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# On the analytic properties of the Landau gauge gluon and quark propagators and related aspects of confinement in the covariant gauge

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Institute of Physics  
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415th WE Heraeus Seminar  
Quarks and Hadrons in Strong QCD



March 20, 2008

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# Outline

## 1 Basic Concepts

- Covariant Gauge Theory
- Kugo–Ojima confinement criterion
- QCD Green functions & Positivity

## 2 Analytic properties of propagators

- Positivity violation for the gluon propagator
- Analytic structure of quark propagator

## 3 Summary and Outlook

# Covariant Gauge Theory

Gauge theory: **Unphysical degrees of freedom!**

**QED:** Physical states obey Lorentz condition.

$$\partial_\mu A^\mu |\Psi\rangle = 0 \quad (\text{Gupta} - \text{Bleuler}).$$

⇒ Two physical massless photons.

Time-like photon (i.e. negative norm state!) cancels

longitudinal photon in S-matrix elements!



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# Covariant Gauge Theory

In S-Matrix

$$A_T + A_0 \xrightarrow{\text{red curved arrow}} A_L = 0$$

$$A_T + A_0 \xrightarrow{\text{red curved arrow}} A_L \neq 0$$

# Covariant Gauge Theory

## QCD in a covariant gauge:



Faddeev–Popov ghosts = anticommuting scalar fields.



Global ghost field as ‘gauge parameter’:

**BRST symmetry of the gauge-fixed action!**

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# Covariant Gauge Theory

Symmetry of the gauge-fixed generating functional:

$$\begin{aligned}\delta_B A_\mu^a &= D_\mu^{ab} c^b \lambda, & \delta_B q &= -igt^a c^a q \lambda, \\ \delta_B c^a &= -\frac{g}{2} f^{abc} c^b c^c \lambda, & \delta_B \bar{c}^a &= \frac{1}{\xi} \partial_\mu A_\mu^a \lambda,\end{aligned}$$

Becchi–Rouet–Stora & Tyutin (BRST), 1975

- Parameter  $\lambda \in$  Grassmann algebra of the ghost fields
- $\lambda$  carries ghost number  $N_{FP} = -1$
- Via Noether theorem: BRST charge operator  $Q_B$
- generates ghost # graded algebra  $\delta_B \Phi = \{iQ_B, \Phi\}$

# Covariant Gauge Theory

BRST algebra:  $Q_B^2 = 0, [iQ_c, Q_B] = Q_B,$

- complete in **indefinite metric** state space  $\mathcal{V}$ .
- generates ghost # graded  $\delta_B \Phi = \{iQ_B, \Phi\}.$
- $\mathcal{L}_{GF} = \delta_B (\bar{c} (\partial_\mu A^\mu + \frac{\alpha}{2} B))$  **BRST exact**.

Positive definite subspace  $\mathcal{V}_{\text{pos}} = \text{Ker}(Q_B)$

(i.e. all states  $|\psi\rangle \in \mathcal{V}$  with  $Q_B|\psi\rangle = 0$ )

contains  $\text{Im } Q_B$  (i.e. all states  $Q_B|\phi\rangle$ ),

c.f. exterior derivative in differential geometry.

Hilbert space: cohomology  $\mathcal{H} = \frac{\text{Ker } Q_B}{\text{Im } Q_B} \simeq \mathcal{V}_s$  **BRST singlet**  
longitudinal & timelike gluons, ghosts : **elementary BRST quartet**  
(c.f. Gupta–Bleuler mechanism in QED)



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# NB: BRST symmetry of Green functions

most convenient device to derive STIs:

## BRST symmetry of Green functions

(C. H. Llewellyn-Smith, 1980)

