coulomb energy vs "True" (Wilson) energy

$$|q\bar{q}\rangle = q^\dagger(R/2\hat{z})\bar{q}^\dagger(-R/2\hat{z})|0\rangle \qquad V_C(R) = \langle q\bar{q}|H|q\bar{q}\rangle$$





$$H|q\bar{q},T\rangle = E(R)|q\bar{q},T\rangle$$

$$|q\bar{q},T\rangle = q^\dagger(R/2\hat{z})\bar{q}^\dagger(-R/2\hat{z})|0\rangle + q^\dagger(R/2\hat{z})\alpha^\dagger\bar{q}^\dagger(-R/2\hat{z})|0\rangle + \cdots$$

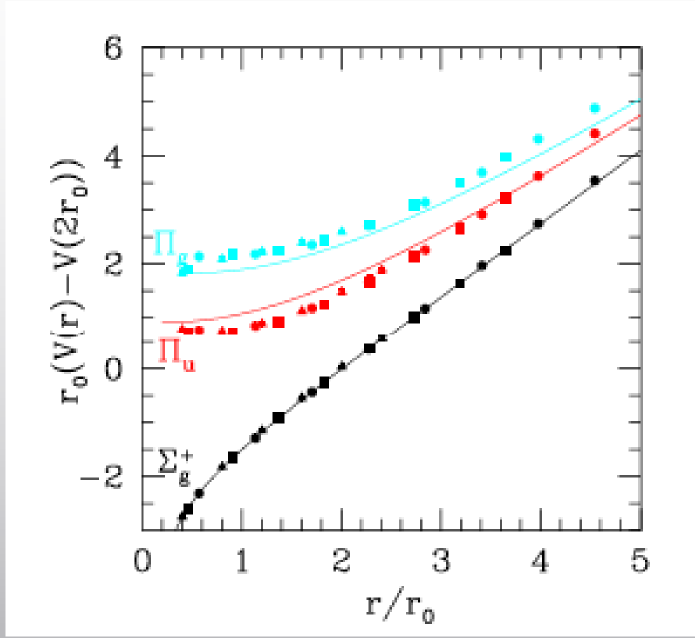
Zwanziger

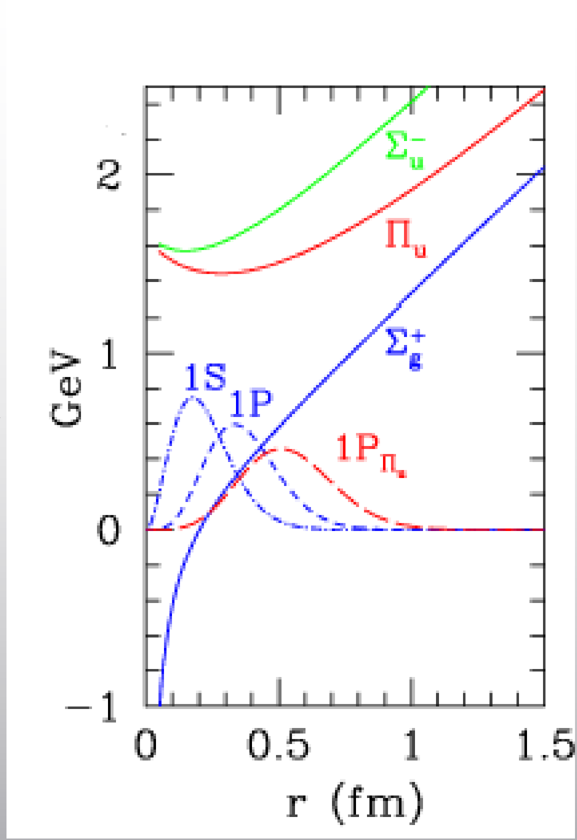
$$V_C(R) > E(R)$$

"No confinement without Coulomb confinement"

Bali
Morningstar, et al.

