Baryons in a covariant Faddeev approach

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March 2008

St. Goar, Germany

Outline

- Baryonic structure: experimental advances and theoretical efforts. (Motivation)

- Back to fundaments: Poincaré covariant Faddeev equations.

  Ingredients: Quark propagator ... DSEs.

  Diquarks ...BSEs.

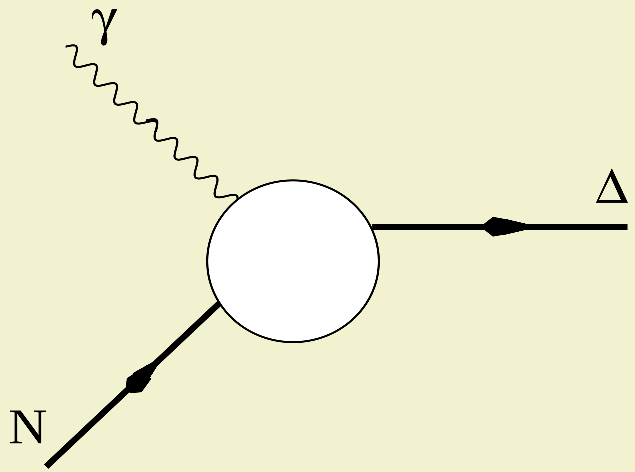
  Nucleon quark-core: solve a quark-diquark system.

- Calculated by now: M_N, nucleon FF, elmag. radii...

- Outlook (M_Δ, $N \to \Delta\gamma$, $\Sigma \to \Lambda\gamma$, π corrections).

Experimental advances

- Baryons are composite objects.
- Electromagnetic interaction:
  present precise tests of the structure.
- Nucleon electromagnetic form factors, polarizabilities,
  strangeness content ($e^- p$ scattering).
- Electro- and photoproduction:
  the proton and its lightest resonance $\Delta(1232)$.



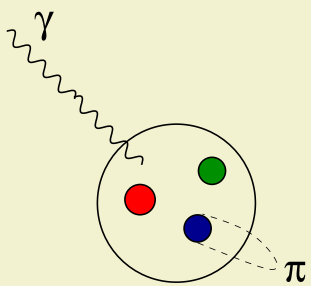
- JLab-CEBAF, MIT-BATES, Mainz-MAMI, Brookhaven-LEGS.

Theoretical efforts

$EMR(\%) = \frac{E2}{M1} = -2.5 \pm 0.5$ (MAMI) $CMR(\%) = \frac{C2}{M1} = -4.81 \pm 0.27$ (LEGS)

Models

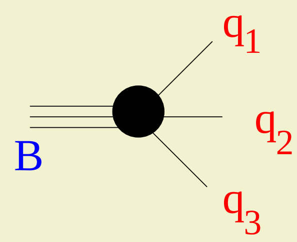
- Constituent quark models, pion cloud models....



- Covariance, link to effective theories,
  unified description of light and heavy baryons.

Do we understand the nucleon?

- Models offer a good description of data.Satisfied???
- NO!!!
  A QCD based solution is desired.
- A correct theoretical understanding
  of the hadron observables begins with the calculation of the Poincaré covariant hadron amplitudes.



The baryon-quark vertex is not trivial!

QCD emergent phenomena: confinement, DCSB ...

Nonperturbative tools

- The nonperturbative solution of the DSEs enable a study of hadrons as composites of dressed-quarks and dressed-gluons.

  C.D. Roberts, M.S. Bhagwat, A. Höll, S.V. Wright, arxiv:0802.0217 (2007).

- BSEs - applied to the meson sector.
  Masses, electromagnetic FF, leptonic decay constants...

  (see Andreas Krassnigg's talk).

- Faddeev equations - the complicated 3-body problem!
  Important steps were made for the nucleon.