Start from the BRST invariance of

$$\delta_B \langle c^a(x) \bar{c}^b(y) \bar{c}^c(z) \rangle = 0$$

yields, when neglecting irreducible ghost-ghost scattering contr.,

$$\begin{aligned} \tilde{Z}_1 \frac{1}{2} g f^{ade} (2\pi)^4 \delta^4(p+q+k) & \left\{ D_G^{eb}(-q) D_G^{dc}(-k) - D_G^{db}(-q) D_G^{ec}(-k) \right\} = \\ &= \frac{1}{\xi} i k_\mu D_{\mu\nu}^{cd}(k) D_G^{ae}(p) G_\nu^{def}(k, p, -q) D_G^{fb}(-q) \\ &\quad - \frac{1}{\xi} i q_\mu D_{\mu\nu}^{bd}(q) D_G^{ae}(p) G_\nu^{def}(q, p, -k) D_G^{fc}(-k) \end{aligned}$$

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# Kugo–Ojima confinement criterion

⇒ Physical states are BRST singlets!

(BRST cohomology: Hilbert space  $\mathcal{H} = \frac{\text{Ker } Q_{BRST}}{\text{Im } Q_{BRST}}$ .)

Time-like and longitudinal gluons (BRST quartet) removed from asymptotic states as in QED, but:

Transverse gluons also BRST quartets?

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# Kugo–Ojima confinement criterion

Realization of Confinement depends on **global gauge structure**:

Globally conserved current ( $\partial^\mu J_\mu^a = 0$ )

$$J_\mu^a = \partial^\nu F_{\mu\nu}^a + \{Q_B, D_\mu^{ab}\bar{c}^b\}$$

with charge

$$Q^a = G^a + N^a.$$

---

**QED:** MASSLESS PHOTON states in both terms.

Two different combinations yield:

unbroken global charge  $\tilde{Q}^a = G^a + \xi N^a$ .

spont. broken displacements (photons as Goldstone bosons).

---

**No massless** gauge bosons in  $\partial^\nu F_{\mu\nu}^a$ :  $G^a \equiv 0$ .

(QCD, e.w. Higgs phase, ...)



# Kugo–Ojima confinement criterion

**QCD:** Unbroken global charge

$$Q^a = N^a = \{ Q_B, \int d^3x D_0^{ab} \bar{c}^b \}$$

well-defined in  $\mathcal{V}$ .

With  $D_\mu^{ab} \bar{c}^b(x) \xrightarrow{x^0 \rightarrow \pm\infty} (\delta^{ab} + u^{ab}) \partial_\mu \bar{\gamma}^b + \dots$

⇒ 2nd Kugo-Ojima Confinement Criterion:

$$u^{ab}(0) = -\delta^{ab}$$

where

$$\begin{aligned} & \int dx e^{ip(x-y)} \langle 0 | T D_\mu c^a(x) g(A_\nu \times \bar{c})^b(y) | 0 \rangle \\ &= (g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}) u^{ab}(p^2), \end{aligned}$$

If fulfilled: **Physical States**  $\equiv$  **BRST singlets**  $\equiv$  **color singlets!**



# Kugo–Ojima confinement criterion

In Landau gauge:

**Ghost propagator more sing. than simple pole**

↓

**Kugo–Ojima criterion**

T. Kugo, hep-th/9511033, Int. Symp. “BRS Symmetry”, Kyoto.

# QCD Green functions & Positivity

Assume: States with transverse gluons violate positivity!

These states do not belong to  $\text{Ker } Q_B$ !

BRST quartet:

transverse gluon, gluon-ghost; gluon-antighost, 2-gluon

→ **Kinematical Gluon Confinement:** Cancelations of quartets of Feynman diagrams s.t. there are **no asymptotic transverse gluons!**

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# NB: Picturing Gluon Confinement

Assume DSE scaling solution to be valid:

- ▶ Gluon propagator vanishes on the light cone, and
- ▶  $n$ -point gluon vertex functions diverge on the light cone!

⇒ Attempts to kick a gluon free (i.e. to produce a real gluon) immediately results in production of infinitely many virtual soft gluons!

⇒ perfect color charge screening

+ quartet cancelation:

Gluon (color charge) confinement!