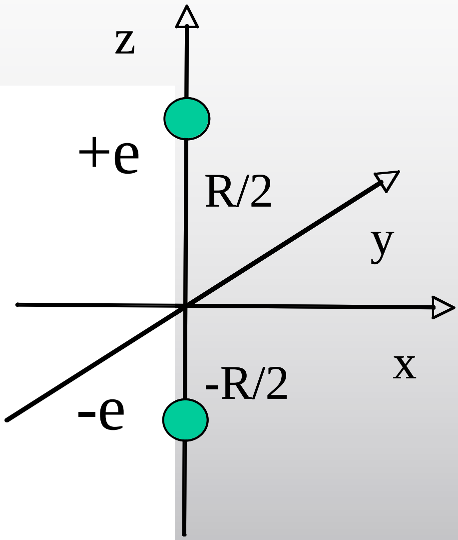
New states ?





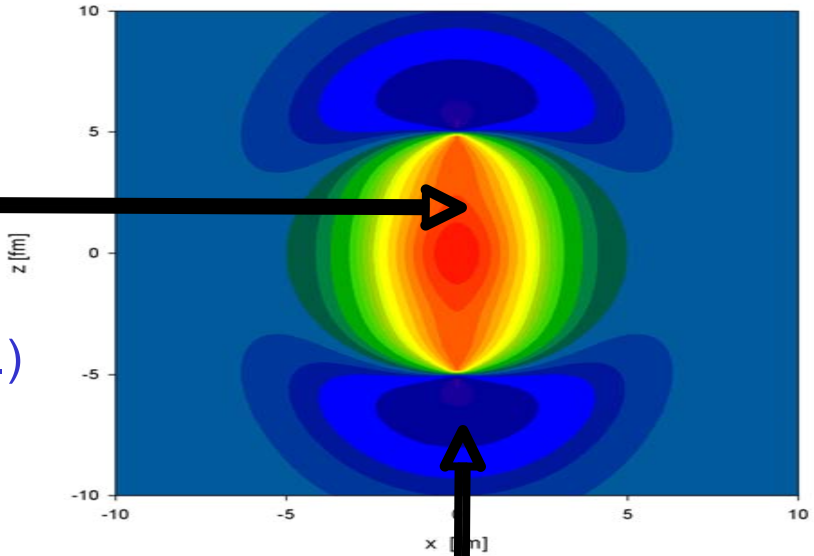
Constituent gluon models : low lying states should reproduce JPC = 1+- (Lg =1)not JPC=1- - (Lg=0) for the lowest gluon state

$$\vec{E}_L^2(x) = \frac{e^2}{4\pi}\left[\frac{\vec{x}-\vec{R}/2}{|\vec{x}-\vec{R}/2|^3} - \frac{\vec{x}+\vec{R}/2}{|\vec{x}+\vec{R}/2|^3}\right]^2$$



$$\vec{E}_L^2(|x|=0) \sim \frac{1}{R^4}$$

(from dimensional anal.)



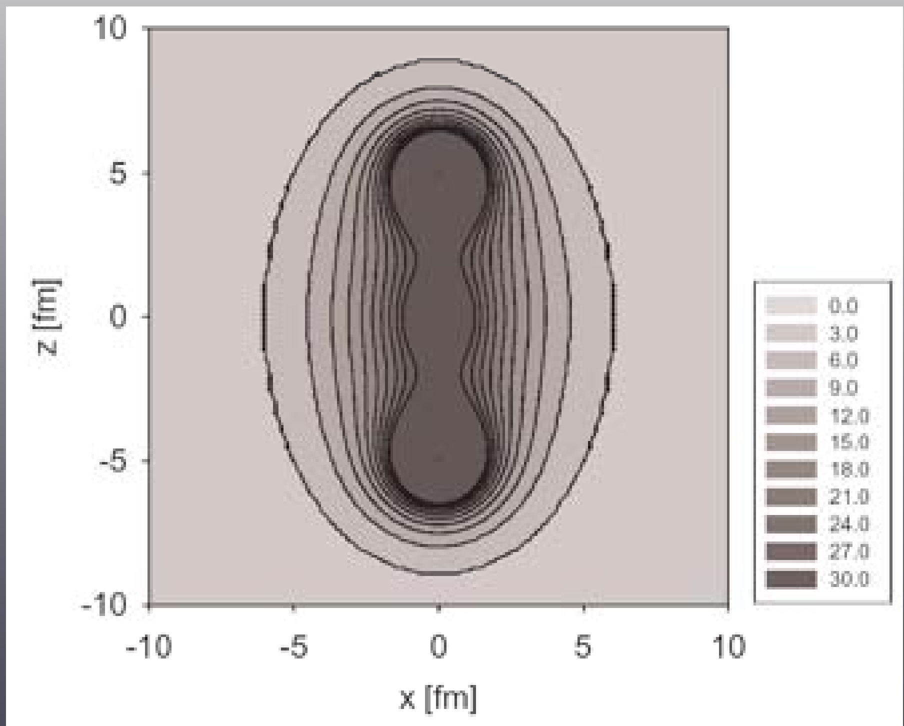
$$\vec{E}_L^2(|x|>>R) \sim \frac{R^2}{x^6}$$