  R. Alkofer, A. Höll, M. Kloker, A. Krassnigg, C.D. Roberts, nucl-th/0412046 (2004).

  M. Oettel, R. Alkofer, L. v. Smekal, nucl-th/0006082 (2000).

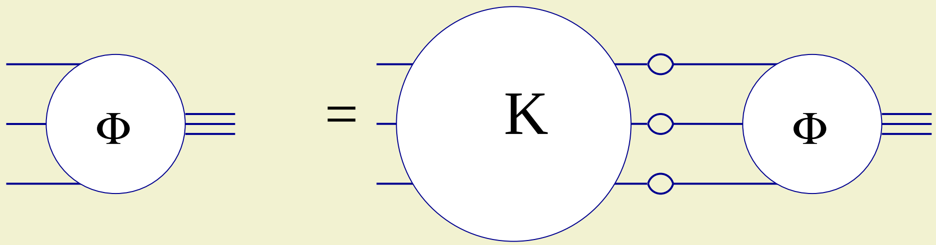
Faddeev equations

- The foundation of FEs:
  observe that the same attractive interaction which describes $1_c^{q\bar{q}}$ mesons also generates $\bar{3}_c^{qq}$ 'diquark' correlations.

  Diquarks do not survive as asymptotic states!
  But qq correlations will always couple as color antitriplet states
  in a color singlet baryon! $\bar{3}_c^{qq} \times 3_c^{q}$
- Exploit this feature $\rightarrow$ view the baryons as composite objects of confined quarks and nonpointlike diquarks.
- The nucleon is bound by an iterative exchange of roles between any of the quarks in the diquark and the dormant quark.
- Numerically: solve a quark-diquark BSE.

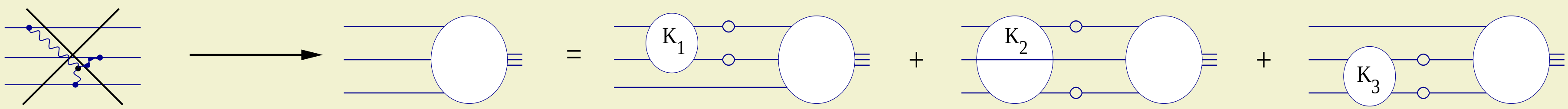
How to do it

- A three-quark bound state appears as a pole in the quark 6-point Green-function.
  $\Longrightarrow$ homogenous integral equations for the baryon amplitudes.
- Solve iteratively if ingredients are known:
  quark propagator, three-quark interaction kernel.



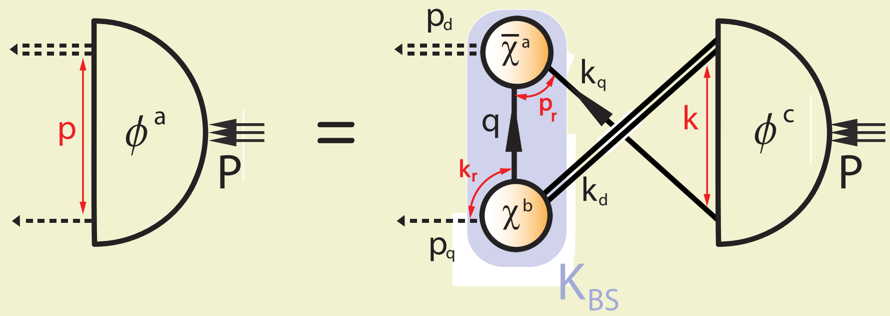
Complicated, but not impossible! Still very complicated!!! So:

- Faddeev approximation:
  retain only 2-particle interaction kernels.



Faddeev equations

- Solve numerically a quark-diquark BSE.



- Ingredients: quark propagator, 2-particle interaction kernel.
- Treat the qq correlations in analogy to the $\bar{q}q$ correlations.

Quark propagator

- The dressed quark propagator is the solution of the DSE.