PS: If you can agree on the mechanism but not on the semantics,  
let's call this differently!?!?



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# QCD Green functions & Positivity

Positivity:  $\langle \Omega | A^\dagger A | \Omega \rangle \geq 0$  for local operator  $A$ .

But spectral sum rule for gluon correlation function:

$$Z_3^{-1} = Z + \int_{m^2}^{\infty} d\kappa^2 \rho(\kappa^2) \quad \text{with} \quad Z_3 = \left( \frac{g^2}{g_0^2} \right)^\gamma$$

in linear covariant gauges.

Antiscreening:  $Z_3^{-1} \rightarrow 0$ , i.e.  $Z_3^{-1} \leq Z$ ,  $\Rightarrow \rho(\kappa^2) \leq 0$ .

Oehme–Zimmermann superconvergence relation:

Antiscr. contradicts positivity of gluon spectral density!

R. Oehme and W. Zimmermann, Phys. Rev. D21 (1980) 471.

Non-perturbatively true?

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# Positivity violation for the gluon propagator

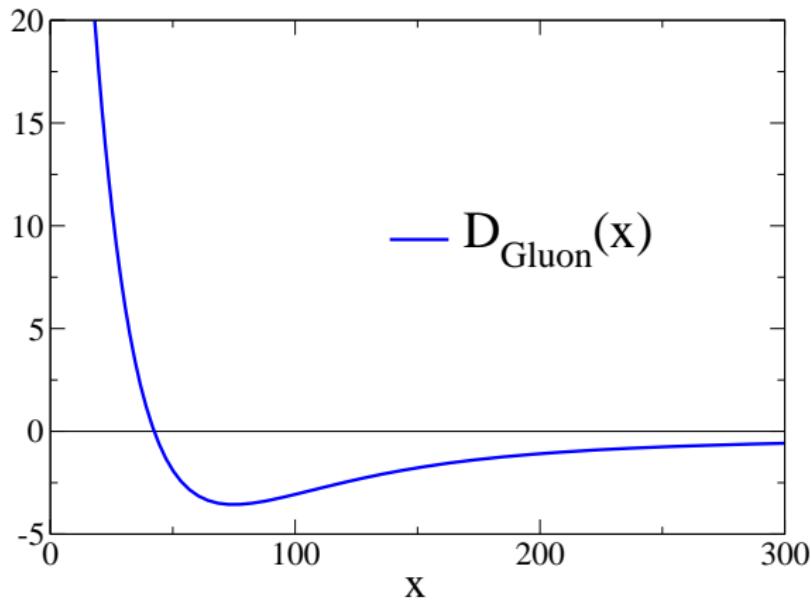
Simple argument [Zwanziger]:  
IR vanishing gluon propagator implies

$$0 = D_{\text{gluon}}(k^2 = 0) = \int d^4x D_{\text{gluon}}(x)$$

⇒  $D_{\text{gluon}}(x)$  has to be negative for some values of  $x$ .

# Positivity violation for the gluon propagator

Fourier transform of DSE result:



Gluon Confinement!

# Positivity violation for the gluon propagator

No Lehmann representation

$$D_{\text{gluon}}(k^2) = \int_0^\infty dm^2 \frac{\rho(m^2)}{k^2 + m^2} \text{ with } \rho(m^2) \geq 0!$$

⇒ Signal for **confined gluons**.

- Non-perturbative realization of  
Oehme–Zimmermann superconvergence relation

# Positivity violation for the gluon propagator

Analytic structure of the gluon propagator:

(R.A., W. Detmold, C.S. Fischer and P. Maris, PRD**70** (2004) 014014)