(Van der Waals)

$$\vec{E}_L^2(\mathbf{x}) = \frac{b_c}{\pi^3}\Big[\frac{\vec{x}_1}{|\vec{x}_1|^2}(1 - j_0(\omega(0)x_1)) - \frac{\vec{x}_1}{|\vec{x}_2|^2}(1 - j_0(\omega(0)x_2))\Big]^2$$

approximate analytical solution with UV suppressed

$$\vec{E}_L^2(x/R \to \infty) \sim \frac{R^2}{x^4}$$



P.Bowman,
Szczepaniak

QCD Quasi-particles:

Choose a physical gauge, e.g. Coulomb gauge

$$A^{\mu,a}(\mathbf{x}) \to \mathbf{A}^a_\perp(\mathbf{x})$$

Compute the QCD Hamiltonian

$$H \to H\left[\mathbf{A}^a_\perp(\mathbf{x}), \mathbf{E}^a_\perp(\mathbf{x}), q_i(\mathbf{x}), q_i^\dagger(\mathbf{x})\right]$$

Diagonalize in a quasi-particle Fock space

$$\mathbf{A}^a_\perp(\mathbf{x}), \mathbf{E}^a_\perp(\mathbf{x}) \to \alpha^a_\perp(\mathbf{k}), \alpha^{\dagger,a}_\perp(\mathbf{k})$$

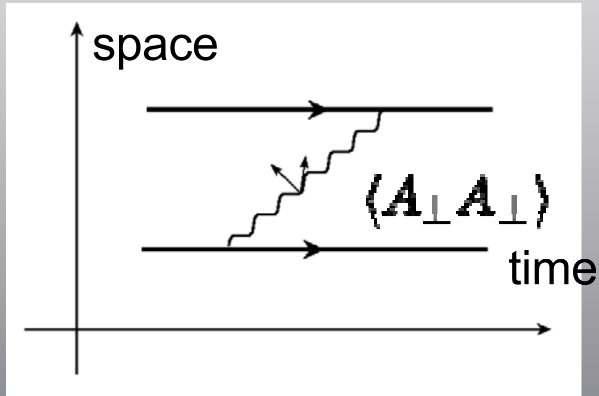
$$\alpha^a_\perp(\mathbf{k})|0\rangle = 0$$