- General form: $S(p) = i \not p \, \sigma_v(p^2) - \sigma_s(p^2) = -\frac{1}{i \not p \, A(p^2) + B(p^2)}$

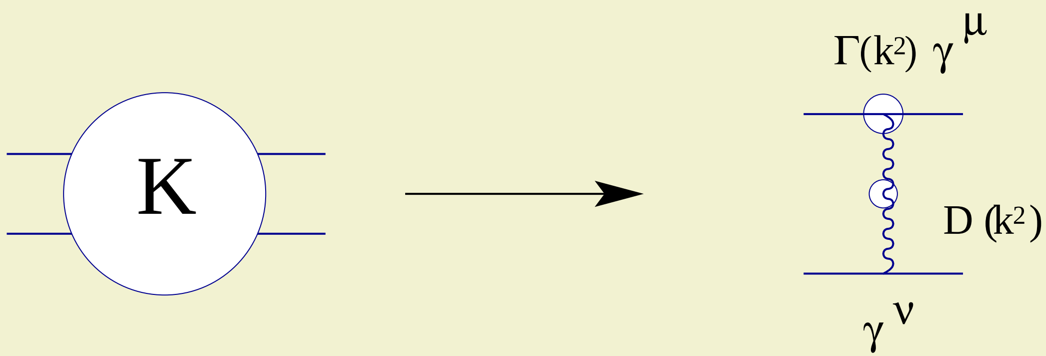
- The quark-mass functions reflects the quark dressing by gluons. $M(p^2) = \frac{B(p^2)}{A(p^2)}$

- Correct UV behaviour $\rightarrow$ perturbative quark propagator.

- Solving of the quark DSE implies the knowledge of the dressed quark-gluon vertex and dressed-gluon propagator
An infinite tower of DSEs should be solved.
For practical calculations truncations are needed!

Rainbow-ladder truncation

- A simple, nonperturbative truncation of DSEs.
  To ensure the implementation of chiral symmetry and its breaking, the kernels of DSE and BSE are related through the Axial-Vector WTI.
  Any truncation should preserve AV WTI!
- Solve the quark DSE using the rainbow truncation:
  quark-gluon vertex ansatz $i\Gamma_\mu = i\gamma_\mu \times \Gamma(k^2)$
- Consistent solution of qq BSE using the ladder truncation:
  one dressed-gluon exchange ansatz.



Rainbow - ladder truncation

- Assumption for the dressed quark-gluon vertex and dressed-gluon propagator: the gluon dressing $D(k^2)$ and the quark-gluon vertex dressing $\Gamma(k^2)$ are absorbed in an effective running coupling

$$\alpha_{eff}(k^2) \sim D(k^2)\,\Gamma(k^2)$$

- Ansatz for the effective coupling [(a)]

  constrained by the correct UV behaviour $\rightarrow \frac{\pi\,\gamma_m}{ln\frac{k^2}{\Lambda^2_{QCD}}}$

  and strong enough in IR to generate DCSB.