Running coupling in minMOM scheme precisely represented by:

$$\begin{aligned}\alpha_{\text{fit}}(p^2) &= \frac{\alpha_s(0)}{1 + p^2/\Lambda_{\text{QCD}}^2} \\ &+ \frac{4\pi}{\beta_0} \frac{p^2}{\Lambda_{\text{QCD}}^2 + p^2} \left( \frac{1}{\ln(p^2/\Lambda_{\text{QCD}}^2)} - \frac{1}{p^2/\Lambda_{\text{QCD}}^2 - 1} \right)\end{aligned}$$

with  $\beta_0 = (11N_c - 2N_f)/3$

- Landau pole subtracted
- analytic in complex  $p^2$  plane except real timelike axis
- logarithm produces cut for real  $p^2 < 0$
- Cutkosky's rule obeyed



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$$D_{\text{gluon}}^{\text{fit}}(p^2) = w \frac{1}{p^2} \left( \frac{p^2}{\Lambda_{\text{QCD}}^2 + p^2} \right)^{2\kappa} \left( \alpha_{\text{fit}}(p^2) \right)^{-\gamma}$$

- IR part: cut for  $-\Lambda_{\text{QCD}}^2 < p^2 < 0$
- $D_{\text{gluon}}^{\text{fit}}$ : cut along negative, i.e. timelike, half-axis!

*Wick rotation possible!*

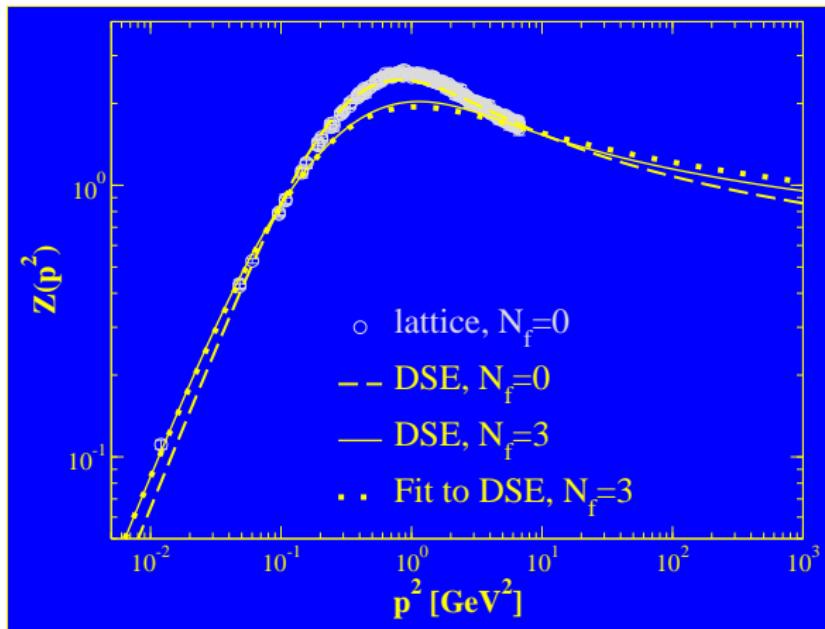
- $w$  arbitrary normalization parameter
- $\kappa = \frac{93 - \sqrt{1201}}{98}$  fixed from IR analysis
- $\gamma = \frac{-13N_c + 4N_f}{22N_c - 4N_f}$  from perturbation theory
- **Effectively one parameter<sup>†</sup>:**  $\Lambda_{\text{QCD}} \approx 500 \text{ MeV!}$

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from fits to lattice data:  $\Lambda_{\text{QCD}} \approx 400 \text{ MeV}$