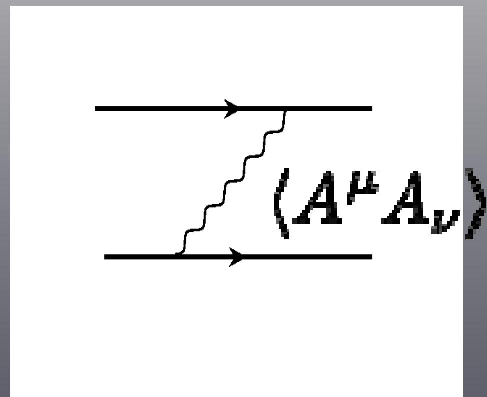
for high momentum transverse gluons

$$E(\mathbf{k}) \to |\mathbf{k}|$$

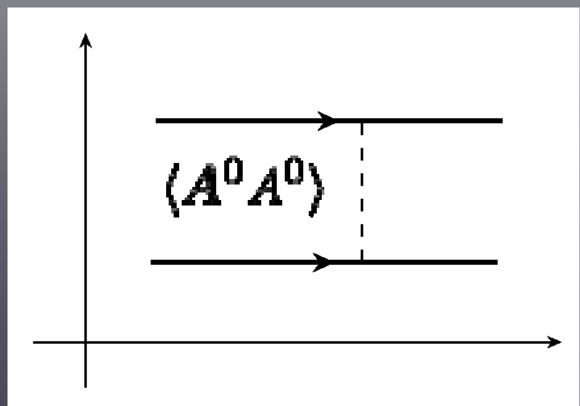
for low momentum transverse gluons

$$E(\mathbf{k}) \to m_g \sim 600 \text{ MeV}$$

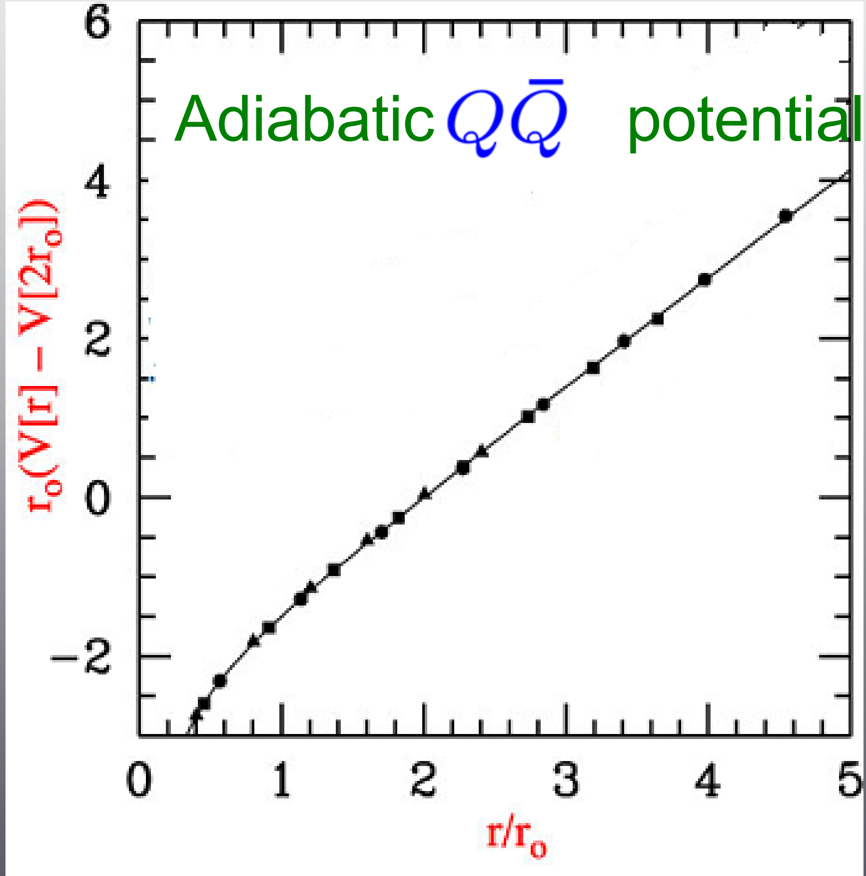




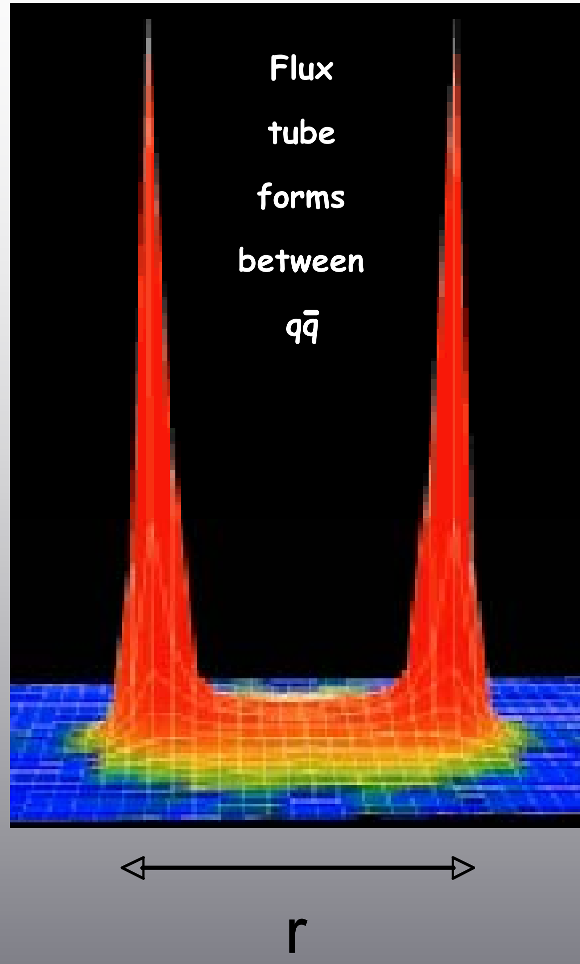
QCD Coulomb interaction leads to confinement (Zwanziger,Greensite,Szczepaniak,Swanson Reinhardt, Feuchter)