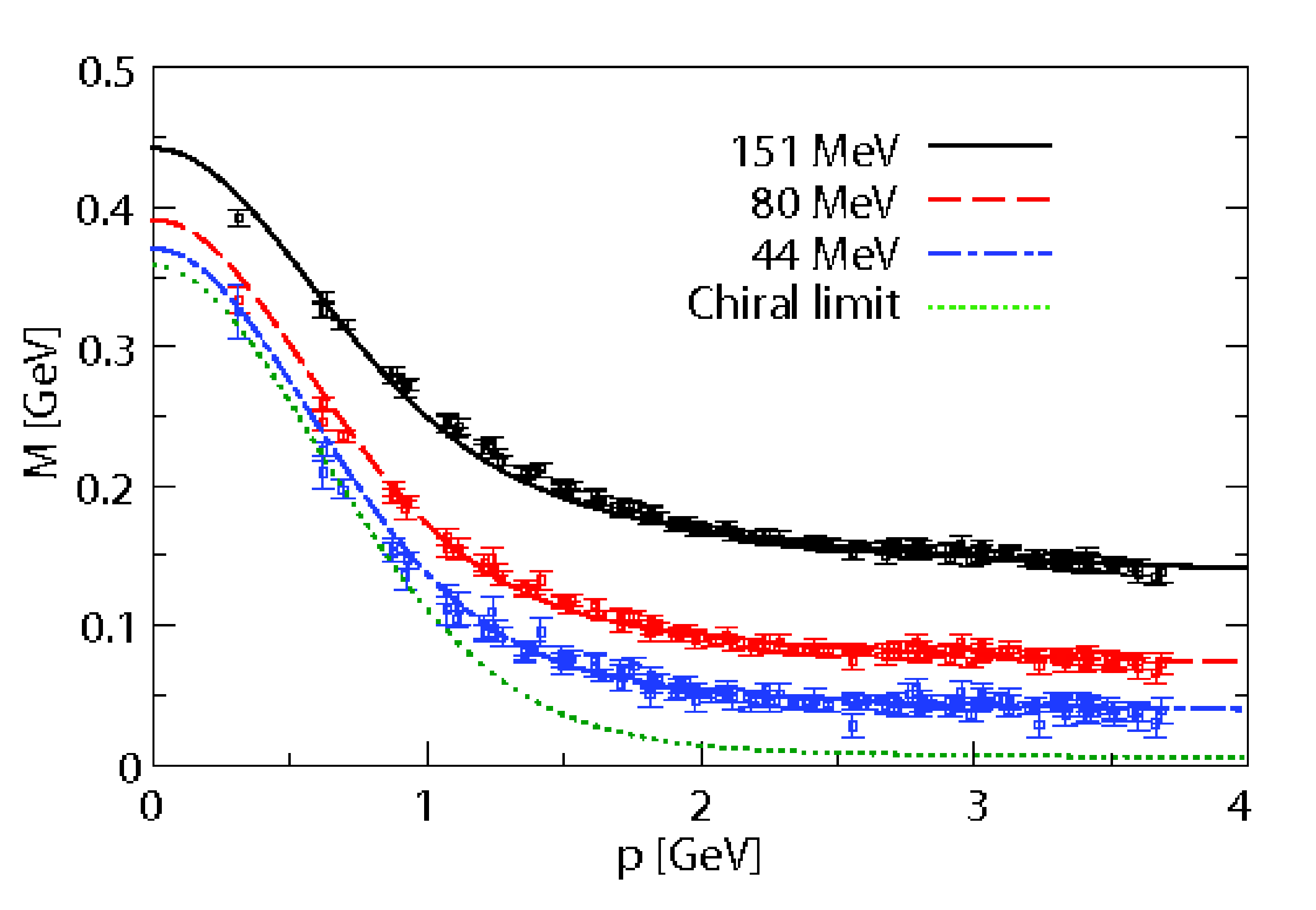
- Dressed quark propagator

$$S^{-1}(p,\mu^2) \sim S_0^{-1}(p,\Lambda^2) - \int_q^{\Lambda} d^4q\;\alpha_{eff}(k^2) D_{\mu\nu}^{free}\gamma_\mu S(q,\mu^2) i\gamma_\nu$$

(a) P. Maris, P.C. Tandy Phys. Rev. C 60, 055214 (1999).

$M(p)$ *in rainbow approximation*

Quark mass function fixed by lattice calculations.
P.O. Bowman *et al.* Phys. Rev. D 66, 014505 (2002).



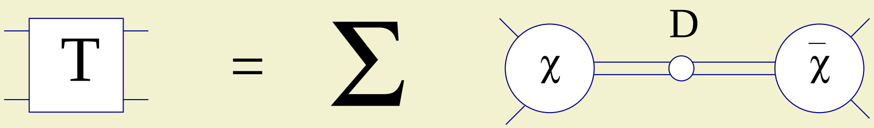
G. Eichman, A. Krassnigg, M. Schwinzerl, R.Alkofer, arxiv:0712.2666

Diquark ingredients

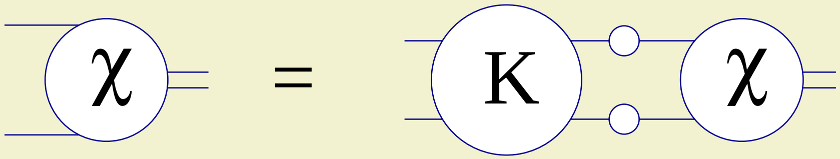
- The two-quark scattering matrix is approximated as a sum over all possible diquark pseudoparticle terms.
  The scalar and the axial-vector correlations are dominant for the ground state octet and decuplet.

$$T_{qq}(p_1, p_2, P) \sim \chi(p_1) D(P) \bar{\chi}(p_2) + \chi^{\mu}(p_1) D^{\mu\nu}(P) \bar{\chi}^{\nu}(p_2)$$

  The definition of diquarks: two-quark separable correlations.