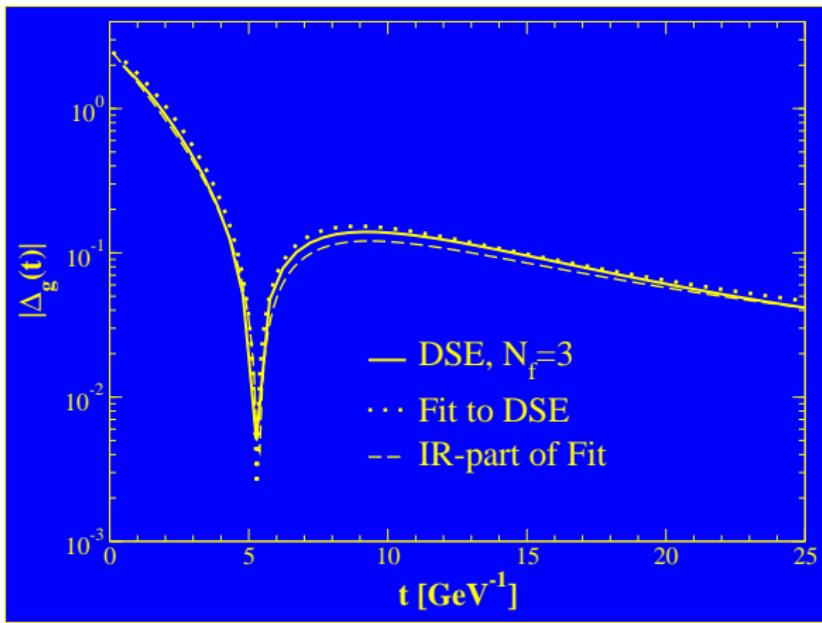
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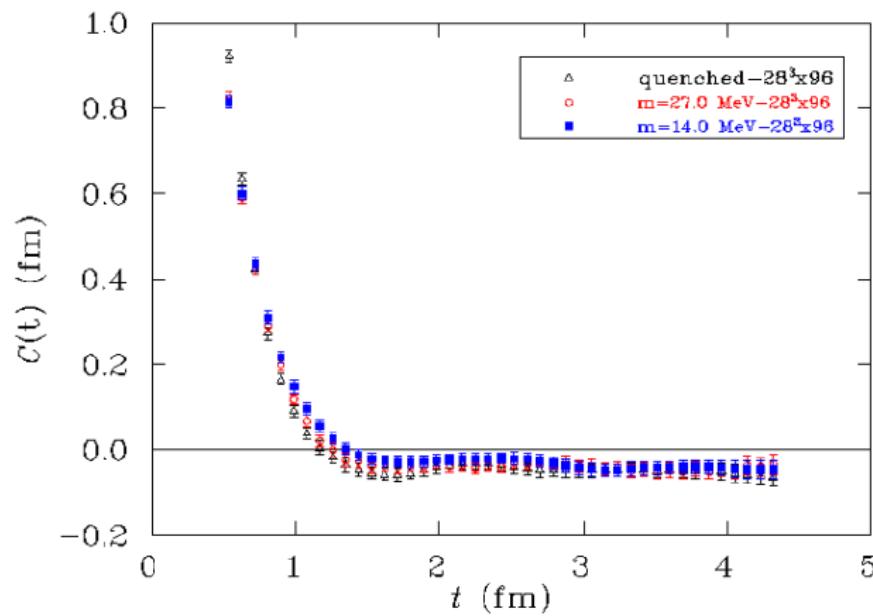
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# Positivity violation for the gluon propagator

Recent corresponding lattice results e.g.

P. Bowman et al., Phys.Rev.D76 (2007) 094505

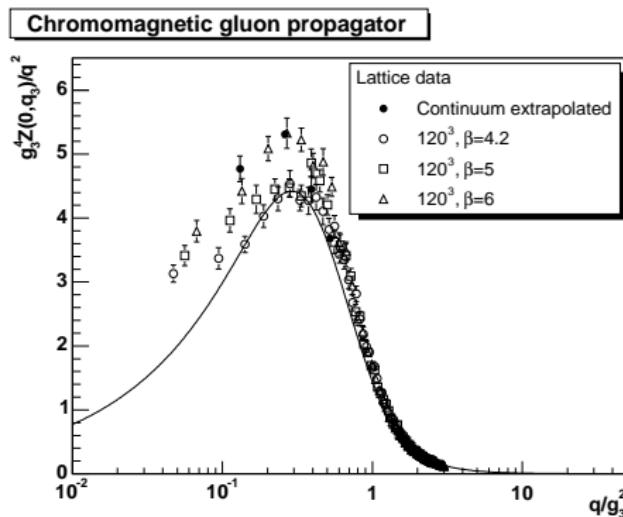


# Positivity violation for the gluon propagator

Gluon propagator at high  $T$ :