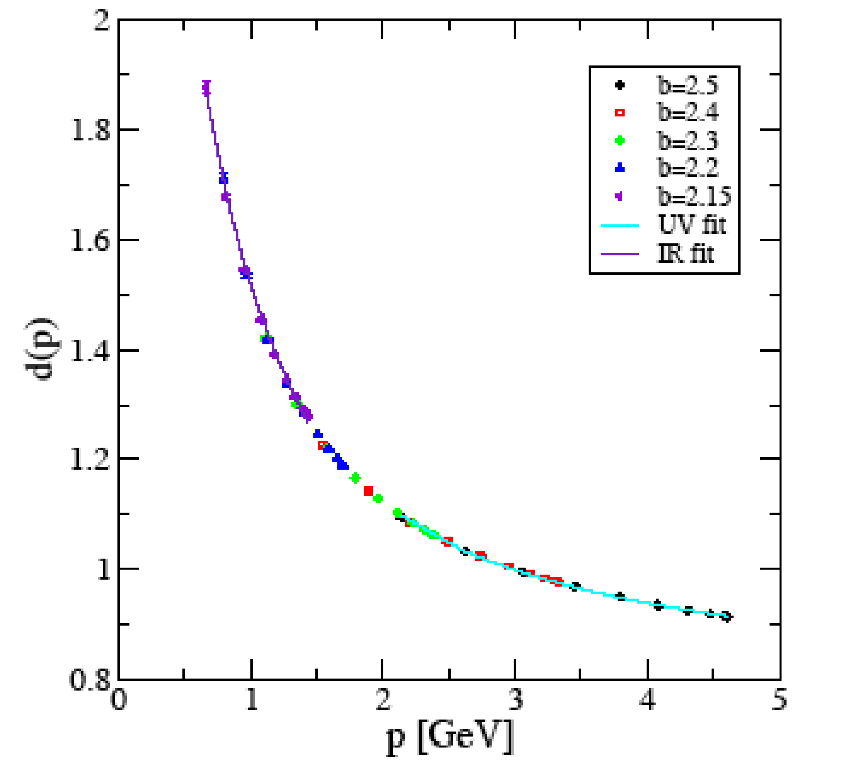


Soft gluons lead to : Confinement



r_0 = 0.5 fm