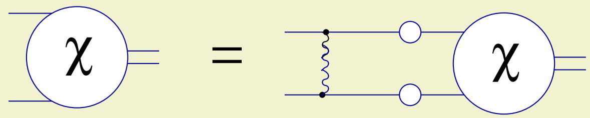


- Diquark amplitudes on the mass-shell are the solutions of the qq homogenous integral BSE.



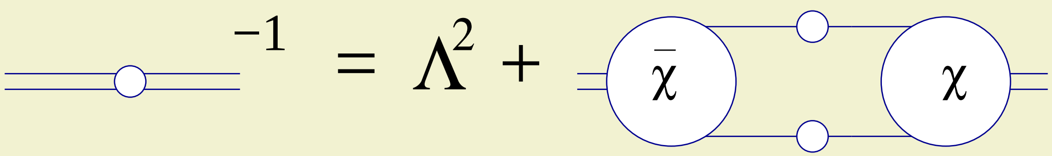
Diquark ingredients

- Solve using rainbow-ladder truncation:
  one gluon exchange $+$ vector-like quark-gluon vertex.



  The off-shell behaviour of the diquark BSA is parametrized following requirements imposed by the UV behaviour of BSA.

- The diquark propagator in ladder approximation.

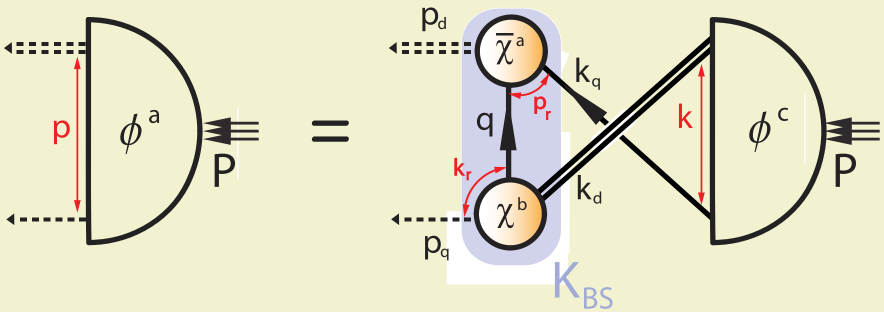


$$D^{-1} = \bar{\chi}\,(K^{-1} - G_0)\,\chi$$

  The on-shell behavior is calculated, the off-shell behaviour is parametrized consistently with
  the off-shell BSA parametrizations.