A. Maas, J. Wambach, RA, EPJ **C37** (2004) 335; **C42** (2005) 93.

A. Cucchieri, T. Mendes and A.R. Taurines, PR **D67** (2003) 091502.



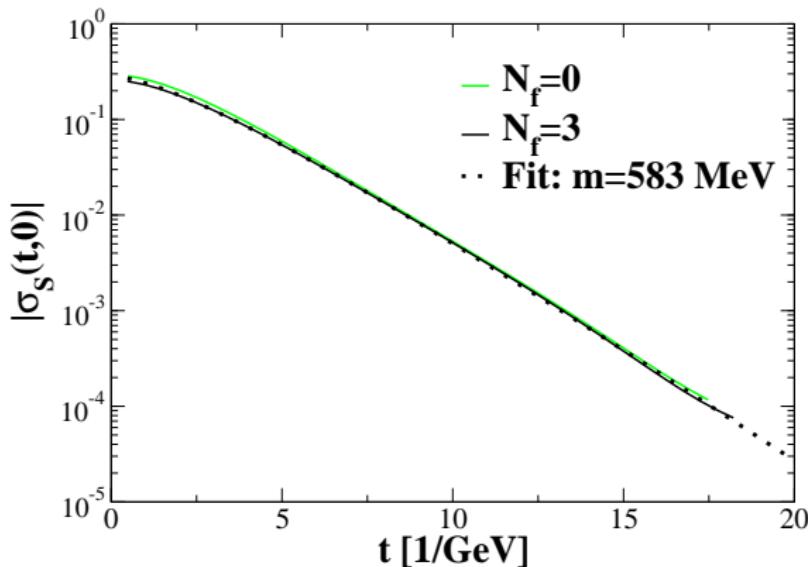
Gribov-Zwanziger / Kugo-Ojima scenario  
applies in confined and deconfined phase!

# Analytic structure of quark propagator

(R.A., W. Detmold, C.S. Fischer and P. Maris, PRD**70** (2004) 014014)

Positivity of the Landau gauge quark propagator from DSE solution with ansatz for quark-gluon vertex:

Quark-gluon vertex: all terms

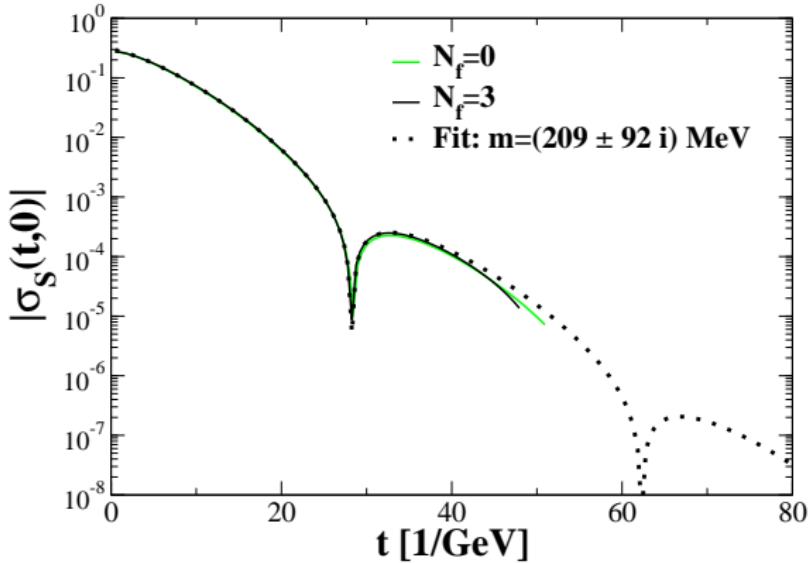


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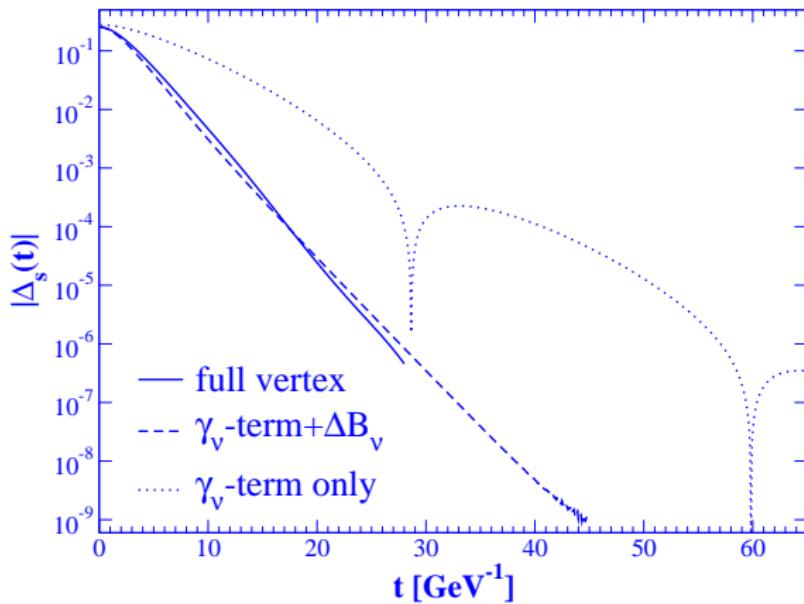
Quark-gluon vertex:  $\gamma_\mu$ -term only



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# Analytic structure of quark propagator

- Result welcome in strong-coupling QED ...
- Puzzling in QCD ...

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, in preparation;  
talks by Christian Fischer and Richard Williams

IR singular Quark-Gluon-Vertex related to

- ▶ Quark confinement,
- ▶ Dynamical Chiral Symmetry Breaking,
- ▶  $U_A(1)$  anomaly

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- Result welcome in strong-coupling QED ...
- Puzzling in QCD ...

R.A., C.S. Fischer, F. Llanes-Estrada, K. Schwenzer, in preparation;  
talks by Christian Fischer and Richard Williams

IR singular Quark-Gluon-Vertex related to

- ▶ Quark confinement,
- ▶ Dynamical Chiral Symmetry Breaking,
- ▶  $U_A(1)$  anomaly

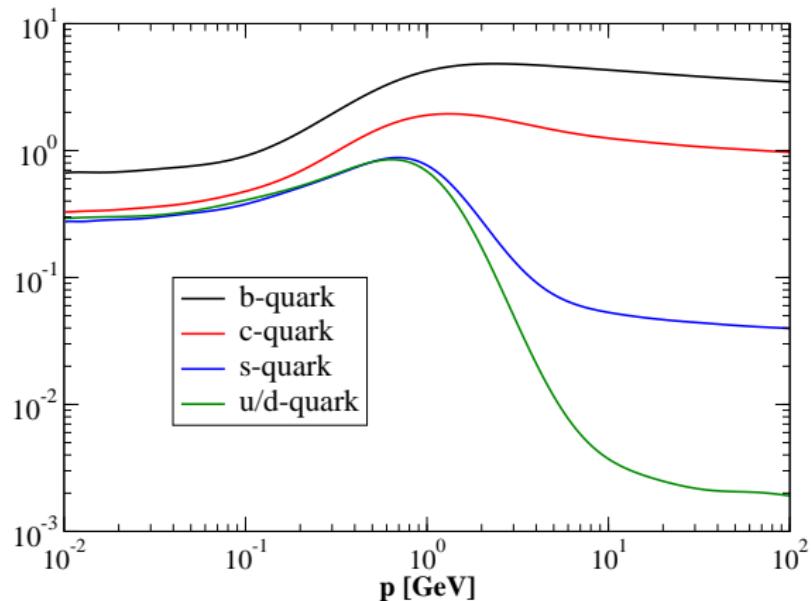
# Analytic structure of quark propagator