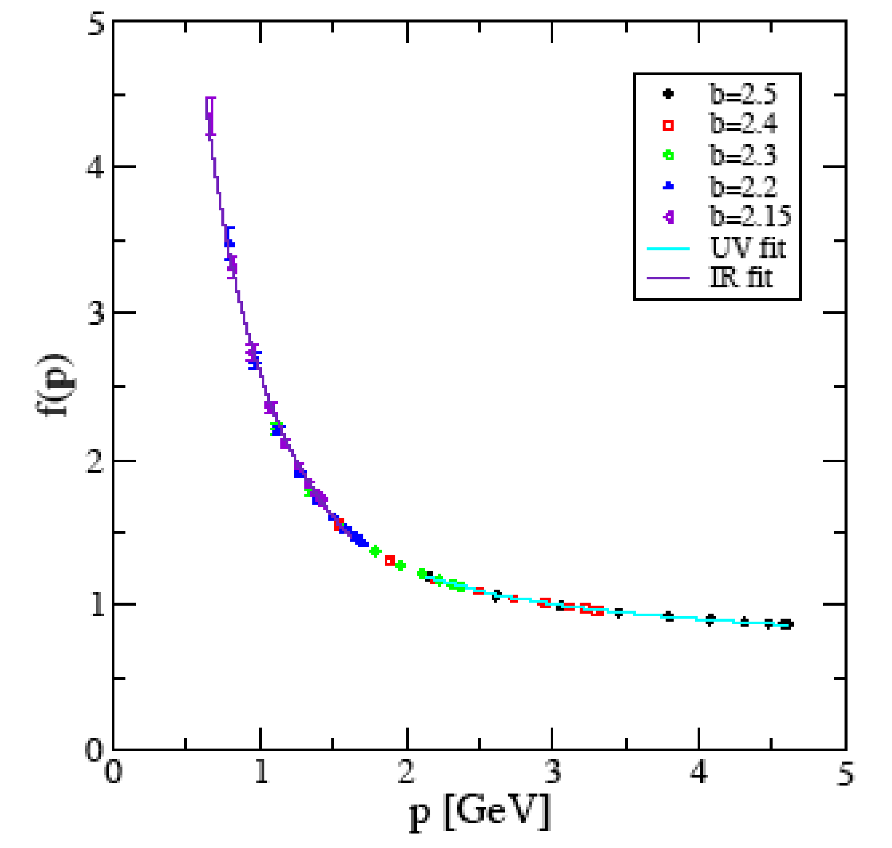
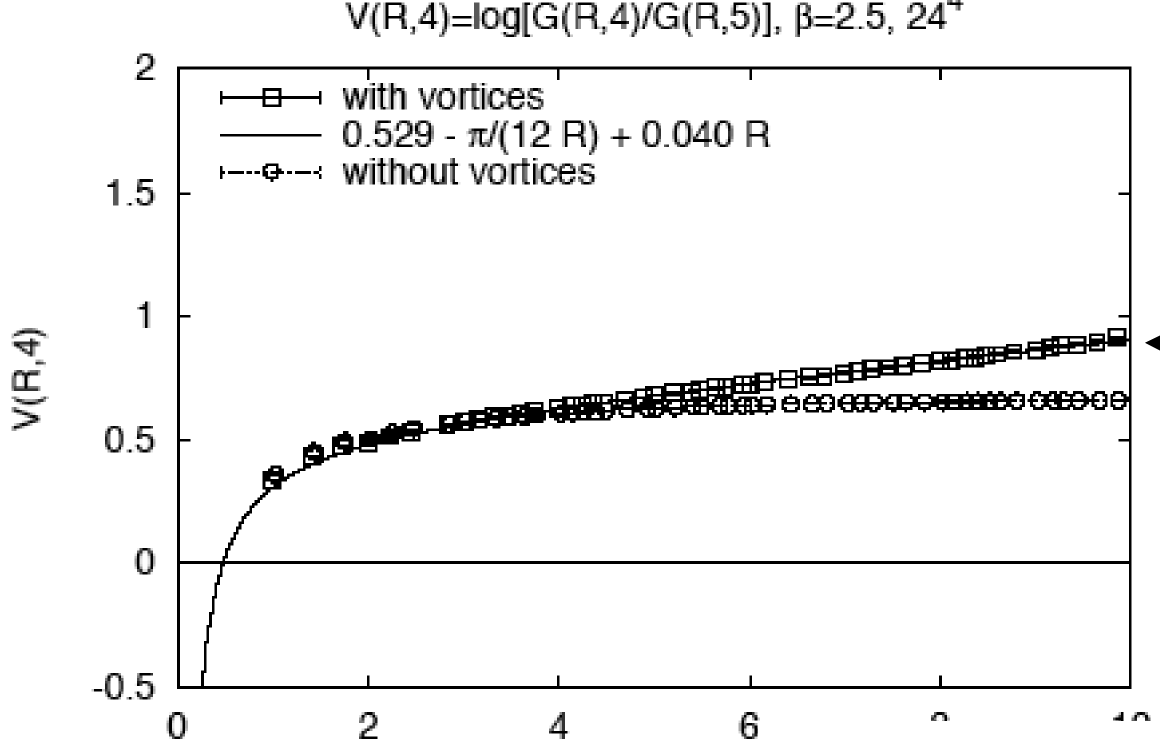


FIG. 4: The factorization function $f(\mathbf{p})$.