The nucleon BSE

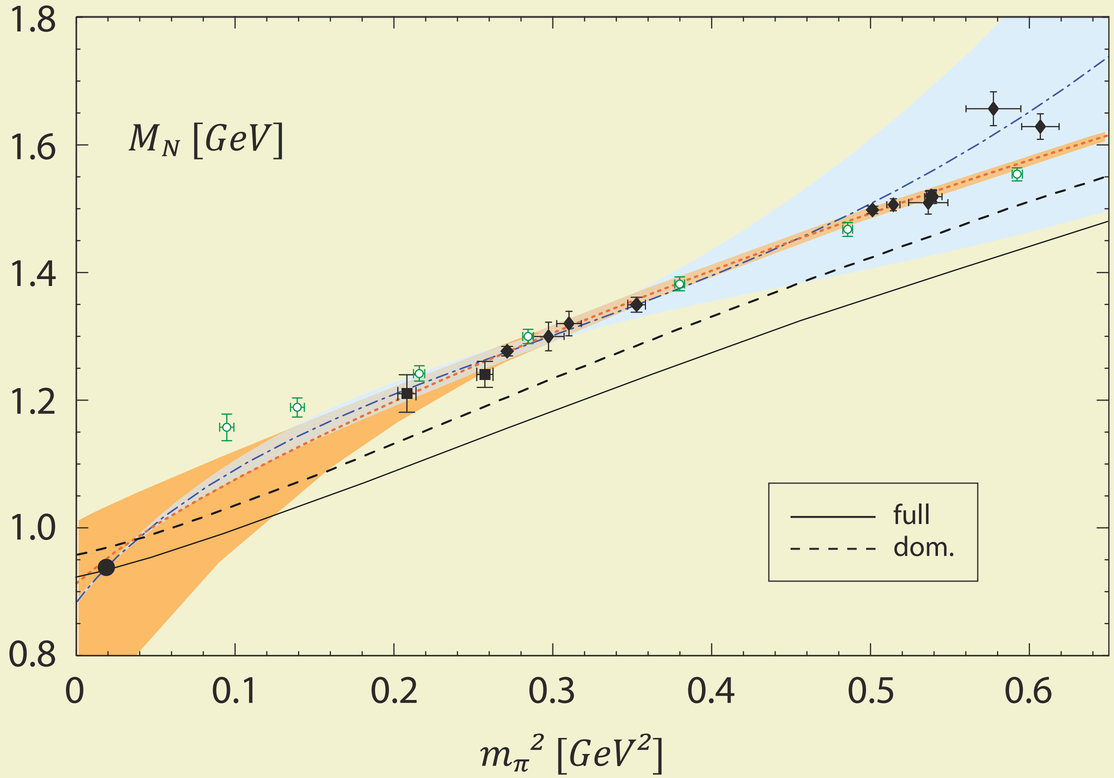
$$\Phi(p,P)^a = \sum_{b,c} \int \frac{d^4k}{(2\pi)^2}\, \chi^b(k_r,k_d)\, S^T(q)\, \bar{\chi}^{aT}(p_r,p_d)\, S(k_q)\, D^{bc}(k_d) \Phi(k,P)$$



- All the ingredients are specified!
  Decomposition of Faddeev amplitudes in Dirac space
  - in explicit calculations use full diquark amplitudes.

Nucleon mass

- $M_N = 0.93$ GeV at physical point $m_\pi = 138MeV$
  $M_N^{exp} = 0.94$ GeV.