Solution of coupled DSEs of Quark Propagator and Quark-Gluon-Vertex:



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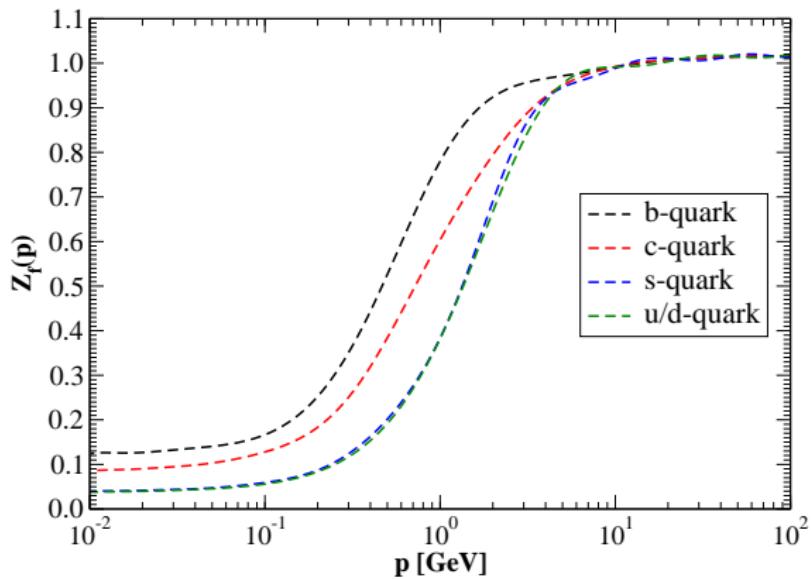
Solution of coupled DSEs of Quark Propagator and Quark-Gluon-Vertex:  
Quark mass functions:



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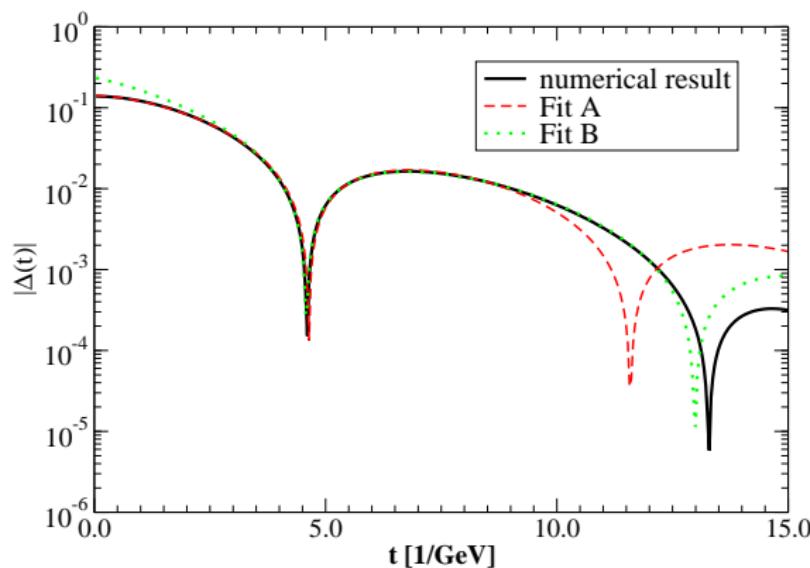
Quark renormalization functions:



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Solution of coupled DSEs of Quark Propagator and Quark-Gluon-Vertex:

Quark Schwinger function (here chiral limit):



# Summary and Outlook

## Analytic properties of Landau gauge QCD propagators:

- ▶ Gluons (“confined by ghosts”): Positivity violated!  
Gluons removed from S-matrix!
- ▶ Analytic structure of gluon propagator:
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(Kugo–Ojima Confinement,  
Oehme–Zimmermann superconvergence, Gribov–Zwanziger  
horizon condition, . . . )
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