The IR fit completely agrees with solutions of Dyson eqs.

lattice : Langfeld and Moyaerts

lines : fit based on Coulomb gauge from Swanson and Szcze

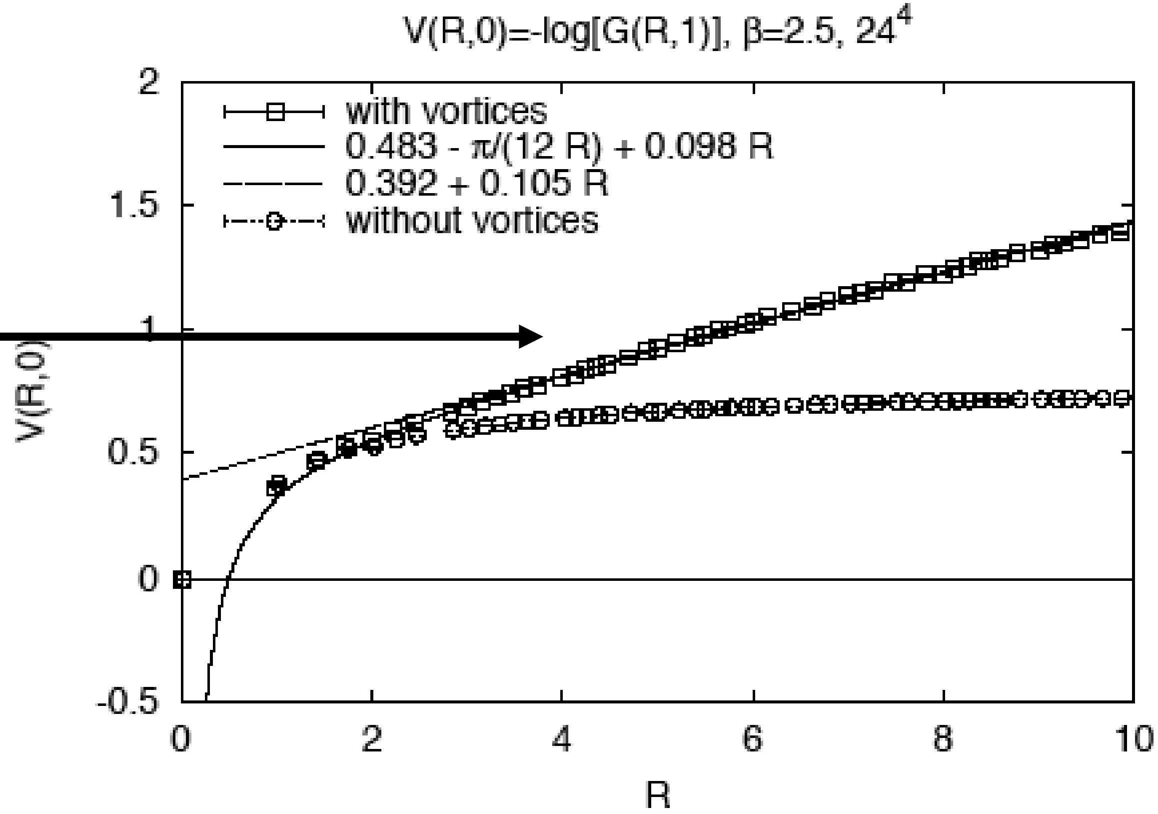


Greensite and Olejnik

$E(R)$

$b_C \sim 3b$

$V_C(R)$



Here |0> is the lattice vacuum state

Discretization Errors

Handles:

Transformation to "lattice" momenta

$$M \rightarrow \nabla^2$$

$$D(k) = \frac{1}{4/a^2 \sum_i \sin^2(k_i a/2)} = \frac{1}{q^2}$$

$$q_i \equiv \frac{2}{a} \sin(k_i a/2)$$

Momentum cut along "diagonal" direction

Vary lattice volume