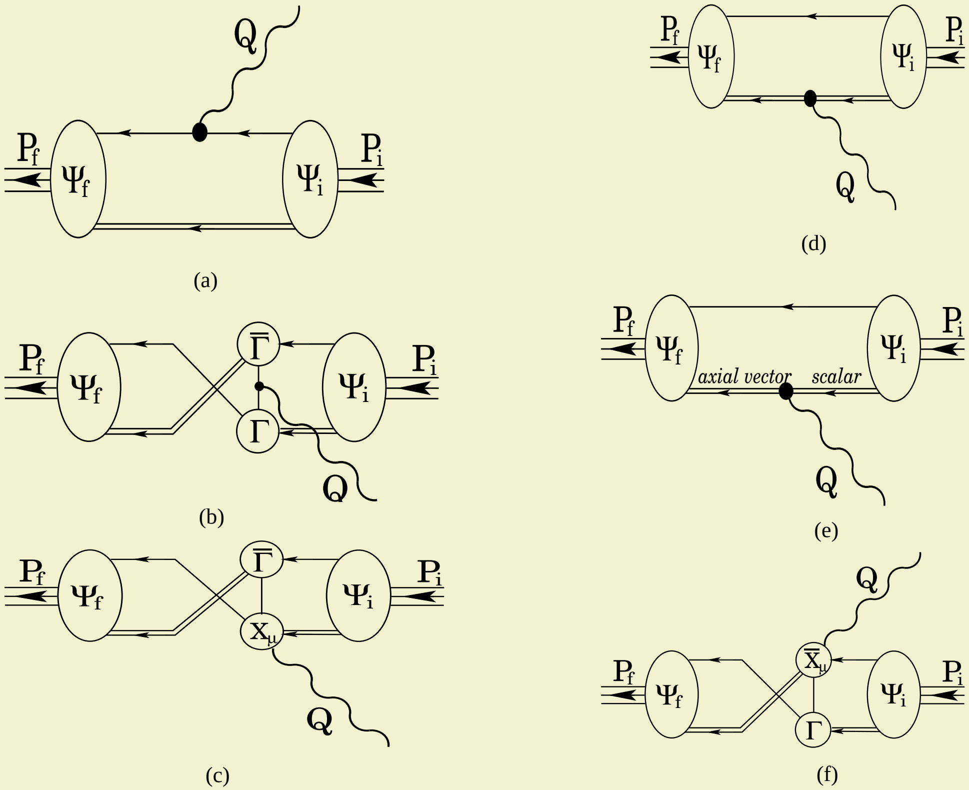


G. Eichman, A. Krassnigg, M. Schwinzerl, R.Alkofer, arxiv:0712.2666

Lattice data; chiral extrapolation methods.

Electromagnetic interaction

- Ward-Takahashi identity: more than impulse approx. is needed.
  M. Oettel, M.A. Pichowsky, L.v. Smekal, nucl-th/9909082 (2000).
- The electromagnetic transition from Ax. diquark to Sc. diquark satisfies gauge invariance alone.



- General current: $< J^\mu >= \int \bar{\Phi}\{D\, S\, \Gamma^\mu_q\, S + D\, \Gamma^\mu_{diq.}\, D\, S + D\, S\, K^\mu\, S\, D\}\Phi$

Results

Electromagnetic radii of nucleon

	r_E^p	r_E^n	r_M^p	r_M^n	[fm]
exp	0.87	0.34	0.86	0.88	
calc.	0.67	0.13	0.58	0.57	

Magnetic moments of nucleon.

	μ_p	μ_n	[n.m]
exp	2.79	-1.91	
calc.	2.52	-1.55	

G. Eichman, A. Krassnigg, M. Schwinzerl, R.Alkofer, arxiv:0712.2666

Summary

- Main message: develop QCD - based understanding of the nucleon structure.

- The Poincaré covariant DSE-BSE approach provides a suitable framework to study hadrons as relativistic bound states of quarks and gluons AND to explore QCD emergent phenomena.

- Too many assumptions, truncations, approximations?
  Yes! BUT!
  Baryons are complicated objects - the quark-diquark picture describes well nucleon quark-core.

- Rainbow-ladder truncation: consistent solutions of DSE and BSE.
  Calculation of nucleon static properties.

  Progress: from ansätze for quark-quark-diquark vertex and quark propagator to exact solutions of DSE and BSE. No baryon observables as input!

Outlook or Why $N \to \Delta\,\gamma$ *?*

- Experimentally - recent high precise data exists!
  Theoretically - highly non trivial!
  The long disputed electromagnetic ratios
  $\frac{E2}{M1}, \frac{C2}{M1}$ are positive and about few %

  The naive quark model fails describing these ratios.
  Improved relativistic versions, one-gluon and one pion-exchange models... do not properly describe the whole body of data (helicity amplitudes, FF).
  Chiral effective theories: good description, limited range of applicability.
- A crucial test for the Faddeev approach.

Outlook or Why $N \to \Delta\,\gamma$?

- First step:
  study the Δ (1232) resonance, a spin$-\frac{3}{2}$ particle, fully flavour symmetric; only the axial-vector correlations will contribute!

  By now: using specific ansätze for the quark propagator and the diquark correlations the obtained mass is
  $M_\Delta = 1.004 \div 1.007$ GeV

  M. Oettel, R. Alkofer, L. von Smekal, Eur. Phys. J. A8: 553-566 (2000)
- Compare to the diagonal $\Sigma \to \Lambda\gamma$ and
  thus underline the importance of the axial-vector correlations.

Thank you

Danke

Gracias

Grazie

Merci

Spasibo

Obrigada

Arigato